Dalitz plot analysis of $D^0 \to K_S^0 \pi^+ \pi^-$ decays in a factorization approach

Analysis done in collaboration with

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Motivation

Studies of the $D^0 \to K_S^0 \pi^+ \pi^-$ reaction are useful in:

1. measurements of the **D^0 - $\bar{D}^0$ mixing** parameters,
2. determination of the **Cabibbo- Kobayashi- Maskawa angle** γ in the decay amplitude $B^\pm \to D K^\pm$, $D \to K_S^0 \pi^+ \pi^-$,
3. description of the **final state** interactions between mesons, in particular in the S-waves,
4. testing theoretical models of meson **form factors**,
5. understanding properties of the meson resonances and their interference effects on the **Dalitz plot**.

Isobar model and its problems

1. Amplitudes in the isobar model are **not unitary** neither in three-body decay channels nor in two-body subchannels.
2. It is **difficult** to distinguish the **S-wave** amplitude from the **background** terms. Their interference is often very strong.
3. Some **branching fractions** extracted in such analyses could be unreliable.
4. The isobar model has many free parameters (at least two fitted parameters for each amplitude component). Recently Belle used 49 fitted parameters and BaBar 43 parameters.

Why unitarity is important?

Unitary model allows for:

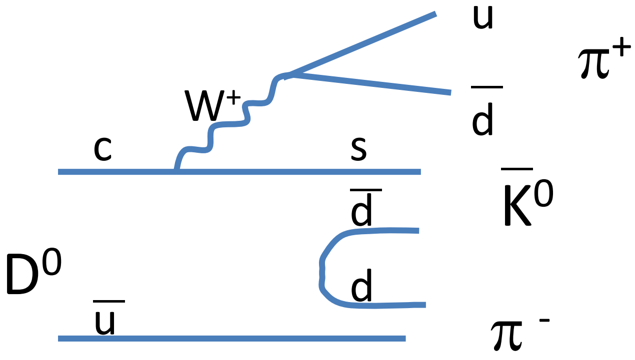
1. proper construction of the D-decay amplitudes,
2. partial wave analyses of final states,
3. explanation of structures seen in Dalitz plots,
4. adequate determination of branching fractions and CP asymmetries for different quasi-two-body decays,
5. extraction of standard model parameters (weak amplitudes),
6. application not only in analyses of D decays but also in studies of other reactions.

Towards a unitary approach

1. Construction of unitary three-body strong interaction amplitudes in a wide range of effective masses is difficult.
2. As **a first step** we attempt to incorpotate **in our model two-body unitarity** into the D-decay amplitudes with final state interactions in the following subchannels:
   a) $K^0 \pi$ S-wave amplitude,
   b) $\pi \pi$ S-wave amplitude,
   c) $\pi \pi$ P-wave amplitude.

Allowed and suppressed tree transitions

Transition $c \to s u \bar{d}$

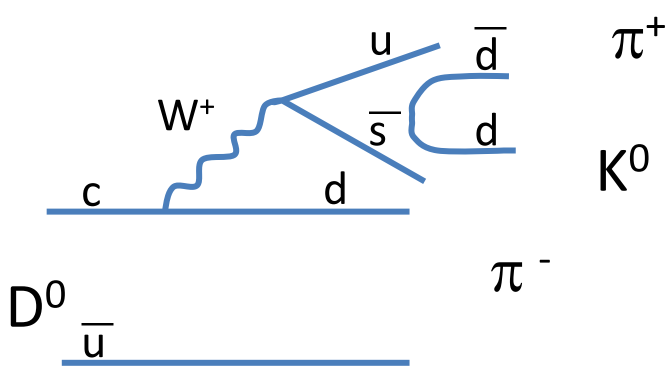


$$O_1 \propto \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} (\bar{s}c)_{V-A} (\bar{u}d)_{V-A}$$

$V_{cs} \approx V_{ud} \approx \cos\theta_C$ θ_C= Cabibbo angle

allowed

Transition $c \to d u \bar{s}$

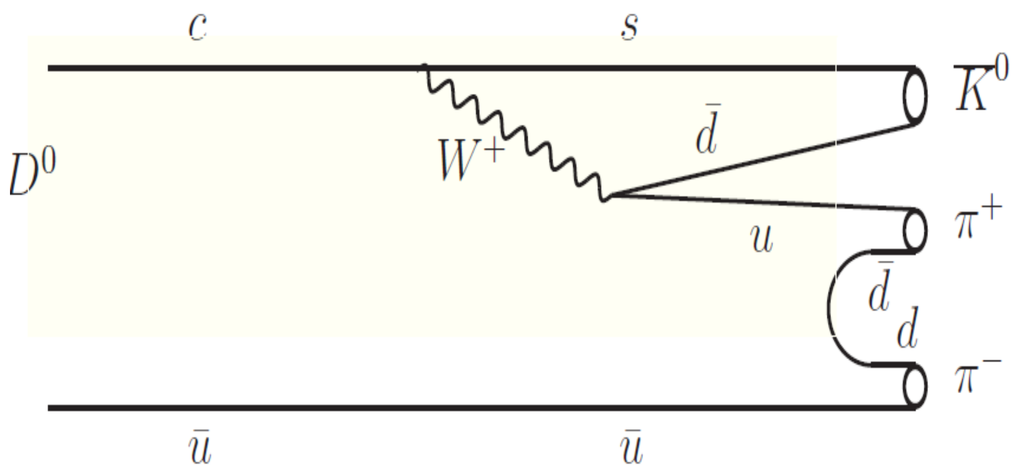


$$O_2 \propto \frac{G_F}{\sqrt{2}} V_{cd}^* V_{us} (\bar{d}c)_{V-A} (\bar{u}s)_{V-A}$$

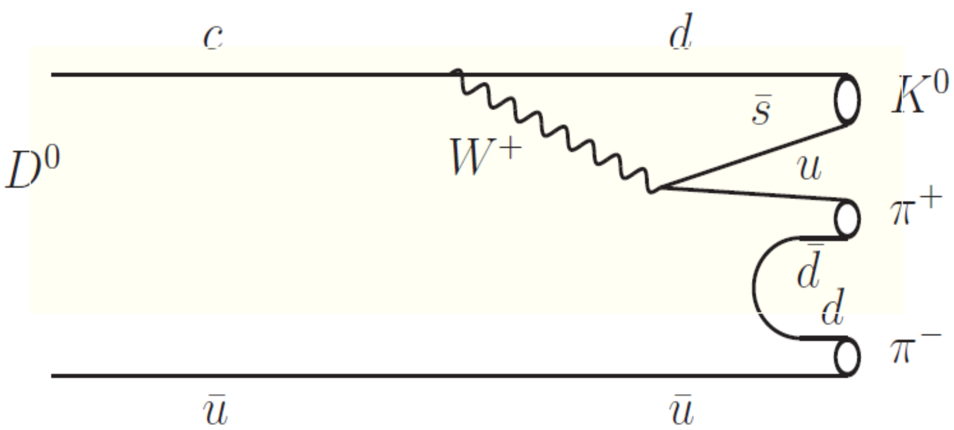
$$V_{cd} \approx -\lambda, \quad V_{us} \approx \lambda, \quad \lambda = \sin\theta_C \approx 0.225$$

