Dispersive Approach to Hadronic Light-by-Light Scattering and the Muon $g - 2$

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Magnetic moment

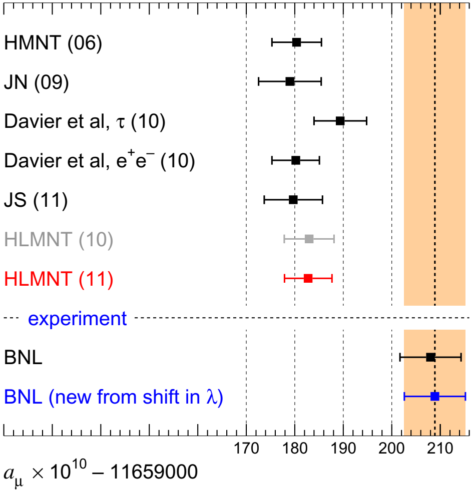
- Relation of spin and magnetic moment of a lepton:

$$\vec{\mu}_\ell = g_\ell \frac{e}{2m_\ell} \vec{s}$$

  g_ℓ: Landé factor, gyromagnetic ratio

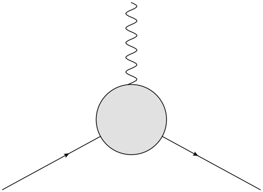
- Dirac's prediction: $g_e = 2$
- Anomalous magnetic moment: $a_\ell = (g_\ell - 2)/2$
- Helped to establish QED and QFT as the framework for elementary particle physics
- Today: probing not only QED but entire SM

a_μ: comparison of theory and experiment



→ Hagiwara et al. 2012

Interaction of a muon with an external electromagnetic field



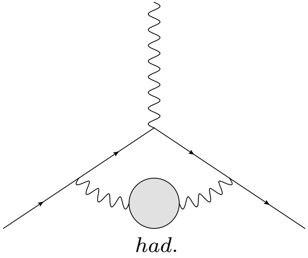
Anomalous magnetic moment given by one particular form factor

QED and electroweak contributions

- Full $\mathcal{O}(\alpha^5)$ calculation by Kinoshita et al. 2012 (involves 12672 diagrams!)
- EW contributions (EW gauge bosons, Higgs) calculated to two loops (three-loop terms negligible)

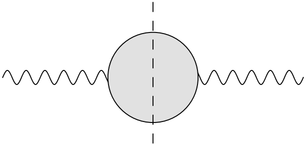
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
QED total	$116\,584\,718.95$	0.08
EW	153.6	1.0
Theory total	$116\,591\,855$	59

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$



- Problem: QCD is non-perturbative at low energies
- First principle calculations (lattice QCD) may become available in the future
- Current evaluations based on dispersion relations and data

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$



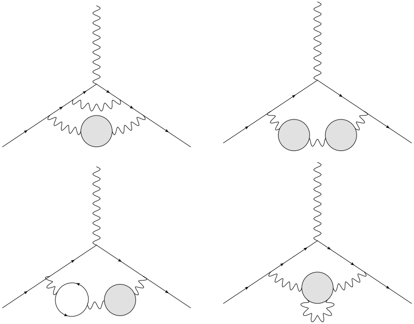
- Basic principles: unitarity and analyticity
- Direct relation to experiment: total hadronic cross section $\sigma_{\rm tot}(e^+e^- \to \gamma^* \to \text{hadrons})$
- One Lorentz structure, one kinematic variable

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$

- At present: dominant theoretical uncertainty
- Theory error due to experimental input
- Can be systematically improved: dedicated e^+e^- program (BaBar, Belle, BESIII, CMD3, KLOE2, SND)

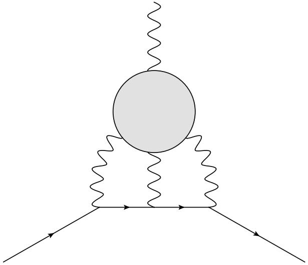
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
LO HVP	$6\,949$	43
Theory total	$116\,591\,855$	59

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$



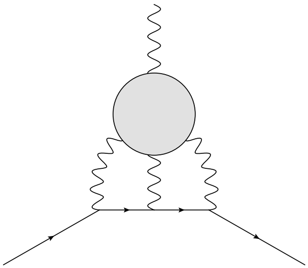
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$
NLO HVP	-98	1
Theory total	$116\,591\,855$	59

