Partial Wave Analysis of $\pi^-\pi^0$ system in VES experiment

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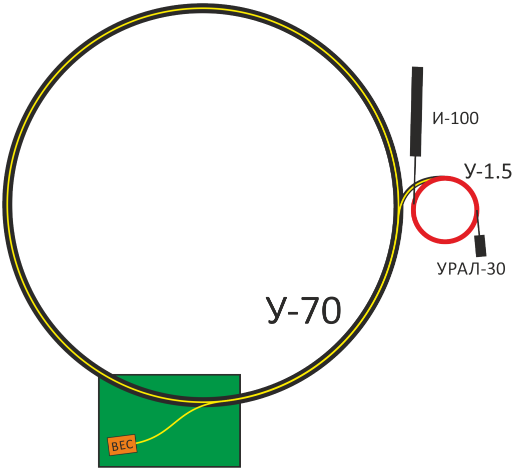
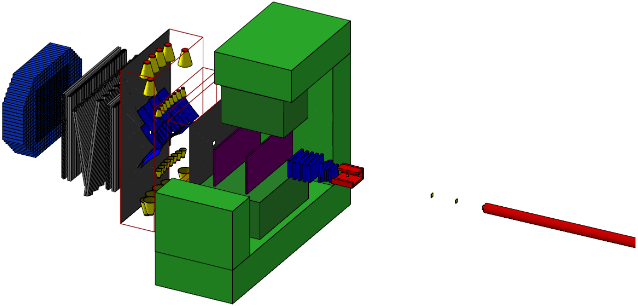
Contents

The experimental facility

- U70 beam - proton 50...70 GeV
- VES beam (27 GeV) - secondary particles
- 98% pions and and 2% kaons, 0.2% antiprotons



VES experiment

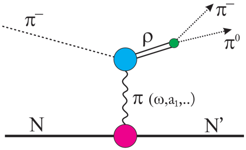


- Beam (π^-, 28 GeV)
- Target(Be $10\%\lambda_I$)
- Detectors $\rightarrow$
- Trigger $\underbrace{S1 \cdot S2 \cdot S3}_{beam} \cdot \underbrace{\bar{K}1 \cdot \bar{K}2}_{interaction} \cdot \underbrace{\bar{A}10 \cdot \bar{A}11}_{!halo} \cdot \underbrace{\bar{G}}_{!veto}$

- Three beam Cerenkov counters
- Wire chambers
- Spectrometer 1 T
- Large Cerenkov counter
- Three station of drift tubes
- EM Calorimeter

The motivation

The reaction $\pi^- + N \to \pi^-\pi^0 + N'$

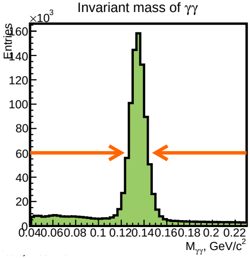


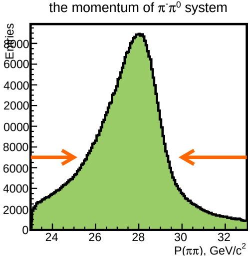
- High statistics. ($> 10^6$)
- $I = 1,\ G = +1 \to$ A few resonances (Odd wave only).
  $\pi^+\pi^-$: S, P, D, F, G, H
  $\pi^-\pi^0$: ..., P, ..., F, ..., H
- High mass ($> 2\,\mathrm{GeV/c^2}$) region has never been studied.
- Low mass ($[0.5 - 1.2]\,\mathrm{GeV/c^2}$): ρ-meson shape, production mechanism were studied at low energy only (till $5\,\mathrm{GeV/c^2}$)

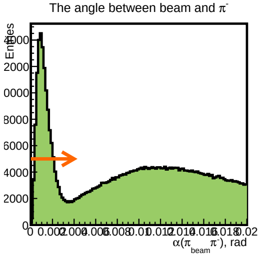
The data

Data-sample

- The topology $(1n + 0p + 2z)$.
- The gamma's energy more then 0.5 GeV
- $\gamma\gamma$ mass cut $(m_\pi \pm 15\,\mathrm{MeV})$
- "Exclusivity" cut $25 - 30\,\mathrm{GeV/c^2}$.
- Vertex Z cut 16 cm, while target length is 4 cm
- The $\pi_{beam} - \pi^-$ angle more 0.003 rad to suppress $\pi^-_{beam}(\gamma\gamma)_{noise}$ events.
- Fiducial cuts for all detectors.

