

Non-perturbative pion dynamics for the $X(3872)$

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in collaboration with

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related articles: PLB 726, 537 (2013), PRD 84, 074029 (2011)

X(3872). Known facts

Discovered by the Belle: $B^\pm \rightarrow K^\pm J/\Psi \pi^+ \pi^-$

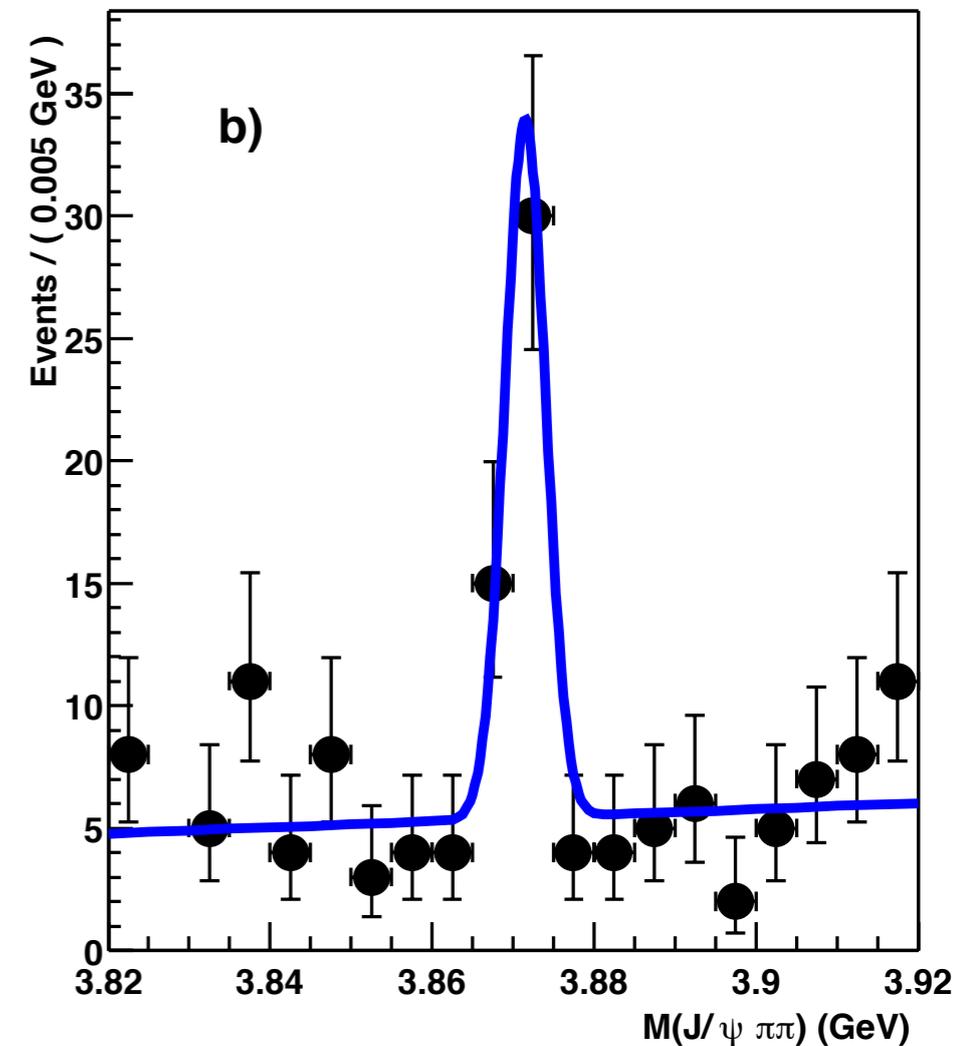
Observed: CDF, D0, BABAR, LHCb, BESIII

$M_X = 3871.68 \pm 0.17$ MeV PDG (2013)

- $\Gamma_X < 2.3$ MeV Belle
- $X \rightarrow J/\Psi \gamma \implies C = +$ Belle
- $Br(X \rightarrow J/\Psi \omega) / Br(X \rightarrow J/\Psi \rho) \approx 0.8 \pm 0.3$ Belle (2003), BABAR(2010)
 \implies isospin violation

• Quantum numbers: 1^{++} and 2^{-+} from BABAR/Belle angular distributions

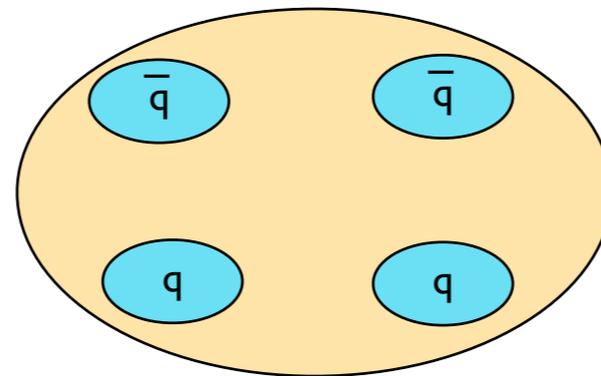
• LHCb (2013): 2^{-+} is rejected with $CL = 8 \sigma \implies X$ is 1^{++}



X(3872). Mechanisms

- Conventional charmonium: $\eta_{c2}(1^1D_2)$ ruled out by LHCb
 $\chi'_{c1}(2^3P_1)$
- Exotics: tetraquark, molecular, mixture....

