# Interactions of light mesons with photons

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Meson 2014, Cracow, May/June 2014

# Collaborators

- Uppsala: Per Engström, Bruno Strandberg (now Glasgow), Hazhar Ghaderi, Carla Terschlüsen
- GSI: Igor Danilkin (now JLAB), Matthias Lutz
- Bonn: Franz Niecknig, Martin Hoferichter (now Bern), Sebastian Schneider, Bastian Kubis

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# Reactions of hadrons with (virtual) photons

Why is it interesting?

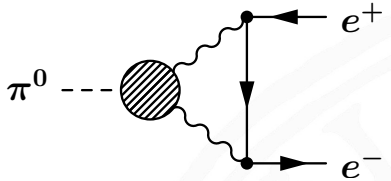
- explore intrinsic structure of hadrons
  - $\rightsquigarrow$ form factors
  - $\rightsquigarrow$ to which extent does vector meson dominance hold?

# Reactions of hadrons with (virtual) photons

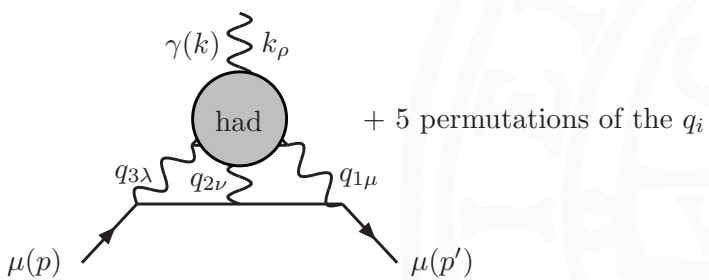
Why is it interesting?

- explore intrinsic structure of hadrons
  - $\rightsquigarrow$ form factors
  - $\rightsquigarrow$ to which extent does vector meson dominance hold?
- background for physics beyond standard model

$\rightsquigarrow$ rare pion decay $\pi^0 \longrightarrow e^+e^-$

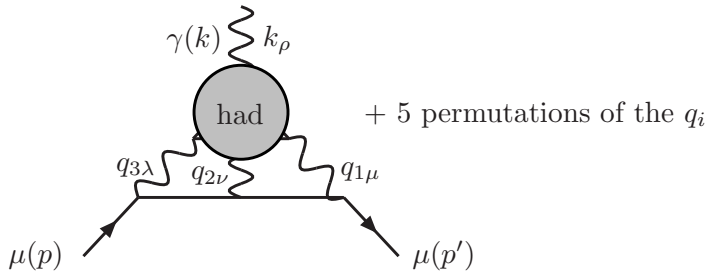



$\rightsquigarrow$ $g-2$ of muon



# Hadronic contribution to $g-2$ of the muon

light-by-light scattering




- $\gamma^*\gamma^* \leftrightarrow$ hadron(s) is not directly accessible by experiment

$\hookrightarrow$ need good theory with reasonable estimate of uncertainty
(ideally an effective field theory)

$\hookrightarrow$ need experiments to constrain such hadronic theories

true for all hadronic contributions:

- the lighter the hadronic system, the more important
(though high-energy contributions not unimportant for light-by-light)

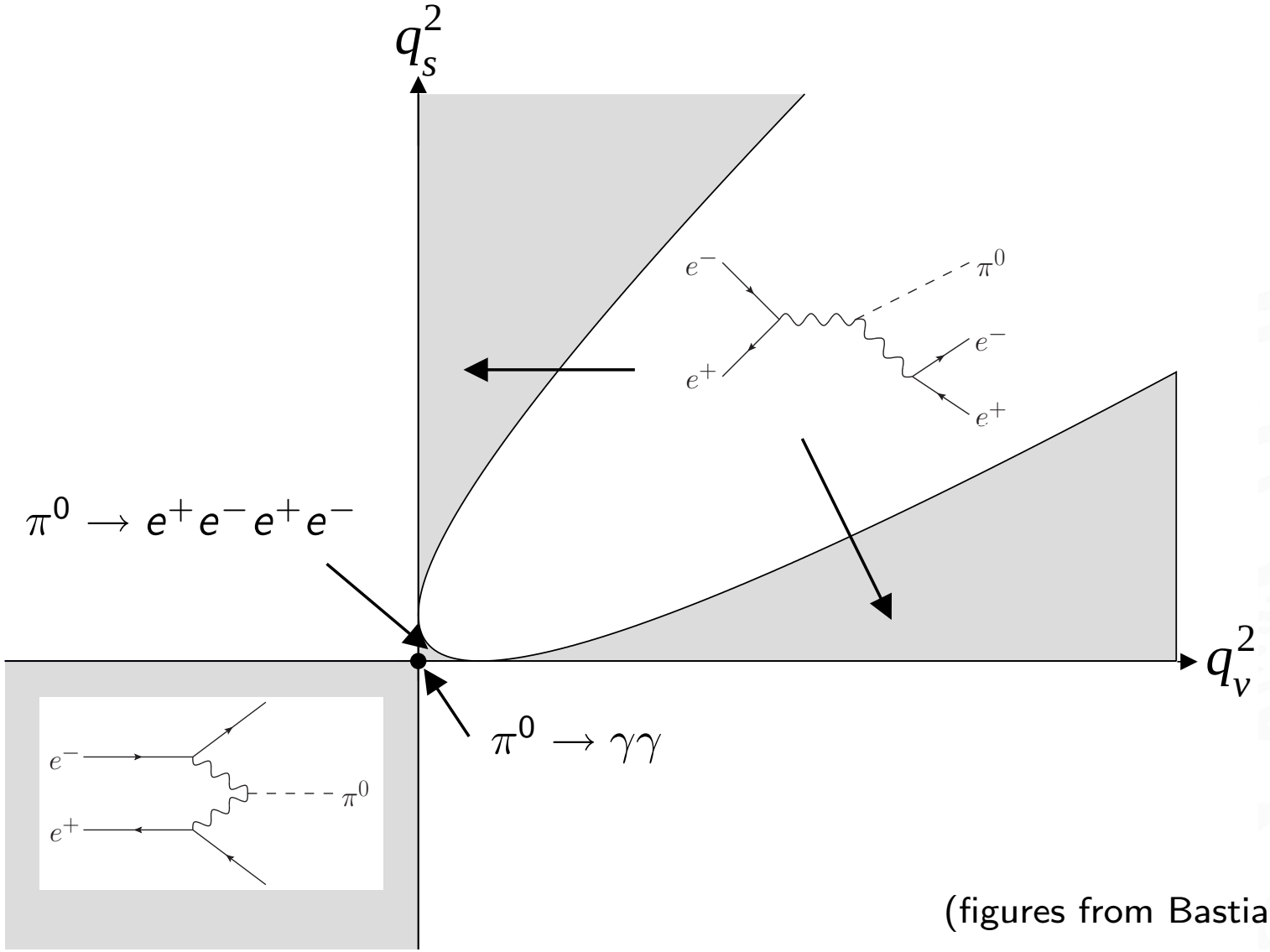
$\hookrightarrow$ $\gamma^{(*)}\gamma^{(*)} \leftrightarrow \pi^0$ (you've seen this before for rare pion decay),
$\gamma^{(*)}\gamma^{(*)} \leftrightarrow 2\pi$, ...

