ANALYSIS OF THE PION SCALAR FORM FACTOR PROVIDES MODEL INDEPENDENT PARAMETERS OF $f_0(500)$ and $f_0(980)$ MESONS.

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Outline

In contrast to other $SU(3)$ known multiplets of hadrons, the **identification of the scalar mesons - long-standing puzzle.** Despite of this fact, **all experimentally established scalar mesons** are now classified:

- into **light scalar nonet** comprising the $f_0(500), K_0^*(800), f_0(980)$ and $a_0(980)$ mesons - **not necessarily to be $q\bar{q}$ states**
- into **regular nonet** consisting of the $f_0(1370), K_0^*(1430), a_0(1450)$ and $f_0(1500)$ (or $f_0(1700)$) mesons.

The $f_0(500)$ or **sigma-meson** is the lightest hadronic resonance with vacuum quantum numbers 0^{++}- to be identical with **glueballs**.

In the past $\sigma - meson$ has been:

- listed in PDG as **"not well established" until 1974**
- **removed from PDG in 1976**
- **listed back in 1996**, after missing more than two decades, although **still with an obscure denotation** $f_0(400-1200)$
- from 2002 as **"well established"** $f_0(600)$, but with conservative estimate of the

mass: $400-1200 MeV$

and **width**: $600-1000 MeV$

A **clarification of this controversial situation** has been achieved in the papers *I.Caprini, G.Colangelo, H.Leutwyler: Phys. Rev. Lett. 96 (2006) 132001*

R.Garcia-Martin, R.Kaminski, J.R.Pelaez, J.Ruiz de Elvira: Phys. Rev. Lett 107 (2011) 072001

In this presentation **we confirm an existence of $f_0(500)$ by the pion scalar FF analysis.**

As for the latter a representation of the **pion scalar form factor is valid in the whole elastic region up to $1GeV^2$**, one can **determine also the $f_0(980)$ meson parameters** as well.

The **pion scalar FF** $\Gamma_\pi(t)$ is defined by the matrix element of the quark density

$$< \pi^i(p_2) \mid \widehat{m}(\bar{u}u + \bar{d}d) \mid \pi^j(p_1) >= \delta^{ij}\Gamma_\pi(t)$$

where $t = (p_2 - p_1)^2$ and $\widehat{m} = \frac{1}{2}(m_u + m_d)$.

NOTE:

$\Gamma_\pi(t)$ is **not directly measurable quantity** and it enters e.g. in the matrix element for the decay of the Higgs particle into two pions.

Properties of $\Gamma_\pi(t)$:

- it is **analytic in the whole complex** t**-plane** besides for a cut along the positive real axis starting at $t = 4m_\pi^2$
- for real values $t < 4m_\pi^2$ $\Gamma_\pi(t)$ is real
  $\Rightarrow$ it implies the so-called **reality condition**
  $\Gamma_\pi^*(t) = \Gamma_\pi(t^*)$
- at $t = 0$ $\Gamma_\pi(t)$ coincides with the **pion sigma term**
  $\Gamma_\pi(0) = (0.99 \pm 0.02)m_\pi^2$,
  however, in our considerations we normalize it
  exactly to m_π^2, i.e.
  $\Gamma_\pi(0) = 1$

- if $\Gamma_\pi(t)$ is evaluated on the **upper boundary of the cut** $\Rightarrow$ the **unitarity condition is obeyed**

$$Im\Gamma_\pi(t) = \sum_n < \pi(p')\pi(p) \mid T \mid n >< n \mid \widehat{m}(\bar{u}u + \bar{d}d) \mid 0 >$$

where the **sum runs over a complete set of allowed states** like $2\pi, 4\pi, ...K\bar{K}$, etc., which create **additional branch cuts** on the positive real axis of the t-plane between $4m_\pi^2$ and ∞.

