# Effects of (axial)vector mesons on the chiral phase transition: initial results

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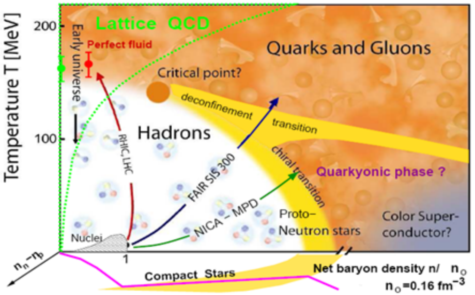
# Overview

# QCD phase diagram

Phase diagram in the $T - \mu_{\rm B} - \mu_{\rm I}$ space



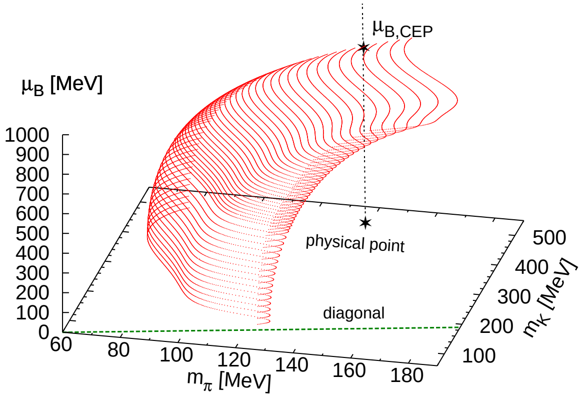

- At $\mu_{\rm B} = 0$
  $T_c = 151(3)$ MeV
  Y. Aoki, *et al.*, PLB **643**, 46 (2006)
- Is there a CEP?
- At $T = 0$ in $\mu_{\rm B}$ where is the phase boundary?
- Behaviour as a function of $\mu_{\rm I}/\mu_{\rm S}$

Details of the phase diagram are heavily studied theoretically (Lattice, EFT), and experimentally (RHIC, LHC, FAIR, NICA)

# Previous results (with linear $\sigma$-model)

## Critical surface and the CEP

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015

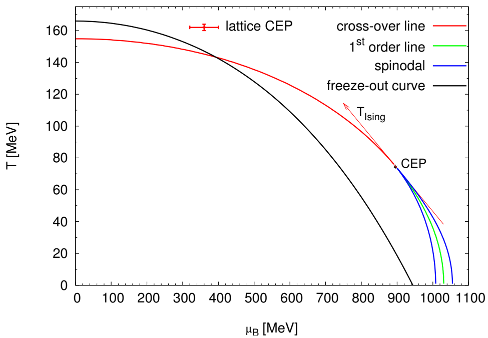



The surface bends towards the physical point $\Longrightarrow$ The CEP must exist

# Previous results (with linear $\sigma$-model)

## The CEP at the physical point of the mass plane

P. Kovács, Zs. Szép: Phys. Rev. D **75**, 025015




effective model

- $T_c(\mu_B = 0) = 154.84$ MeV
  $\Delta T_c(x\chi) = 15.5$ MeV
- $T_{CEP} = 74.83$ MeV
  $\mu_{B,CEP} = 895.38$ MeV
- $T_c \frac{d^2 T_c}{d\mu_B^2}\Big|_{\mu_B=0} = -0.09$

lattice

- $T_c(\mu_B = 0) = 151(3)$ MeV
  $\Delta T_c(\chi_{\bar{\psi}\psi}) = 28(5)$ MeV
  Y. Aoki,*et al.*, PLB **643**, 46 (2006)
- $T_{CEP} = 162(2)$ MeV
  $\mu_{B,CEP} = 360(40)$ MeV
- $-0.058(2)$
  Z. Fodor,*et al.*, JHEP 0404 (2004) 050

# Addressed problems

- By adding more degrees of freedom to our model how does the phase boundary change?
- More specifically adding (axial)vector mesons to the model how does the position of the CEP change?
- What is the effect of the medium on the various masses?
- Results will be closer to the Lattice?