doubly Cabibbo suppressed

Tree diagrams with internal W lines

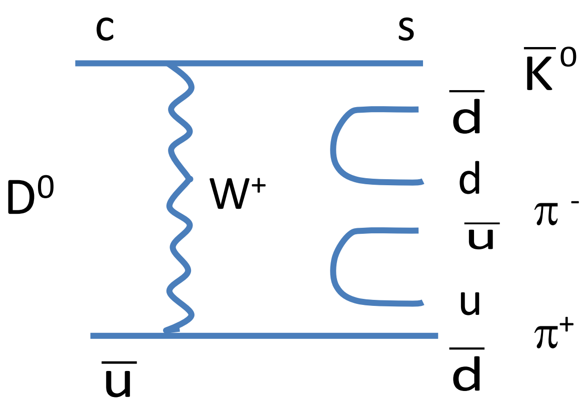


allowed



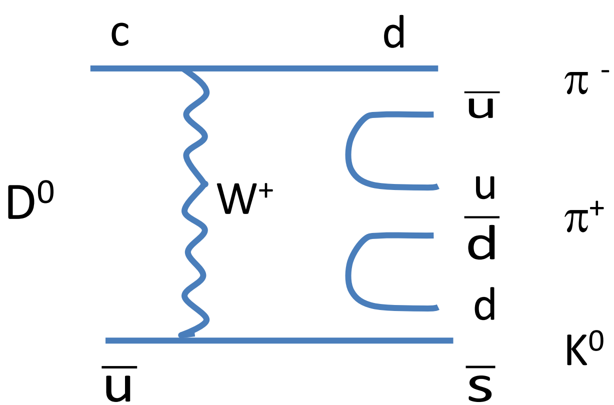
doubly Cabibbo suppressed

Annihilation decay amplitudes



Transition $c \rightarrow s u \bar{d}$

allowed



Transition $c \rightarrow d u \bar{s}$

doubly Cabibbo **suppressed**

Factorization approach

Quark currents: $j_1 = (\bar{s}c)_{V-A}\,,\, j_2 = (\bar{u}d)_{V-A}\,,\quad j_1' = (\bar{u}c)_{V-A}\,,\, j_2' = (\bar{s}d)_{V-A}$

main part of the effective **Hamiltonian:** $H \propto G_F/\sqrt{2}\; V_{cs}^{*}V_{ud}\, j_1 \otimes j_2$

Factorization:

$$\langle \bar{K}^0\pi^-\pi^+ | j_1 \otimes j_2 | D^0\rangle \approx \langle \bar{K}^0\pi^- | j_1 | D^0\rangle\langle \pi^+ | j_2 | 0\rangle$$

$$+ \langle \pi^-\pi^+ | j_1' | D^0\rangle\langle \bar{K}^0 | j_2' | 0\rangle$$

$$+ \langle 0 | j_1' | D^0\rangle\langle \bar{K}^0\pi^-\pi^+ | j_2' | 0\rangle$$

$\langle \pi^+ | j_2^{\mu} | 0\rangle = i f_\pi p_\pi^{\mu}$ — f_π - pion decay constant

$\langle \bar{K}^0 | j_2'^{\mu} | 0\rangle = i f_K p_K^{\mu}$ — f_K - kaon decay constant

$\langle 0 | j_1'^{\mu} | D^0\rangle = -i f_D p_D^{\mu}$ — f_D - D decay constant

Types of decay amplitudes

27 amplitudes for the $D^0 \to K_S^0 \pi^+ \pi^-$ decay:

a) **7 allowed** tree amplitudes,

b) **6** doubly Cabibbo **suppressed** tree amplitudes,

c) **14 annihilation** (W-exchange) amplitudes
(7 allowed and 7 doubly Cabibbo suppressed).

Seven partial wave amplitudes:

1. S-, P- and D- wave amplitudes in the $K\pi$ subsystem,
2. S-, P- and D- wave amplitudes in the $\pi^+ \pi^-$ subsystem, including in addition the $\omega \to \pi^+ \pi^-$ P- wave transition .

Resonances in decay amplitudes

Channel:	wave:	name:
$\overline{K}^0\pi^-$	S	$K_0^*(800)^-$ or κ^-, $K_0^*(1430)^-$
	P	$K^*(892)^-, K^*(1410)^-, K^*(1680)^-$
	D	$K_2^*(1430)^-$
$K^0\pi^+$		same list as above but with pion charge +
$\pi^+\pi^-$	S	$f_0(500)$ or σ, $f_0(980)$, $f_0(1400)$
	P	$\rho(770), \rho(1450), \omega(782)$
	D	$f_2(1270)$

Very rich resonance spectrum $\rightarrow$ **complexity of final state interactions**

Selected formulae of decay amplitudes

$$D^0 \to K_S^0 \pi^+ \pi^- \qquad |K_S^0\rangle \approx \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

Allowed transitions with $K_S^0\pi^-$ final state interactions $\Lambda_1 = V_{cs}^* V_{ud}$

$m_\mp$ eff. masses of $K_S^0\pi^\mp$, $m_0 - \pi^+\pi^-$ eff. mass a$_1$ - effective Wilson coefficient

S-wave:

$$A_{1S} = -\frac{G_F}{2}\Lambda_1 a_1 f_\pi (m_D^2 - m_\pi^2)) F_0^{DK_0^{*-}}(m_\pi^2) F_0^{\overline{K}_0\pi^-}(m_-^2)$$

$F_0^{DK_0^{*-}}(m_\pi^2)$ - D to K$_0^*$ transition scalar form factor

$F_0^{\overline{K}_0\pi^-}(m_-^2)$ - K$_0$π scalar form factor $m_-^2 = (p_{\pi^-} + p_{K^0})^2$