- in the elastic region $4m_\pi^2 \leq t \leq 16m_\pi^2$ **only the first term in the unitarity condition contributes**, then

$$Im\Gamma_\pi(t) = \Gamma_\pi M_0^{0*}$$

where M_0^0 is $I = J = 0$ $\pi\pi$ **scattering amplitude**

$$M_0^0 = e^{i\delta_0^0} sin\delta_0^0;$$

with δ_0^0 the S-**wave isoscalar** $\pi\pi$ **phase shift**.

- $\Rightarrow$ the **elastic unitarity condition** is

$$Im\Gamma_\pi(t) = \Gamma_\pi e^{-i\delta_0^0} sin\delta_0^0$$

  from where the **identity**
  $\delta_\Gamma \equiv \delta_0^0$
  follows, where δ_Γ is **phase of the pion scalar FF**.

  NOTE:
  However, the **phenomenological analysis of the** $\pi\pi$ **interactions reveals** that this **identity is valid well above** $t = 4m_K^2 \approx 1 GeV^2$, where the inelastic two-body channel $\pi\pi \to K\bar{K}$ is opened

- The **asymptotic behavior** of $\Gamma_\pi(t)$ is predicted to be $\Gamma_\pi(t)_{|t|\to\infty} \sim 1/t$.

- Starting from S-**wave isoscalar** $\pi\pi$ **scattering amplitude unitarity condition**
  $ImM_0^0 = - \mid M_0^0 \mid^2$
  one can do **analytic continuation of** M_0^0 **through the upper and lower boundaries of the elastic unitary cut** and to come to
  $M_0^{0II} = \frac{M_0^{0I}}{1-2iM_0^{0I}}$
  which **reveals the singularity at** $t = 4m_\pi^2$ **to be a square root branch point**
- the **same is valid** for $\Gamma_\pi(t)$
  $\Gamma_\pi^{II} = \frac{\Gamma_\pi^I}{1-2iM_0^{0I}}$
  Moreover, **they have identical denominators!**

NOTE:

From this identity of denominators it follows:
If $f_0(500)$ and $f_0(980)$ resonances appear as **poles on the II. Riemann sheet of M_0^0**
$\Rightarrow$ they have to appear also as the **poles on the II. Riemann sheet of $\Gamma_\pi(t)$.**

For the model independent identification of these poles we use **phase representation of $\Gamma_\pi(t)$.**

Now, by an application of the **conformal mapping**

$$q = [(t-4)/4]^{1/2}, \quad m_\pi = 1 \tag{1}$$

two-sheeted Riemann surface of $\Gamma_\pi(t)$ is mapped into one absolute valued pion c.m. three-momentum q-plane and the **elastic cut disappears.**

Neglecting all higher branch points, there are only poles and zeros of $\Gamma_\pi(t)$ in q-plane
$\Rightarrow$ $\Gamma_\pi(t)$ can be **represented by a Pad'e-type approximation**

$$\Gamma_\pi(t) = \frac{\sum_{n=0}^{M} a_n q^n}{\prod_{i=1}^{N}(q - q_i)}.$$

Because $\Gamma_\pi(t)$ is a **real analytic function**,
$\Rightarrow$ **coefficients** a_n **with** M **even**(odd) **real** (pure imaginary).

The **poles** q_i can appear **on the imaginary axis** or they are **placed always two of them symmetrically according to it.**

If one multiplies both, the **numerator and the denominator** by the **complex conjugate factor** $\prod_{i=1}^{N}(q-q_i)^*$
$\Rightarrow$ **new denominator is a polynomial with real coefficients** and

$$tan\,\delta_\Gamma(t) = \frac{Im[\prod_{i=1}^{N}(q-q_i)^* \sum_{n=1}^{M} a_n q^n]}{Re[\prod_{i=1}^{N}(q-q_i)^* \sum_{n=1}^{M} a_n q^n]}.$$

By **using the identity** $\delta_\Gamma = \delta_0^0$ and the **threshold behavior of** δ_0^0, the following parametrization

$$tan\,\delta_0^0(t) = \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \ldots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \ldots}$$

or

$$\delta_0^0(t) = \frac{1}{2i} ln \frac{(1+A_2q^2+A_4q^4+A_6q^6+..)+i(A_1q+A_3q^3+A_5q^5+A_7q^7+...)}{(1+A_2q^2+A_4q^4+A_6q^6+..)-i(A_1q+A_3q^3+A_5q^5+A_7q^7+...)}$$

is obtained , **with A_i new real coefficients.**

NOTE:

The **parameter** A_1 is exactly equal to the S-**wave iso-scalar** $\pi\pi$ **scattering length** a_0^0**.**

One can see **directly from** $tan\, \delta_0^0(t)$ that **if the degree of the numerator is higher than the degree of its denominator** then

$$\lim_{q\to\infty} \delta_0^0(t) = \frac{\pi}{2}.$$

However, **if the degree of the numerator is lower than the degree of its denominator** then

$$\lim_{q \to \infty} \delta_0^0(t) = 0.$$

These **asymptotic behaviors can not be solved beforehand**.

Only a comparison of *tan* $\delta_0^0(t)$ **with existing data on** $\delta_0^0(t)$ can decide - **what type of pion scalar FF phase representation** derived:

- either from the **dispersion relation with one subtraction,**
- or from the **dispersion relation without subtractions,**

will be **the most suitable in our further considerations.**

As we are interested only for scalar meson resonances below $1GeV^2$, **we have collected slightly scattered 66 experimental points** in the latter region and **tried to find their best description** by the

$$\delta_0^0(t) = arctan\frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + ...}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + ...}$$

parametrization to be equivalent to

$$\delta_0^0(t) = \frac{1}{2i} ln\frac{(1+A_2 q^2+A_4 q^4+A_6 q^6+..)+i(A_1 q+A_3 q^3+A_5 q^5+A_7 q^7+...)}{(1+A_2 q^2+A_4 q^4+A_6 q^6+..)-i(A_1 q+A_3 q^3+A_5 q^5+A_7 q^7+...)}.$$

The **analysis of the data has been carried out successively.**

The results are summarized in this Table:

Number of A_i	χ^2/ndf
1	17.75
2	1.66
3	1.60
4	1.49
5	1.41
6	1.44
7	1.50

The **minimum of χ^2/ndf is achieved with 5 coefficients**,

$$A_1 = 0.25684 \pm 0.0107;$$

$$A_3 = 0.14547 \pm 0.01620; A_5 = -.01217 \pm 0.00070$$

$$A_2 = 0.02274 \pm 0.02830; A_4 = -.01537 \pm 0.00480$$

which give a **description** of the data on S_0^0 in Fig. **by full line.**

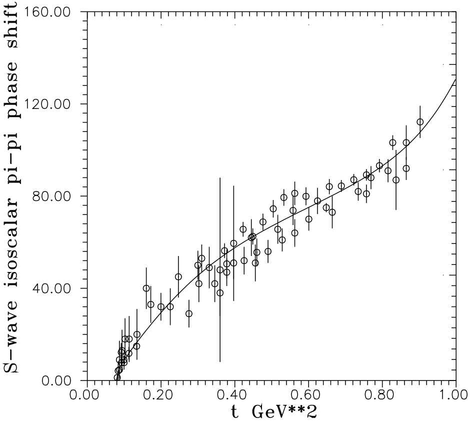

Figure : Description of the S-wave iso-scalar $\pi\pi$ phase shift by the [5/4] Pad‘e type approximation

A description of S_0^0 by the **[5/4] Pad'e type approximation** is enough to conclude - **one has to start construction of the pion scalar FF by the dispersion relation with one subtraction** at $t = 0$

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi} \int_{4m_\pi^2}^{\infty} \frac{Im\Gamma_\pi(t')}{t'(t'-t)} dt'.$$

to be **derived by an application of the Cauchy formula** to the function

$$\frac{\Gamma_\pi(t) - \Gamma_\pi(0)}{t - 0}.$$

Substitution of the **elastic unitarity condition**

$$Im\Gamma_\pi(t) = \Gamma_\pi e^{-i\delta_0^0} sin\delta_0^0$$

into the dispersion relation with one subtraction leads to the **Omnes-Muskelishvili integral equation**

$$\Gamma_\pi(t) = 1 + \frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\Gamma_\pi(t)e^{-i\delta_0^0} sin\delta_0^0}{t'(t'-t)} dt'.$$

Its solution is just the **phase representation** of $\Gamma_\pi(t)$

$$\Gamma_\pi(t) = P_n(t)exp[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty} \frac{\delta_0^0(t')}{t'(t'-t)} dt']$$

with one subtraction, where $P_n(t)$ **is an arbitrary polynomial** to be restricted with $P_n(0) = 1$.