# Shopping list for hadron theory and experiment

- transition form factors of pseudoscalars $\gamma^{(*)}\gamma^{(*)} \leftrightarrow P$ with $P = \pi^0, \eta, \eta', \ldots$

$\hookrightarrow$ several interesting kinematical regions $\rightsquigarrow$ next slide (for pion)

# $\pi^0 \to \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor




(figures from Bastian Kubis)

# Shopping list for hadron theory and experiment

- transition form factors of pseudoscalars $\gamma^{(*)}\gamma^{(*)} \leftrightarrow P$ with $P = \pi^0, \eta, \eta', \ldots$
- if invariant mass of dilepton around mass of a vector meson:

$\rightsquigarrow$ relation to
transition form factors of vector to pseudoscalar mesons
$V \leftrightarrow P\gamma^{(*)}$ with $V = \rho^0, \omega, \phi, \ldots$

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- "two-gamma physics" $\gamma\gamma \longrightarrow \pi^+\pi^-, \pi^0\pi^0, \pi^0\eta, K\bar{K}, \ldots$
(cross relation to polarizability of the pion)

$\hookrightarrow$ has triggered a lot of experimental activity,
in particular MesonNet (WASA, KLOE, MAMI, HADES, ...)

# Two complementary approaches

- Lagrangian approach
  - use only hadrons which are definitely needed
    (here: lowest nonets of pseudoscalar and vector mesons)
  - sort interaction terms concerning importance,
    essentially based on large-$N_c$
  - include causal rescattering/unitarization for reactions
    (I. Danilkin, L. Gil, M. Lutz, Phys.Lett. B703, 504 (2011))
  - long-term goal: obtain sensible estimates of uncertainties
- dispersive approach
  - include most important hadronic inelasticities
  - use measured (and dispersively improved) phase shifts (2-body)
  - use Breit-Wigner plus background for narrow resonances
    ($n$-body, $n > 2$)
  - error estimates from more vs. less subtracted dispersion relations

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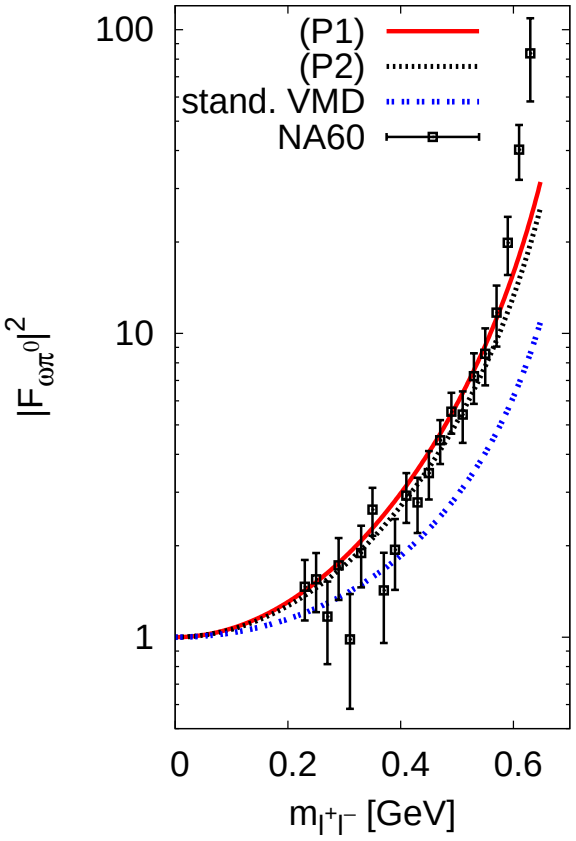
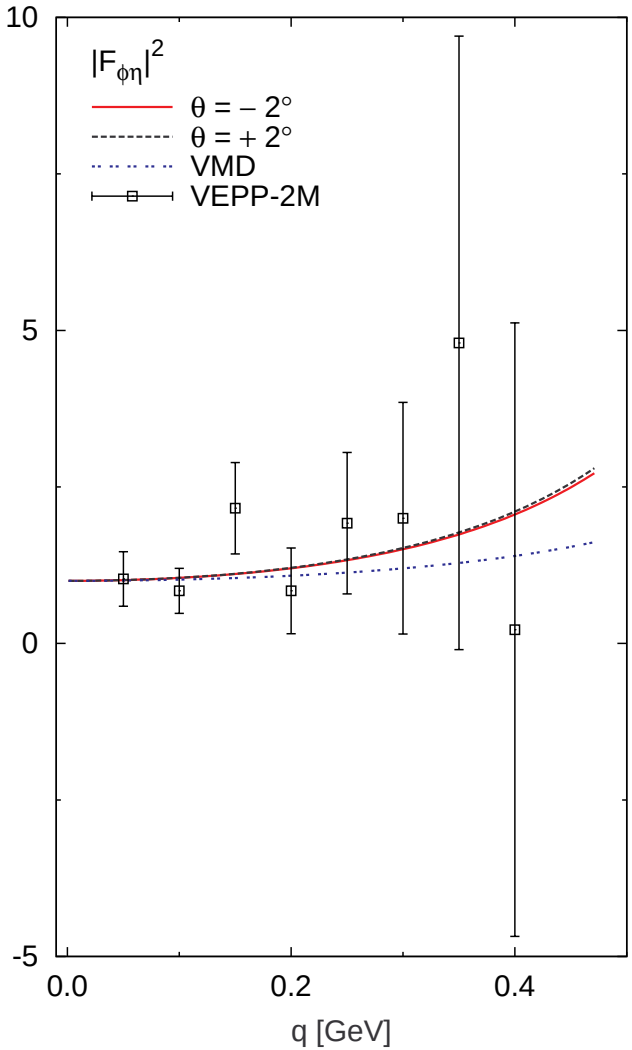
# Transition form factor $\omega \longrightarrow \pi^0 +$ dilepton




- data and our Lagrangian approach show strong deviations from vector-meson dominance (VMD)
- our approach describes data fairly well except for large invariant masses close to phase-space limit (log plot!)
- second experimental confirmation desirable

C. Terschlüsen, S.L., Phys. Lett. B691, 191 (2010)

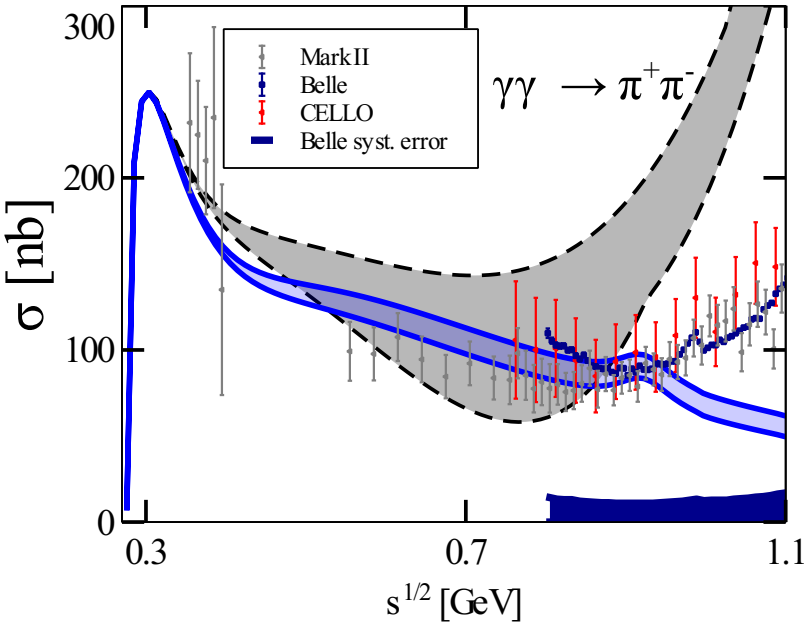
# Transition form factor $\phi \longrightarrow \eta +$ dilepton

