Pseudoscalar Transition Form Factors from Rational Approximants

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Work in collaboration with R. Escribano and P. Masjuan

[Phys.Rev. D86 (2012) 094021, Phys.Rev. D89 (2014) 034014, ...]

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SFB 1044
THE LOW-ENERGY FRONTIER
OF THE STANDARD MODEL

PRiSMA

JG|U
JOHANNES GUTENBERG
UNIVERSITÄT MAINZ

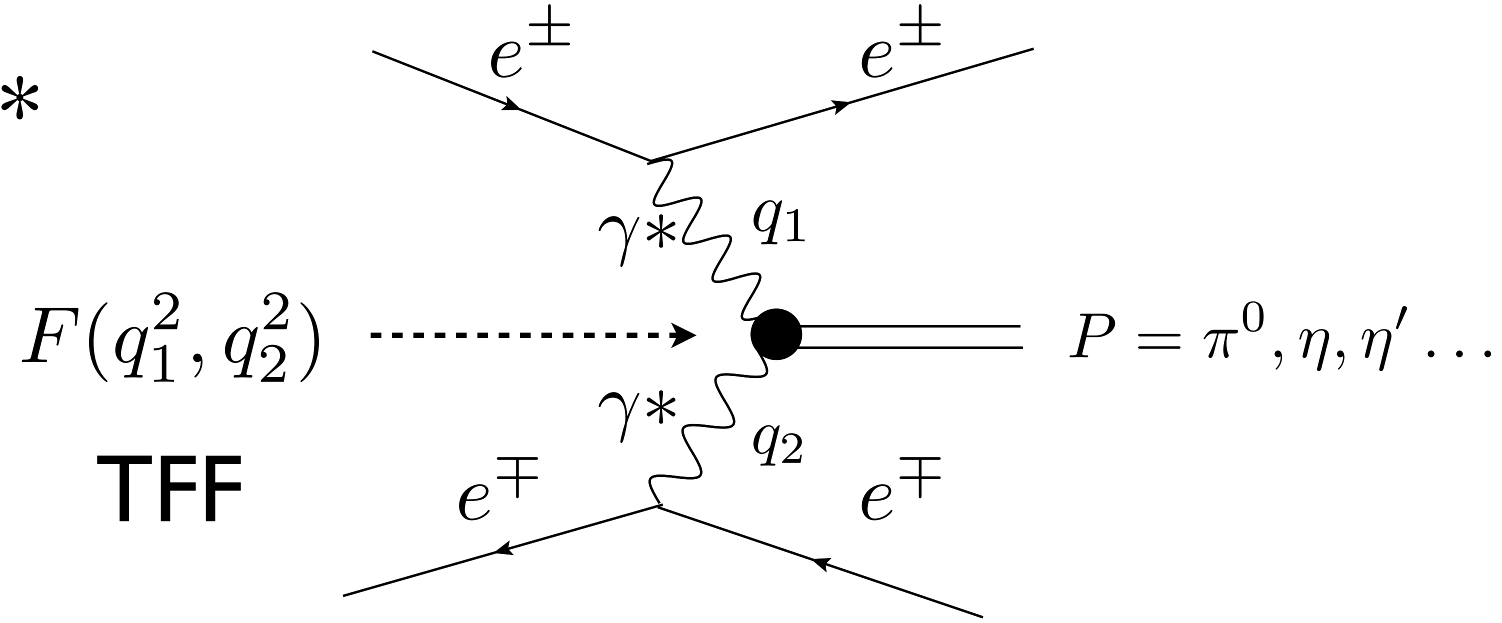
Outline

Pseudoscalar Transition Form Factors

- Pseudoscalar Transition Form Factors
- Parameterization using Rational Approximants
- Results from space-like data alone
  - η-η' mixing
- Implications from space-like + time-like data
- Conclusions and outlook

Pseudoscalar Transition Form Factors

- Study of ee→ee$\gamma^*\gamma^*$ with $\gamma^*\gamma^*\to\pi,\eta,\eta'$



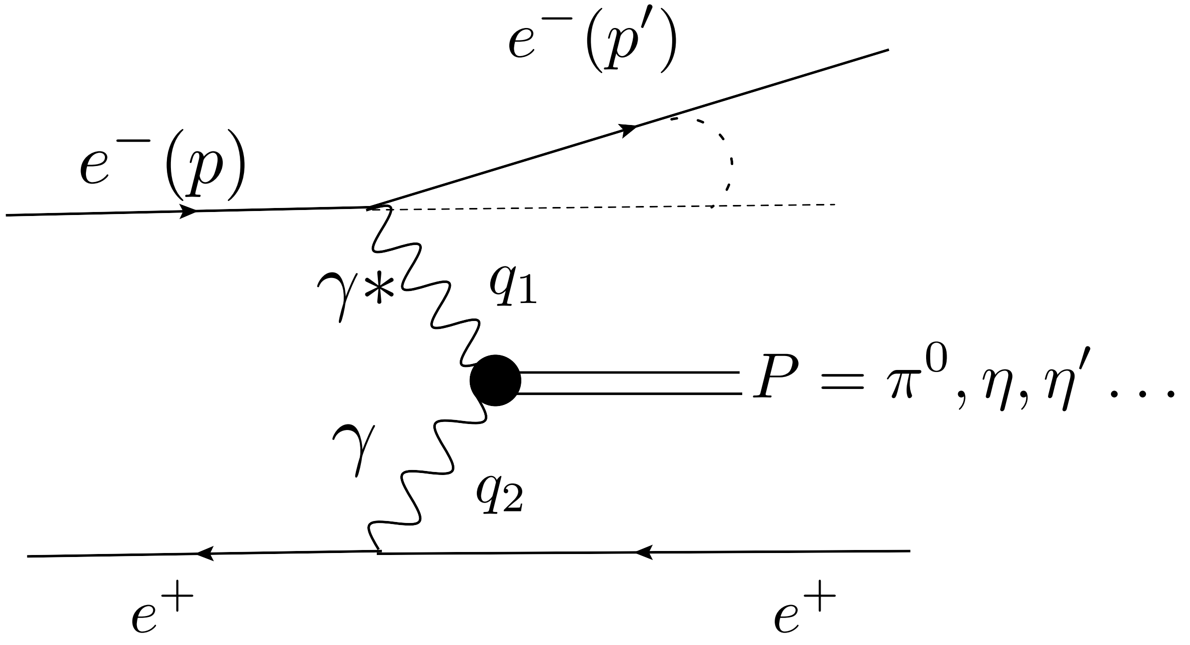
- Meson Structure
  - Transition Form Factors (TFF) give access to Meson Distribution Amplitudes
- Precision Tests of the Standard Model
  - Relation to mixing parameters and muon anomaly $(g\text{-}2)_\mu$

How do we do that?

- Single Tag Method can access the Meson Transition Form Factor

Selection criteria

- 1 e^- detected
- 1 e^+ along beam axis
- Meson full reconstructed



Momentum transfer

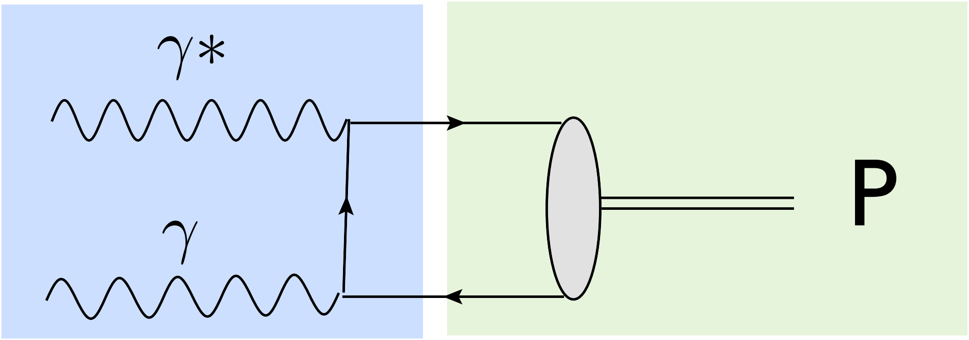
- tagged: $Q^2 = -q_1^2 = -(p-p')^2$
⇒highly virtual space-like (SL) photon

- untagged: $Q^2 = -q_2^2 \sim 0\ \mathrm{GeV}^2$
⇒quasi-real photon

How do we do that?

Cross section for P production depends only on $F(q_1^2, q_2^2)$

With the Single Tag Method: $F(q_1^2, q_2^2) \rightarrow F(Q^2)$



$$F(Q^2) = \int T_H(x, Q^2)\Phi_P(x, \mu_F)\mathrm{d}x$$

$$T_H(\gamma^*\gamma \rightarrow q\bar{q}) \quad \Phi_P(q\bar{q} \rightarrow P)$$

- μ_F is scale between soft and hard
- x-dependence of $\Phi_P(x,Q^2)$

not known from first principles → model

- Experimental data on $F(Q^2)$ is needed

- $F(0)$ from χPT
- $F(Q^2) \sim Q^{-2}$ at high Q^2 from pQCD

convolution of perturbative and non-perturbative regimes

Our proposal
use Padé Approximants

$$F_{P\gamma*\gamma}(Q^2,0) = \underset{\substack{\nearrow\\ \Gamma_{P\to\gamma\gamma}}}{a_0^P}\left(1 + \underset{\substack{\uparrow\\ \text{slope}}}{b_P}\,\frac{Q^2}{m_P^2} + \underset{\substack{\uparrow\\ \text{curvature}}}{c_P}\,\frac{Q^4}{m_P^4} + \ldots\right)$$

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$$F_{P\gamma*\gamma}(Q^2,0) = \underset{\substack{\nearrow \\ \Gamma_{P\to\gamma\gamma}}}{a_0^P}\left(1 + \underset{\substack{\uparrow \\ \text{slope}}}{b_P}\frac{Q^2}{m_P^2} + \underset{\substack{\uparrow \\ \text{curvature}}}{c_P}\frac{Q^4}{m_P^4} + \dots\right)$$

We have published space-like data for $Q^2 F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \dots$$

Our proposal
use Padé Approximants

$$F_{P\gamma*\gamma}(Q^2,0) = a_0^P\left(1 + b_P\frac{Q^2}{m_P^2} + c_P\frac{Q^4}{m_P^4} + \ldots\right)$$