The **logarithmic representation** of $\delta_0^0(t)$ with 5 nonzero above-mentioned coefficients leads to the expression

$$\Gamma_\pi(t) = P_n(t) exp \frac{(q^2+1)}{\pi i} \int_0^\infty \frac{q' ln \frac{(1+A_2 q'^2 + A_4 q'^4) + i(A_1 q' + A_3 q'^3 + A_5 q'^5)}{(1+A_2 q'^2 + A_4 q'^4) - i(A_1 q' + A_3 q'^3 + A_5 q'^5)}}{(q'^2+1)(q'^2-q^2)} dq',$$

in which $m_\pi = 1$ is assumed.

As the **integrand is even function of its argument**, i.e. it is invariant under the transformation $q' \to -q'$, the **latter expression can be transformed** into the final integral form

$$\Gamma_\pi(t) = P_n(t) exp \frac{(q^2+1)}{2\pi i} \int_{-\infty}^\infty \frac{q' ln \frac{(1+A_2 q'^2 + A_4 q'^4) + i(A_1 q' + A_3 q'^3 + A_5 q'^5)}{(1+A_2 q'^2 + A_4 q'^4) - i(A_1 q' + A_3 q'^3 + A_5 q'^5)}}{(q'^2+1)(q'^2-q^2)} dq'$$

where the **integral is suitable to be calculated by means of the theory of residua.**

In order to **carry out this programm** one needs to know **roots of polynomials** in numerator and denominator under the logarithm, which generate **branch points** in q-plane.

NOTE:
The **roots of denominator are complex conjugate roots of numerator !**

$\Rightarrow$ enought to investigate numerator
$(1 + A_2 q'^2 + A_4 q'^4) + i(A_1 q' + A_3 q'^3 + A_5 q'^5) = 0.$
In order to have equation with real coefficients one substitutes
$q' = ix$
$\Rightarrow 1 - A_1 x - A_2 x^2 + A_3 x^3 + A_4 x^4 - A_5 x^5 = 0$
or
$-x^5 + \frac{A_4}{A_5} x^4 + \frac{A_3}{A_5} x^3 - \frac{A_2}{A_5} x^2 - \frac{A_1}{A_5} x + \frac{1}{A_5} = 0$

Solutions of the latter equation are:

$$
\begin{aligned}
x_1 &= -1.8633297 \\
x_2 &= 0.2832535 - i3.5830748 \\
x_3 &= 1.2800184 - i1.3328447 \\
x_4 &= 0.2832535 + i3.5830748 \\
x_5 &= 1.2800184 + i1.3328447
\end{aligned}
$$

from where one finds **roots of numerator and denominator** under the logarithm of integrand $\phi(q', q)$

$$
\begin{aligned}
q_1 &= -i1.8633297 \\
q_2 &= -3.5830748 + i0.2832535 \\
q_3 &= -1.3328447 + i1.2800184 \\
q_4 &= 3.5830748 + i0.2832535 \\
q_5 &= 1.3328447 + i1.2800184
\end{aligned}
$$

and

$$\begin{aligned} q_1^* &= -q_1 \\ q_2^* &= -q_4 \\ q_3^* &= -q_5 \\ q_4^* &= -q_2 \\ q_5^* &= -q_3 \end{aligned}$$

respectively.
Then **integral in previous expression**, considering the case $q^2 < 0$ i.e. $q = i\sqrt{\frac{4-t}{4}} \equiv ib$, is transformed into

$$I = \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q'-q_1)(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{(q'-q_1^*)(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq'$$

with all **singularities of its integrant** presented in Fig. 2.