# Chiral symmetry

If the quark masses are zero (chiral limit) $\Longrightarrow$ QCD invariant under the following global transformation (chiral symmetry):

$$U(3)_L \times U(3)_R \simeq U(3)_V \times U(3)_A = SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$$

$U(1)_V$ term $\longrightarrow$ baryon number conservation

$U(1)_A$ term $\longrightarrow$ broken through axial anomaly

$SU(3)_A$ term $\longrightarrow$ broken down by any quark mass

$SU(3)_V$ term $\longrightarrow$ broken down to $SU(2)_V$ if $m_u = m_d \neq m_s$
$\longrightarrow$ totally broken if $m_u \neq m_d \neq m_s$ (realized in nature)

Since QCD is very hard to solve $\longrightarrow$ low energy effective models can be set up $\longrightarrow$ reflecting the global symmetries of QCD $\longrightarrow$ degrees of freedom: observable particles instead of quarks and gluons

Linear realization of the symmetry $\longrightarrow$ linear sigma model
(nonlinear representation $\longrightarrow$ chiral perturbation theory (ChPT))

# Lagrangian (2/1)

$$\begin{aligned}
\mathcal{L}_{\mathrm{Tot}} = {} & \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2 \\
& - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) + \mathrm{Tr}\left[\left(\frac{m_1^2}{2} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)] \\
& + c_2(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\
& + \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger). \\
& + g_3[\mathrm{Tr}(L_\mu L_\nu L^\mu L^\nu) + \mathrm{Tr}(R_\mu R_\nu R^\mu R^\nu)] + g_4[\mathrm{Tr}\,(L_\mu L^\mu L_\nu L^\nu) \\
& + \mathrm{Tr}\,(R_\mu R^\mu R_\nu R^\nu)] + g_5\mathrm{Tr}\,(L_\mu L^\mu)\;\mathrm{Tr}\,(R_\nu R^\nu) + g_6[\mathrm{Tr}(L_\mu L^\mu)\,\mathrm{Tr}(L_\nu L^\nu) \\
& + \mathrm{Tr}(R_\mu R^\mu)\,\mathrm{Tr}(R_\nu R^\nu)] + \bar\Psi\,(i\,\partial\!\!\!/ - g_F\Phi_5)\,\Psi + \mathcal{L}_{\mathrm{Polyakov}}
\end{aligned}$$

where

$$D^\mu\Phi = \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA^\mu[T_3, \Phi]$$

# Lagrangian (2/2)

$$\Phi = \sum_{i=0}^{8}(\sigma_i + i\pi_i)T_i, \quad H = \sum_{i=0}^{8} h_i T_i \qquad T_i : U(3) \text{ generators}$$

$$R^\mu = \sum_{i=0}^{8}(\rho_i^\mu - b_i^\mu)T_i, \quad L^\mu = \sum_{i=0}^{8}(\rho_i^\mu + b_i^\mu)T_i$$

$$L^{\mu\nu} = \partial^\mu L^\nu - ieA^\mu[T_3, L^\nu] - \{\partial^\nu L^\mu - ieA^\nu[T_3, L^\mu]\}$$

$$R^{\mu\nu} = \partial^\mu R^\nu - ieA^\mu[T_3, R^\nu] - \{\partial^\nu R^\mu - ieA^\nu[T_3, R^\mu]\}$$

$$\bar{\Psi} = (\bar{u}, \bar{d}, \bar{s})$$

non strange – strange base:

$$\varphi_N = \sqrt{2/3}\varphi_0 + \sqrt{1/3}\varphi_8,$$

$$\varphi_S = \sqrt{1/3}\varphi_0 - \sqrt{2/3}\varphi_8, \qquad \varphi \in (\sigma, \pi, h)$$

broken symmetry: non-zero condensates $\langle\sigma_N\rangle, \langle\sigma_S\rangle \longleftrightarrow \phi_N, \phi_S$