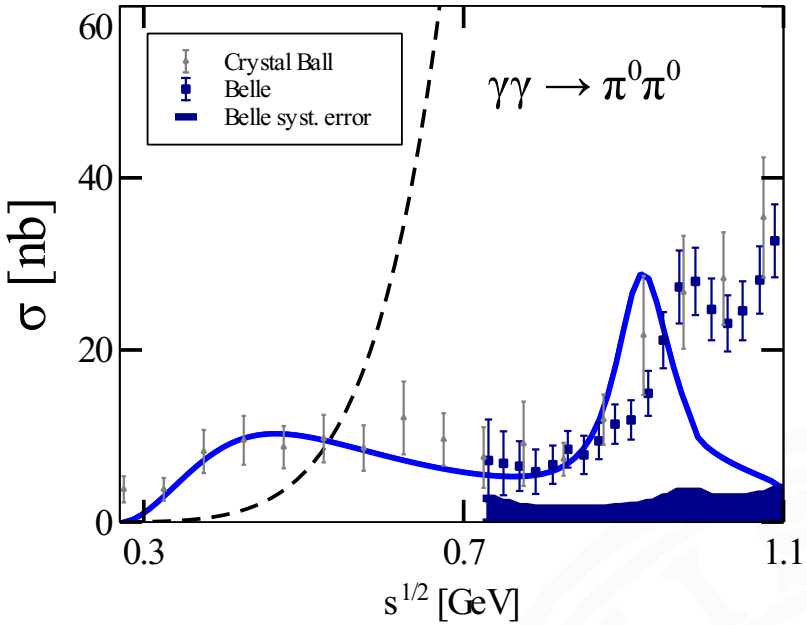



- our Lagrangian approach deviates from VMD
- new data from KLOE will come soon

C. Terschlüsen, S.L., M.F.M. Lutz, Eur.Phys.J. A48, 190 (2012)

# $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$



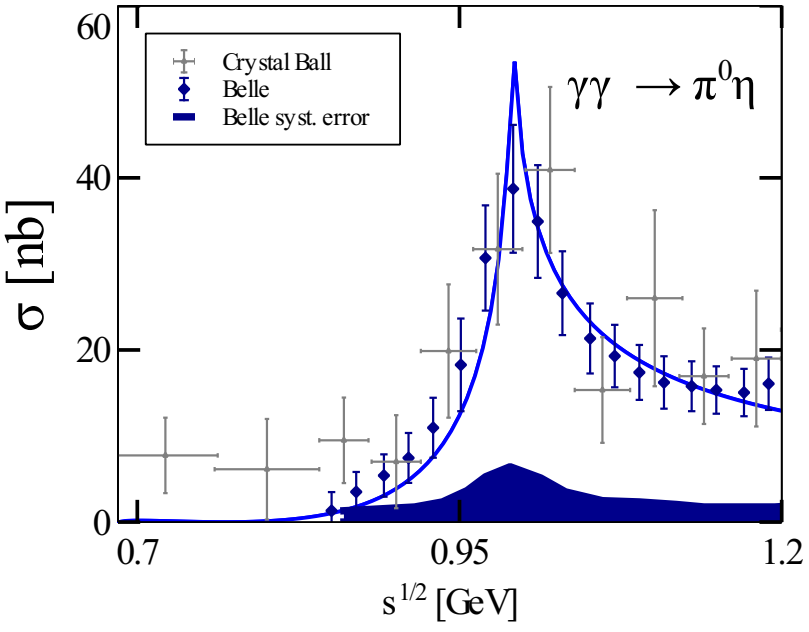



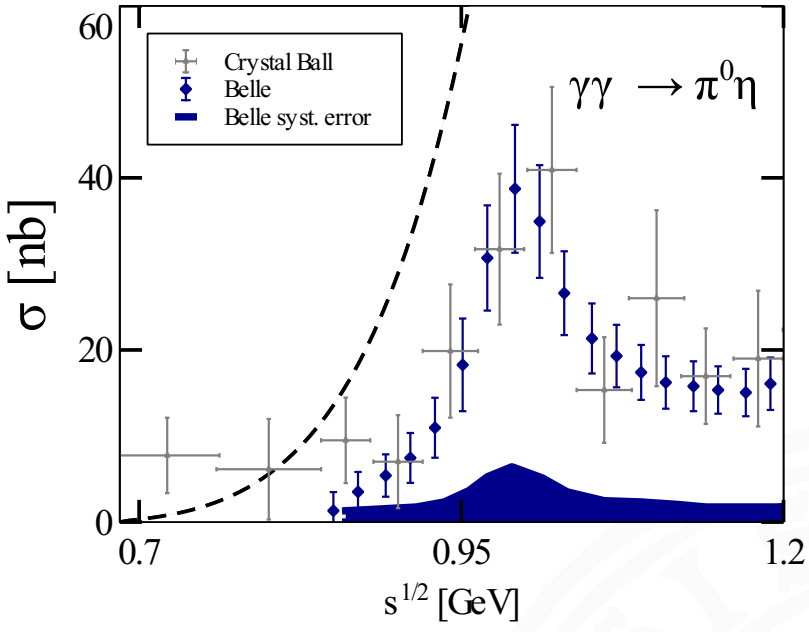

- dashed black lines: tree level,
  blue lines: with coupled-channel rescattering of two pseudoscalar mesons
- overall good description, room for improvement concerning $f_0(980)$
- at high energies spin-2 mesons are missing

I.V. Danilkin, M.F.M. Lutz, S.L., C. Terschlüsen,

Eur.Phys.J. C73, 2358 (2013)

# $\gamma\gamma \to \pi^0\eta$



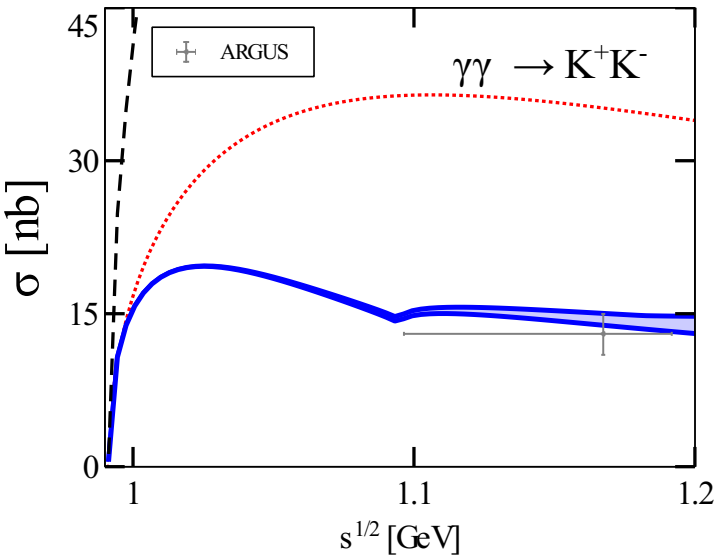



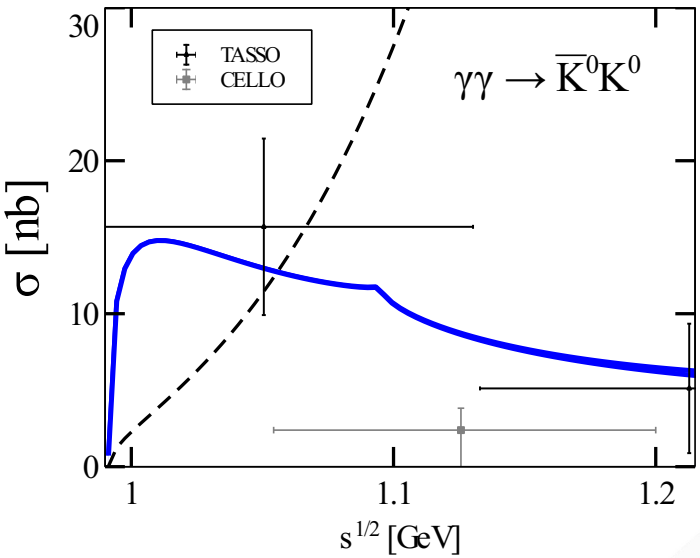

- dashed black line: tree level,
  blue line: with coupled-channel rescattering of two pseudoscalar mesons
- $a_0(980)$ dynamically generated