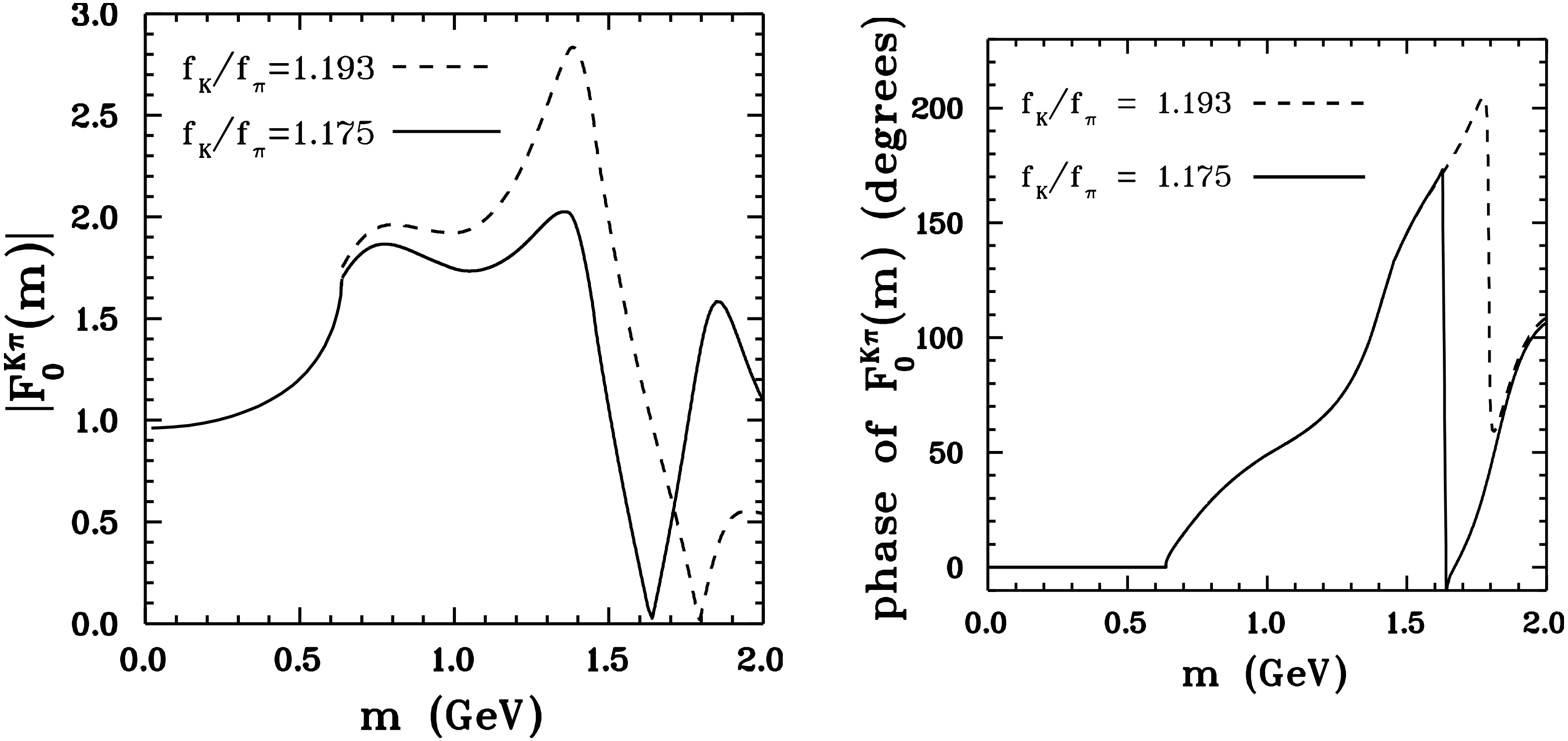
P-wave:

$$A_{1P} = -\frac{G_F}{2}\Lambda_1 a_1 \frac{f_\pi}{f_\rho}[m_0^2 - m_+^2 + \frac{(m_D^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_-^2}] A_0^{DK^{*-}}(m_\pi^2) F_1^{\overline{K}_0\pi^-}(m_-^2)$$

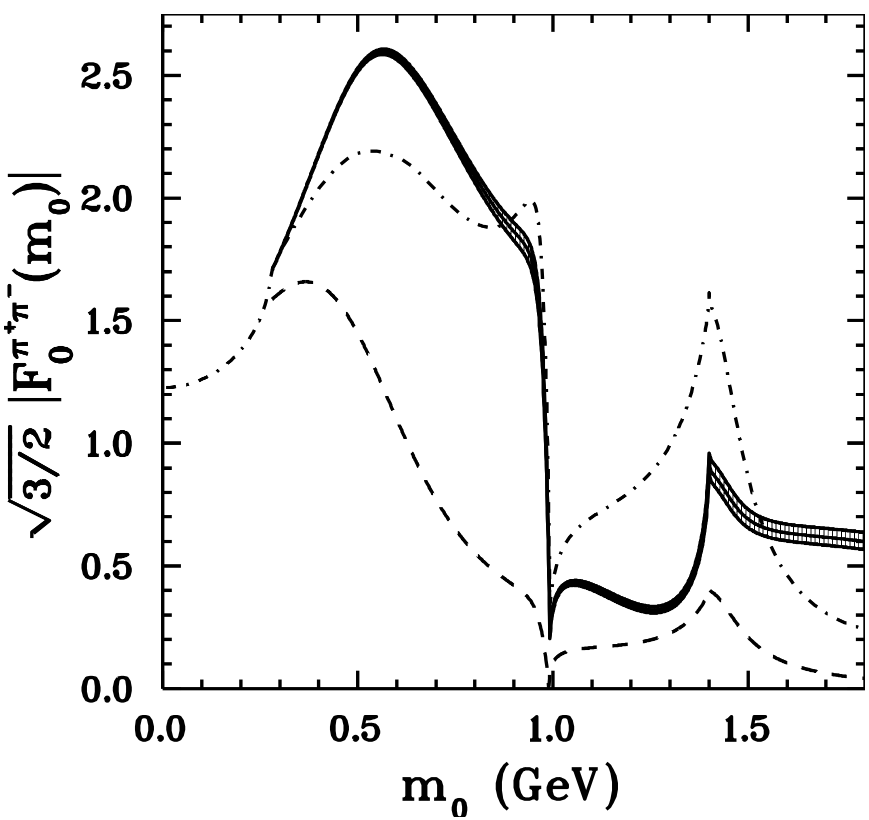
$A_0^{DK_0^{*-}}(m_\pi^2)$ - D to K$_0^*$ transition vector form factor $m_+^2 = (p_{\pi^+} + p_{K^0})^2$