↗ $\Gamma_{P\to\gamma\gamma}$ ↑ slope ↑ curvature

We have published space-like data for $Q^2F_{P\gamma*\gamma}(Q^2,0)$

$$Q^2F_{P\gamma*\gamma}(Q^2,0) = a_0Q^2 + a_1Q^4 + a_2Q^6 + \ldots$$

Use Padé Approximants instead (better convergence properties)

$$P_M^N(Q^2) = \frac{T_N(Q^2)}{R_M(Q^2)} = a_0Q^2 + a_1Q^4 + a_2Q^6 + \cdots + \mathcal{O}((Q^2)^{N+M+1})$$

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use Padé Approximants

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Use Padé Approximants instead (better convergence properties)

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - \frac{a1}{a0}Q^2}$$

Our proposal
use Padé Approximants

$$F_{P\gamma*\gamma}(Q^2,0) = \underset{\underset{\Gamma_{P\to\gamma\gamma}}{\nearrow}}{a_0^P}\left(1 + \underset{\underset{\text{slope}}{\uparrow}}{b_P}\frac{Q^2}{m_P^2} + \underset{\underset{\text{curvature}}{\uparrow}}{c_P}\frac{Q^4}{m_P^4} + \ldots\right)$$

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$$Q^2 F_{P\gamma*\gamma}(Q^2,0) = a_0 Q^2 + a_1 Q^4 + a_2 Q^6 + \ldots$$

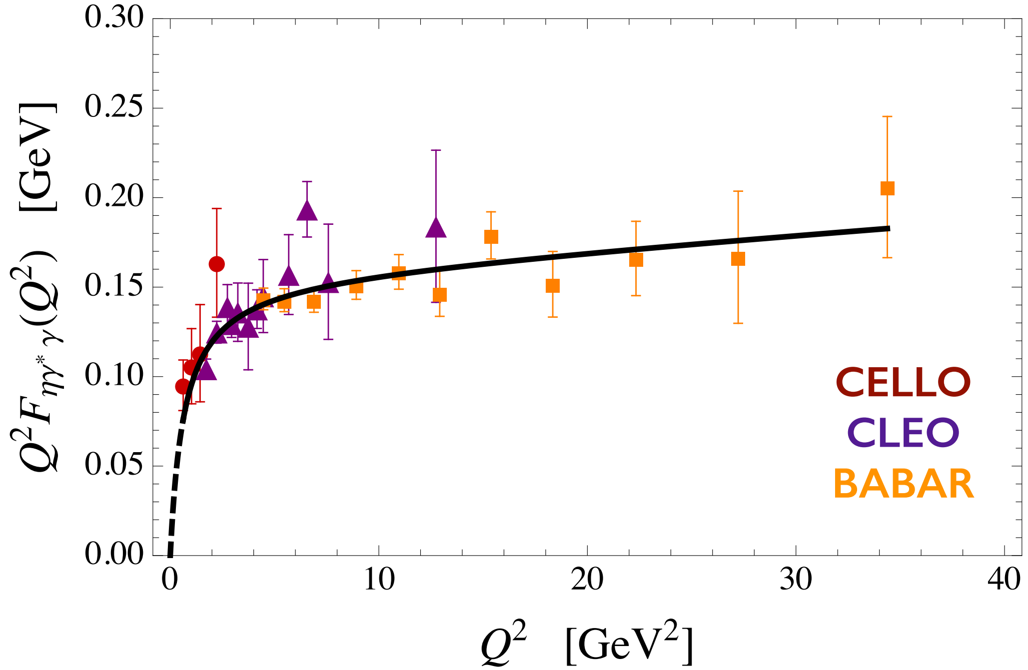
Use Padé Approximants instead (better convergence properties)

$$P_1^1(Q^2) = \frac{a_0 Q^2}{1 - \frac{a1}{a0}Q^2} \longrightarrow \begin{array}{l} P_1^N(Q^2) = P_1^1(Q^2), P_1^2(Q^2), P_1^3(Q^2), \ldots \\ P_N^N(Q^2) = P_1^1(Q^2), P_2^2(Q^2), P_3^3(Q^2), \ldots \end{array}$$

η-TFF

Fit to Space-like data: CELLO’91, CLEO’98, BABAR’11

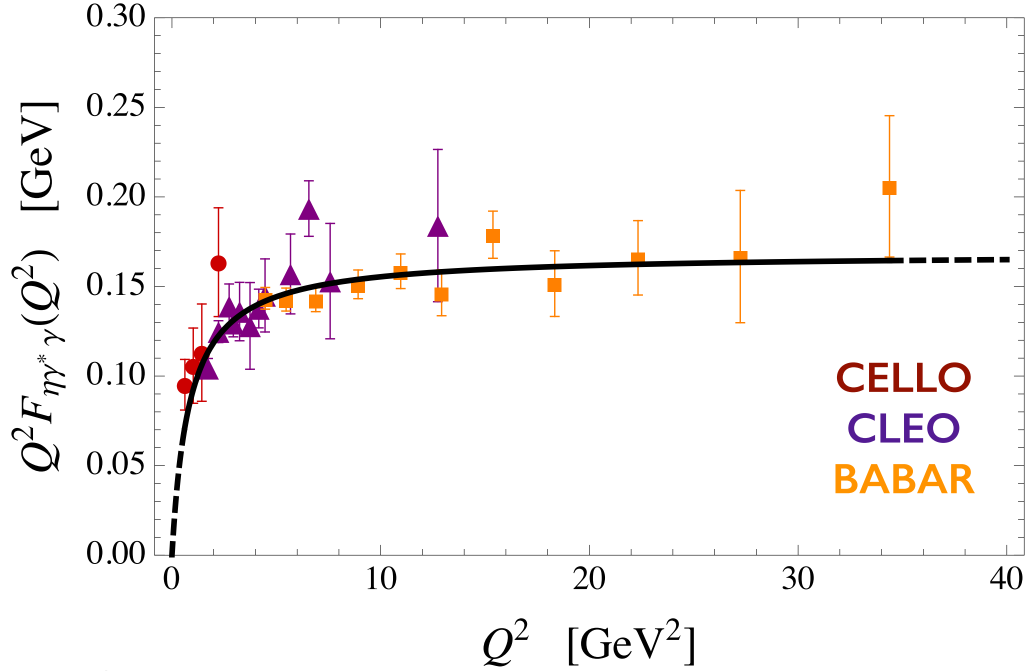
[Phys.Rev. D89 (2014) 034014]



$$P_1^N(Q^2)$$ up to N=2

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.38 \pm 0.17)keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.516 \pm 0.018)keV$$