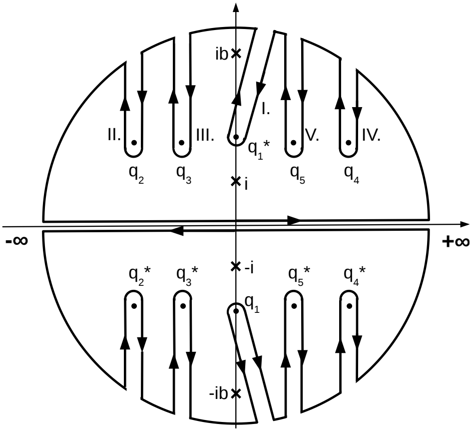

Figure : Poles ($\times$) and branch points ($\bullet$) of the integrands $\phi_1(q')$ and $\phi_2(q')$ with contours of integrations in the upper and lower half-planes, respectively.

Further it is convenient to **split the integral into sum of two integrals**

$$I = \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{(q'-q_1^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq' +$$

$$+ \int_{-\infty}^{\infty} \frac{q' \ln \frac{(q'-q_1)}{(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq' = I_1 + I_2$$

according to **singularities to be placed in the upper half-plane or in the lower half-plane**, respectively.

Let us start to calculate the **first integral by the theory of residues**

$$\oint \frac{q' ln \frac{(q'-q_2)(q'-q_3)(q'-q_4)(q'-q_5)}{(q'-q_1^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq' = 2\pi i \sum_n Res_n$$

where the **contour of integration is closed in the upper half-plane** (see Fig. 2).

As the integral on the half-circle is 0 then

$$I_1 = \int_{-\infty}^{\infty} \phi_1(q')dq' = 2\pi i \sum_{n=1}^{2} Res_n - [-\int_{1^*} + \int_2 + \int_3 + \int_4 + \int_5]$$

where the **integrals on the right-hand side represent contributions of the cuts** generated by the branch points $q_1^*, q_2, q_3, q_4, q_5$ in Fig. 2.
The **residua at the poles** are straightforward to calculate ($ib = q$)

$$Res\phi_1(i,q) = -\frac{1}{2(q^2+1)} ln\frac{(i-q_2)(i-q_3)(i-q_4)(i-q_5)}{(i-q_1^*)},$$

$$Res\phi_1(ib,q) = \frac{1}{2(q^2+1)} ln\frac{(q-q_2)(q-q_3)(q-q_4)(q-q_5)}{(q-q_1^*)}.$$

Now the **contributions of the cuts**

$$\int_{1^*} = \int_{\infty}^{q_1^*} \frac{q' ln_+(q' - q_1^*)}{(q'^2+1)(q'^2+b^2)} dq' + \int_{q_1^*}^{\infty} \frac{q' ln_-(q' - q_1^*)}{(q'^2+1)(q'^2+b^2)} dq' =$$

$$= \int_{q_1^*}^{\infty} \frac{q'}{(q'^2+1)(q'^2+b^2)} [ln_-(q' - q_1^*) - ln_+(q' - q_1^*)] dq' =$$

$$= -2\pi i \int_{q_1^*}^{\infty} \frac{q'}{(q'^2+1)(q'^2+b^2)} dq' =$$

$$= -\frac{\pi i}{(b^2-1)} ln \frac{(q_1^{*2}+b^2)}{(q_1^{*2}+1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2+1)} ln \frac{(q_1^{*2}-q^2)}{(q_1^{*2}+1)}.$$

Similarly

$$\int_j = -\frac{\pi i}{(b^2-1)} ln \frac{(q_j^2+b^2)}{(q_j^2+1)} \equiv \frac{1}{2}\frac{2\pi i}{(q^2+1)} ln \frac{(q_j^2-q^2)}{(q_j^2+1)}; \qquad j = 2,3,4,5.$$

Then the **sum of all these partial results gives** the final result for I_1

$$I_1 = \frac{1}{2}\frac{2\pi i}{(q^2+1)} ln \frac{(q+q_1^*)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)} \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i+q_1^*)}.$$

Similarly one can calculate also the **second integral** I_2 by means of the theory of residua

$$\oint \frac{q' ln \frac{(q'-q_1)}{(q'-q_2^*)(q'-q_3^*)(q'-q_4^*)(q'-q_5^*)}}{(q'+i)(q'-i)(q'+ib)(q'-ib)} dq' = 2\pi i \sum_{n=1}^{2} Res_n$$

where the **contour of integration is closed in the lower half-plane** (see Fig. 2).