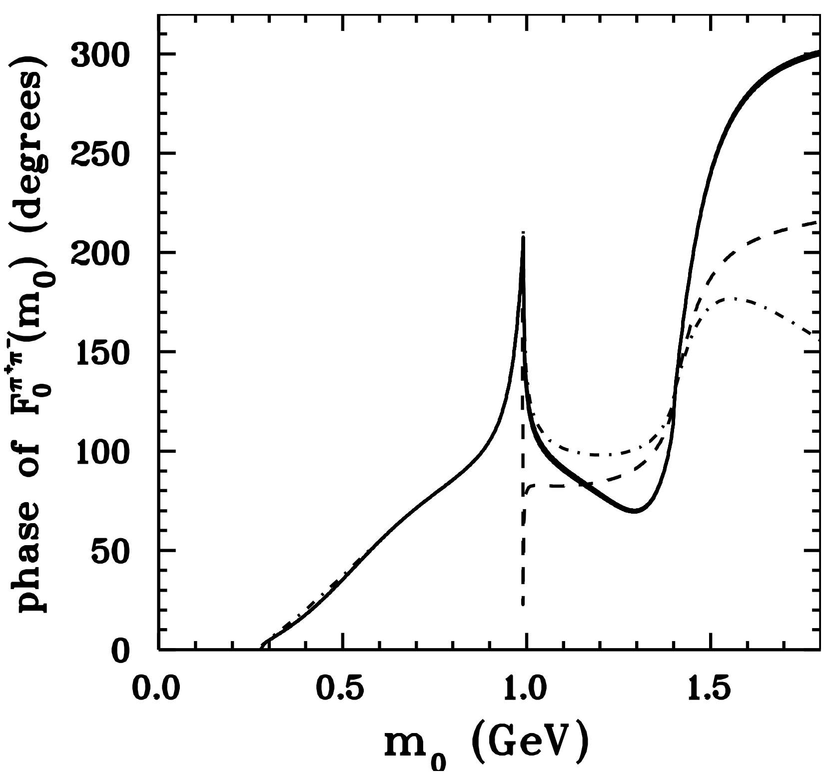
$F_1^{\overline{K}_0\pi^-}(m_-^2)$ - K$_0$π vector form factor $m_0^2 = (p_{\pi^-} + p_{\pi^+})^2$

kaon-pion scalar form factor



pion scalar form factor





χ^2 joint fit

$$\chi^2 = \chi_{D^0}^{\;2} + \chi_{\tau}^{\;2} + \chi_{Br}^{\;2}$$

Data for:

1. $D^0 \to K_S^0 \pi^+ \pi^-$ decays, A. Poluektov et al. (Belle Coll.), Phys. Rev. D 81, 112002 (2010),
2. $\tau^- \to K_S^0 \pi^- \nu_\tau$ decays, D. Epifanov et al. (Belle Coll.), Phys. Lett. B 654, 65 (2008),
3. total branching fraction $Br^{exp} = (2.82 \pm 0.19)$ %.

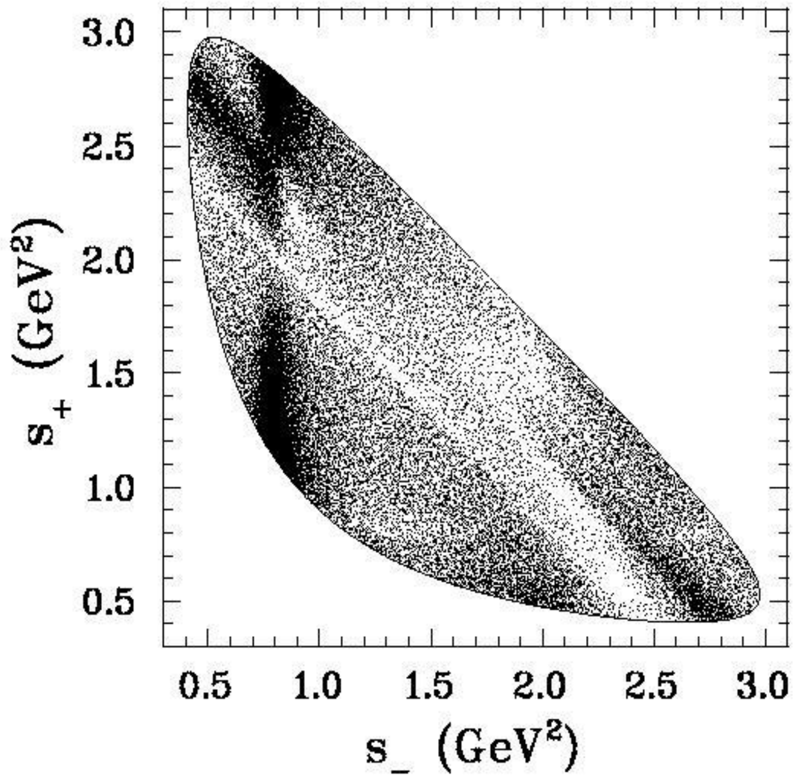
Number of degrees of freedom:

ndf= 6321 + 89 + 1 – 33 free model param.= 6378.

Result: $\chi^2 = 9451$ which gives $\chi^2/\,ndf = 1.48$.

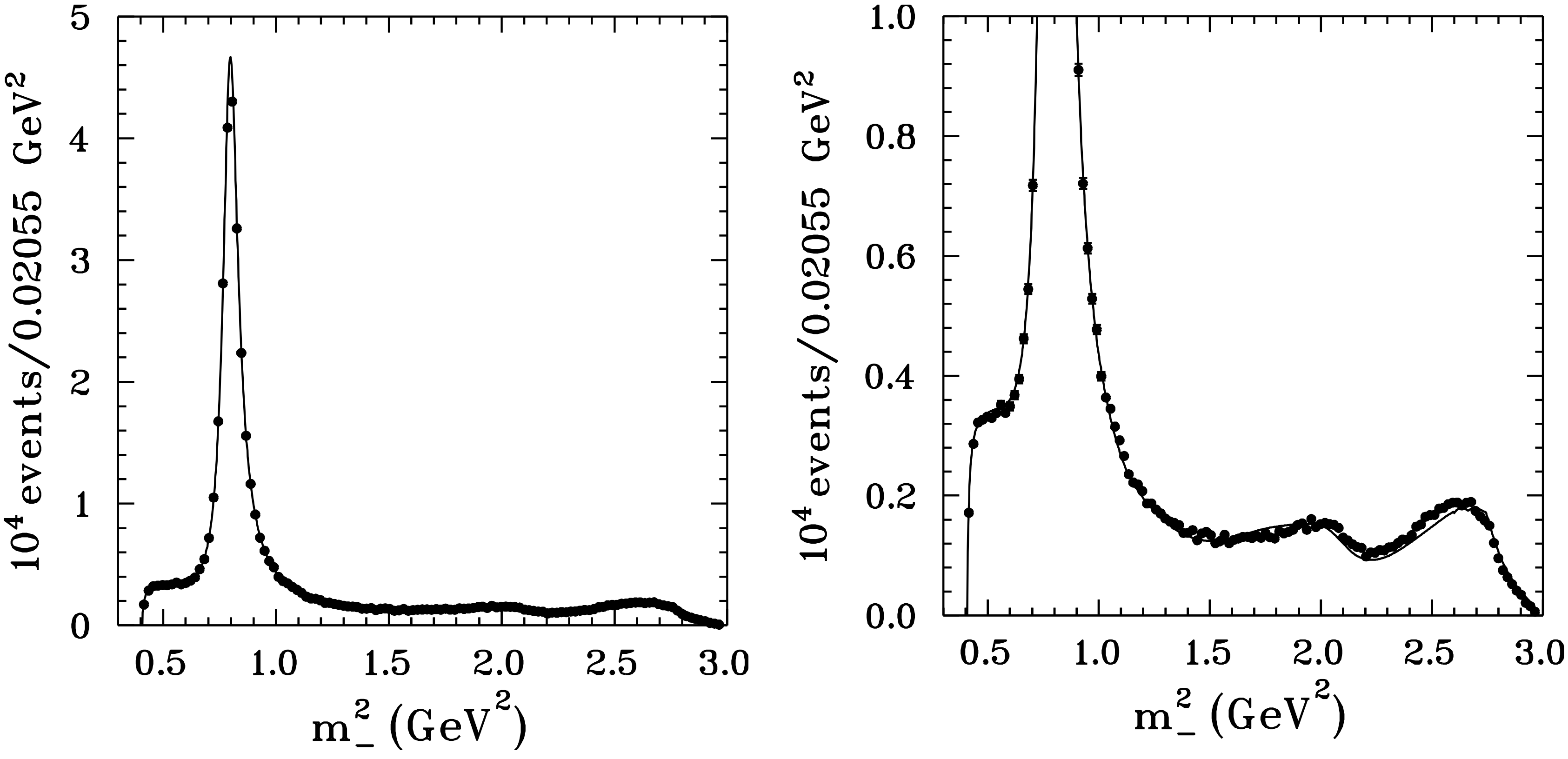
Dalitz plot density distribution for the $D^0 \to K_S^0 \pi^+ \pi^-$ decay