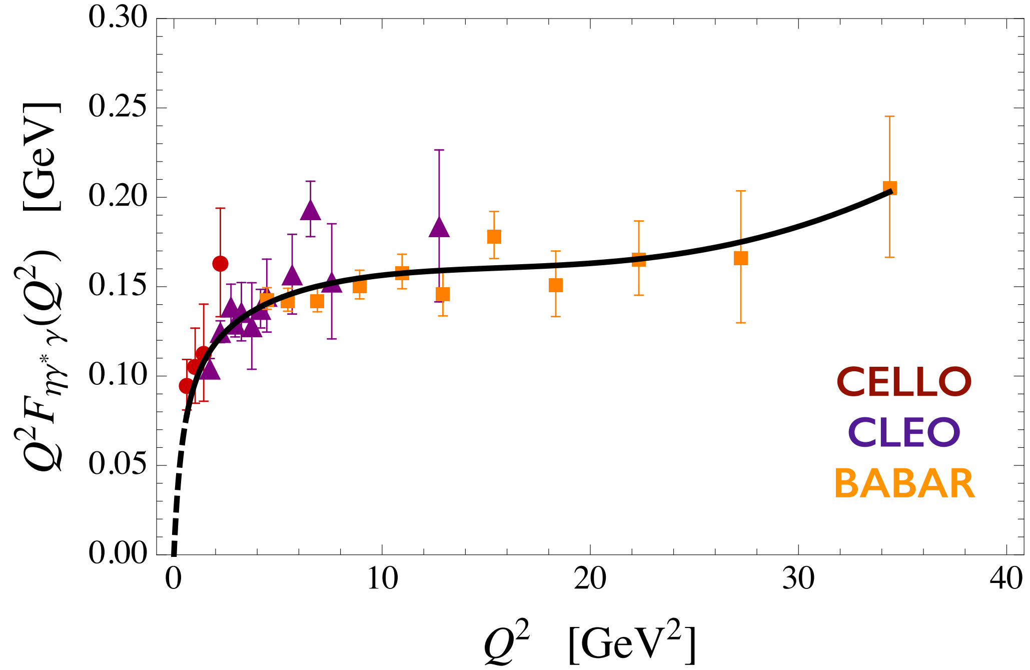
$$P_N^N(Q^2)$$ up to N=1

$$\lim_{Q^2\to\infty} Q^2F_{\eta\gamma*\gamma}(Q^2,0) = 0.17(6)GeV$$

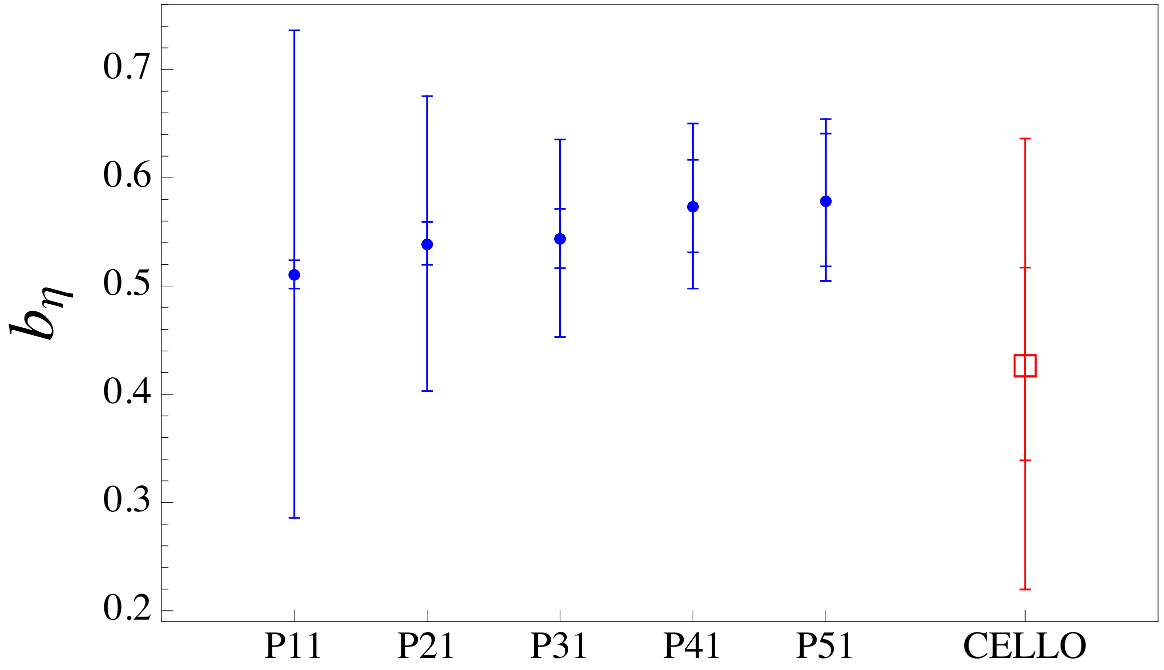
η-TFF

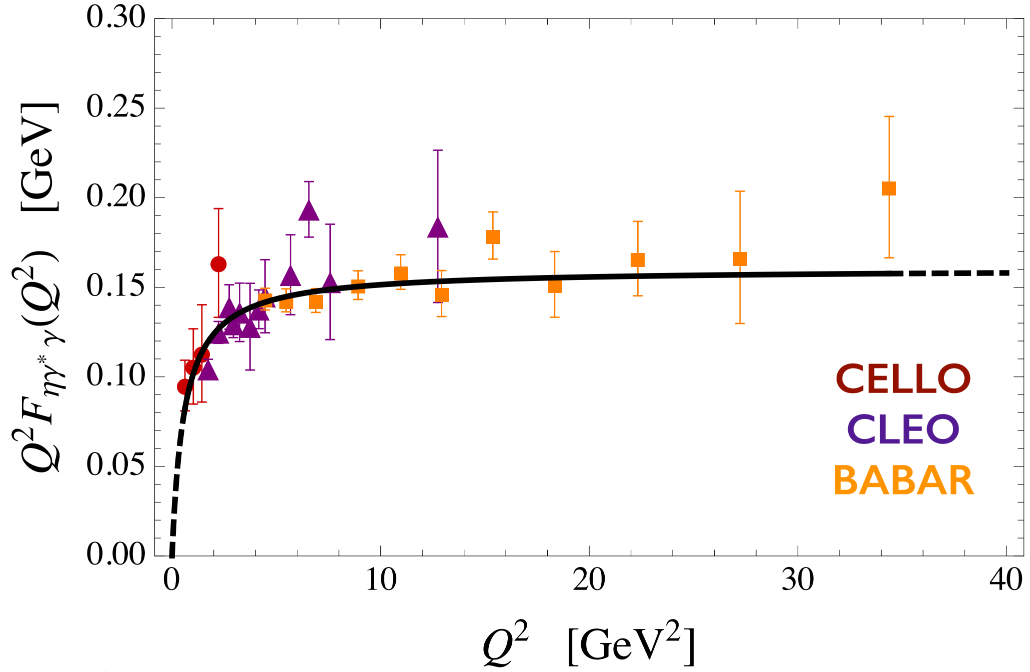
Fit to Space-like data: CELLO’91, CLEO’98, BABAR’11+ $\Gamma_{\eta\to\gamma\gamma}$

[Phys.Rev. D89 (2014) 034014]



$P_1^N(Q^2)$ up to N=5





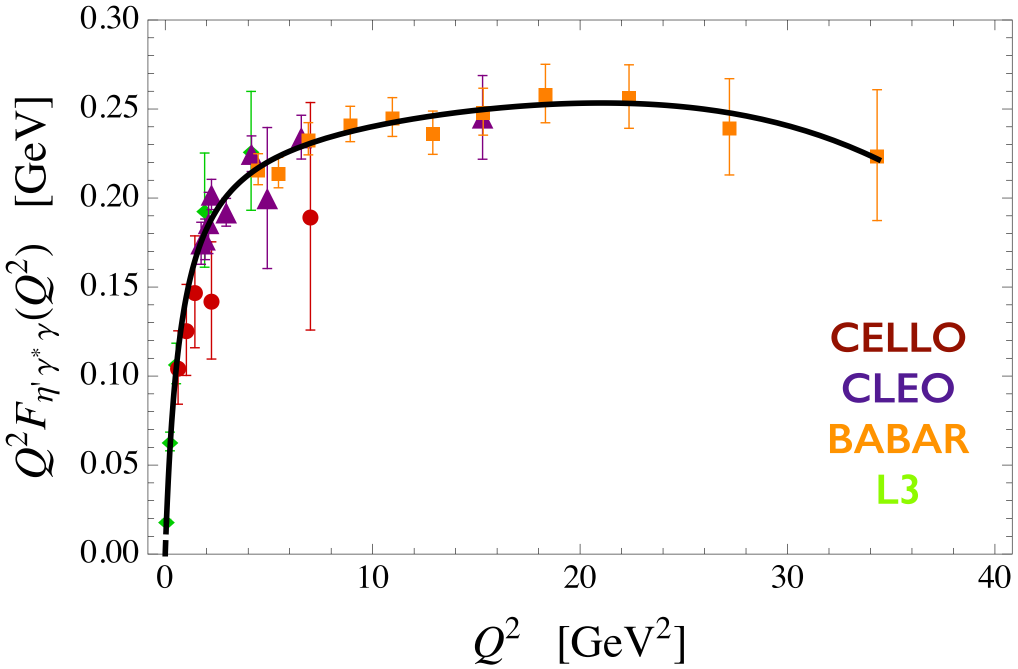
$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.160(24)GeV$$

η’-TFF

Fit to Space-like data: CELLO’91, CLEO’98, L3’98, BABAR’11

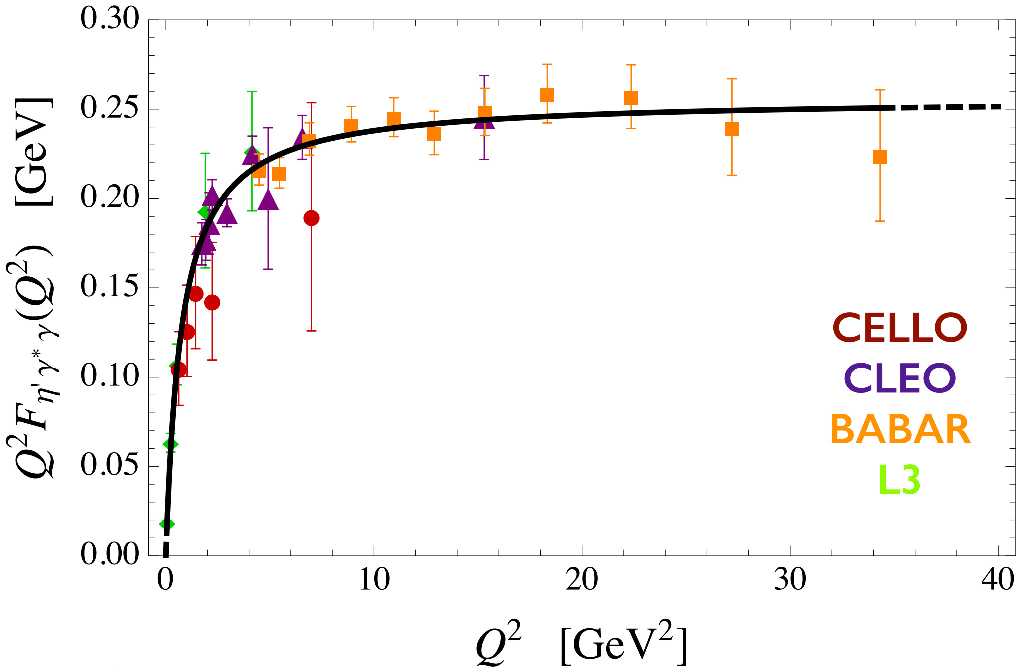
[Phys.Rev. D89 (2014) 034014]



$P_1^N(Q^2)$ up to N=5

$$\Gamma^{pred}_{\eta'\to\gamma\gamma} = (4.21 \pm 0.43)keV$$

$$\Gamma^{PDG}_{\eta'\to\gamma\gamma} = (4.34 \pm 0.14)keV$$



$P_N^N(Q^2)$ up to N=1

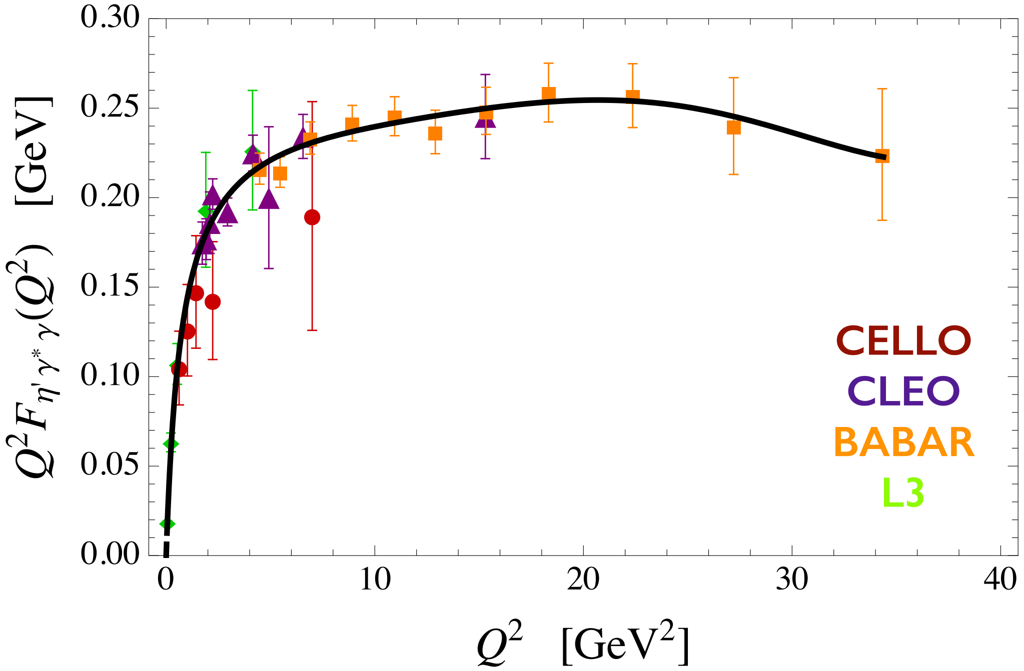
$$\lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma*\gamma}(Q^2,0) = 0.256(4)GeV$$