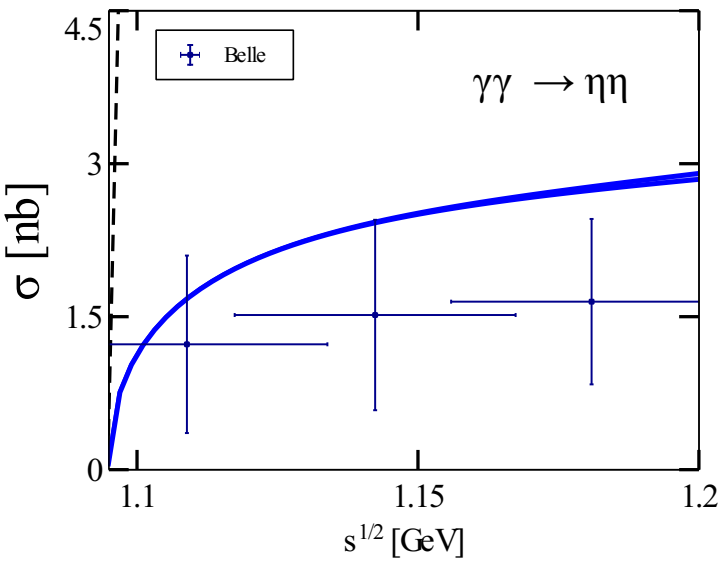
I.V. Danilkin, M.F.M. Lutz, S.L., C. Terschlüsen,

Eur.Phys.J. C73, 2358 (2013)

# $\gamma\gamma \to K^+K^-, K^0\bar{K}^0, \eta\eta$ (pure predictions)



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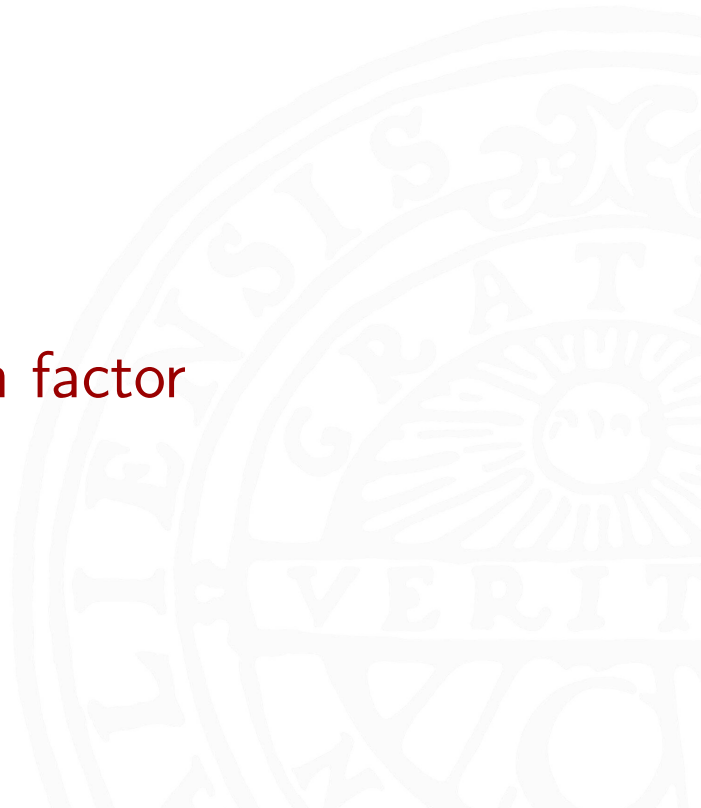
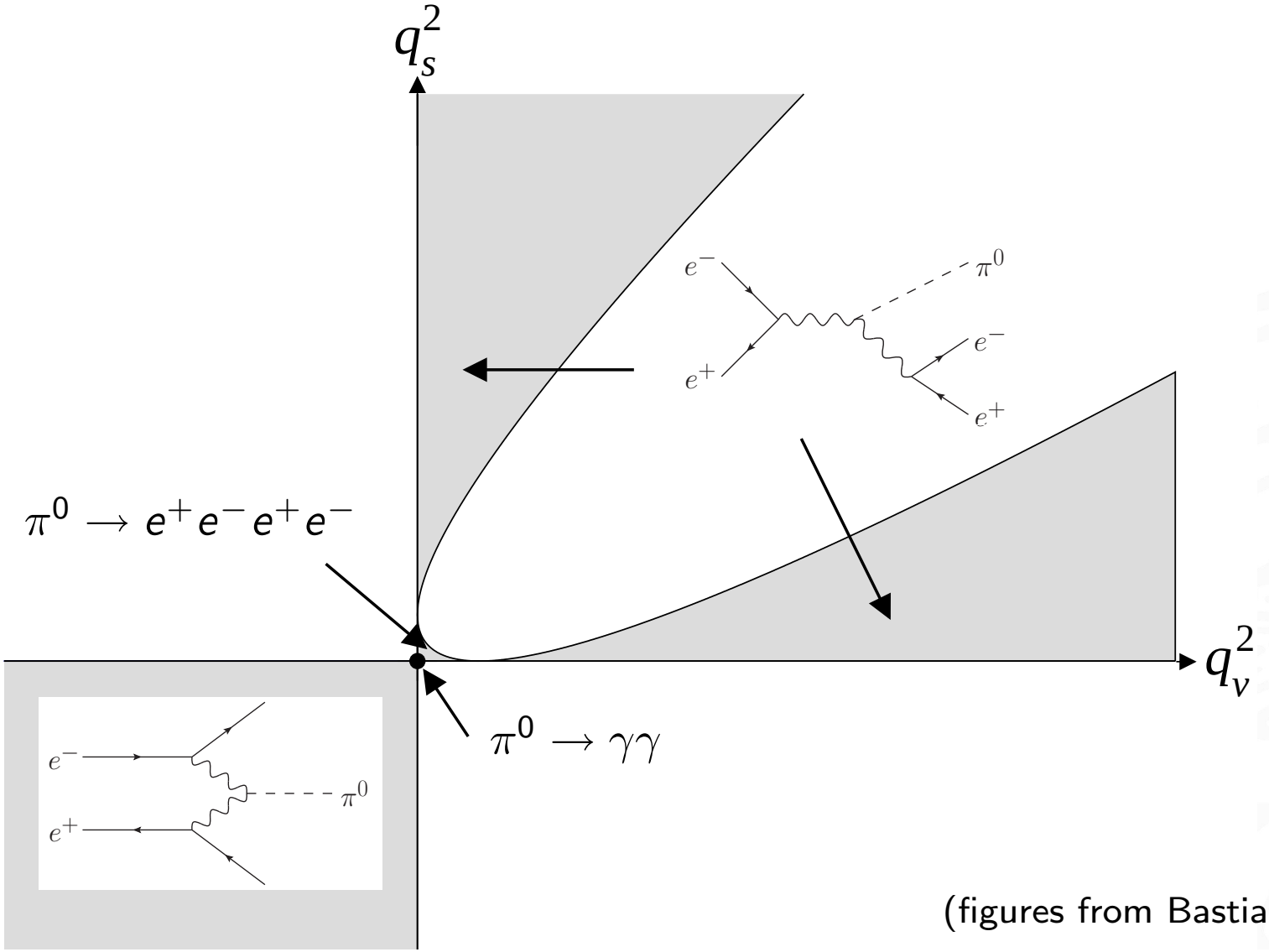
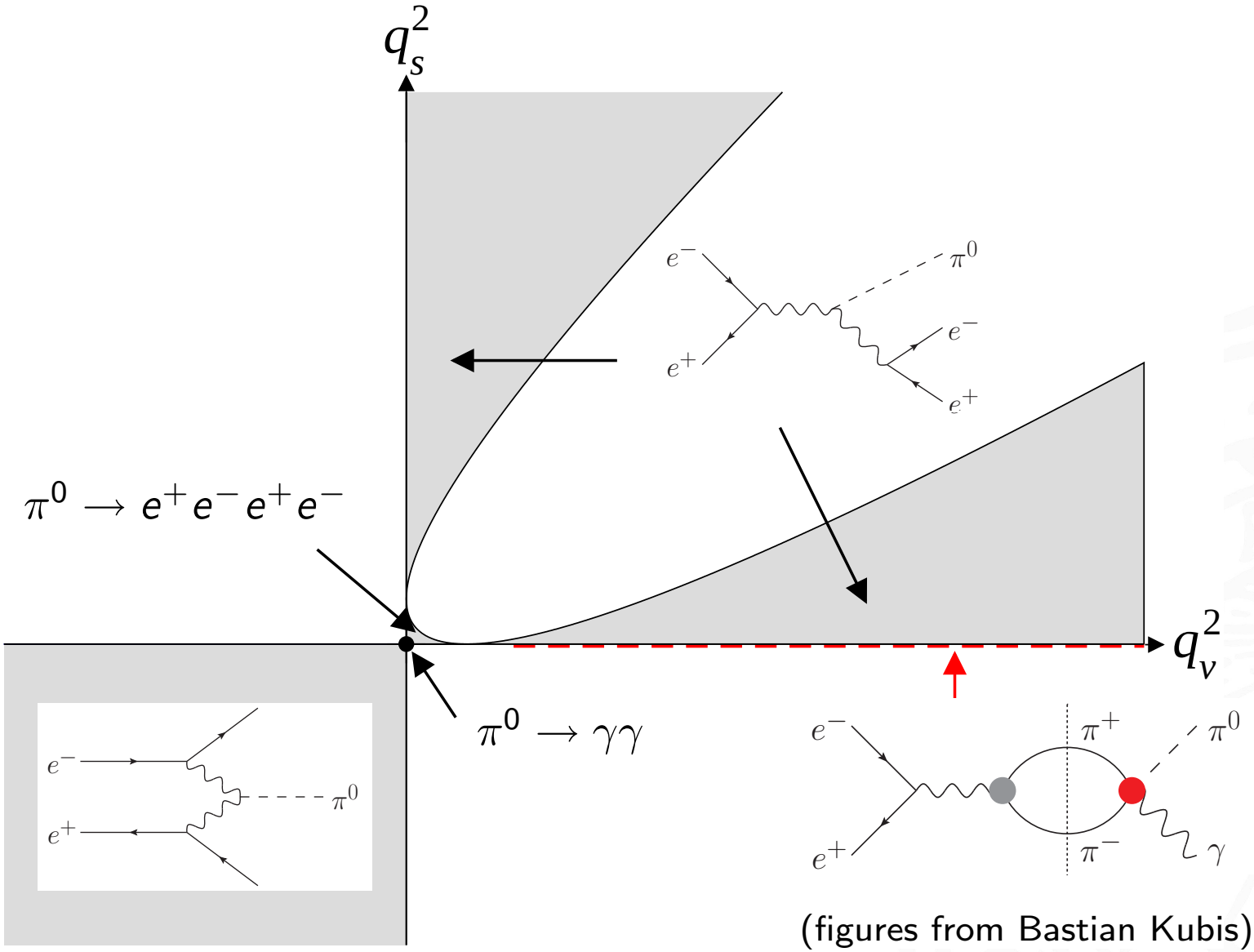