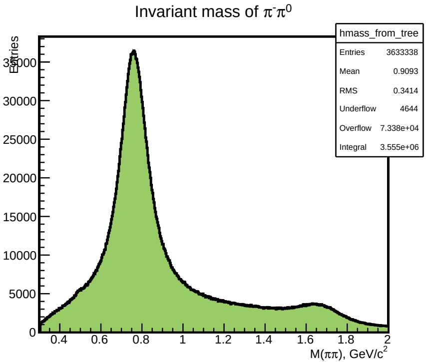


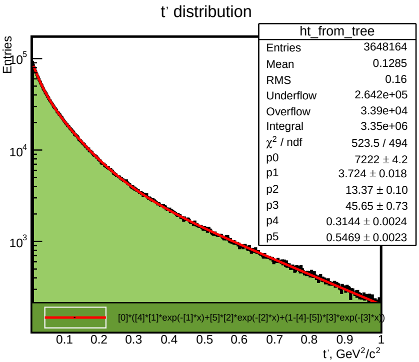




Data-sample

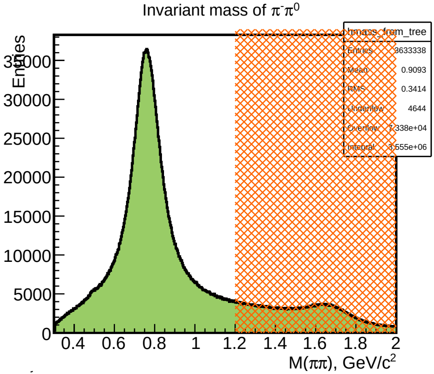
Mass spectrum and t' distributions in details:

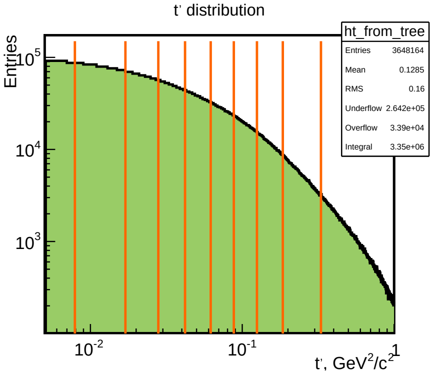




Data-sample

The intervals where studies were performed: inv.mass from 0.3 GeV to 1.2 GeV, the mass bin is 50 MeV; t' in 10 intervals.



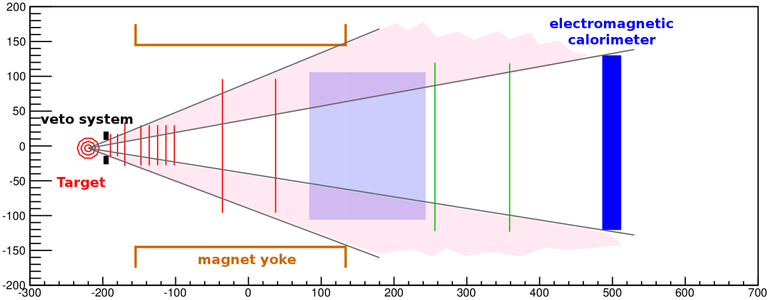


The background

The experimental setup

- The experimental setup is not hermetic.
- 186 - 404 mrad is the hole for neutral particles.
- It is critical if there is reaction with additional neutrals(γ) and large cross section.

The VES setup: top view.