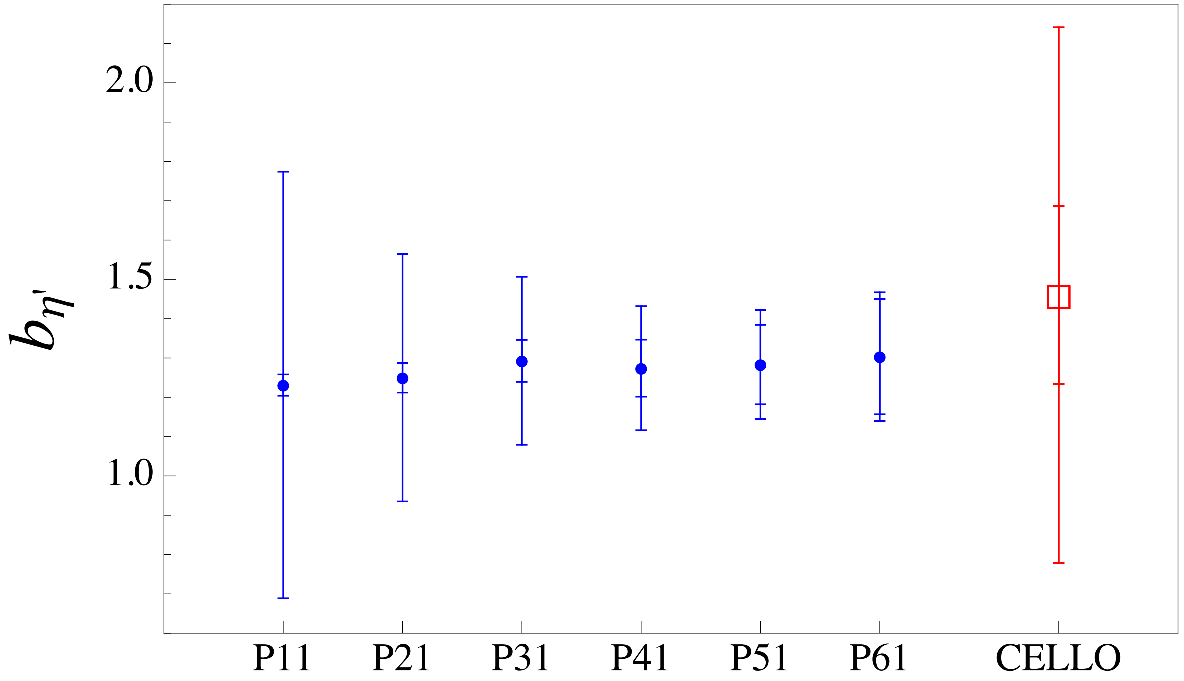
η'-TFF

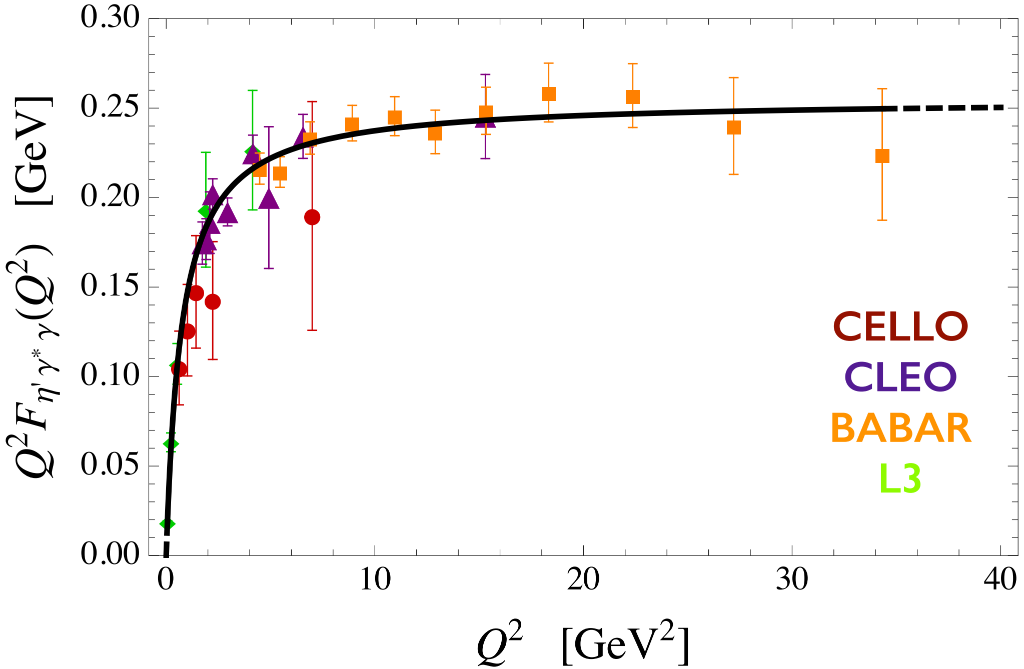
Fit to Space-like data: CELLO'91, CLEO'98, L3'98, BABAR'11+ $\Gamma_{\eta'\to\gamma\gamma}$

[Phys.Rev. D89 (2014) 034014]

$P_1^N(Q^2)$ up to N=6







$P_N^N(Q^2)$ up to N=1

$$\lim_{Q^2\to\infty} Q^2F_{\eta'\gamma^*\gamma}(Q^2,0) = 0.255(4)GeV$$

η-η' mixing

η-η' mixing in the flavor basis

$$\left(\begin{array}{cc} F_{\eta}^{q} & F_{\eta}^{s} \\ F_{\eta'}^{q} & F_{\eta'}^{s} \end{array}\right) = \left(\begin{array}{cc} F_q \cos\phi & -F_s \sin\phi \\ F_q \sin\phi & F_s \cos\phi \end{array}\right)$$

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From the TFFs we can determine F_q, F_s, ϕ

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}cos\phi - \frac{\hat{c}_s}{F_s}sin\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q cos\phi - \hat{c}_s F_s sin\phi)$$

$$F_{\eta'\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}sin\phi + \frac{\hat{c}_s}{F_s}cos\phi\right) \qquad \lim_{Q^2\to\infty} Q^2 F_{\eta'\gamma^*\gamma}(Q^2) = 2(\hat{c}_q F_q sin\phi + \hat{c}_s F_s cos\phi)$$

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η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} F^q_\eta & F^s_\eta \\ F^q_{\eta'} & F^s_{\eta'} \end{pmatrix} = \begin{pmatrix} F_q \cos\phi & -F_s \sin\phi \\ F_q \sin\phi & F_s \cos\phi \end{pmatrix}$$

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[Phys.Rev. D89 (2014) 034014]

$$F_q = 1.06(1)F_\pi, \quad F_s = 1.56(24)F_\pi, \quad \phi = 40.3(1.8)^\circ$$

Update of Frere-Escribano '05 with PDG12 using 9 inputs

$$F_q = 1.07(1)F_\pi, \quad F_s = 1.63(3)F_\pi, \quad \phi = 39.6(0.4)^\circ$$

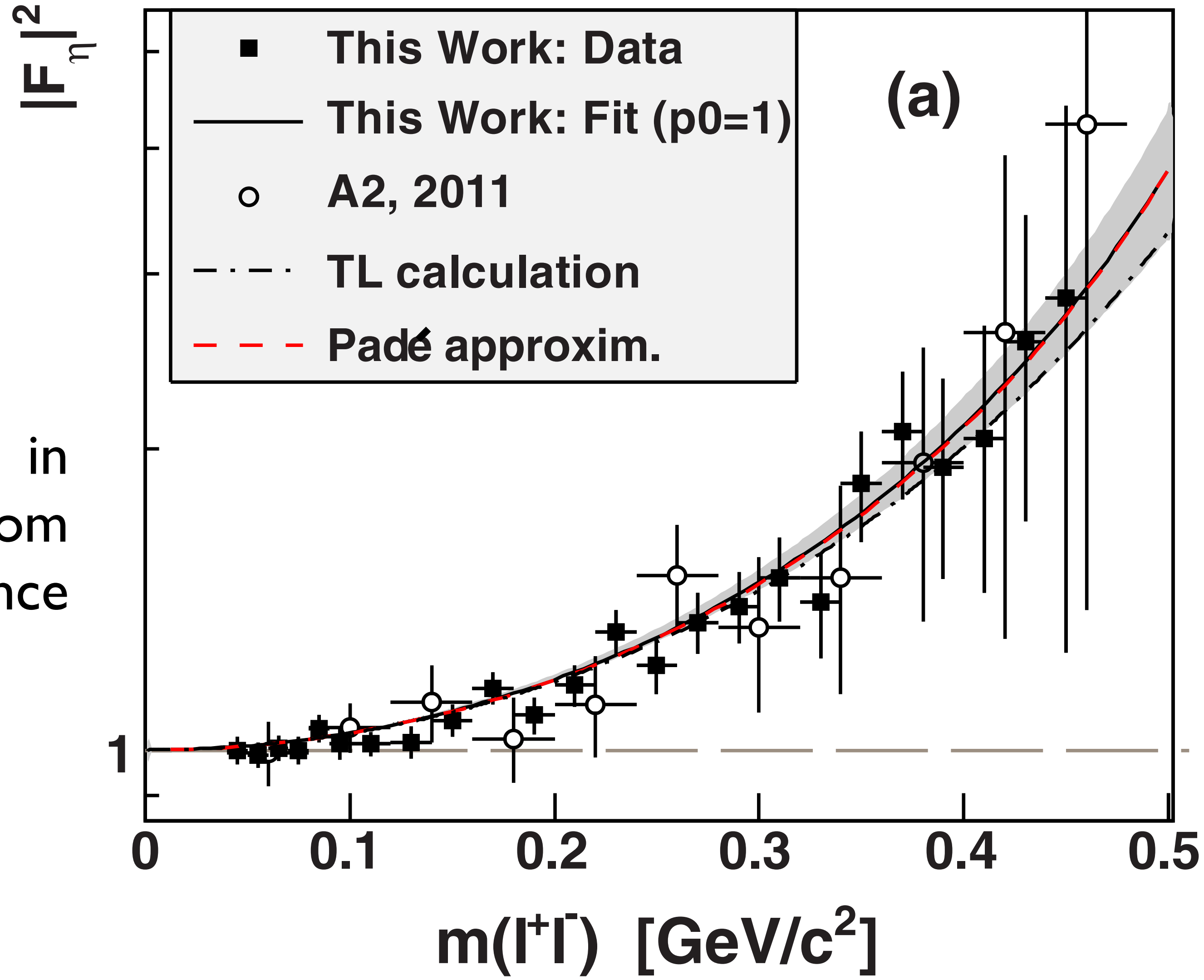
Pseudoscalar Transition Form Factors

- Study Dalitz decays

  $\eta \to \gamma^* \gamma \to e^+ e^- \gamma$

- Our predictions from SL data are in excellent agreement with latest results from A2 Coll. as expected below resonance region.