# $\pi^0 \to \gamma^*(q_v^2)\gamma^*(q_s^2)$ transition form factor



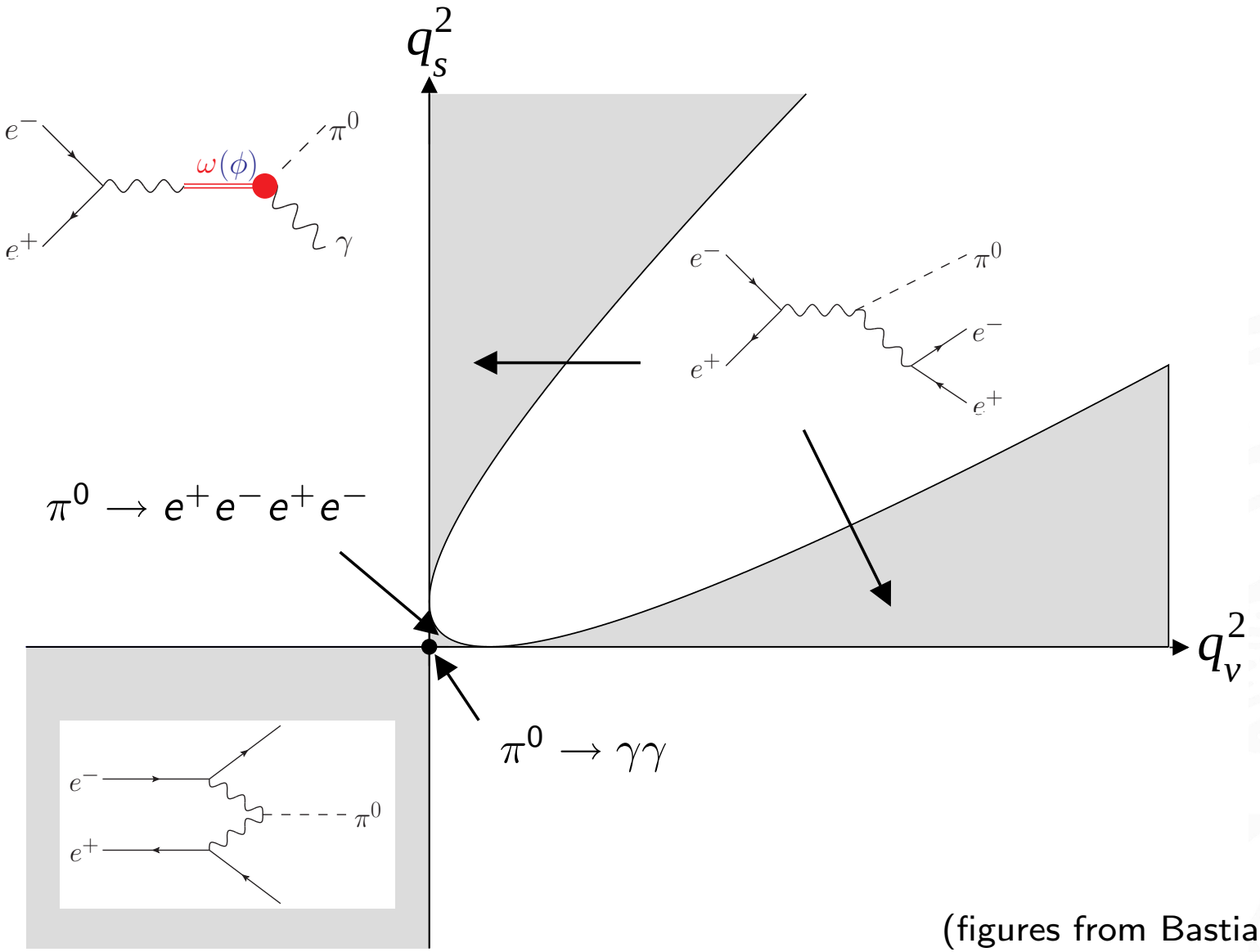

(figures from Bastian Kubis)

# $\pi^0 \longrightarrow \gamma^*(q_v^2)\gamma$ transition form factor




(figures from Bastian Kubis)

# $\pi^0 \longrightarrow \gamma\gamma^*(q_s^2)$ transition form factor




(figures from Bastian Kubis)

# Pion transition form factor — dispersive approach

- want prediction for $e^+e^- \to \pi^0\gamma$ (up to $\approx 1\,\text{GeV}$)

$\hookrightarrow$ dominant inelasticities:

  - $I = 1$: $e^+e^- \to \pi^+\pi^- \to \pi^0\gamma$
  - $I = 0$: $e^+e^- \to \pi^0\pi^+\pi^- \to \pi^0\gamma$

- required input for $I = 1$:
  - pion phase shift and pion form factor $\rightsquigarrow$ measured
  - strength of amplitude $\pi^+\pi^- \to \pi^0\gamma$ $\rightsquigarrow$ chiral anomaly

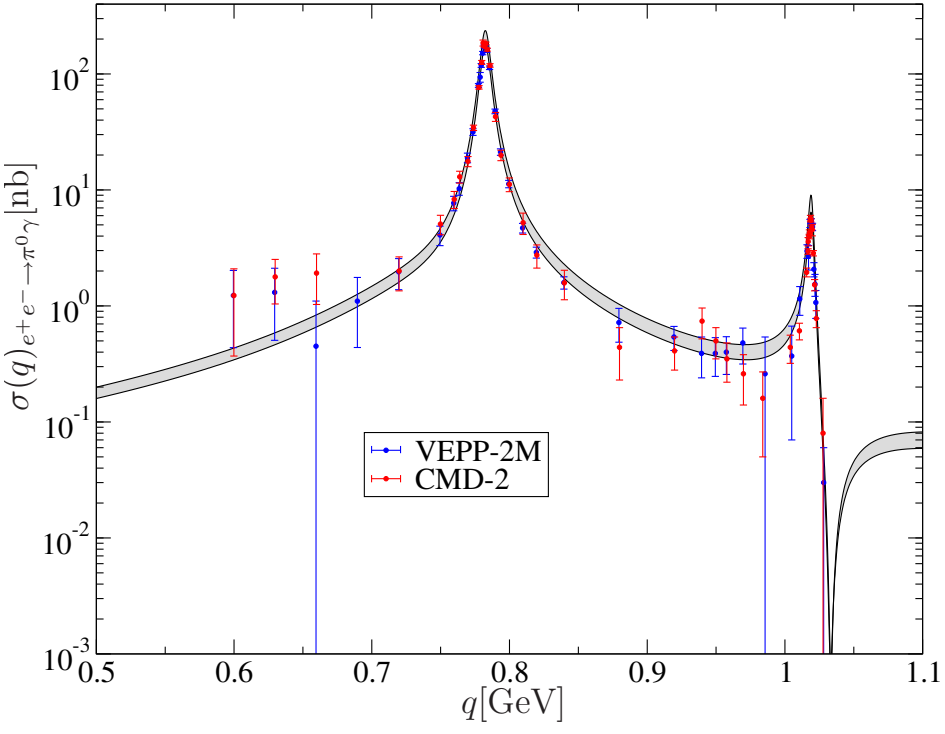
    (M. Hoferichter, B. Kubis, D. Sakkas, Phys.Rev. D86 (2012) 116009)

- input for $I = 0$ (three-body!):
  - dominated by narrow resonances $\omega$, $\phi$

  $\hookrightarrow$ use Breit-Wigners plus background for amplitude

  $\hookrightarrow$ fit to $e^+e^- \to \pi^+\pi^-\pi^0$

# Pion transition form factor ($e^+e^- \longrightarrow \pi^0\gamma$)




- unsubtracted dispersion relation
- uncertainty estimate from quality of $\omega/\phi \longrightarrow \pi^0\gamma$

  Schneider et al., PRD86, 054013
- can be extended to decay region $\pi^0 \longrightarrow \gamma\, e^+e^-$ and to spacelike region
- final aim: double virtual transition form factor

$\hookrightarrow$ relevant for $g-2$ and $\pi^0 \longrightarrow e^+e^-$

M. Hoferichter, B. Kubis, S.L., F. Niecknig and S. P. Schneider, in preparation

# Summary

- meson (transition) form factors and two-photon reactions allow access to intrinsic structure of hadrons

$\hookrightarrow$ quark structure, polarizabilities, ...

- in addition input for standard-model baseline calculations for rare decays ($\pi^0$) and high-precision determinations (muon's $g-2$)

$\hookrightarrow$ we are sharpening our theory tools to improve the accuracy of predictions

# Instead of an outlook

From two- to three-gamma physics

- yet another contribution to light-by-light scattering:

$$\gamma^* \longrightarrow \omega \longrightarrow 3\gamma^{(*)}$$

$\hookrightarrow$ related to scattering amplitude (dispersion theory)

$$\gamma\,\omega \longrightarrow \pi\,\pi \longrightarrow \gamma\,\gamma$$

i.e. to decays

$$\omega \longrightarrow \gamma\,\pi^+\,\pi^-\,, \qquad \omega \longrightarrow \gamma\,\pi^0\,\pi^0$$

- more (differential) data needed
- and also $\phi$ instead of $\omega$ (better data situation)

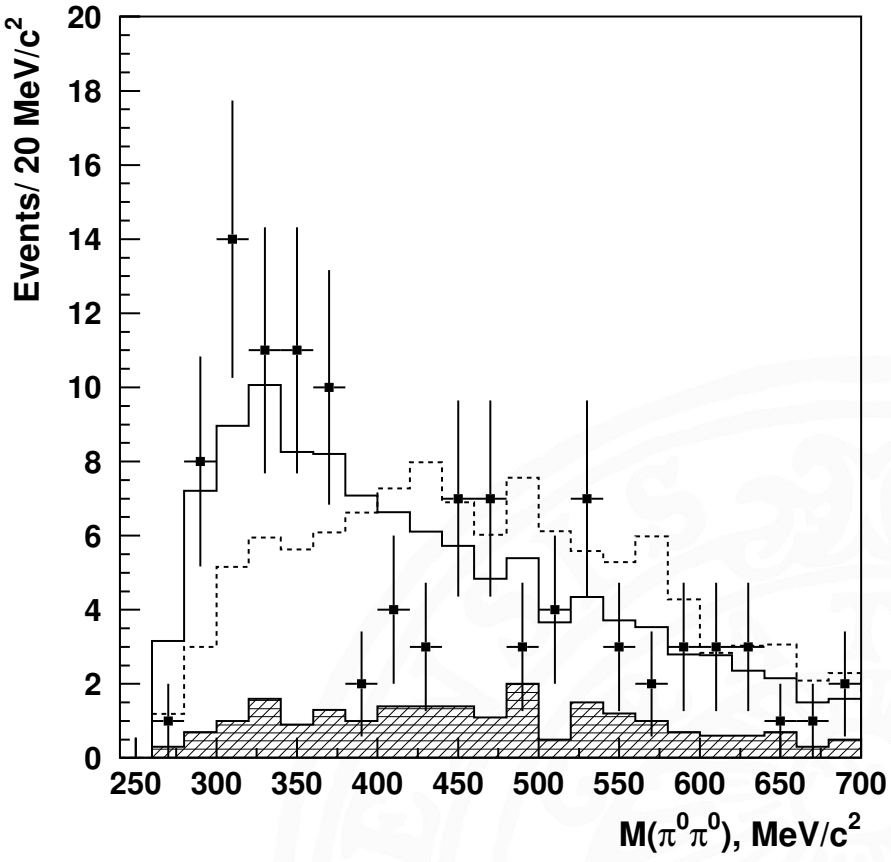
# Rare $\omega$ decays into $2\pi\,\gamma$

- $\omega \longrightarrow \pi^+\pi^-\gamma$:
  only upper limit

- $\omega \longrightarrow \pi^0\pi^0\gamma$:

$\hookrightarrow$ branching ratio: $6.6 \cdot 10^{-5}$

$\hookrightarrow$ differential data from CMD2
(Akhmetshin et al.,
Phys.Lett.B580, 119 (2004))




histograms are simulations with an intermediate rho (full) or sigma meson (dotted)