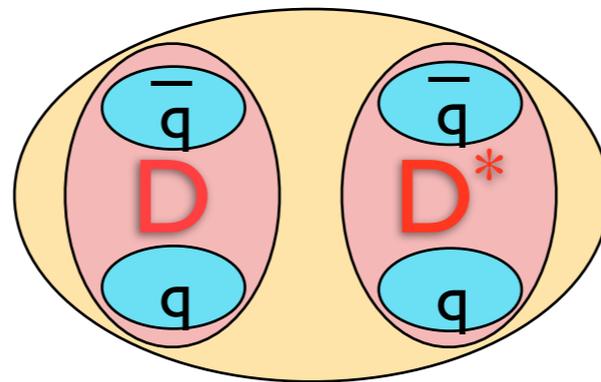
tetraquarks Maiani,...



compact state

molecular

Okun, Voloshin (1976), many works



Strong hadronic interactions with large s-wave scatt. length

natural effective range

specific analytic properties:

$\bar{D}D^*$ unitarity cut

Weinberg (1963-65)

$$M_X = 3871.68 \pm 0.17 \text{ MeV};$$

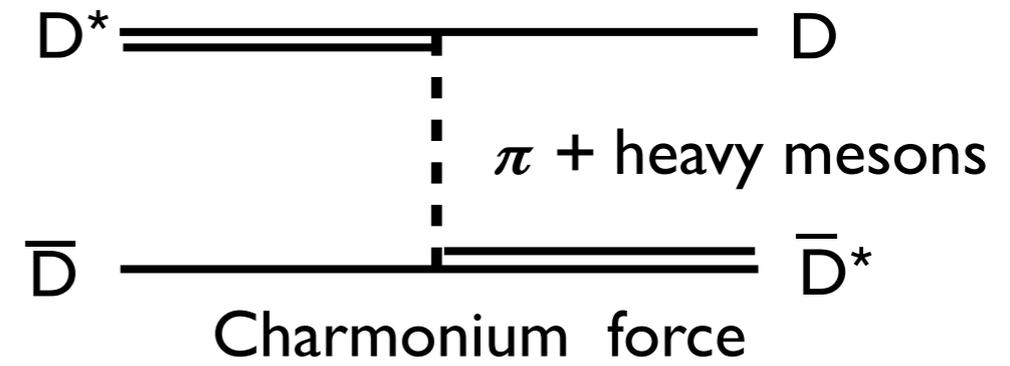
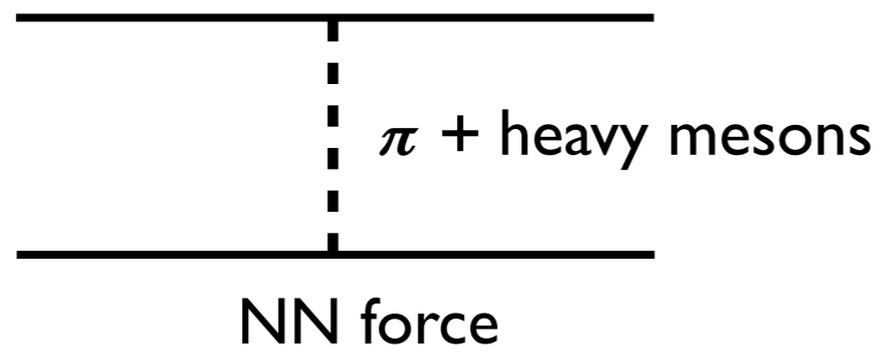
$$M_{D^0\bar{D}^{0*}} = 3871.80 \pm 0.35 \text{ MeV}$$

\implies X is the **S-wave bound state** of $D\bar{D}^*$ with $E_B = 0.12 \pm 0.26 \text{ MeV}$

From NN system to charm sector

Prediction of the $\bar{D}D^*$ molecular state in analogy to the deuteron

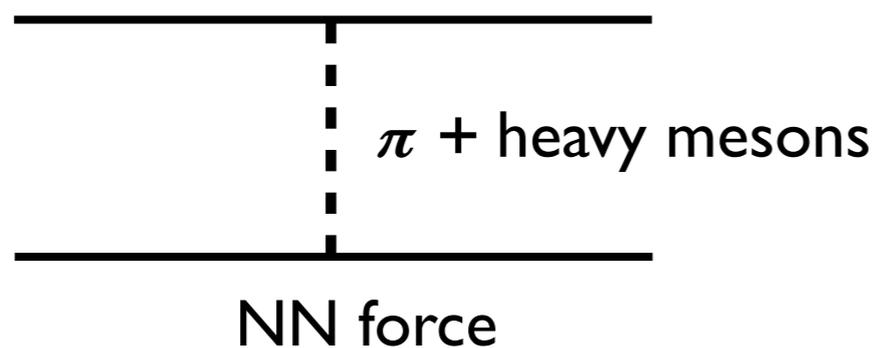
Okun, Voloshin (1976), Törnqvist (1991)