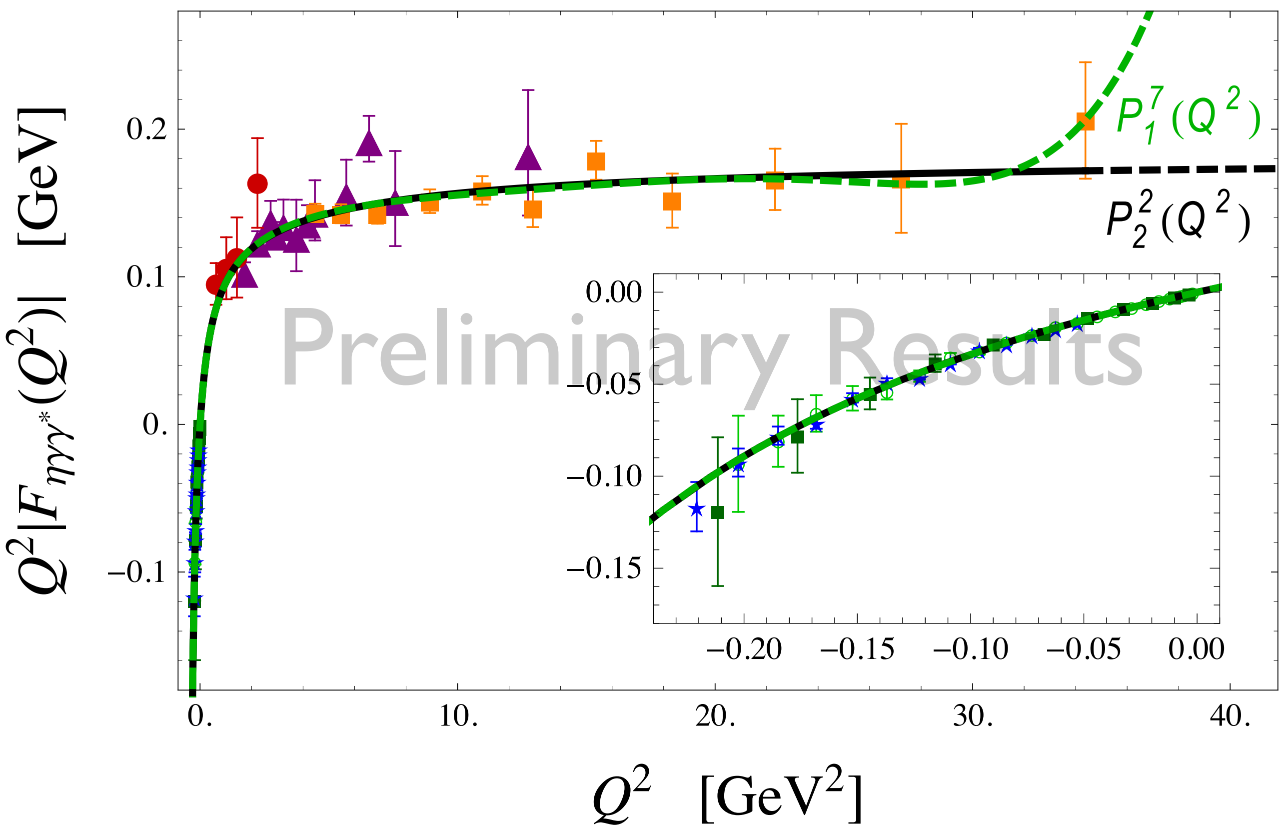
Result from A2 Coll. [Phys.Rev. C89 (2014) 044608]



η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

[R.Escribano, P.M., P. Sanchez-Puertas, '14]



$P_1^N(Q^2)$ up to N=7

$$\Gamma^{pred}_{\eta\to\gamma\gamma} = (0.42 \pm 0.10)keV$$

$$\Gamma^{PDG}_{\eta\to\gamma\gamma} = (0.518 \pm 0.018)keV$$

$P_N^N(Q^2)$ up to N=2

$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2, 0) = 0.169(14)GeV$$

η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
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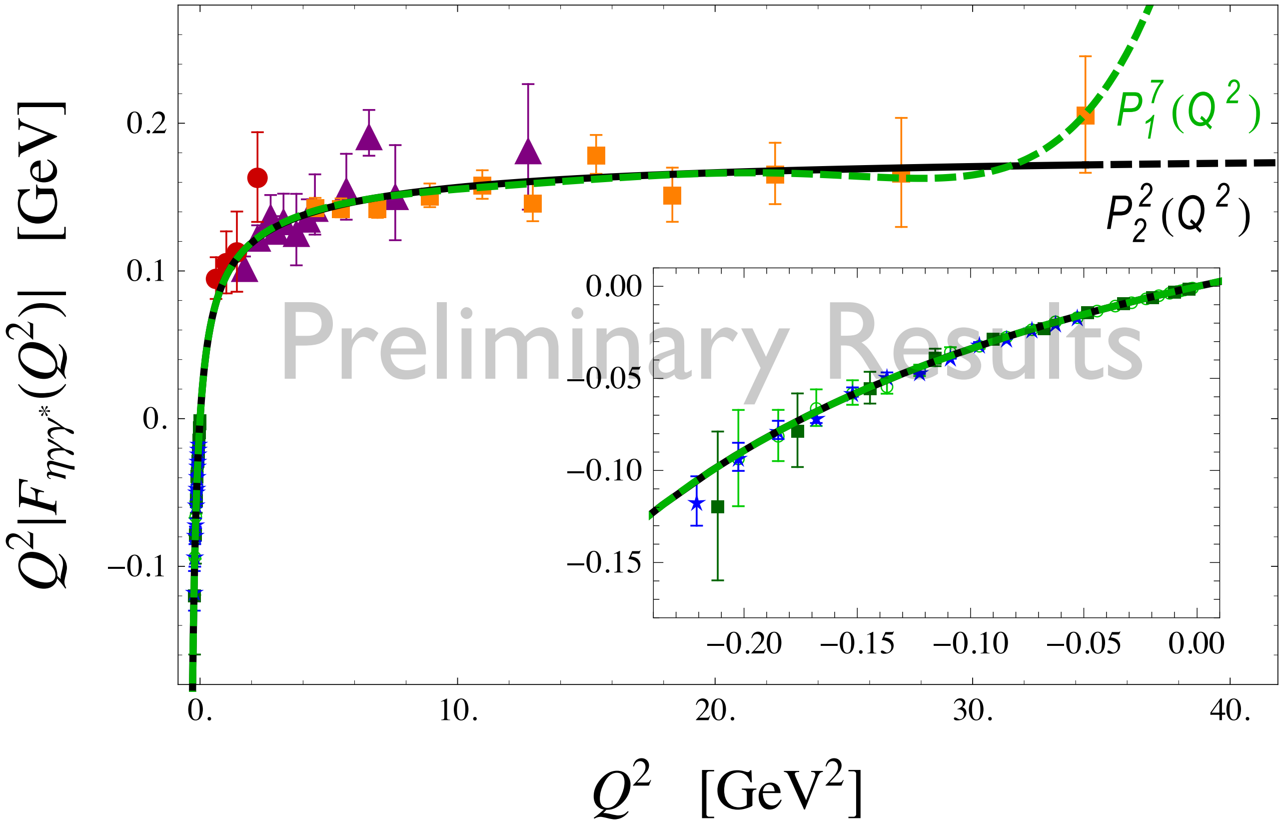
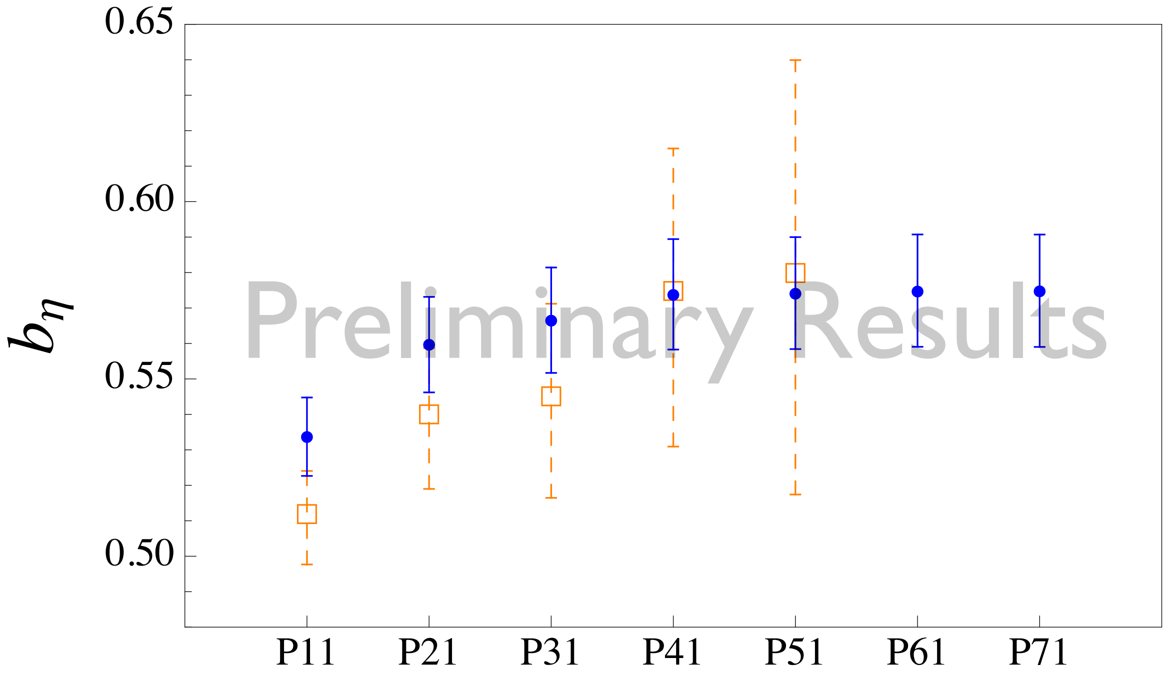