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$ Hadronic light-by-light (HLbL) scattering



- Hadronic matrix element of four EM currents
- Up to now, only model calculations
- Lattice QCD not yet competitive

Higher order hadronic contributions: $\mathcal{O}(\alpha^3)$
Hadronic light-by-light (HLbL) scattering



- Uncertainty estimate based rather on consensus than on a systematic method
- Will dominate theory error in a few years
- "Dispersive treatment impossible"

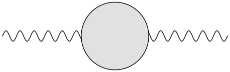
	$10^{11} \cdot a_\mu$	$10^{11} \cdot \Delta a_\mu$	
BNL E821	$116\,592\,091$	63	→ PDG 2013
QED total	$116\,584\,718.95$	0.08	→ Kinoshita et al. 2012
EW	153.6	1.0	
LO HVP	$6\,949$	43	→ Hagiwara et al. 2011
NLO HVP	-98	1	→ Hagiwara et al. 2011
NNLO HVP	12.4	0.1	→ Kurz et al. 2014
LO HLbL	116	40	→ Jegerlehner, Nyffeler 2009
NLO HLbL	3	2	→ Colangelo et al. 2014
Hadronic total	6982	59	
Theory total	$116\,591\,855$	59	

a_μ: Theory vs. Experiment

- Theory error completely dominated by hadronic effects
- Discrepancy between Standard Model and experiment $\sim 3\sigma$
- Hint to new physics?
- New experiments (FNAL, J-PARC) aim at reducing the experimental error by a factor of 4

Leading hadronic contribution: $\mathcal{O}(\alpha^2)$

Photon vacuum polarisation function:



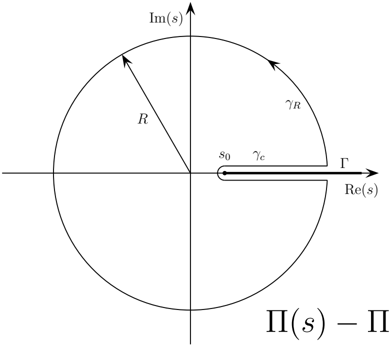
$$= -i(q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

Unitarity of the S-matrix implies the optical theorem:

$$\mathrm{Im}\Pi(s) = \frac{s}{e(s)^2}\sigma_{\mathrm{tot}}(e^+e^- \to \gamma^* \to \mathrm{hadrons})$$

Dispersion relation

Causality implies analyticity:



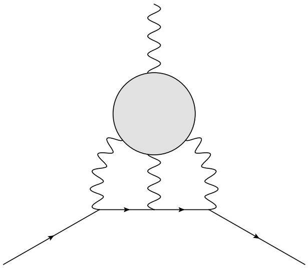
Cauchy integral formula:

$$\Pi(s) = \frac{1}{2\pi i} \oint_\gamma \frac{\Pi(s')}{s' - s} ds'$$

Deform integration path:

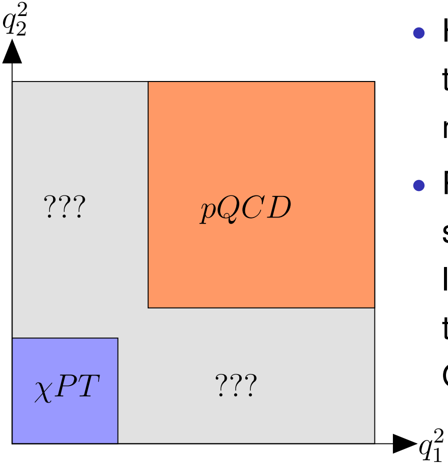
$$\Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4M_\pi^2}^\infty \frac{\mathrm{Im}\Pi(s')}{(s' - s - i\epsilon)s'} ds'$$

How to improve HLbL calculation?