Main background: Reaction $\pi^- N \to \pi^- 2\pi^0 N'$

$\pi^- 2\pi^0$: 1 track + 4 gammas $\to$ 1 track + 2 gammas : $\pi^-\pi^0$

$$\frac{\sigma(\pi^- N \to \pi^- 2\pi^0 N')}{\sigma(\pi^- N \to \pi^- \pi^0 N')} \sim 20...50, \quad \text{for our energy}$$

The leakage study:

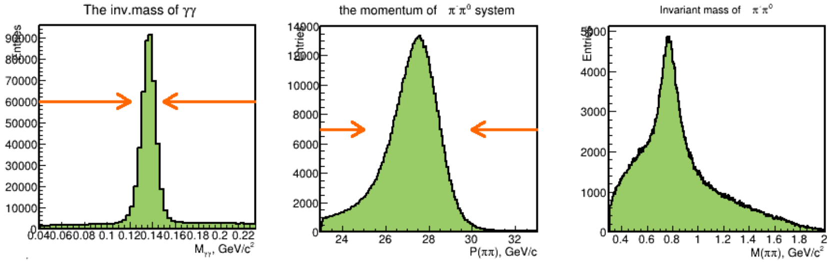
- The $\pi^- 2\pi^0$ PWA result is used as physical generator.
- Geant4 (or factMC) - for event simulation.

The $\pi^- 2\pi^0$ PWA model: (see report from VES at Hadron-2013)

- The isobar model
- *m*- and *t*- independent analysis

The selected background spectra

- The same cuts (like for the data) are applied.
- The leakage is about 5 % of original $\pi^-2\pi^0$ data sample.
- The invariant mass also has ρ-meson peak.
- The “exclusivity” is good, because soft gammas were lost.



The estimation of the background fraction

The background contribution is evaluated by the fit of mass distribution.

The Signal is Breit-Wigner shape with dynamical width

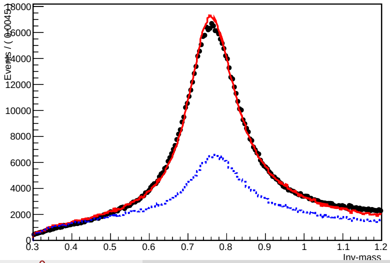
$$\mathbf{BW(m)} = \frac{M\Gamma(m)}{(m^2 - M^2)^2 + (M\Gamma(m))^2}, \quad \Gamma(m) = \Gamma_0 (\frac{p}{p_0})^3 \frac{M}{m}$$

$$p = \frac{1}{2}\sqrt{m^2 - (m_{\pi 0} + m_{\pi -})^2}, \quad p_0 = \frac{1}{2}\sqrt{m_0^2 - (m_{\pi 0} + m_{\pi -})^2}$$

The background is the PWA-based $\pi^- 2\pi^0$-leakage

The **BG** fraction $\sim 50\,\%$ and varies a little bit with t'.

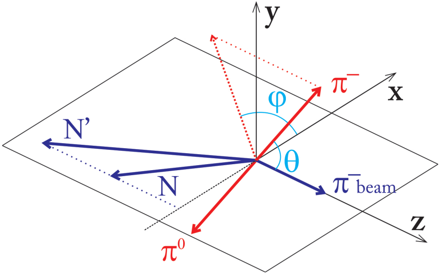
Fit inv.mass for (t_prime>0.103)*(t_prime<1.0)



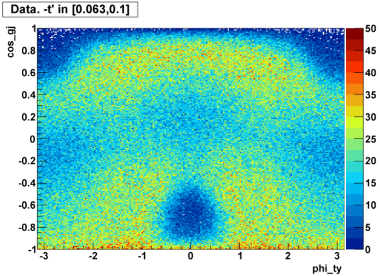
The analysis of angular distributions

Data distribution over angles

We want to know how ρ-meson is produced, and what angular distribution the final particles have (I_{prod}).



The **observed** distribution over $\cos\theta_{GJ} \times \phi_{TY}$



To unfold I_{prod}, the acceptance should be known.

PWA scheme.