$P_1^N(Q^2)$ up to N=7



$P_N^N(Q^2)$ up to N=2

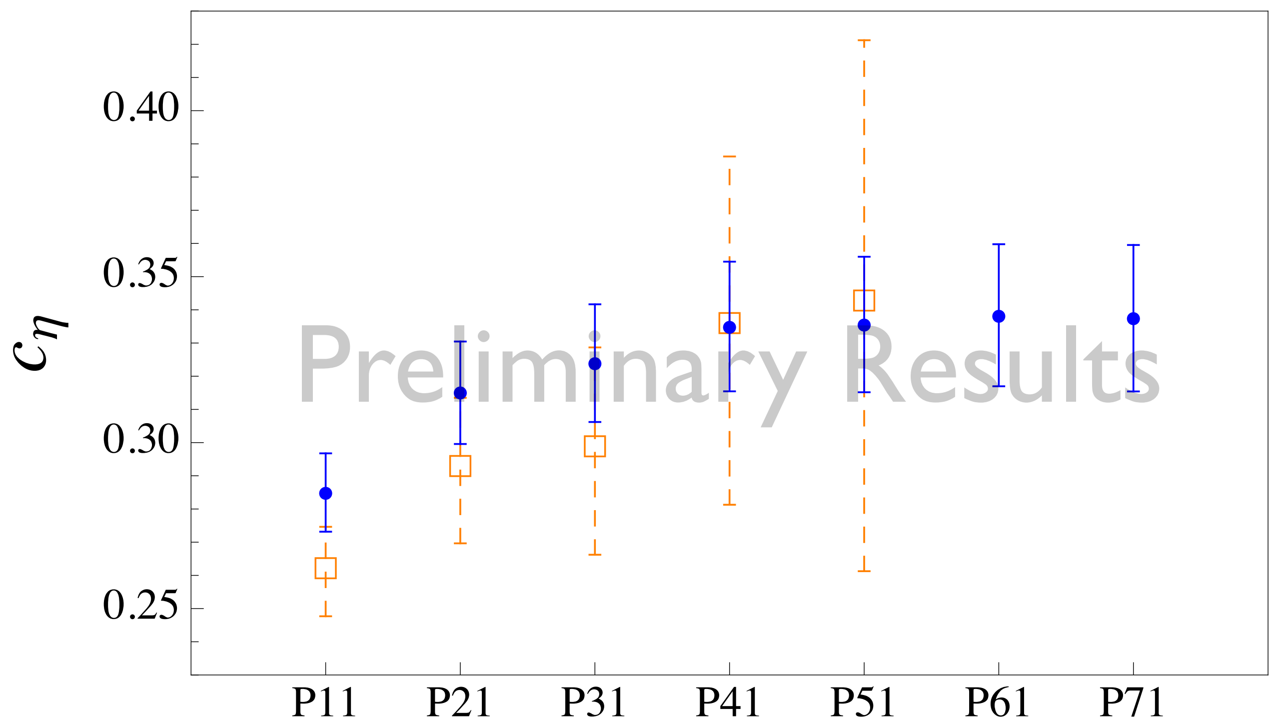
$$\lim_{Q^2\to\infty} Q^2 F_{\eta\gamma*\gamma}(Q^2,0) = 0.177(15)GeV$$

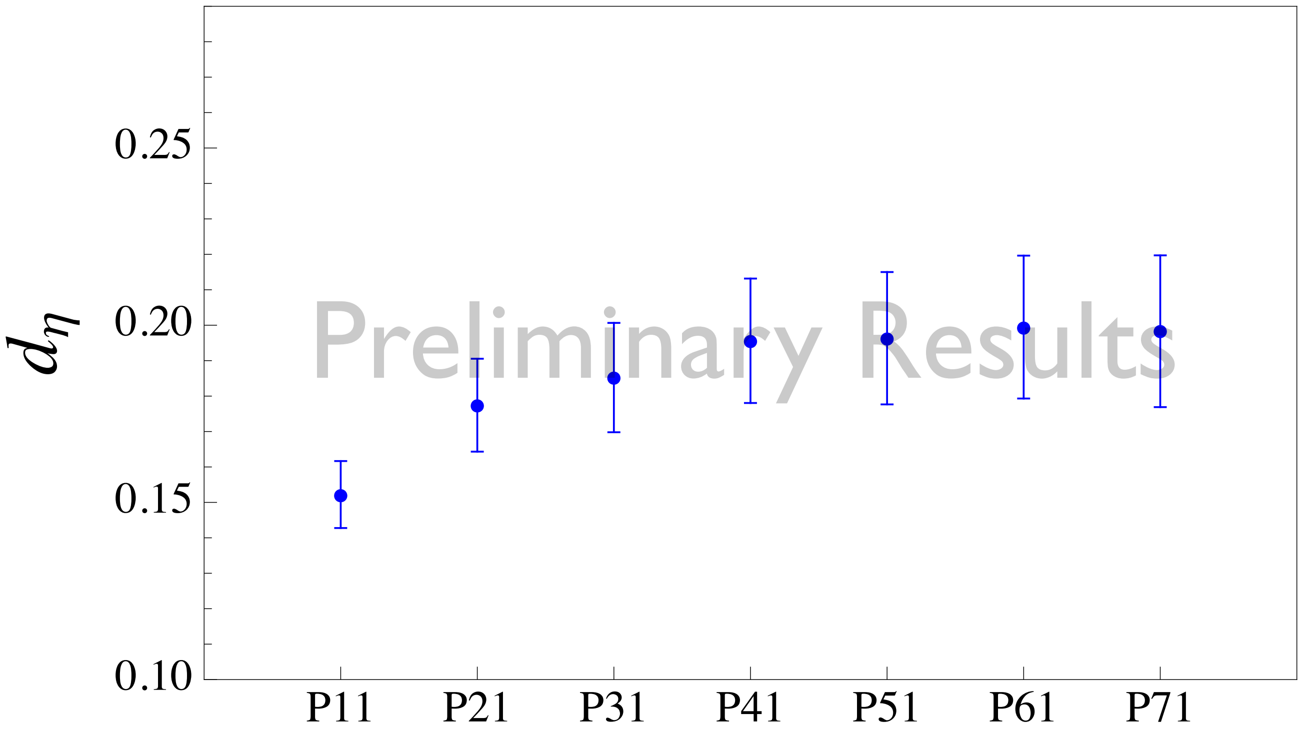
η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
+ Time-like data [NA60'09, A2'11, A2'13]

$P_1^N(Q^2)$ up to N=7

[R.Escribano, P.M., P. Sanchez-Puertas, '14]





slope for the η-TFF

Fit to Space-like data [CELLO'91, CLEO'98, BABAR'11]+$\Gamma_{\eta\to\gamma\gamma}$
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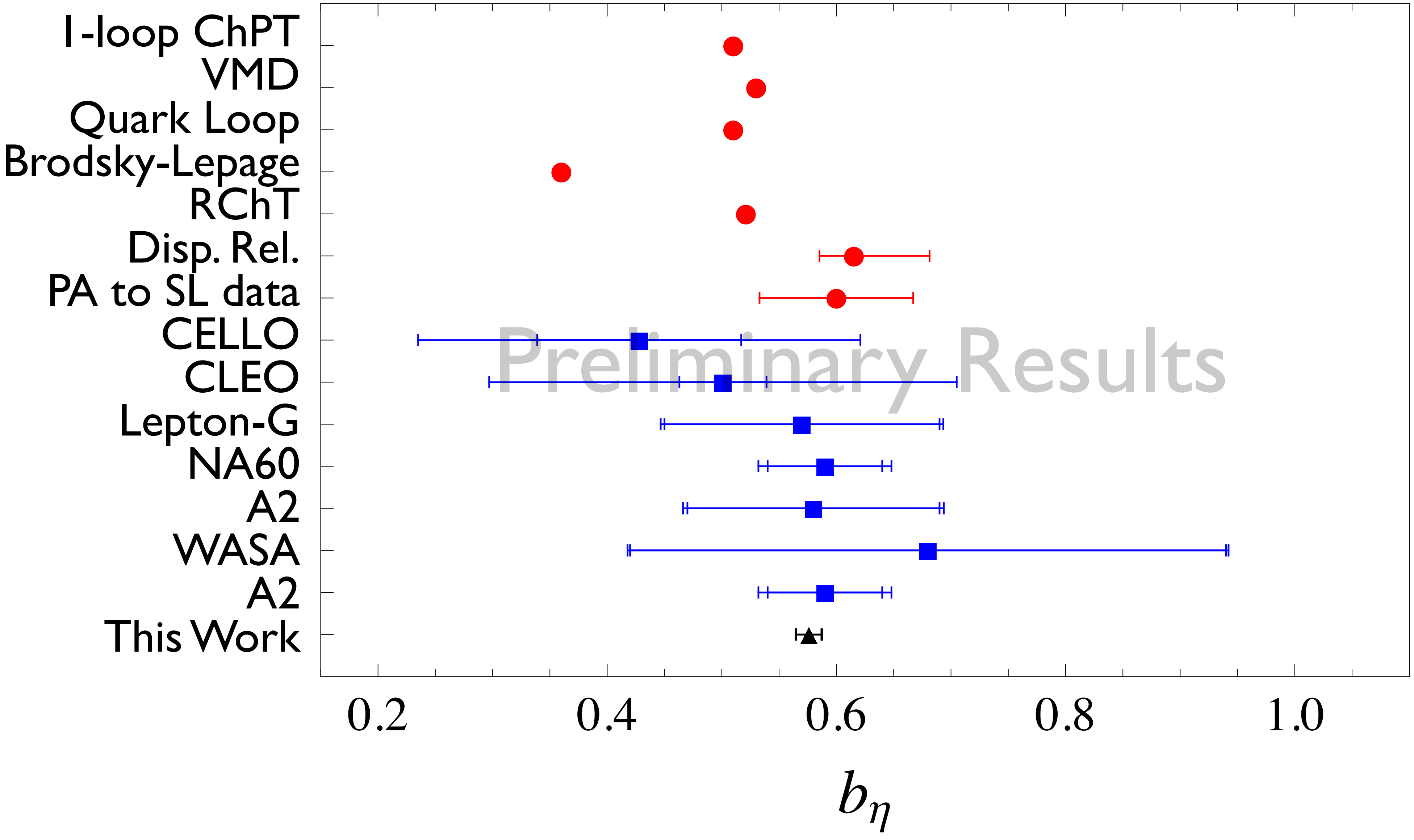