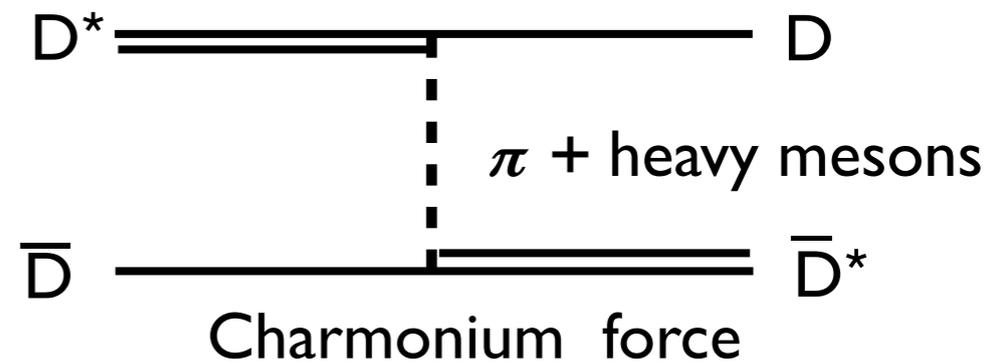
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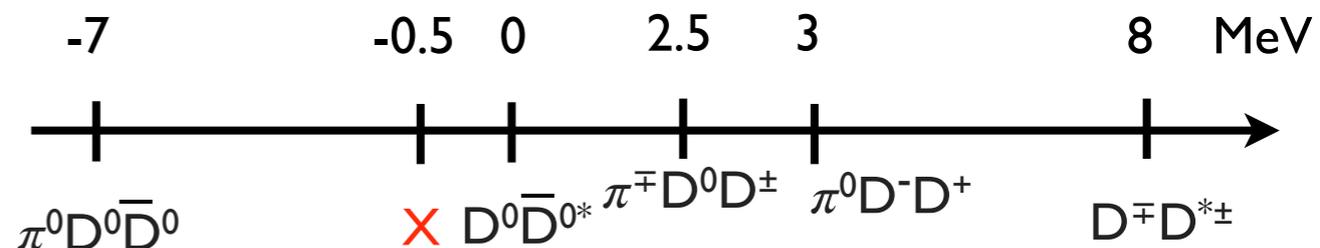
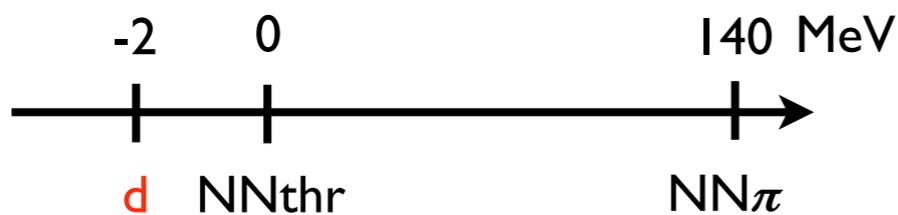


NN



X(3872)

Thresholds



binding momentum

45 MeV

20-30 MeV

range of interaction

m_π

$\pi^0 D^0 \bar{D}^0$ can go on shell \implies Im part
 $\mu = \sqrt{2m_\pi(m_{D^*} - m_D - m_\pi)} \simeq 45$ MeV

static OPE

good approximation

questionable \implies 3-body unitarity is spoiled

Small scales $\ll m_\pi \implies$ NR kinematics

$X(3872)$ as a $\bar{D}D^*$ bound state. Status.

Pionless:

- Asymptotic behavior of the X w.f. Voloshin (2004)
- Contact theory AlFiky et al. (2006), Nieves, Valderrama (2012)

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Pionful:

- X-EFT: resummation of LO contact operators + perturbative pions
—similar to KSW in NN sector
Flemming et al. (2007), Hammer et al. (2014), Mehen, Braaten et al. (2010-2011)
- Phenomenological deuteron-like studies with nonperturb. static OPE and formfactors
For example: Liu et al. (2008), Thomas and Close (2008), Törnqvist (2004)

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Goals of our study

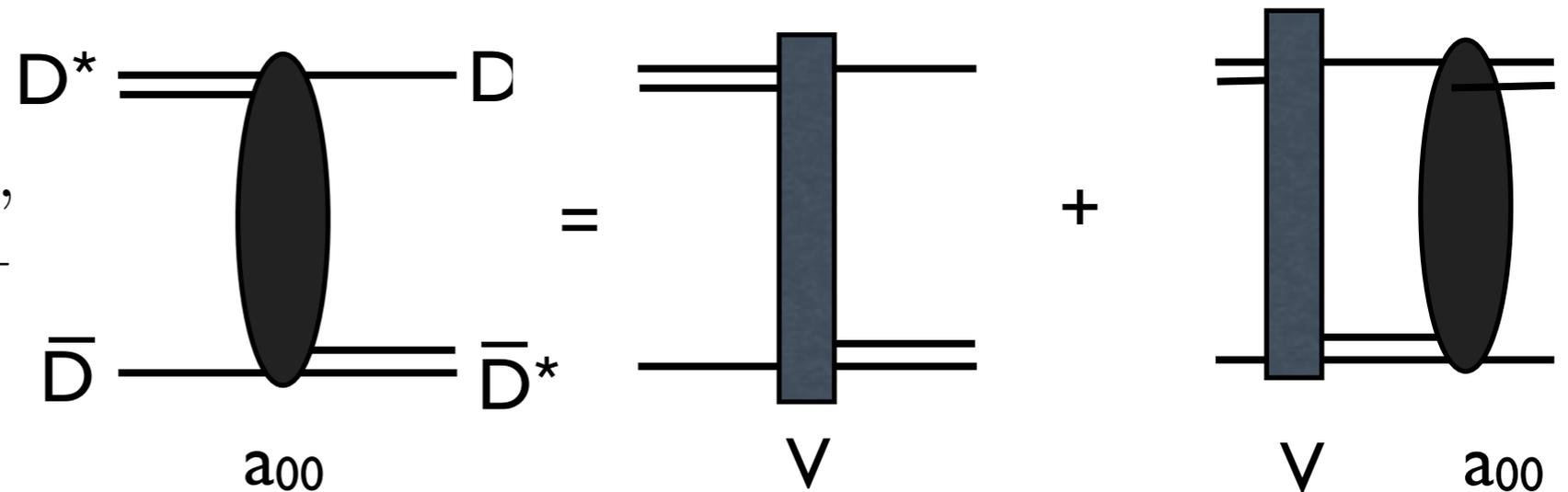
- Investigate the role of 3-body $\pi\bar{D}D$ dynamics on a near threshold resonance
- Check the validity of the static pion approximation
- Investigate the role of coupled channel effects: $D^0\bar{D}^{*0}, D^+\bar{D}^{*-}$
Needed to explain isospin violation: $Br(J/\psi\rho) \approx Br(J/\psi\omega)$ Gamermann, Oset (2009)
- Study the dependence of the X binding energy on the light quark masses (chiral extrapolations)