$I_{prod}(\Omega|a_i)$ is density function for **produced** data ($N_{prod} = \int I_{prod}\, d\Omega$).
$I_{obs}(\Omega|a_i)$ is density function for **observed** data ($N_{obs} = \int I_{obs}\, d\Omega$).

$$I_{obs}(\Omega|a_i) = A \circ I_{prod} \approx A_0(\Omega)\, I_{prod}(\Omega|a_i),$$

$\{data\} = \{\Omega_i\}$ is **observed** data sample.

$$\mathbb{L}_0 = \prod_{data} \frac{I_{obs}(\Omega_i)}{\int I_{obs}\, d\Omega} \quad \text{is Likelihood function.} \quad \int I_{obs}(\Omega|a_i)\, d\Omega = \mu(a_i)$$

$\mathbb{L}_{ext} = Pois(N_{data}|\mu)\cdot\mathbb{L}_0$ is extended Likelihood function. $\mathbb{P}_0 \equiv -\log \mathbb{L}_{ext}$.

$$\mathbb{P}_0 = -\log\left(\frac{\mu^N}{N!} e^{-\mu} \prod_{data} \frac{A_0 I_{prod}}{\mu}\right), \quad \text{where } \mu = \int A \circ I_{prod}\, d\Omega$$

$$\mathbb{P}_0 \simeq -\sum_{data} \log\left(A_0\, I_{prod}\right) + \int A \circ I_{prod}\, d\Omega, \quad \text{is minimized.}$$

The waves-representation

The intensity is the sum of two non-interfering blocks.

$$I_{prod}(\Omega) = \left| {}^{(+)}Y_{1,1} P_+ \right|^2 + \left| {}^{(-)}Y_{1,0} P_0 + {}^{(-)}Y_{1,1} P_- e^{i\phi_P} + {}^{(-)}Y_{0,0} S_0 e^{i\phi_S} \right|^2$$

${}^{\varepsilon}Y_{l,m}(\cos\theta)$ are spherical functions in the real basis ("naturality basis"):

"Natural" exchange

$${}^{(+)}Y_{1,1} = -\sqrt{\frac{3}{4\pi}} \sin\theta \sin\phi$$

"Unnatural" exchange

$${}^{(-)}Y_{1,0} = \sqrt{\frac{3}{4\pi}} \cos\theta$$

$${}^{(-)}Y_{1,1} = \sqrt{\frac{3}{4\pi}} \sin\theta \cos\phi$$

$${}^{(-)}Y_{0,0} = \sqrt{\frac{1}{4\pi}}$$

A minimum $\mathbb{P}$ gives consistent and efficient estimation for p:

$$\mathbb{P} = -\sum_{data} \log I_{prod}(\Omega_i|p) + \int A \circ I_{prod}(\Omega'|p)\, d\Omega, \quad p = (a_i, \phi_P, \phi_S)$$

The moments representation

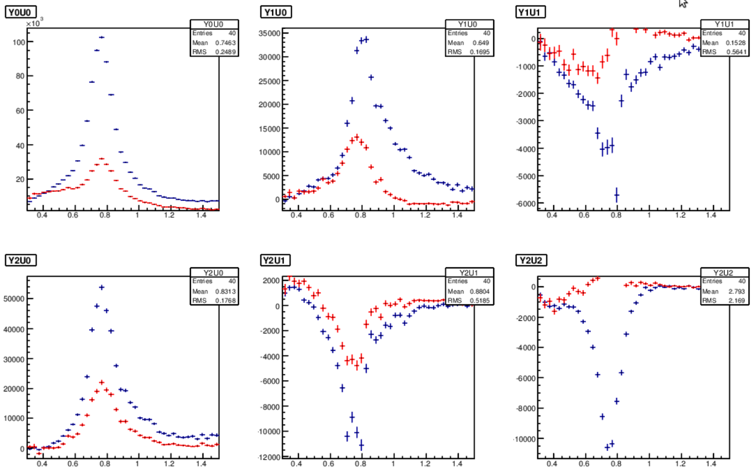
$$I(\theta,\phi) = \sum_{l=0}^{l<L} \sum_{m<l} H_{l,m}\,{}^{\varepsilon}Y_{l,m}(\cos\theta,\phi) =$$
$$= \tilde{H}_{0,0} + \tilde{H}_{1,0}\cos\theta + \tilde{H}_{1,1}\sin\theta\cos\phi +$$
$$+\tilde{H}_{2,0}(3\cos^2\theta - 1) + \tilde{H}_{2,1}\sin\theta\cos\theta\cos\phi + \tilde{H}_{2,2}\sin^2\theta\cos(2\phi)$$