- "Dispersive treatment impossible": No!
- Relate HLbL to experimentally accessible quantities
- Make use of unitarity, analyticity, gauge invariance and crossing symmetry

Mixed scales



- HLbL ‘blob’ inside loops: two independent loop momenta
- Problem of mixed scales: neither low-energy effective theory nor perturbative QCD works

HLbL tensor: properties

- Object in question: $\Pi^{\mu\nu\lambda\sigma}(q_1, q_2, q_3)$
- Basis: 29 independent structures contribute to a_μ
- Five dynamical variables, e.g. two Mandelstam variables

$$s = (q_1 + q_2)^2, t = (q_1 + q_3)^2$$

  and three photon virtualities q_1^2, q_2^2, q_3^2
- Much more complicated analytic structure than HVP

Mandelstam representation

- We limit ourselves to intermediate states of at most two pions
- Writing down a double-spectral (Mandelstam) representation allows us to split up the HLbL tensor:

$$\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0\text{-pole}} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \bar{\Pi}_{\mu\nu\lambda\sigma} + \dots$$

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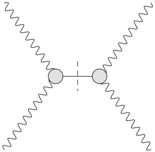
One-pion intermediate state:



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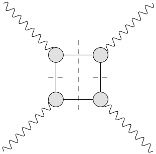
Two-pion intermediate state in both channels:



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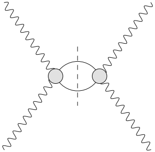
Two-pion intermediate state in first channel: 

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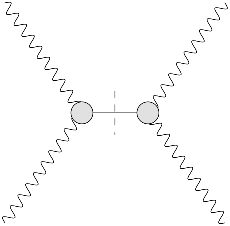
Neglected: higher intermediate states

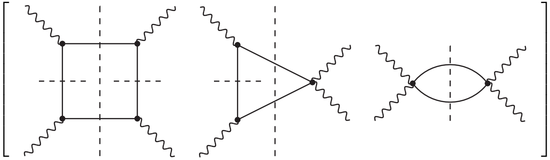
Pion pole



- Input the doubly-virtual and singly-virtual pion transition form factors $\mathcal{F}_{\gamma^*\gamma^*\pi^0}$ and $\mathcal{F}_{\gamma^*\gamma\pi^0}$
- Dispersive analysis of transition form factors in progress

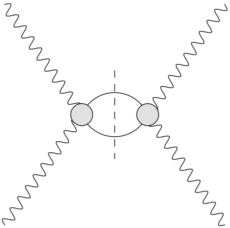
  → B. Kubis, Amherst workshop 2014

FsQED

$F_\pi^V(q_1^2)F_\pi^V(q_2^2)F_\pi^V(q_3^2) \times$ 

- Simultaneous two-pion cuts in two channels
- Analytic properties correspond to sQED box diagram
- Gauge invariance requires triangle and bulb diagrams
- q^2-dependence given by multiplication with pion vector form factor $F_\pi^V(q^2)$ for each off-shell photon

Remainder

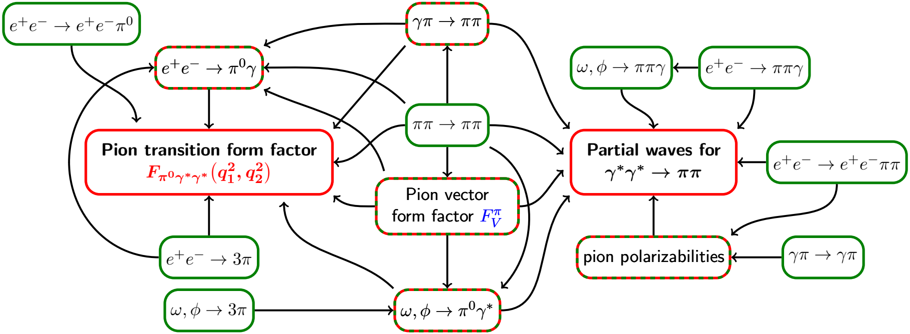


- Two-pion cut in only one channel $\Rightarrow$ scalar functions have only a right-hand cut
- Expand into partial waves
- Unitarity relates it to the helicity amplitudes of the subprocess $\gamma^*\gamma^{(*)} \to \pi\pi$
- Dispersive integrals over the imaginary parts give $\bar{\Pi}_{\mu\nu\lambda\sigma}$

Summary

- Our dispersive approach to HLbL scattering is based on fundamental principles: unitarity, analyticity, crossing, gauge invariance
- We take into account the lowest intermediate states: π^0-pole and $\pi\pi$-cuts
- Relation to experimentally accessible (or again with data dispersively reconstructed) quantities
- A step towards a model-independent calculation of a_μ
- Numerical evaluation is work in progress

A roadmap for HLbL



→ Flowchart by M. Hoferichter