# Included fields - pseudoscalar and scalar meson nonets

$$\Phi_{PS} = \sum_{i=0}^{8} \pi_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix} \quad (\sim \bar{q}_i \gamma_5 q_j)$$

$$\Phi_{S} = \sum_{i=0}^{8} \sigma_i T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_S^0 \\ K_S^- & \bar{K}_S^0 & \sigma_S \end{pmatrix} \quad (\sim \bar{q}_i q_j)$$

Particle content:

Pseudoscalars: $\pi(138), K(495), \eta(548), \eta'(958)$

Scalars: $a_0(980 \text{ or } 1450), K_0^\star(800 \text{ or } 1430)$,

$(\sigma_N, \sigma_S)$ : 2 of $f_0(500, 980, 1370, 1500, 1710)$

# Included fields - vector meson nonets

$$V^\mu = \sum_{i=0}^{8} \rho_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^\mu$$

$$A_V^\mu = \sum_{i=0}^{8} b_i^\mu T_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+ \\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0 \\ K_1^- & \bar{K}_1^0 & f_{1S} \end{pmatrix}^\mu$$

Particle content:

Vector mesons: $\rho(770), K^\star(894), \omega_N = \omega(782), \omega_S = \phi(1020)$

Axial vectors: $a_1(1230), K_1(1270), f_{1N}(1280), f_{1S}(1426)$

# Polyakov loops in Polyakov gauge

Polyakov loop variables: $\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}$ and $\bar{\Phi}(\vec{x}) = \frac{\mathrm{Tr}_c \bar{L}(\vec{x})}{N_c}$ with

$$L(x) = \mathcal{P} \exp\left[i \int_0^\beta d\tau A_4(\vec{x}, \tau)\right]$$

$\longrightarrow$ signals center symmetry ($\mathbb{Z}_3$) breaking at the deconfinement transition

low $T$: confined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle = 0$
high $T$: deconfined phase, $\langle\Phi(\vec{x})\rangle, \langle\bar{\Phi}(\vec{x})\rangle \neq 0$

Polyakov gauge: the temporal component of the gauge field is time independent and can be gauge rotated to a diagonal form in the color space

$$A_{4,d}(\vec{x}) = \phi_3(\vec{x})\lambda_3 + \phi_8(\vec{x})\lambda_8; \quad \lambda_3, \lambda_8 \text{ : Gell-Mann matrices.}$$

In this gauge the Polyakov loop operator is

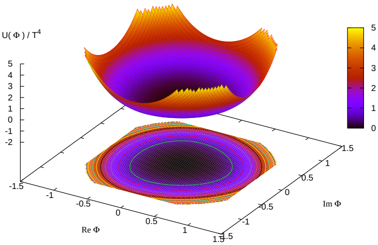
$$L(\vec{x}) = \mathrm{diag}(e^{i\beta\phi_+(\vec{x})}, e^{i\beta\phi_-(\vec{x})}, e^{-i\beta(\phi_+(\vec{x})+\phi_-(\vec{x}))})$$

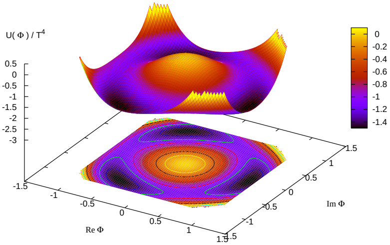
where $\phi_\pm(\vec{x}) = \pm\phi_3(\vec{x}) + \phi_8(\vec{x})/\sqrt{3}$

# Polyakov loop potential

"Color confinement"
$\langle\Phi\rangle = 0 \longrightarrow$ no breaking of $\mathbb{Z}_3$
one minimum

"Color deconfinement"
$\langle\Phi\rangle \neq 0 \longrightarrow$ spontaneous breaking of $\mathbb{Z}_3$
minima at $0, 2\pi/3, -2\pi/3$
one of them spontaneously selected




from H. Hansen et al., PRD75, 065004 (2007)