# backup slides

# How we sort interactions/diagrams

- without assigning importance to anything:
  - infinitely many interaction terms (with more and more derivatives)
  - infinitely many loop diagrams
- large-$N_c$ framework ($N_c$ = number of colors)
  - $\hookrightarrow$ loops are suppressed
    - note: we resum loops from rescattering, $s$-channel
  - $\hookrightarrow$ sorting scheme applies to scattering kernel (potential), not to scattering amplitude
- for interaction terms:
  - ensure appropriate $N_c$ scaling by dimensionful decay constant $f \sim \sqrt{N_c}$
  - to ensure pertinent dimension of interaction term in Lagrangian:
  - $\hookrightarrow$ assume large scale $\Lambda_{\mathrm{hard}} \gg m_V$ in denominator
  - $\hookrightarrow$ expansion in derivatives/momenta over $\Lambda_{\mathrm{hard}}$
    - depends on chosen representation

# Examples for interaction terms

- relevant, e.g., for $\omega \longrightarrow 3\pi$ and $\omega \longrightarrow \pi\gamma^*$
- both can proceed directly or via $\pi\rho^*$
- some unsuppressed interaction terms

$$\varepsilon_{\mu\nu\alpha\beta}\,\mathrm{tr}\left(\left\{V^{\mu\nu},\nabla_\lambda V^{\lambda\alpha}\right\}u^\beta\right)\,,$$

$$i\,f\,\mathrm{tr}\big(V_{\mu\nu}\,[u^\mu,u^\nu]\big)\,,\qquad f\,\mathrm{tr}\left(V^{\mu\nu}\,f^+_{\mu\nu}\right)$$

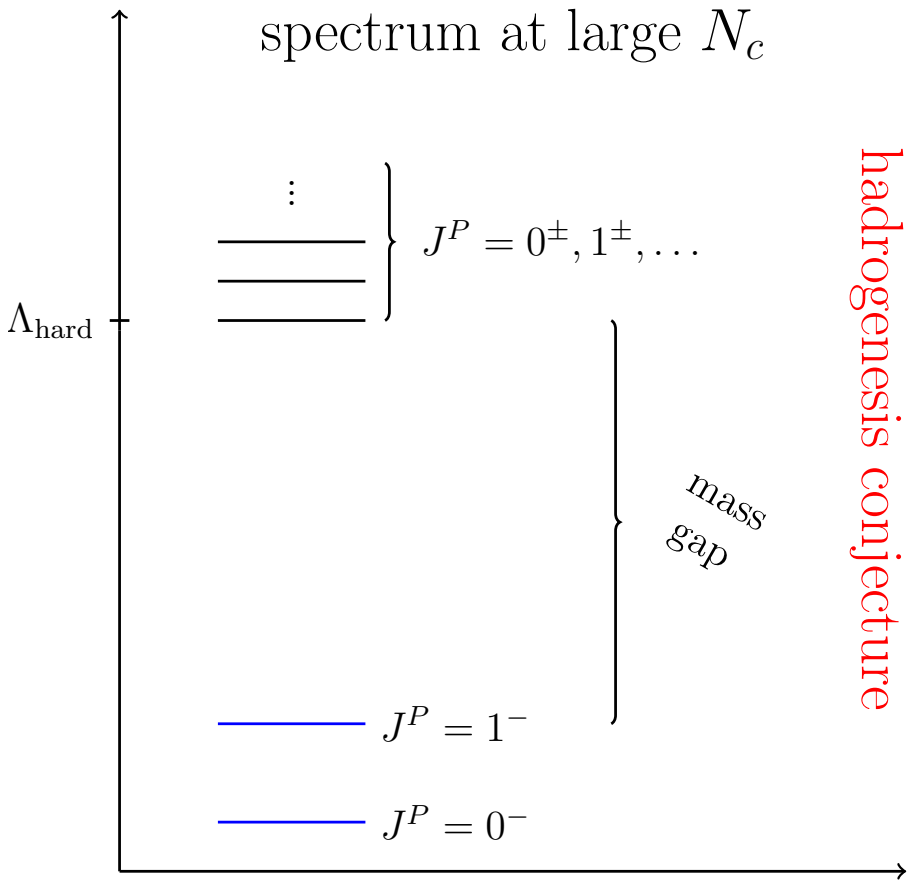
- some suppressed interaction terms (the direct ones)

$$\frac{f}{\Lambda^2_{\mathrm{hard}}}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\big(\nabla^\lambda V_{\lambda\mu}\,u_\nu\,u_\alpha\,u_\beta\big)\,,$$

$$\frac{f}{\Lambda^2_{\mathrm{hard}}}\,\varepsilon^{\mu\nu\alpha\beta}\,\mathrm{tr}\big(\{\nabla^\lambda V_{\lambda\mu},f^+_{\nu\alpha}\}\,u_\beta\big)\,.$$

- $\Lambda_{\mathrm{hard}}$: hadrogenesis gap or
  (here also O.K.) mass of excited vector mesons

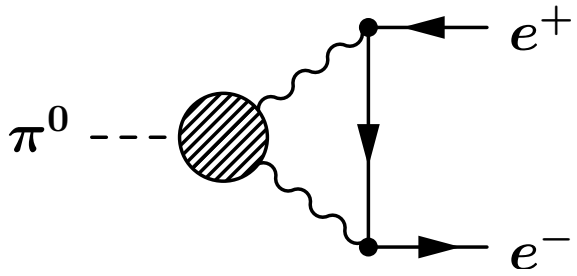
# hadrogenesis conjecture




other observed mesons below $\Lambda_{\rm hard}$ are supposed to be dynamically generated, i.e. meson molecules

C. Terschlüsen, S.L., M.F.M. Lutz, Eur.Phys.J. A48, 190 (2012)

# Rare pion decay — status




- $B(\pi^0 \to e^+e^-) = (6.46 \pm 0.33) \cdot 10^{-8}$ (KTeV, 2007)
- 3 $\sigma$ deviation between experiment and standard model