$$s_+ = (p_{K_S^0} + p_{\pi^+})^2$$

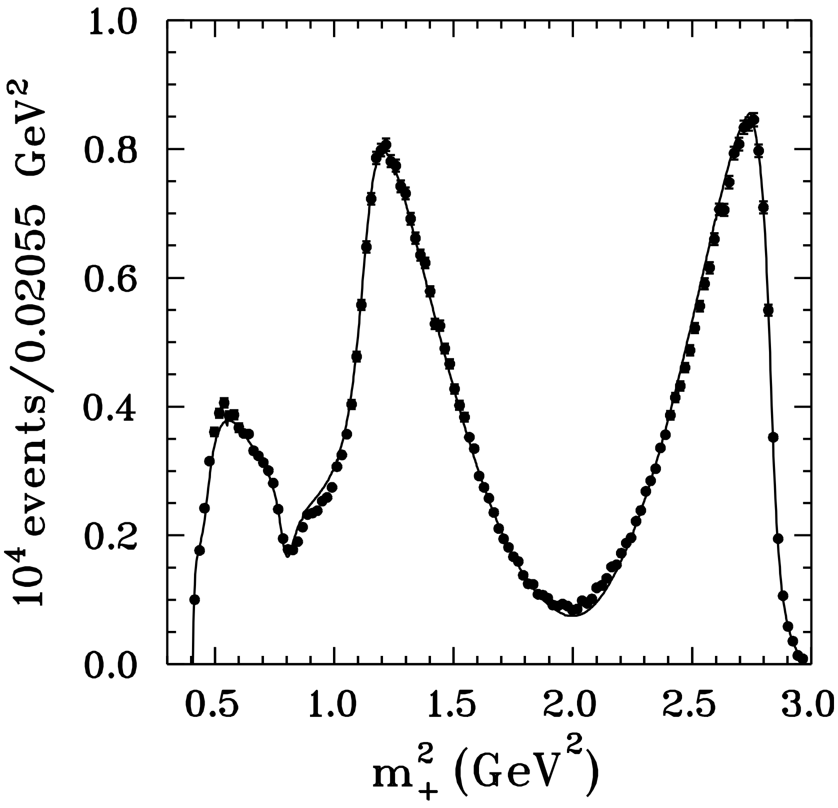


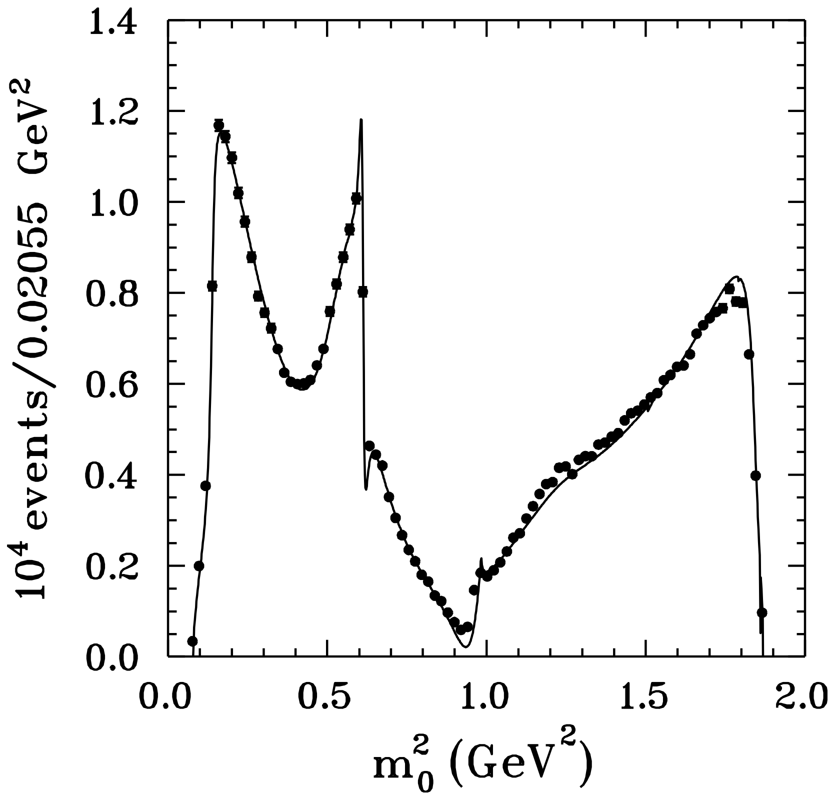
$$s_- = (p_{K_S^0} + p_{\pi^-})^2$$

Comparison of the $K_S^0 \pi^-$ effective mass squared distributions with the Belle data



Comparison of the $K_S^0\,\pi^+$ and $\pi^+\pi^-$ effective mass squared distributions with the Belle data