η-η' mixing

η-η' mixing in the flavor basis

$$\begin{pmatrix} F_\eta^q & F_\eta^s \\ F_{\eta'}^q & F_{\eta'}^s \end{pmatrix} = \begin{pmatrix} F_q \cos\phi & -F_s \sin\phi \\ F_q \sin\phi & F_s \cos\phi \end{pmatrix}$$

From the TFFs we can determine F_q, F_s, ϕ

$$F_{\eta\gamma\gamma}(0) = \frac{1}{4\pi^2}\left(\frac{\hat{c}_q}{F_q}cos\phi - \frac{\hat{c}_s}{F_s}sin\phi\right)$$

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[Phys.Rev. D89 (2014) 034014]

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η-η’ mixing in the flavor basis

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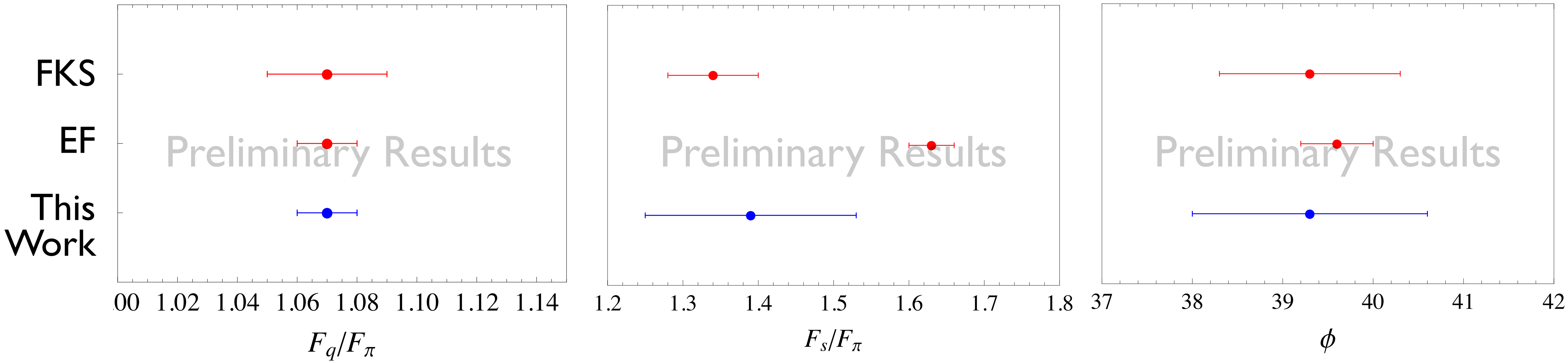


FKS: Feldmann, Kroll, Stech, PRD 58, 114006 (1998)
EF: Escribano, Frere, JHEP 0506, 029 (2005) updated in Escribano, P.M, Sanchez-Puertas, 2013.

Conclusions

- Transition Form Factors are a good laboratory to study meson properties

- Need for a model independent approach: we use Padé App.

- Considering Space-like and time-like data
  - provides very accurate LECs and asymptotic limits
  - provides insight in mixing scheme and meson structure

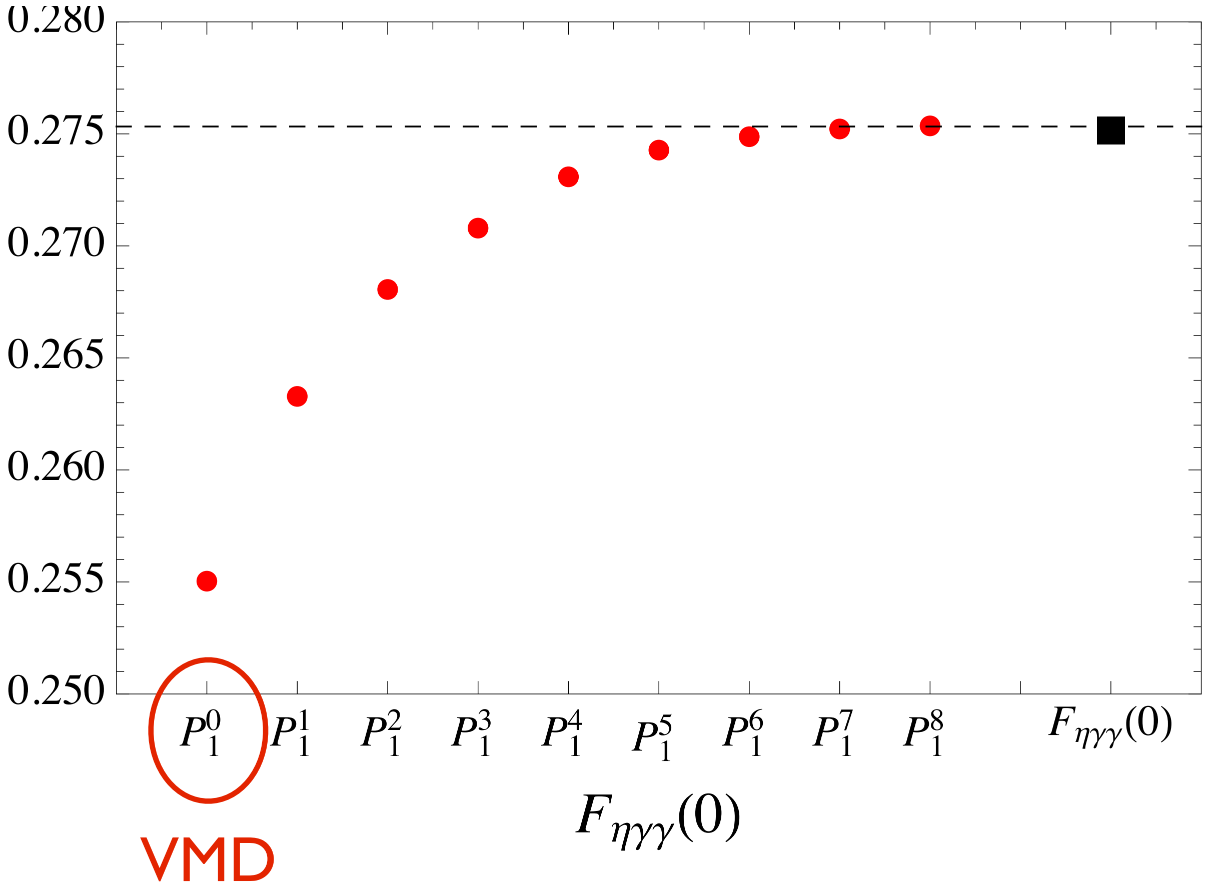
- Many more applications
  - Predicts VPγ, J/Ψ, rare decays ($P \to e^+e^-(\mu^+\mu^-)$), $(g-2)_\mu$...

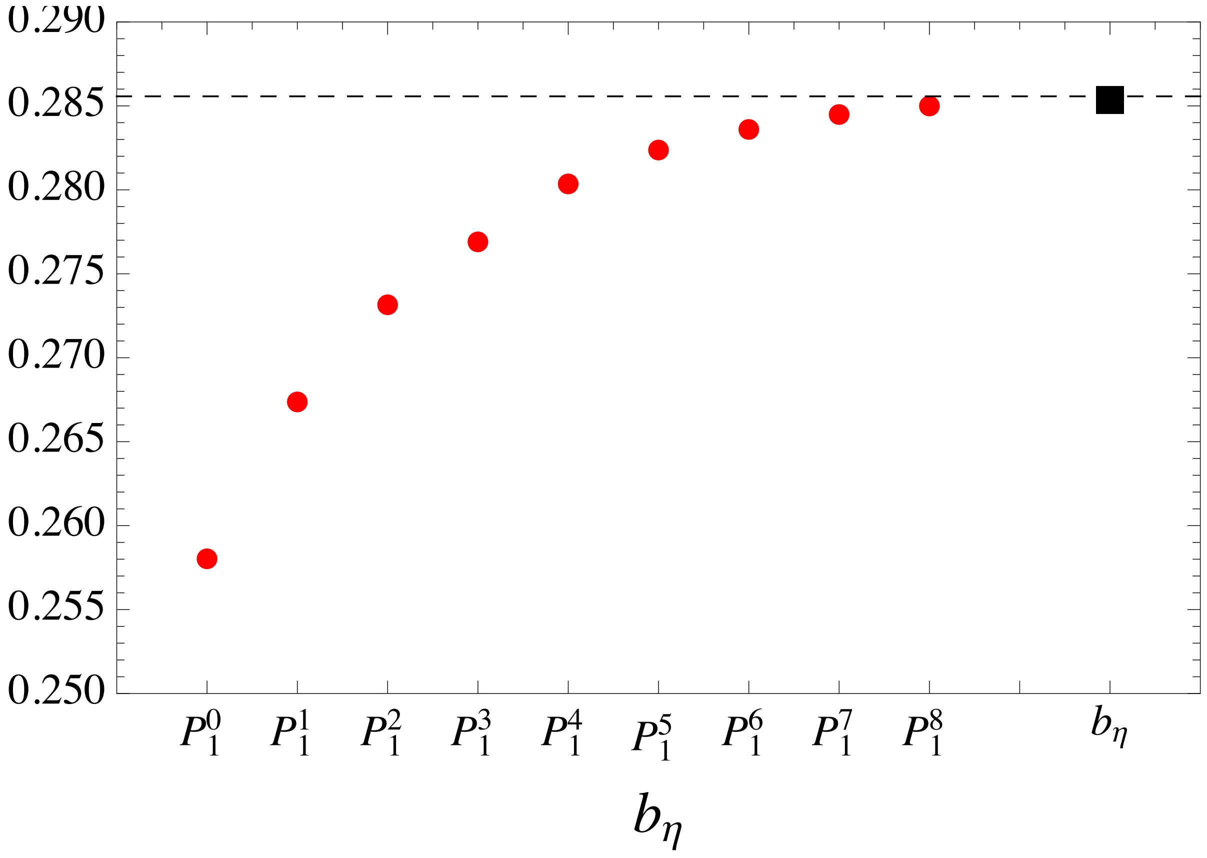
Padé Approximants' method is easy, systematic, deals with syst. errors and can be improved upon by including new data

Thank you!