The relationships between moments and waves

$$H_{0,0} = \sqrt{\frac{1}{4\pi}}(S_0^2 + P_0^2 + P_-^2 + P_+^2), \quad H_{1,0} = \sqrt{\frac{1}{\pi}} S_0 P_0 \cos\phi_S,$$
$$H_{1,1} = \sqrt{\frac{1}{\pi}} S_0 P_- \cos(\phi_S - \phi_P), \quad H_{2,0} = \sqrt{\frac{1}{60\pi}}(2P_0^2 - P_-^2 - P_+^2)$$
$$H_{2,1} = \sqrt{\frac{3}{5\pi}} P_0 P_- \cos\phi_P, \quad H_{2,2} = \sqrt{\frac{3}{20\pi}}(P_-^2 - P_+^2)$$

The produced moments

The blue points are moments for the data. The red ones are moments for the background (scaled according to evaluated contribution).

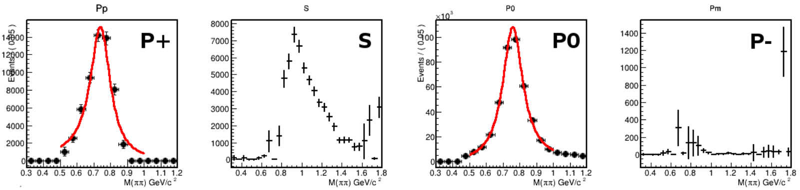


From the moments to waves

Moments to Waves

The "function space" of moments is wider then one of waves expantion. χ^2-like fit is used to find waves (for each bin).

$$\chi^2 = (\bar{M}_i - M_i(w))E_{ij}^{-1}(\bar{M}_j - M_j(w))$$

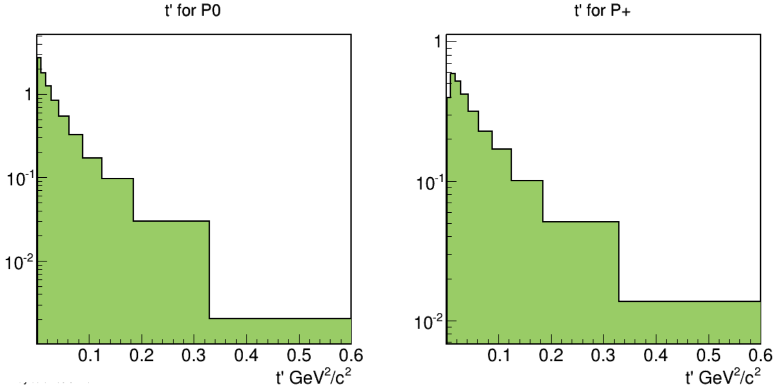


The contribution extraction

For significant waves (P_0, P_+) the amount of event is extracted by the BW shape fit.

t' dependancies

The analysis was performed at 10 t' intervals.



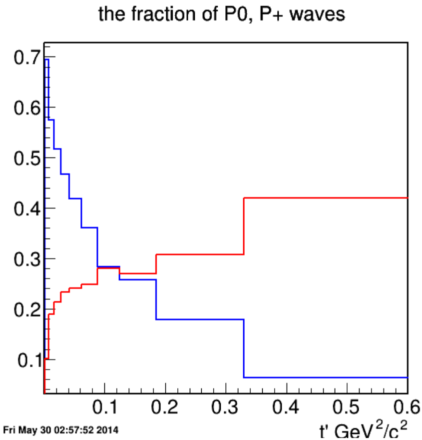
Conclusion

- The data for the reaction $\pi^- N \to \pi^- \pi^0 N'$ were collected at the VES experiment with tree order higher statistics then in previous studies.
- The significant background ($\pi^- 2\pi^0$ leakage) is irremovable. The background contribution was evaluated ($\sim 45 - 55\%$) and taken into accont using the subtruction procedure.
- The $\pi^- \pi^0$ system from ρ-meson decay is observed in P_0, P_+ waves, presumably with dominance of π and ω exchanges respectively.
- The t' dependencies for waves intensities were extracted with no interpetation. We will welcome any help from theoretics.