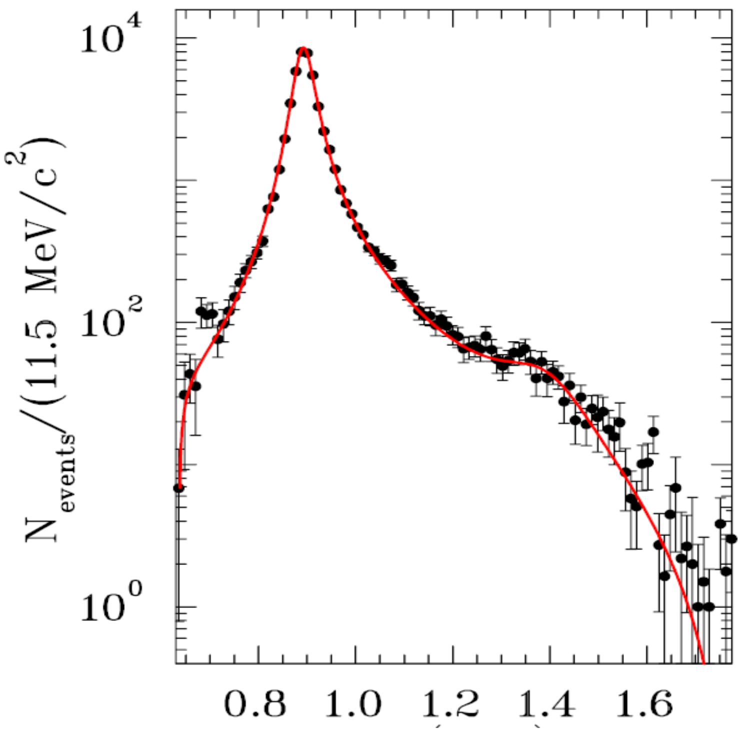




$K_S^0\,\pi^+$

$\pi^+\,\pi^-$

Comparison with the Belle data on the $\tau\tau^- \to K_S^0 \pi^- \nu_\tau$ decay



$m_{K\pi}$ (GeV)

Branching fractions

Channel	Br (%)	Br (tree)	**Annihil. low. limit**
$[K_S^0 \pi^-]_S \pi^+$	25.0 ± 3.6	8.2 ± 0.1	7.9 ± 0.1
$K_S^0 [\pi^- \pi^+]_S$	16.9 ± 1.3	14.7 ± 0.2	2.9 ± 0.1
$[K_S^0 \pi^-]_P \pi^+$	62.7 ± 4.5	24.7 ± 5.7	8.7 ± 3.0
$K_S^0 [\pi^- \pi^+]_P$	22.0 ± 1.6	4.4 ± 0.1	6.7 ± 0.04

Exp.

$Br(K_S^0 \rho) = (21.2 \pm 0.5)\ \%$

$Br(K^*(892)^+ \pi^-) = (62.9 \pm 0.8)\ \%$

Summary

1. The $D^0 \to K_S^0 \pi^+ \pi^-$ decays are analysed using the factorization approximation.
2. The annihilation (via W-echange) amplitudes are added to the weak-decay tree amplitudes.
1. The strong interactions between kaon-pion and pion-pion pairs in the S-, P- states are described in terms of the corresponding **form factors**. For D-waves we use relativistic Breit-Wigner formulae.
2. The kaon-pion and pion-pion scalar form factors are constrained using unitarity, analyticity and chiral symmetry and by the present Dalitz plot analysis.
5. A good agreement with the **Belle and BABAR Dalitz plot** density distributions and with the $\tau^- \to K_S^0 \pi^- \nu_\tau$ decay data is achieved.
6. The lower-limit values of the branching fractions of the **annihilation** amplitudes are **significant.**

parameter	modulus	phase (deg)
χ_1	$5.43 \pm 0.22 \pm 0.00$	$248.1 \pm 1.3 \pm 2.0$
χ_2	$32.50 \pm 1.21 \pm 0.09$	$221.9 \pm 0.9 \pm 0.7$
$\tilde{F}_0^{\pi^+ R_S[\overline{K}^0 \pi^-],}(m_{D^0}^2)$	$1.94 \pm 0.03 \pm 0.00$	$245.6 \pm 1.1 \pm 1.1$
$\tilde{F}_0^{\overline{K}^0 R_S[\pi^- \pi^+]}(m_{D^0}^2)$	$1.36 \pm 0.02 \pm 0.00$	$37.7 \pm 0.4 \pm 0.2$
$\tilde{A}_0^{\pi^+ R_P[\overline{K}^0 \pi^-]}(m_{D^0}^2)$	$0.95 \pm 0.05 \pm 0.06$	$294.2 \pm 2.2 \pm 11.9$
$\tilde{A}_0^{\overline{K}^0 R_P[\pi^- \pi^+]}(m_{D^0}^2)$	$0.66 \pm 0.04 \pm 0.01$	0.0 (fixed)
$\tilde{A}_0^{\overline{K}^0 \omega}(m_{D^0}^2)$	$1.23 \pm 0.04 \pm 0.03$	$319.1 \pm 1.1 \pm 0.2$
q_6	$1.44 \pm 0.07 \pm 0.15$	$26.2 \pm 1.6 \pm 3.8$
s_6	$1.84 \pm 0.09 \pm 0.16$	$199.2 \pm 1.3 \pm 1.5$
q_7	$0.68 \pm 0.03 \pm 0.02$	$245.9 \pm 1.6 \pm 4.9$
s_7	$1.01 \pm 0.05 \pm 0.03$	$102.3 \pm 1.7 \pm 4.1$
z_8	$2.09 \pm 0.12 \pm 0.04$	$206.1 \pm 3.1 \pm 3.5$
z_9	$1.64 \pm 0.09 \pm 0.31$	$135.3 \pm 1.9 \pm 0.3$
q_{10}	$23.19 \pm 1.26 \pm 3.10$	$220.8 \pm 3.1 \pm 15.6$
s_{10}	$24.26 \pm 1.33 \pm 3.74$	$40.3 \pm 3.0 \pm 14.5$
c (GeV^{-4})	$0.29 \pm 0.02 \pm 0.02$	
κ (MeV)	$305.61 \pm 2.74 \pm 1.33$	
$m_{K^{*\mp}}$ (MeV)	894.74 ± 0.08	
Γ_{K^*} (MeV)	46.98 ± 0.18	