# Form of the potential

I.) Simple polynomial potential invariant under $\mathbb{Z}_3$ and charge conjugation: R.D.Pisarski, PRD 62, 111501

$$\frac{\mathcal{U}_{\text{poly}}(\Phi,\bar{\Phi})}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}\left(\Phi^3 + \bar{\Phi}^3\right) + \frac{b_4}{4}\left(\bar{\Phi}\Phi\right)^2$$

with $$b_2\left(T\right) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2} + a_3\frac{T_0^3}{T^3}$$

II.) Logarithmic potential coming from the $SU(3)$ Haar measure of group integration K. Fukushima, Phys. Lett. **B591**, 277 (2004)

$$\frac{\mathcal{U}_{\log}(\Phi,\bar{\Phi})}{T^4} = -\frac{1}{2}a(T)\Phi\bar{\Phi} + b(T)\ln\left[1 - 6\Phi\bar{\Phi} + 4\left(\Phi^3 + \bar{\Phi}^3\right) - 3\left(\Phi\bar{\Phi}\right)^2\right]$$

with $$a(T) = a_0 + a_1\frac{T_0}{T} + a_2\frac{T_0^2}{T^2}, \qquad b(T) = b_3\frac{T_0^3}{T^3}$$

$\mathcal{U}\left(\Phi,\bar{\Phi}\right)$ models the free energy of a pure gauge theory
$\longrightarrow$ the parameters are fitted to the pure gauge lattice data

# Effects of Polyakov loops on FD statistics

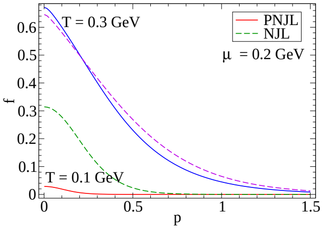
Inclusion of the Polyakov loop modifies the Fermi-Dirac distribution function

$$f(E_p-\mu_q) \longrightarrow f_\Phi^+(E_p) = \frac{\left(\bar\Phi + 2\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}{1+3\left(\bar\Phi+\Phi e^{-\beta(E_p-\mu_q)}\right) e^{-\beta(E_p-\mu_q)} + e^{-3\beta(E_p-\mu_q)}}$$

$$f(E_p+\mu_q) \longrightarrow f_\Phi^-(E_p) = \frac{\left(\Phi + 2\bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}{1+3\left(\Phi+\bar\Phi e^{-\beta(E_p+\mu_q)}\right) e^{-\beta(E_p+\mu_q)} + e^{-3\beta(E_p+\mu_q)}}$$

$\Phi,\bar\Phi \to 0 \Longrightarrow f_\Phi^\pm(E_p) \to f(3(E_p\pm\mu_q))$ $\Phi,\bar\Phi \to 1 \Longrightarrow f_\Phi^\pm(E_p) \to f(E_p\pm\mu_q)$

three-particle state appears: mimics confinement of quarks within baryons




the effect of the Polyakov loop is more relevant for $T < T_c$

at $T=0$ there is no difference between models with and without Polyakov loop: $\Theta(3(\mu_q - E_p)) \equiv \Theta((\mu_q - E_p))$

H. Hansen et al., PRD75, 065004

# $T/\mu_B$ dependence of the Polyakov-loops (EoS)

By deriving the grand canonical potential for Polyakov loops ($\Omega$) according to $\Phi$ and $\bar{\Phi}$

$$-\frac{d}{d\Phi}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right)+\frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^-(p)}}{g_q^-(p)}+\frac{e^{-2\beta E_q^+(p)}}{g_q^+(p)}\right)=0$$

$$-\frac{d}{d\bar{\Phi}}\left(\frac{U(\Phi,\bar{\Phi})}{T^4}\right)+\frac{2N_c}{T^3}\sum_{q=u,d,s}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\left(\frac{e^{-\beta E_q^+(p)}}{g_q^+(p)}+\frac{e^{-2\beta E_q^-(p)}}{g_q^-(p)}\right)=0$$

$$g_q^+(p) = 1+3\left(\bar{\Phi}+\Phi e^{-\beta E_q^+(p)}\right)e^{-\beta E_q^+(p)}+e^{-3\beta E_q^+(p)}$$

$$g_q^-(p) = 1+3\left(\Phi+\bar{\Phi} e^{-\beta E_q^-(p)}\right)e^{-\beta E_q^-(p)}+e^{-3\beta E_q^-(p)}$$

$$E_q^{\pm}(p)=E_q(p)\mp\mu_B/3,\;\; E_{u/d}(p)=\sqrt{p^2+m_{u/d}^2},\, E_s(p)=\sqrt{p^2+m_s^2}$$