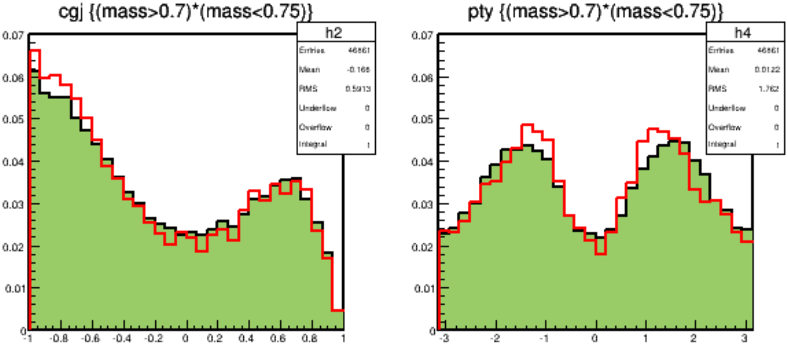
The end

Thank you.

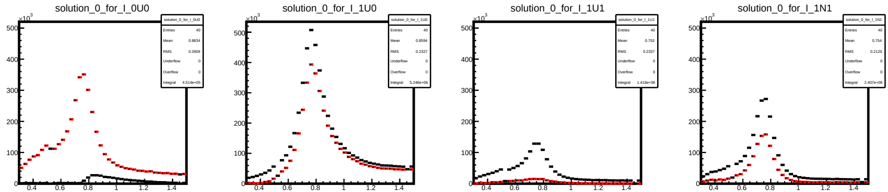
The fraction of the P_0 and P_+ waves



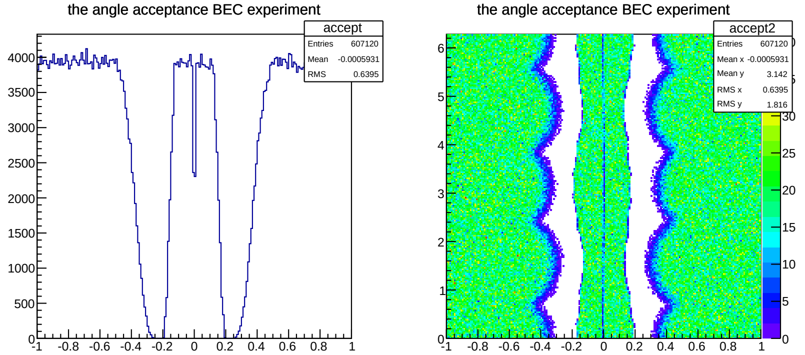
The predictions



PWA solution



BackSlide, the acceptance for gammas



PWA technical details

Need to construct:

$$I_{prod}(\Omega) = \left|\mathbf{a_{P_+}} F_{P+}(\Omega)\right|^2 + \left|\mathbf{a_{P_0}} F_{P0}(\Omega) + \mathbf{a_{P_-}} e^{i\phi_P} F_{P-}(\Omega) + \mathbf{a_S} e^{i\phi_S} F_{S0}(\Omega)\right|^2,$$

where $\Omega = (\cos\theta, \phi)$ – variables.

Easier to use `Re-` and `Im-` part as independent parameters.

$$W(\Omega|a) = \left|\mathbf{a}_{\{\mathbf{Y1N1}\}} F_{P+}\right|^2 + \left|\mathbf{a}_{\{\mathbf{Y1U0}\}} F_{P0} + \mathbf{a}_{\{\mathbf{Y1U1}\}} F_{P-} + \mathbf{a}_{\{\mathbf{Y0U0}\}} F_S\right|^2$$

$$I_{prod}(\Omega) = W(\Omega|a_{RE}) + W(\Omega|a_{IM}), \quad \Rightarrow \{a_i\}_{i=1..8} \text{ - real parameters.}$$

A "minimum" of NLL is degenerated and allows continuous transformation in this case (solution is to fix two parameters.)