As the integral on the half-circle is 0 then

$$I_2 = \int_{-\infty}^{\infty} \phi_2(q')dq' = -2\pi i \sum_{n=1}^{2} Res_n + [+\int_1 - \int_{2^*} - \int_{3^*} - \int_{4^*} - \int_{5^*}].$$

where the **integrals on the right-hand side represent contributions of the cuts** generated by the branch points $q_1, q_2^*, q_3^*, q_4^*, q_5^*$ in Fig. 2.
The **residua at the poles** take the form($ib = q$)

$$Res\phi_2(-i, q) = -\frac{1}{2(q^2+1)} ln \frac{(-i - q_1)}{(-i - q_2^*)(-i - q_3^*)(-i - q_4^*)(-i - q_5^*)},$$

$$Res\phi_2(-ib, q) = \frac{1}{2(q^2+1)} ln \frac{(-q - q_1)}{(-q - q_2^*)(-q - q_3^*)(-q - q_4^*)(-q - q_5^*)}.$$

The **contributions of the cuts**

$$\int_1 = \int_{\infty}^{q_1} \frac{q' ln_+(q'-q_1)}{(q'^2+1)(q'^2+b^2)} dq' + \int_{q_1}^{\infty} \frac{q' ln_-(q'-q_1)}{(q'^2+1)(q'^2+b^2)} dq' =$$

$$= \int_{q_1}^{\infty} \frac{q'}{(q'^2+1)(q'^2+b^2)} [ln_-(q'-q_1) - ln_+(q'-q_1)] dq' =$$

$$= -2\pi i \int_{q_1}^{\infty} \frac{q'}{(q'^2+1)(q'^2+b^2)} dq' =$$

$$= -\frac{\pi i}{(b^2-1)} ln \frac{(q_1^2+b^2)}{(q_1^2+1)} \equiv \frac{1}{2} \frac{2\pi i}{(q^2+1)} ln \frac{(q_1^2-q^2)}{(q_1^2+1)}.$$

Similarly

$$\int_{j^*} = -\frac{\pi i}{(b^2-1)} ln \frac{(q_{j^*}^2 + b^2)}{(q_{j^*}^2 + 1)} \equiv \frac{1}{2}\frac{2\pi i}{(q^2+1)} ln \frac{(q_{j^*}^2 - q^2)}{(q_{j^*}^2 + 1)};$$
$$j^* = 2^*, 3^*, 4^*, 5^*.$$

Then the **sum of all these partial results gives**

$$I_2 = \frac{1}{2}\frac{2\pi i}{(q^2+1)}[ln \frac{(q+q_1)}{(q+q_2^*)(q+q_3^*)(q+q_4^*)(q+q_5^*)} \frac{(i+q_2^*)(i+q_3^*)(i+q_4^*)(i+q_5^*)}{(i+q_1)}$$
$$+ ln \frac{q_1^2 - q^2}{q_1^2+1} - ln \frac{q_2^{*2}-q^2}{q_2^{*2}+1} - ln \frac{q_3^{*2}-q^2}{q_3^{*2}+1} - ln \frac{q_4^{*2}-q^2}{q_4^{*2}+1} - ln \frac{q_5^{*2}-q^2}{q_5^{*2}+1}]$$

By using the relations between q_i **and** q_j^* finally one gets

$$I_2 = \frac{1}{2}\frac{2\pi i}{(q^2+1)} ln \frac{(q+q_1^*)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)} \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i+q_1^*)}.$$

The sum $I_1 + I_2$ represents the total integral

$$I = \frac{2\pi i}{(q^2+1)} ln \frac{(q-q_1)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)} \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i-q_1)}.$$

The **substitution of this logarithmic form** into the pion scalar FF final integral representation **gives**

$$\Gamma_\pi(t) = P_n(t) \frac{(q-q_1)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)} \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i-q_1)},$$

an **explicit form for the pion scalar FF** to be **graphically presented** in Fig. 3.