Amplitude	channel	Br	tree	ann. low
$\mathcal{M}_1$	$[K_S^0\pi^-]_S\pi^+$	$25.03 \pm 3.61 \pm 0.18$	8.24 ± 0.10	7.88 ± 0.11
$\mathcal{M}_2$	$K_S^0[\pi^-\pi^+]_S$	$16.92 \pm 1.27 \pm 0.02$	14.70 ± 0.17	2.92 ± 0.09
$\mathcal{M}_3$	$[K_S^0\pi^-]_P\pi^+$	$62.72 \pm 4.45 \pm 0.15$	24.69 ± 5.65	8.74 ± 2.97
$\mathcal{M}_4$	$K_S^0[\pi^-\pi^+]_P$	$21.96 \pm 1.55 \pm 0.06$	4.36 ± 0.06	6.74 ± 0.04
$\mathcal{M}_5$	$K_S^0\omega$	$0.79 \pm 0.07 \pm 0.04$	0.24 ± 0.01	0.16 ± 0.02
$\mathcal{M}_6$	$[K_S^0\pi^-]_D\pi^+$	$1.41 \pm 0.11 \pm 0.04$		
$\mathcal{M}_7$	$K_S^0[\pi^-\pi^+]_D$	$2.15 \pm 0.19 \pm 0.10$		
$\mathcal{M}_8$	$[K_S^0\pi^+]_S\pi^-$	$0.56 \pm 0.07 \pm 0.03$	0.07 ± 0.00	0.29 ± 0.02
$\mathcal{M}_9$	$[K_S^0\pi^+]_P\pi^-$	$0.64 \pm 0.06 \pm 0.02$	0.77 ± 0.15	0.01 ± 0.01
$\mathcal{M}_{10}$	$[K_S^0\pi^+]_D\pi^-$	$0.63 \pm 0.07 \pm 0.11$	0	0.63 ± 0.11

Lower limit of annihilation amplitudes

$M = \sum_{i=1}^{10} M_i\,;\quad M_i = \mathrm{T_i} + \mathrm{A_i}$ $\mathrm{T_i}$ – tree ampl. $\mathrm{A_i}$ – annihilation ampl.

$$\frac{d^2 Br_i}{ds_- ds_+} = c\,|M_i|^2 = c\,|\overline{M}_i|^2;$$ fitted ampl. $\overline{M}_i = e^{-i\rho} M_i$

$$\frac{d^2 Br_i^{tree}}{ds_- ds_+} = c\,|T_i|^2; \qquad \frac{d^2 Br_i^{ann.}}{ds_- ds_+} = c\,|A_i|^2$$

$$A_i = e^{i\rho}\,\overline{M}_i - T_i$$

ρ = phase of the K_S^0 ρ amplitude

Lower limit of the annihilation branching fraction:

$$Br_i^{ann.\ low} = Br_i + Br_i^{tree} - 2\iint ds_- ds_+\,|\overline{M}_i|\,|T_i|$$

Transition matrix elements (1)

Two mesons form a resonance R=h_2h_3

$$\langle h_2(p_2)h_3(p_3)|\,j\,|D^0(p_D)\rangle \approx G_{Rh_2h_3}(s_{23})\langle R(p_2+p_3)|\,j\,|D^0(p_D)\rangle$$

Example: $D^0(p_D)\to\pi^+(p_1)\overline{K}^0(p_2)\pi^-(p_3)$ $R=K^*(892)^-\to\overline{K}^0\pi^-$

$p_D=p_1+p_2+p_3,\;\; s_{23}=(p_2+p_3)^2,\;\; p_1{}^2=m_\pi{}^2$ $j=(\bar{s}c)_{V-A}$

$$\langle R(p_2+p_3)|\,j\,|D^0(p_D)\rangle = -i2m_{K^*}\frac{\varepsilon^*\cdot p_D}{p_1^2}\,p_1^\mu A_0^{DK^*}(m_\pi^2)+3\text{ other terms}$$

ε - K^* polarization $A_0^{DK^*}(m_\pi^2)$ **D to K* transition form factor**

Vertex function:

$$G_{K^{*-}\overline{K}^0\pi^-}(s_{23})=\varepsilon\cdot(p_2-p_3)\frac{1}{m_{K^*}f_{K^*}}F_1^{\overline{K}^0\pi^-}(s_{23})$$

$F_1^{\overline{K}^0\pi^-}(s_{23})$ - **kaon-pion** transition **vector form factor**

Transition matrix elements (2)

$$\langle h_1(p_1)h_2(p_2)h_3(p_3)|\,j'|0\rangle \approx G_{Rh_2h_3}(s_{23})\langle h_1(p_1)R(p_2+p_3)|\,j'|0\rangle$$

Example: $h_1=\overline{K}^0, \quad R=f_0\to\pi^+\pi^-$

$$p_D=p_1+p_2+p_3,\;\; s_{23}=(p_2+p_3)^2, \qquad j'=(\bar{s}d)_{V-A}$$

$$\langle\overline{K}^0(p_1)f_0(p_2+p_3)|\,j'^{\mu}|0\rangle=-i\frac{m_{K^0}^2-s_{23}}{p_D^2}p_D^{\mu}F_0^{\overline{K}^0 f_0}\left(m_D^2\right)+$$ 2nd term

$F_0^{\overline{K}^0 f_0}(m_D^2)$ - **kaon to f_0 transition form factor** (complex number)

$$G_{f_0\pi^+\pi^-}(s_{23})\approx\chi_2F_0^{\pi^+\pi^-}(s_{23})$$

$F_0^{\pi^+\pi^-}(s_{23})$ - **pion scalar form factor**, χ_2 - constant

Selected formulae of decay amplitudes (1)

$D^0 \rightarrow K_S^0 \pi^+ \pi^-$ $\qquad |K_S^0\rangle \approx \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle)$

Allowed transitions with $K_S^0 \pi^-$ final state interactions $\qquad \Lambda_1 = V_{cs}^* V_{ud}$

$m_{\mp}$ eff. masses of $K_S^0 \pi^{\mp}$, $m_0 - \pi^+ \pi^-$ eff. mass $\qquad$ a_1 - effective Wilson coefficient

S-wave:

$$A_{1S} = -\frac{G_F}{2}\Lambda_1 a_1 f_\pi (m_D^2 - m_\pi^2)) F_0^{DK_0^{*-}}(m_\pi^2) F_0^{\bar{K}_0 \pi^-}(m_-^2)$$

$F_0^{DK_0^{*-}}(m_\pi^2)$ - D to K_0^* transition scalar form factor

P-wave:

$$A_{1P} = -\frac{G_F}{2}\Lambda_1 a_1 \frac{f_\pi}{f_\rho}[m_0^2 - m_+^2 + \frac{(m_D^2 - m_\pi^2)(m_K^2 - m_\pi^2)}{m_-^2}] A_0^{DK^{*-}}(m_\pi^2) F_1^{\bar{K}_0 \pi^-}(m_-^2)$$

D- wave:

$$A_{1D} = -\frac{G_F}{2}\Lambda_1 a_1 f_\pi F^{DK_2^*}(m_-^2) \frac{G_{K_2^* K_S^0 \pi} D(m_+^2, m_-^2)}{m_{K_2^*}^2 - m_-^2 - i m_{K_2^*} \Gamma_{K_2^*}}$$

$F^{DK_2^{*-}}(m_-^2)$ - combination of D to $K_2^{*-}(1430)$ transition form factors

$G_{K_2^* K_S^0 \pi}$ - coupling constant, $D(m_+^2, m_-^2)$ = D-wave angular distribution function

Selected formulae of decay amplitudes (2)

Annihilation (W-exchange) transitions with $\pi^+\pi^-$ **final state interactions**