# $T/\mu_B$ dependence of the condensates ($\phi_{N/S}$)

Equation of state: $\left\langle \frac{\partial \mathcal{L}_{\text{Tot}}}{\partial \sigma_{N/S}} \right\rangle_T = 0$

Hybrid approach at $T = 0$: fermions at one-loop, mesons at tree-level (their effects are much smaller)

At $T \neq 0$: first approximation $\longrightarrow$ only fermion thermal loops

$$m_0^2 \phi_N + \left(\lambda_1 + \frac{1}{2}\lambda_2\right) \phi_N^3 + \lambda_1 \phi_N \phi_S^2 - h_N + \frac{g_F}{2} N_c \left(\langle u\bar{u}\rangle_T + \langle d\bar{d}\rangle_T\right) = 0$$

$$m_0^2 \phi_S + (\lambda_1 + \lambda_2)\, \phi_S^3 + \lambda_1 \phi_N^2 \phi_S - h_S + \frac{g_F}{\sqrt{2}} N_c \langle s\bar{s}\rangle_T = 0$$

$$\langle q\bar{q}\rangle_T = -4 m_q \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_q(p)} \left(1 - f_\Phi^-(E_q(p)) - f_\Phi^+(E_q(p))\right)$$

# Determination of the parameters of the Lagrangian

14 unknown parameters $\longrightarrow$ Determined by the min. of $\chi^2$:

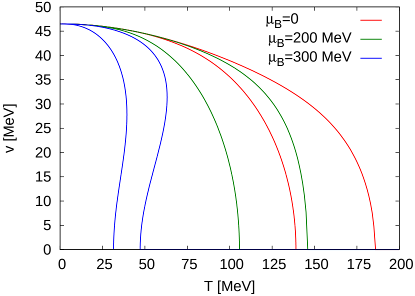
$$\chi^2(x_1,\dots,x_N)=\sum_{i=1}^{M}\left[\frac{Q_i(x_1,\dots,x_N)-Q_i^{\exp}}{\delta Q_i}\right]^2,$$

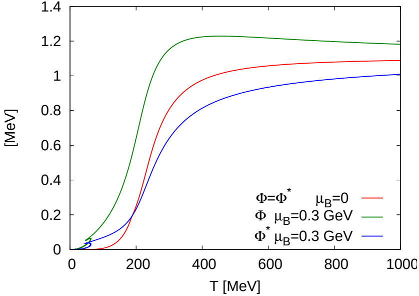
where $(x_1,\dots,x_N)=(m_0,\lambda_1,\lambda_2,\dots)$, $Q_i(x_1,\dots,x_N)$ calculated from the model, while $Q_i^{\exp}$ taken from the PDG

multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\rightarrow$ 2 physical quantities: $f_\pi, f_K$
- Tree-level masses $\rightarrow$ 16 physical quantities: $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\rightarrow$ 12 physical quantities: $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$

# Behaviour of the order parameters



- An extended linear $\sigma$ - model was shown with constituent quarks and Polyakov - loops
- We used hybrid approach at $T = 0$: only fermion loops, since it has the largest contribution
- At finite $T/\mu_B$ there was 4 coupled equations for the 4 order parameters

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$\rightarrow$ To do . . .

$\rightarrow$ Finalize program code

$\rightarrow$ Explore the phase diagram especially the CEP

$\rightarrow$ Investigate the effect of meson thermal loops

$\rightarrow$ Calculate medium dependence of the meson masses

# Thank you for your attention!