The background subtraction

The list of background subtraction methods, we tried:

1. The PWA$_{Ln(\varepsilon S+B)}$: parametrized **BG**-density and parametrised efficiency.
2. The PWA$_{Ln(S+B/\varepsilon)}$: parametrized restored **BG-produced** density.
3. The PWA$_{LnS-LnB}$: rescaling likelihood function.
4. The PMA and moments subtaction.

All methods base on good knowledge of the acceptance and background.
Most of methods required the background fraction to be known.

I. The PWA: $\log(\varepsilon S + B)$

The classical scheme:

$$I_{obs}(\Omega) = A \circ I_{prod} \approx A_0(\Omega)\, I_{prod}(\Omega),$$

$$\mathbb{P}_0 \simeq -\sum_{data} \log\left(A_0\, I_{prod}\right) + \int A \circ I_{prod}\, d\Omega$$

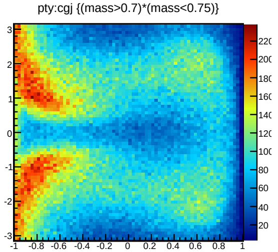
It is possible to take into account the background:

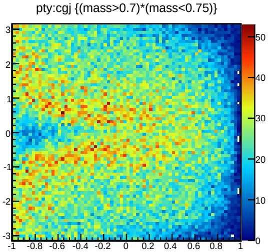
$$I_{obs}(\Omega) \approx A_0\, I_{prod} + I_{back} = A_0(\Omega)\, I_{prod}(\Omega) + N_b\, P_{back}(\Omega)$$

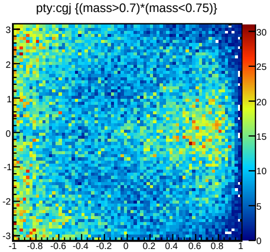
It can be included to extNLL scheme:

$$\mathbb{P}_{s+b} = -\sum_{data} \log\left(A_0\, I_{prod} + N_b\, P_{back}\right) + \int A \circ I_{prod}\, d\Omega + N_b$$

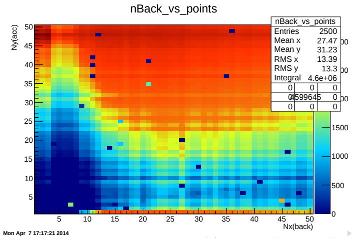
I. The PWA: $\log(\varepsilon S + B)$







- The background contribution can be evaluated.
- Result depents on parametrization quality strongly.



II. Rescaling likelihood function log DATA − log BG

The classical scheme:

$$\mathbb{L}_0 = \prod_{data} \rho_{obs}(\Omega_i), \qquad \mathbb{P}_0 \simeq -\sum_{data} \log\left(A_0\, I_{prod}\right) + \int A \circ I_{prod}\, d\Omega$$

Take into account the background event by event:

$$\mathbb{L}_s = \prod_{data} \rho_{obs}(\Omega_i) / \prod_{bg} \rho_{obs}(\Omega_i),$$

$$\mathbb{P}_0 \simeq -\sum_{data} \log\left(A_0\, I_{prod}\right) + \sum_{bg} \log\left(A_0\, I_{prod}\right) + \int A \circ I_{prod}\, d\Omega$$

Realistic (MC-limited) functional:

$$\mathbb{P}_0 \simeq -\sum_{data} \log\left(A_0\, I_{prod}\right) + \frac{N_b}{N_{MC}} \sum_{bg} \log\left(A_0\, I_{prod}\right) + \int A \circ I_{prod}\, d\Omega$$

II. Rescaling likelihood function log DATA − log BG

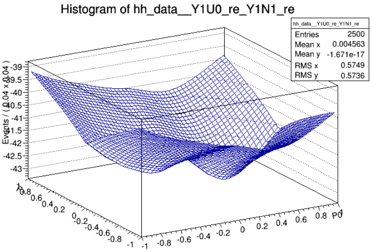
Advantages

Event by event accounting of **BG**.
The right normalized result.

$$\int A \circ I_{prod}\, d\Omega = N_{data} - N_{bg}$$

Disadvantages

It works if a background contribution is small.



The ρ-meson waves

The S, P-waves are plotted in the diffrent t' ranges.