Formalism. Faddeev-type integral Eqs. for $1^{++} \bar{D}D^*$

Channels:

$$|0\rangle = D^0 \bar{D}^{*0}, \quad |\bar{0}\rangle = \bar{D}^0 D^{*0},$$

$$|c\rangle = D^+ D^{*-}, \quad |\bar{c}\rangle = D^- D^{*+}$$



$$a_{00}^{nn'}(\mathbf{p}, \mathbf{p}', E) = \lambda_0 V_{00}^{nn'}(\mathbf{p}, \mathbf{p}') - \sum_{i=0,c} \lambda_i \int \frac{d^3 s}{\Delta_i(s)} V_{0i}^{nm}(\mathbf{p}, \mathbf{s}) a_{i0}^{mn'}(\mathbf{s}, \mathbf{p}', E)$$

$$a_{c0}^{nn'}(\mathbf{p}, \mathbf{p}', E) = \lambda_c V_{c0}^{nn'}(\mathbf{p}, \mathbf{p}') - \sum_{i=0,c} \lambda_i \int \frac{d^3 s}{\Delta_i(s)} V_{ci}^{nm}(\mathbf{p}, \mathbf{s}) a_{i0}^{mn'}(\mathbf{s}, \mathbf{p}', E),$$

\swarrow
 isospin coefficients

- Partial waves of DD^* : ${}^3S_1, {}^3D_1 \implies$ projection operators
- Δ_0 and Δ_c — $D\bar{D}^*$ propagators of the states $|0\rangle$ and $|c\rangle$
- $a_0 = (a_{00} - a_{c0})/2$ — the X-amplitude

$D\bar{D}^*$ potential within chiral EFT

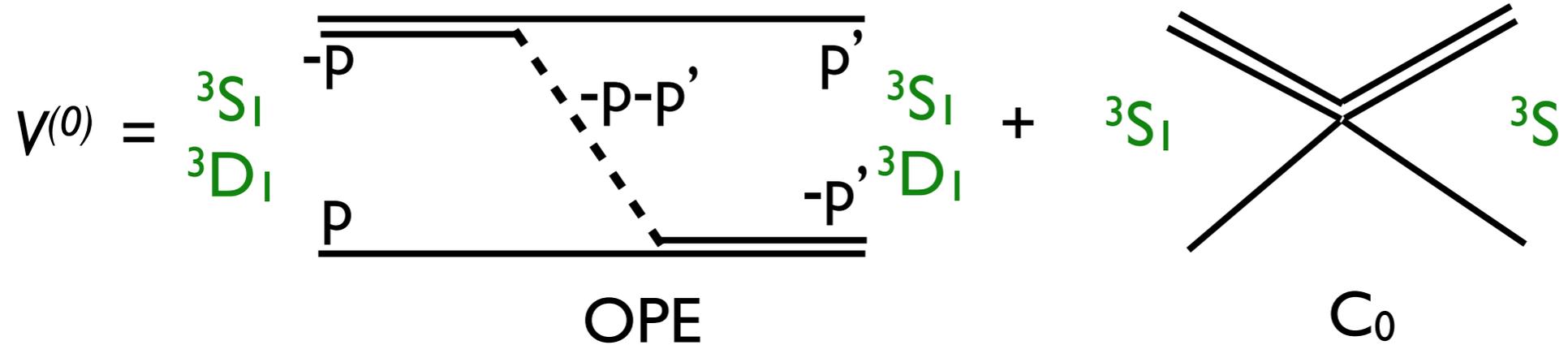
small scale: bind. momentum, range

$$V_{EFT} = V^{(0)} + \chi V^{(1)} + \chi^2 V^{(2)} + \dots$$

$$\chi = \frac{q}{\Lambda_{\chi PT}}$$

large scale: m_ρ

LO:



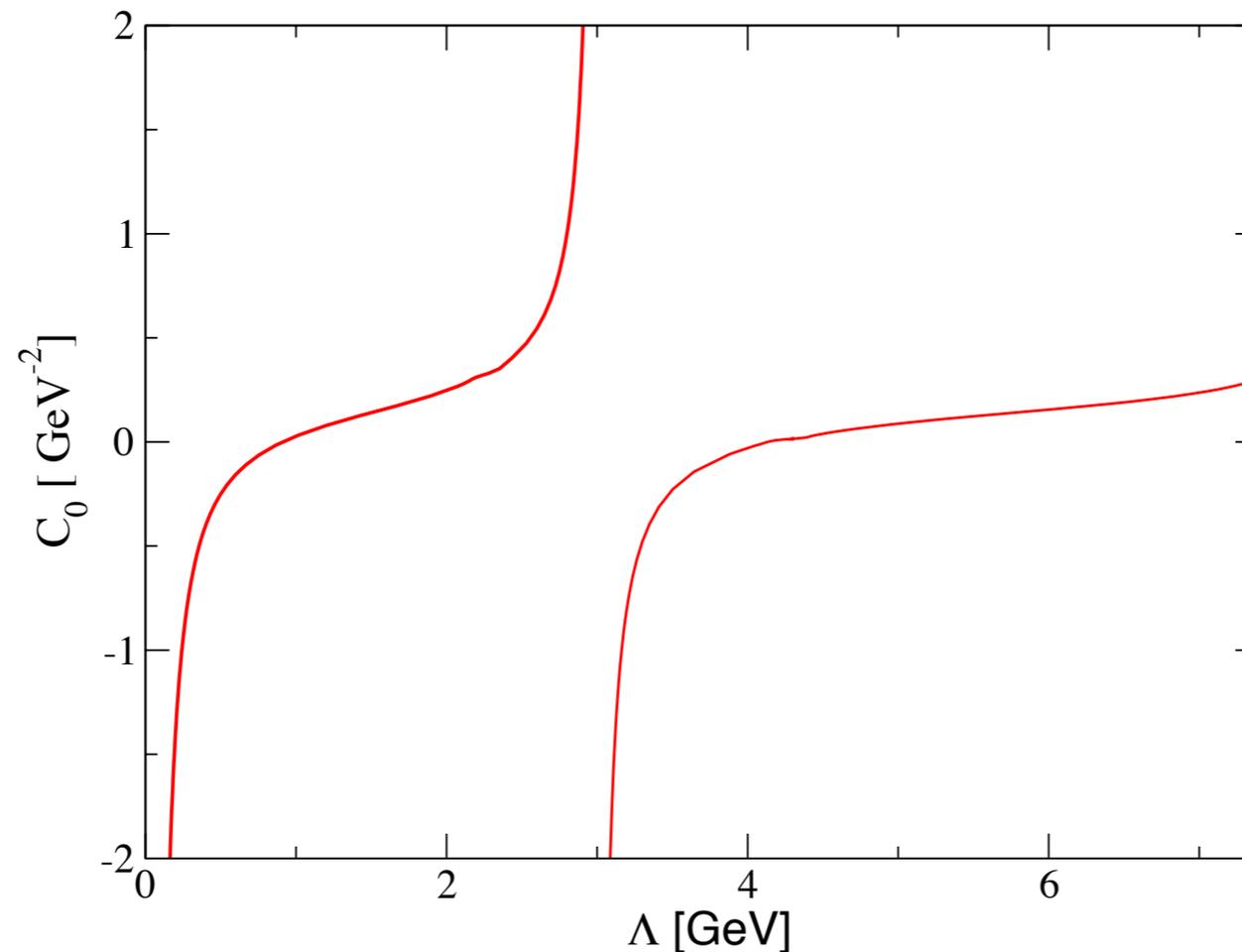
$$V_{OPE}(\vec{p}, \vec{p}') = g^2 \frac{(\vec{p} \cdot \vec{\varepsilon}_d)(\vec{p}' \cdot \vec{\varepsilon}_d)}{M - (2m_D + m_\pi) - \frac{(\vec{p} + \vec{p}')^2}{2m_\pi} - \frac{p^2}{2m_D} - \frac{p'^2}{2m_D} + i0}$$

3-body $D\bar{D}\pi$ prop.

- Coupling constant g from $\Gamma(D^{*0} \rightarrow D^0 \pi^0) = 42 \text{ KeV}$
- Same 3-body cut appears due to dressing the D^* propagator by πD loops

Renormalization of OPE

- Similar to NN [Lepage \(1997\)](#), [Nogga et al. \(2005\)](#), [Epelbaum et al. \(2006-2009\)](#),
- $V_{\text{OPE}} \xrightarrow{p \rightarrow \infty} \text{const} \implies \text{divergent integrals} \implies \text{regularize, e.g., with cutoff } \Lambda \implies \text{renormalize tuning } C_0(\Lambda) \text{ to reproduce the binding energy}$
- Renormalization group limit cycle [Nogga et al. \(2005\)](#), [Bedaque et al. \(1999\)](#), [Braaten and Phillips \(2004\)](#)