$m_0 = \pi^+\pi^-$ effective mass a$_2$ - effective Wilson coefficient

S-wave:

$$An_{2S} = -\frac{G_F}{2}\Lambda_1 a_2 \chi_2 f_D (m_K^2 - m_0^2) F_0^{\bar{K}^0 f_0}(m_D^2) F_0^{\pi^+\pi^-}(m_0^2)$$

$F_0^{\bar{K}^0 f_0}(m_D^2)$ - **$\bar{K}^0$ to f_0 scalar transition form factor**

P-wave:

$$An_{2P} = \frac{G_F}{2}\Lambda_1 a_2 \frac{f_D}{f_\rho}(m_-^2 - m_+^2) A_0^{\rho\bar{K}^0}(m_D^2) F_1^{\pi^+\pi^-}(m_0^2)$$

$A_0^{\rho\bar{K}^0}(m_D^2)$ - **ρ to $\bar{K}^0$ transition form factor**

D-wave:

$$An_{2D} = \frac{G_F}{2}\Lambda_1 a_2 f_D F^{Df_2}(m_0^2) \frac{G_{f_2\pi\pi} D(m_+^2, m_0^2)}{m_{f_2}^2 - m_0^2 - i m_{f_2}\Gamma_{f_2}(m_0^2)}$$

$F^{Df_2}(m_0^2)$ - combination of D to $f_2(1270)$ transition form factors

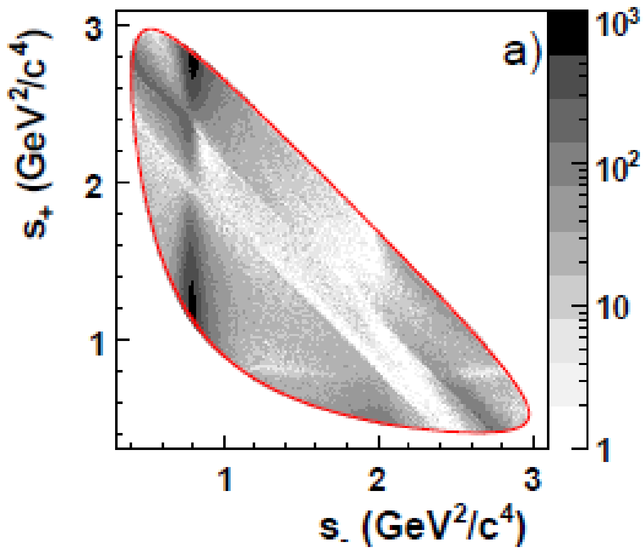
$G_{f_2\pi\pi}$ - coupling constant, $D(m_+^2, m_0^2)$ - D-wave angular distribution function

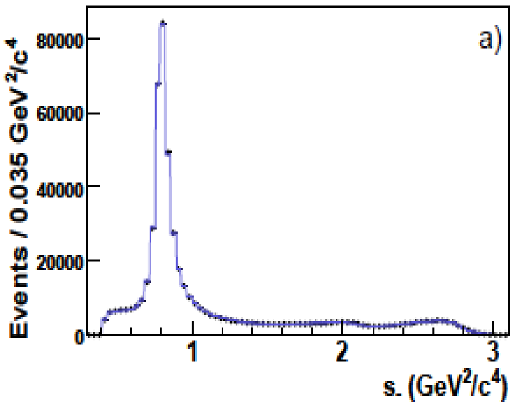
Amplitude	Quasi two-body	Dominant	
	channel	resonances	a
$\mathcal{M}_1$	$[K^0_S\,\pi^-]_S\,\pi^+$	$K^*_0(800)^-$, $K^*_0(1430)^-$	
$\mathcal{M}_2$	$K^0_S\,[\pi^+\pi^-]_S$	$f_0(500)$, $f_0(980)$, $f_0(1400)$	
$\mathcal{M}_3$	$[K^0_S\,\pi^-]_P\,\pi^+$	$K^*(892)^-$	
$\mathcal{M}_4$	$K^0_S\,[\pi^+\pi^-]_P$	$\rho(770)$	
$\mathcal{M}_5$	$K^0_S\,[\pi^+\pi^-]_\omega$	$\omega(782)$	
$\mathcal{M}_6$	$[K^0_S\,\pi^-]_D\,\pi^+$	$K^*_2(1430)^-$	
$\mathcal{M}_7$	$K^0_S\,[\pi^+\pi^-]_D$	$f_2(1270)$	
$\mathcal{M}_8$	$[K^0_S\,\pi^+]_S\,\pi^-$	$K^*_0(800)^+$, $K^*_0(1430)^+$	
$\mathcal{M}_9$	$[K^0_S\,\pi^+]_P\,\pi^-$	$K^*(892)^+$	
$\mathcal{M}_{10}$	$[K^0_S\,\pi^+]_D\,\pi^-$	$K^*_2(1430)^+$	

Experimental data on $D^0 \to K_S^0 \pi^+ \pi^-$ decay

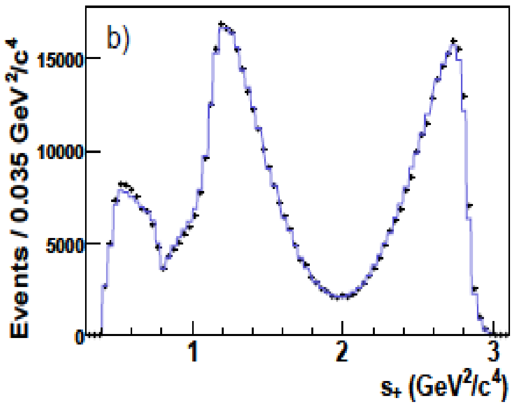
a) A. Poluektov et al. (Belle Coll.), Phys. Rev. D 81, 112002

b) P. del Amo Sanchez et al. (BaBar Coll.), Phys. Rev. Lett. 105 (2010) 081803

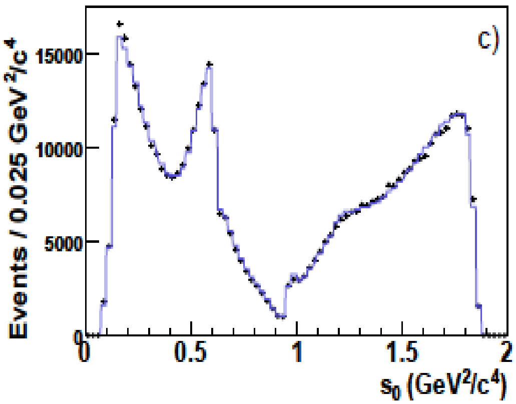




$$s_- = (p_{K_S^0} + p_{\pi^-})^2$$



$$s_+ = (p_{K_S^0} + p_{\pi^+})^2$$



$$s_0 = (p_{\pi^+} + p_{\pi^-})^2$$