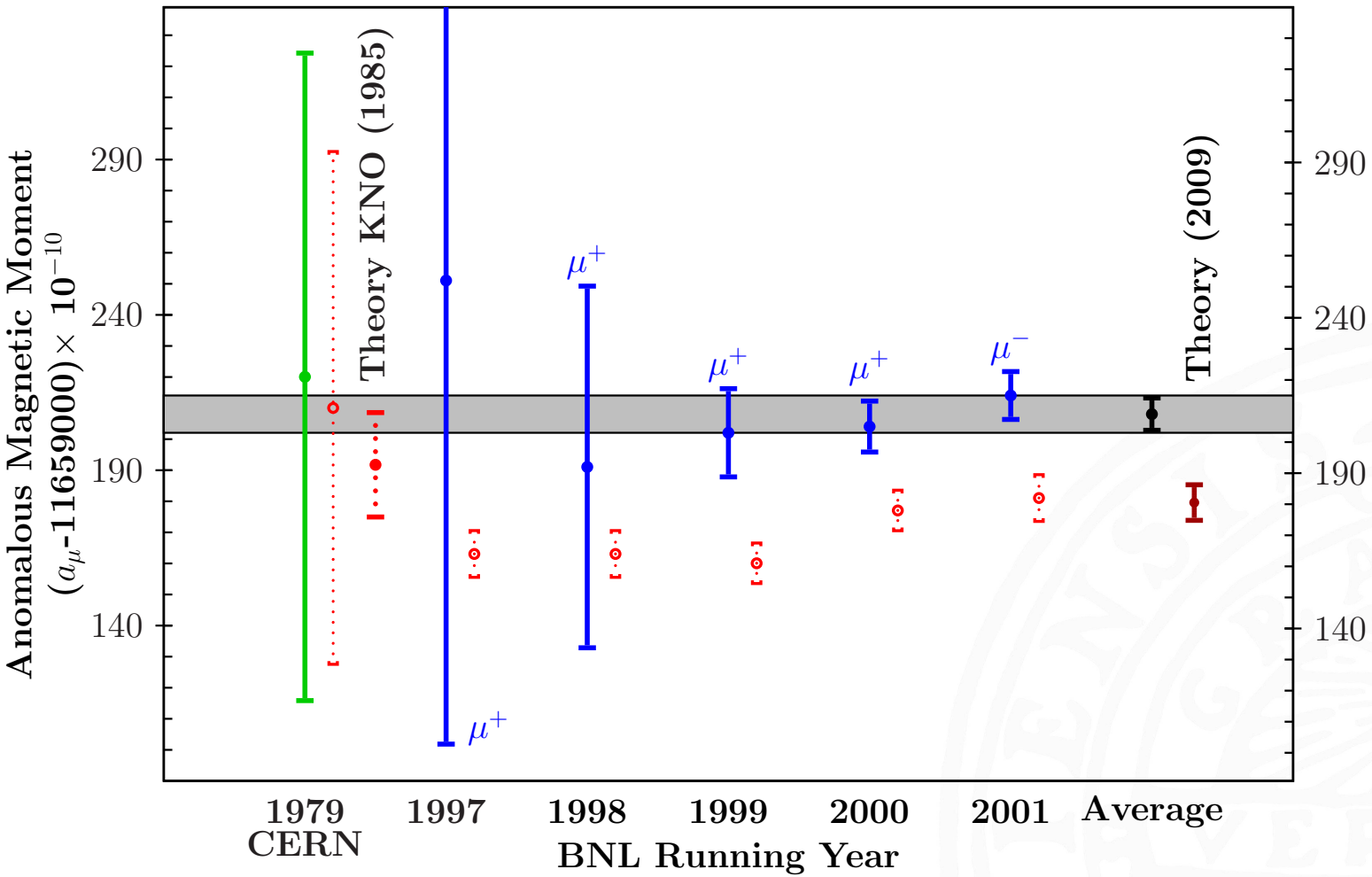
  Dorokhov/Ivanov, Phys. Rev. D75, 114007 (2007)

  (but controversial among theorists!)

- for point-like pion QED loop is divergent

$\hookrightarrow$ process is sensitive to hadronic transition form factor of pion $\pi^0 \leftrightarrow \gamma^{(*)}\gamma^{(*)}$

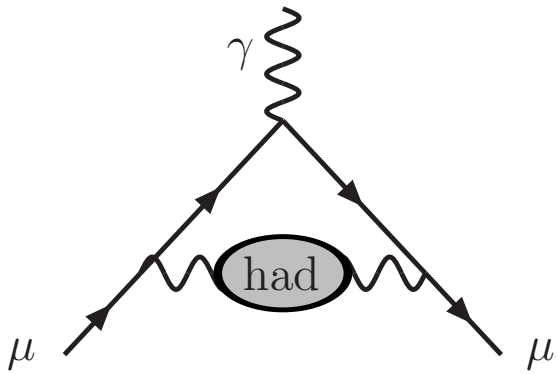
# $g-2$ of the muon — status




Jegerlehner/Nyffeler, Phys. Rept. 477, 1 (2009)

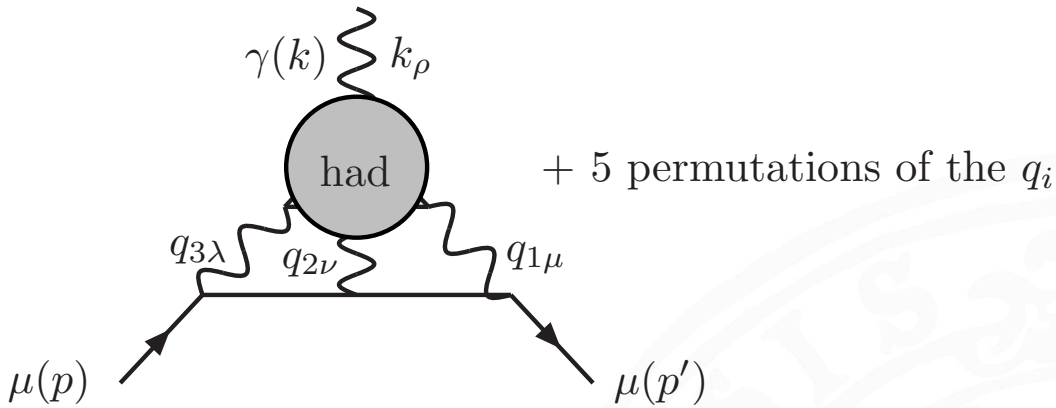
# $g-2$ of the muon — theory

Largest uncertainty of standard model: hadronic contributions




vacuum polarization

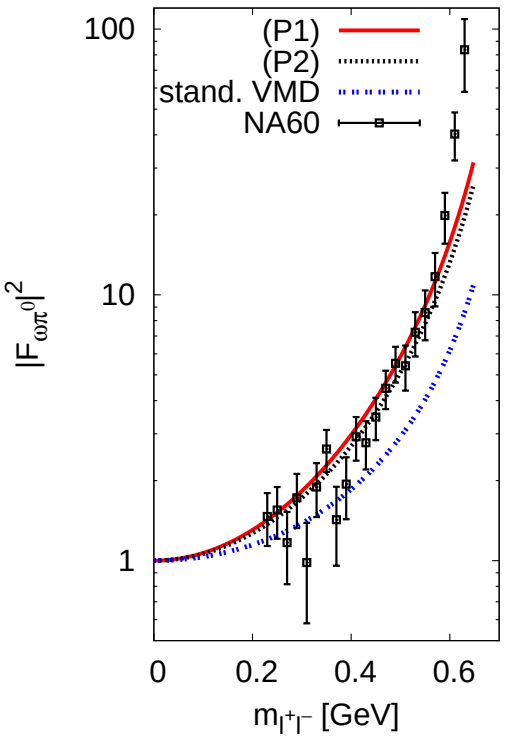
$\sim \alpha^2$



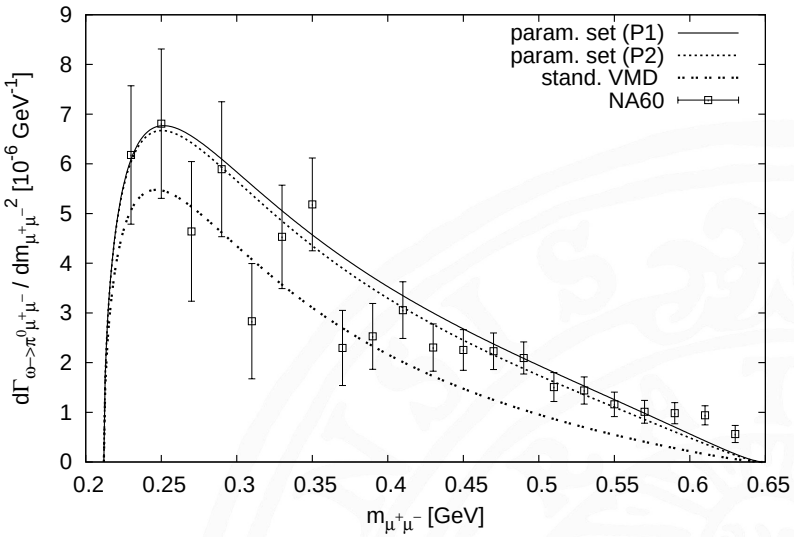

light-by-light scattering

$\sim \alpha^3$

# Transition form factor $\omega \longrightarrow \pi^0 + \mu^+\mu^-$




corresponding differential decay rate:




theory: C. Terschlüsen, S.L., Phys. Lett. B691, 191 (2010)

data: NA60, Phys. Lett. B 677, 260 (2009)