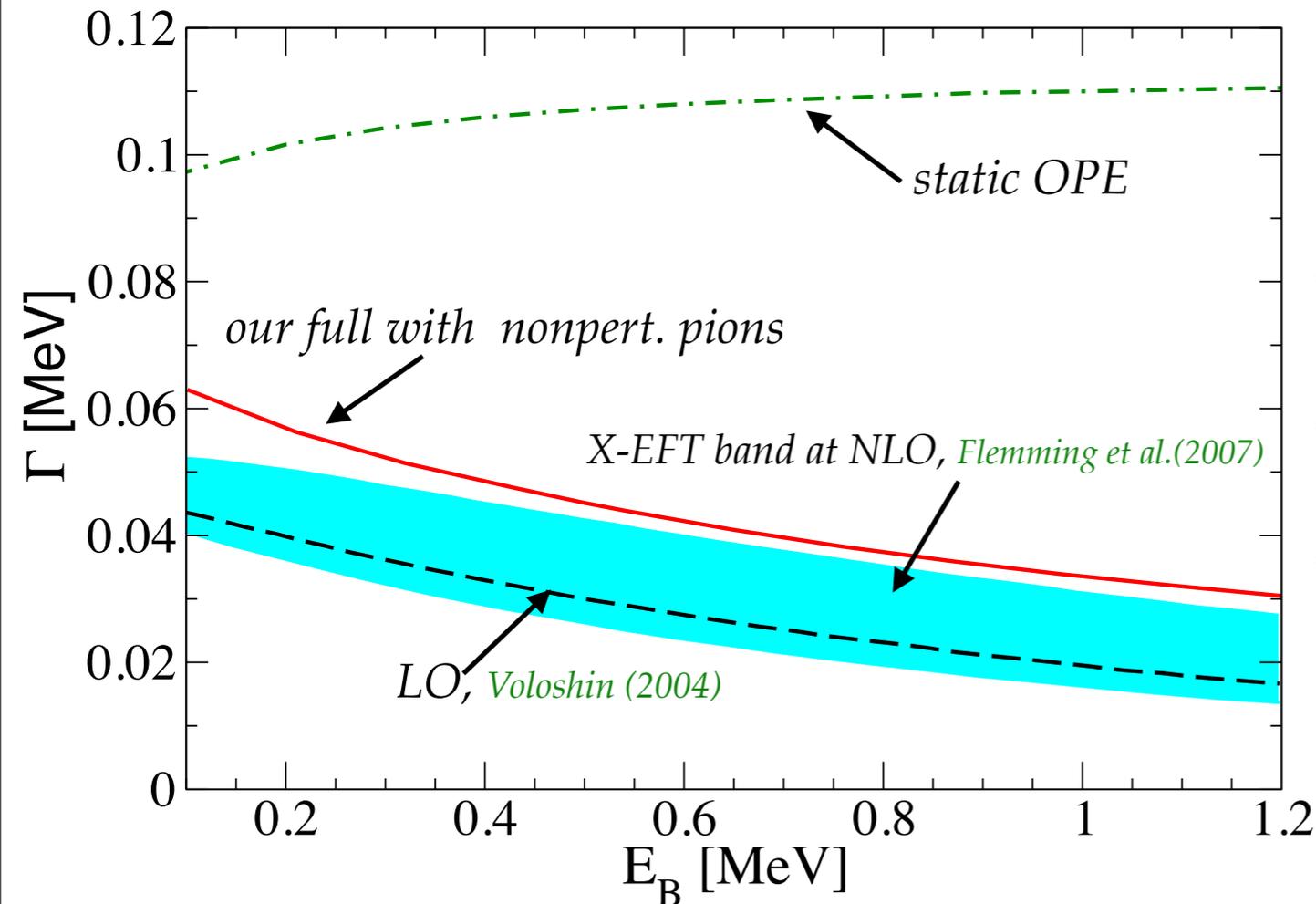
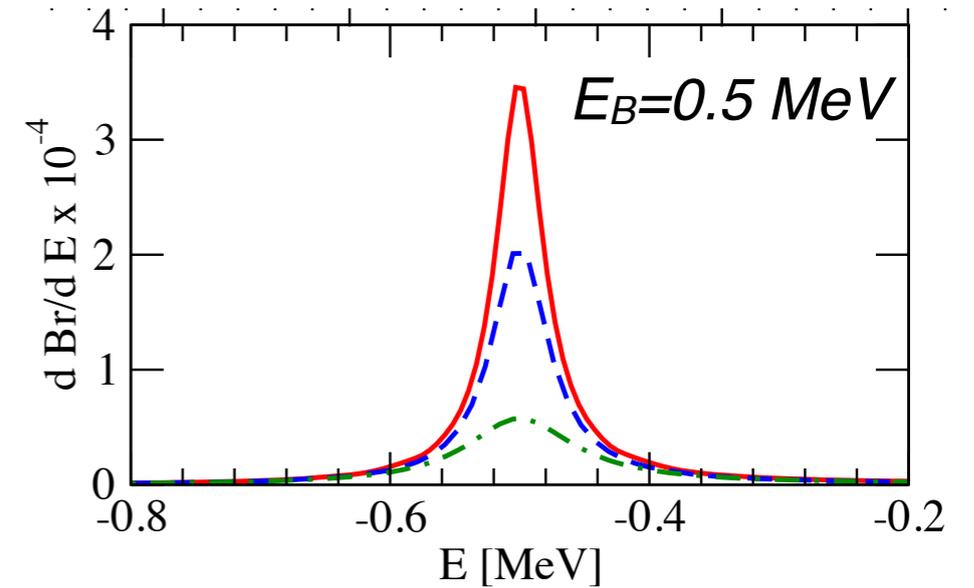
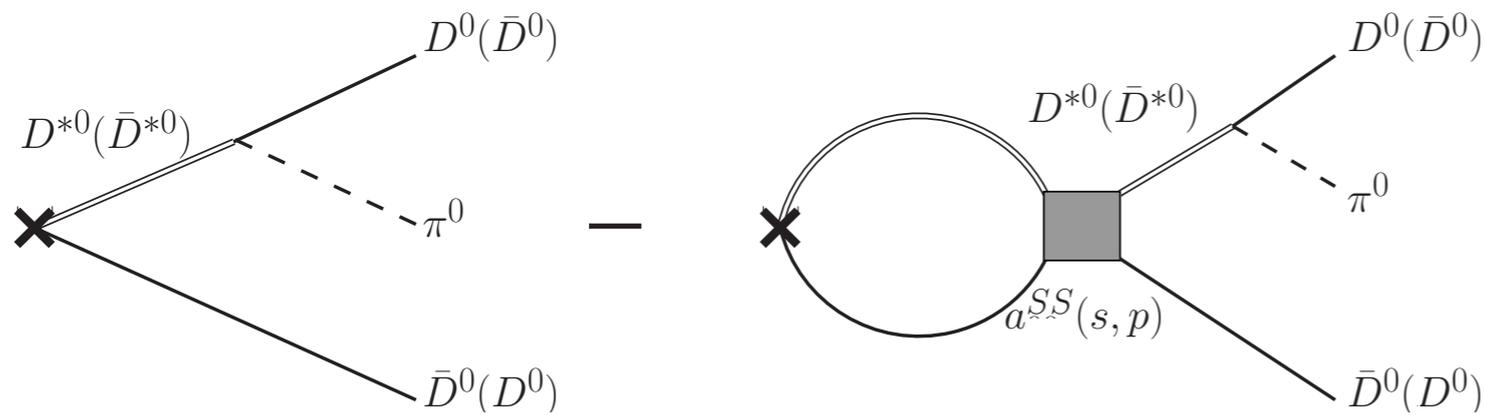


Once renormalized, observables should not depend on Λ within the range of applicability!