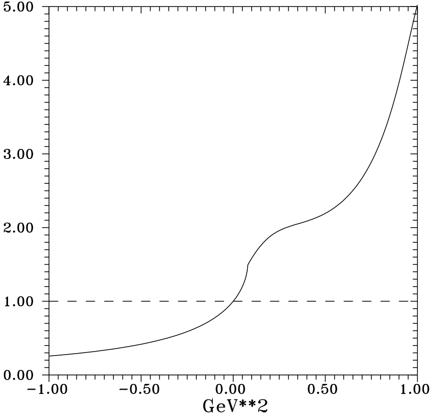

Figure : Behavior of the pion scalar form factor in $-1GeV^2 < t < 1GeV^2$ region.

The $-q_3$ **and** $-q_2$ **poles** of $\Gamma_\pi(t)$ on the second Riemann sheet in t-variable **correspond to** $f_0(500)$ **and** $f_0(980)$ **scalar meson resonances**, respectively.

Their **masses and widths** are determined to be
$m_{f_0(500)} = (360 \pm 33)MeV, \quad \Gamma_{f_0(500)} = (587 \pm 85)MeV,$
$m_{f_0(980)} = (957 \pm 72)MeV, \quad \Gamma_{f_0(980)} = (164 \pm 142)MeV,$
where the **errors correspond to the transferred errors of the coefficients** $A_1, ...A_5$**.**

If, however, other local minimum with the same value of χ^2/ndf is found, then the **masses and widths** are found to be
$m_{f_0(500)} = (431 \pm 47)MeV, \quad \Gamma_{f_0(500)} = (621 \pm 96)MeV,$
$m_{f_0(980)} = (1041 \pm 81)MeV, \quad \Gamma_{f_0(980)} = (167 \pm 139)MeV.$

From the obtained results one comes to the conclusion:

more precise data on δ_0^0 are **highly required** to be measured

in order to determine unambiguous parameters of the lowest scalar mesons by means of the presented method.

Other determinations of $f_0(500)$ for comparison:

I.Caprini, G.Colangelo, H.Leutvyller, Phys. Rev. Lett. 96 (2006) 132001

$m_\sigma = 441 MeV \quad \Gamma_\sigma = 544 MeV$

R.Garcia, J.R.Pelaez, F.J.Yndurain, Phsy. Rev. D76 (2007) 074034

$m_\sigma = 474 MeV \quad \Gamma_\sigma = 508 MeV$

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$m_\sigma = 463 MeV \quad \Gamma_\sigma = 508 MeV$

J.A.Oller, Nucl. Phys. A727 (2003) 353

$m_\sigma = 443 MeV \quad \Gamma_\sigma = 432 MeV$

G.Mennesier, S.Narison, X.-G.Wang, Phys. Lett. B696 (2011) 40

$m_\sigma = 452 MeV \quad \Gamma_\sigma = 520 MeV$

J.R.Pelaez, G.Rios, Phys. Rev. D82 (2010) 114002

$m_\sigma = 453 MeV \quad \Gamma_\sigma = 542 MeV$

R.Garcia-Martinez, R.Kaminski, J.R.Pelaez, J.Ruiz de Elvira, Phys. Rev. Lett. 107 (2011) 072001

$m_\sigma = 457 MeV \quad \Gamma_\sigma = 558 MeV$

- new **method for a prediction of the pion scalar FF behavior in elastic region** has been developed
- by using only δ_0^0 **parametrization obtained from general considerations and experimental data on it** in elastic region, the **parameters of** $f_0(500)$ **and** $f_0(980)$ **are determined** in a model independent way.