Partial width of the X(3872) due to the decay to $\pi^0 D^0 \bar{D}^0$

VB, Filin, Hanhart, Kalashnikova, Kudryavtsev, Nefediev (2011)

$\pi^0 D^0 \bar{D}^0$ production rate near the X(3872) pole



Conclusions:

- Perturbative inclusion of pions is justified
- Static approx. with nonpert. pions fails!
- Keeping 3-body dynamics in pionfull schemes is mandatory

for a study of the $D\bar{D}$ FSI see Guo et al (2014)

Chiral Extrapolations of the $X(3872)$ with m_π

VB, Epelbaum Filin, Hanhart, Meißner, Nefediev (2013)

Strategy: vary all quantities with m_π , expand around physical pion mass m_π^{ph}

- $f(\Lambda)$ is needed to absorb extra Λ -dependence when $m_\pi \neq m_\pi^{\text{ph}}$

$$C_0(\Lambda, m_\pi) = C_0(\Lambda) \left(1 + f(\Lambda) \frac{m_\pi^2 - m_\pi^{\text{ph}2}}{M^2} \right) + O\left(\frac{\delta m_\pi^4}{M^4}\right)$$

- could be fixed if we knew the slope $(\partial E_B / \partial m_\pi) \big|_{m_\pi = m_\pi^{\text{ph}}}$ from lattice

- Estimate $f(\Lambda)$: from two-pion exchange $\implies |f(\Lambda)| \sim 1$, more conservatively: $f(\Lambda) \in [-5, 5]$

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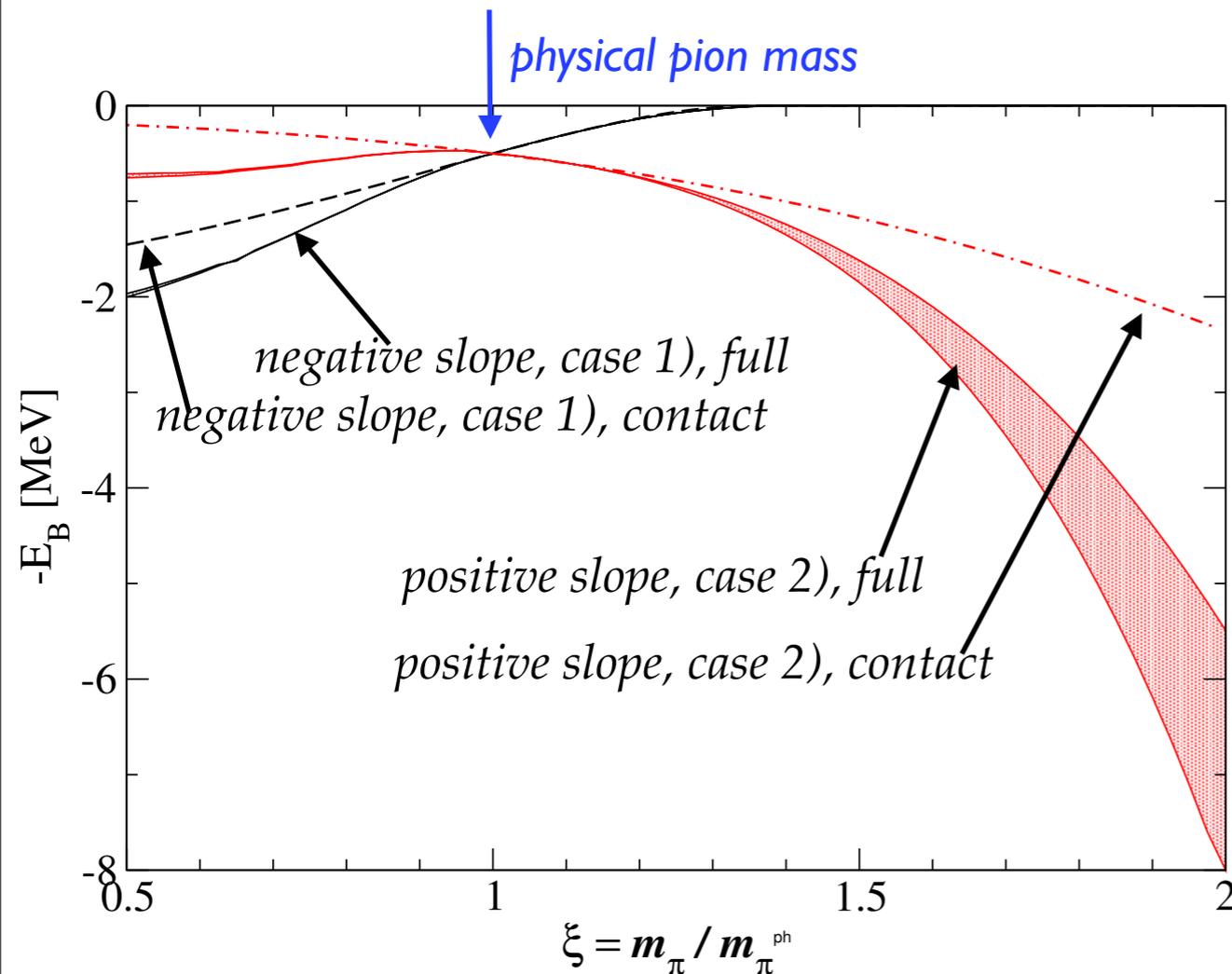
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Conclusions:

1) If $(\partial E_B / \partial m_\pi)|_{m_\pi = m_\pi^{\text{ph}}} < 0 \implies f(\Lambda) < 0 \implies$
bound state disappears quickly with m_π
 growth. **Dominated by short-range physics**

2) If $(\partial E_B / \partial m_\pi)|_{m_\pi = m_\pi^{\text{ph}}} > 0 \implies f(\Lambda) > 0 \implies$

The X is **bound deeper** with m_π growth.
Strong influence of pion dynamics

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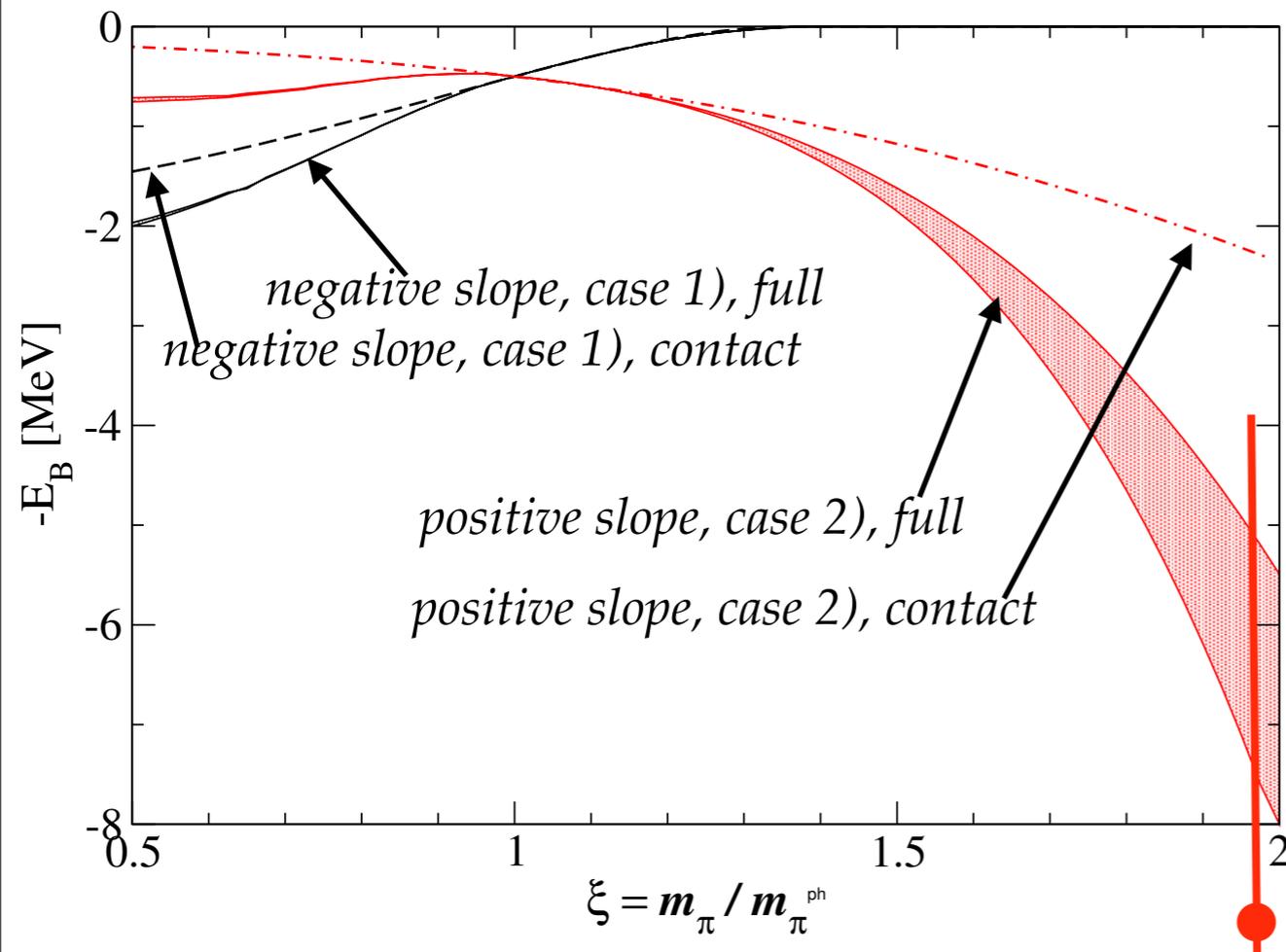
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Conclusions ctd :

- First lattice measurement of the X(3872)
Prelovsek and Leskovec PRL(2013), talk on Saturday

$$E_B = -11 \pm 7 \text{ MeV} \quad m_\pi = 266(4) \text{ MeV}$$

\implies scenario 2) seems to be preferred

Comparison to other EFT studies

- chiral extrapol. of the binding energy with m_π to NLO in X-EFT M.Jansen et al. (2013)

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Claims:

- 1) *OPE alone is sufficient to bind $D^0\bar{D}^{*0}$*
- 2) *Short range contr'n (CT) — a small correction*
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Our opinion

- **All claims are scheme dependent.** OPE due to its short ranged part is always tuned with CT

Example:

in cutoff regularization X has the **same E_B** if $C_0(\Lambda)=0$ ($\Lambda \approx 1$ GeV) or $C_0(\Lambda)=\infty$ ($\Lambda \approx 3$ GeV)

- 3) is in conflict with recent lattice results Prelovsek and Leskovec PRL(2013)

Conclusions

- We studied the role of nonperturbative long-range $\pi D\bar{D}$ dynamics on the $X(3872)$ within a molecular picture
 - ▶ *3-body effects are important in pionful approaches but pion is essentially perturbative*
 - ▶ *static pion approximation is not appropriate*
 - ▶ $\Gamma(X \rightarrow D^0 \bar{D}^0 \pi^0) = 44 \text{ KeV}$ for $E_B=0.5 \text{ MeV}$
- The dependence of the X -binding energy on m_π is highly nontrivial — may provide insights on the binding mechanism
 - ▶ *the X bound state may either disappear or get more bound depending on the interplay of long- and short-range forces*
 - ▶ *First lattice results: more bound*
 - ▶ *More accurate lattice data with better controlled finite volume corrections would be important*