A word on systematics

- Consider a model for η TFF
- Generate a pseudodata set emulation the physical situation (SL+TL)
- Build up your PA sequence
- Fit and compare





η-η' mixing

From the TFFs we can determine F_q, F_s, ϕ

and the VPγ and J/Ψ decays used in FKS and EF as inputs

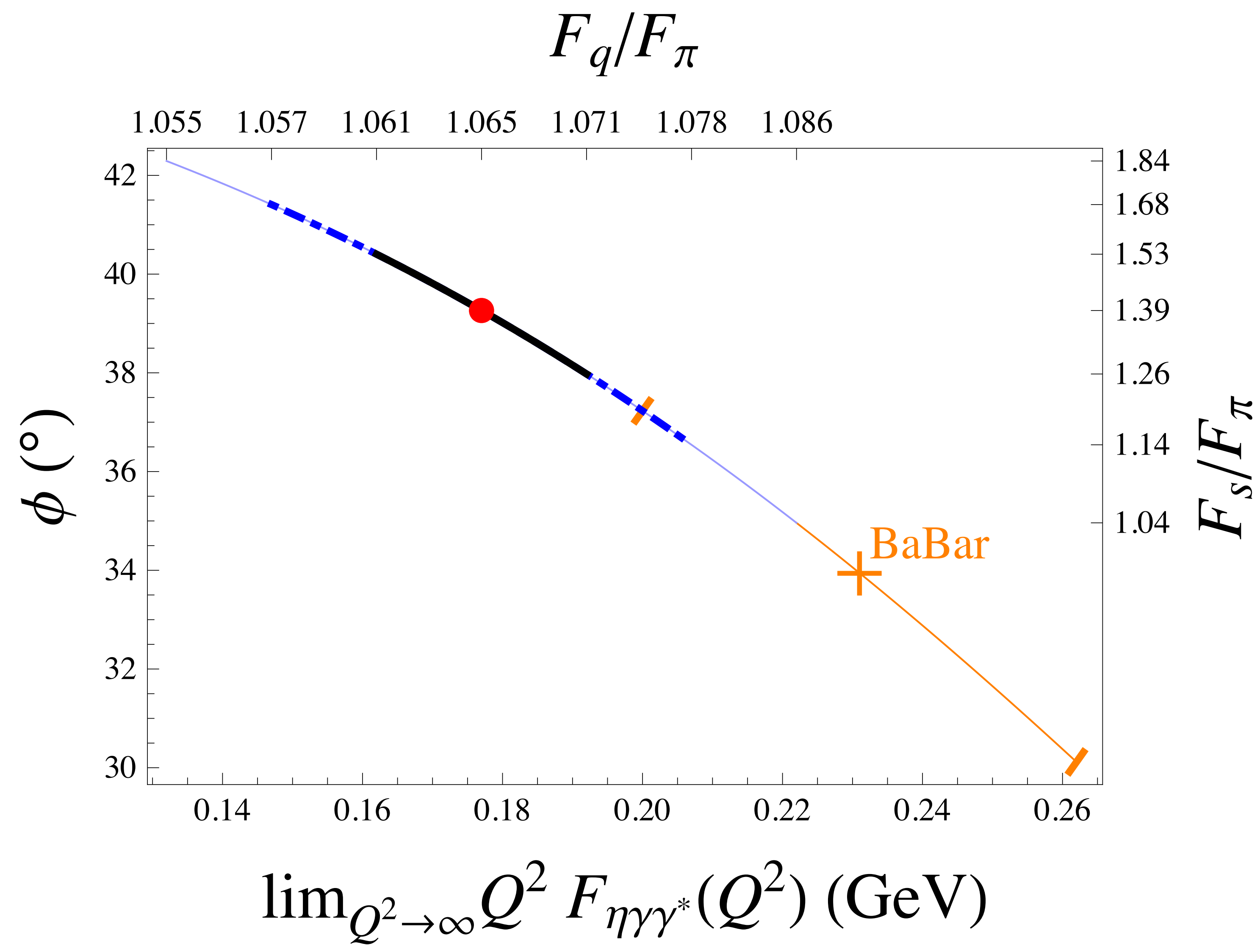
(using $F_{\pi^0} = 131.5 \pm 1.4$ MeV instead of $F_{\pi^-} = 92.21 \pm 0.14$ MeV)

	Our predictions	Experimental determinations
$g_{\rho\eta\gamma}$	1.46(3)	1.58(5)
$g_{\rho\eta'\gamma}$	1.20(4)	1.32(3)
$g_{\omega\eta\gamma}$	0.56(2)	0.45(2)
$g_{\omega\eta'\gamma}$	0.55(2)	0.43(2)
$g_{\phi\eta\gamma}$	–0.78(8)	–0.69(1)
$g_{\phi\eta'\gamma}$	0.88(10)	0.72(1)
$\frac{J/\Psi \to \eta'\gamma}{J/\Psi \to \eta\gamma}$	5.09(47)	4.67(20)

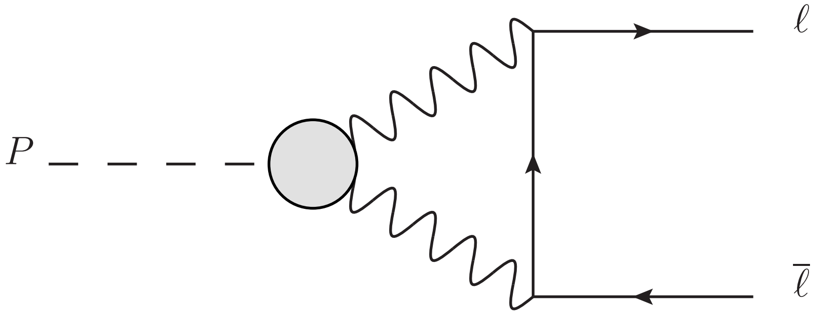
η-η’ mixing

η-η’ mixing in the flavor basis

From the TFFs we can determine F_q, F_s, ϕ



Dissection of η→l⁺l⁻



$$\frac{BR(P \to \bar{\ell}\ell)}{BR(P \to \gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2$$

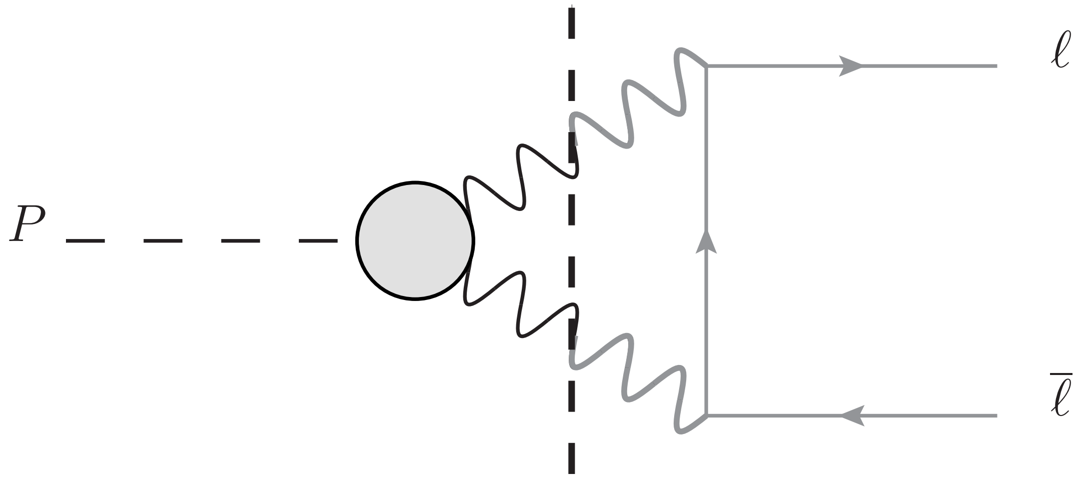
The only unknown $\mathcal{A}(m_P^2)$ from loop calculation where the TFF enters.

$$\mathcal{A}(q^2) = \frac{2i}{\pi^2 q^2}\int d^4k \frac{(q^2k^2 - (q\cdot k)^2)F_P(k^2,(q-k)^2)}{(k^2+i\epsilon)((q-k)^2+i\epsilon)((p-k)^2-m^2+i\epsilon)}$$

Dissection of η→l^+l^-

As model independent as possible:

Cutcosky rules provides the imaginary part



Dissection of η→l⁺l⁻

As model independent as possible:

Cutcosky rules provides the imaginary part

$$Im\mathcal{A}(q^2) = \frac{\pi}{2\beta_l(q^2)} ln\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right); \quad \beta_l(q^2) = \sqrt{1-\frac{4m_l^2}{q^2}}$$

$$q^2 = m_P^2$$

Dissection of η→l⁺l⁻

As model independent as possible:

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$$q^2 = m_P^2$$

Use dispersion relations to get the real part

$$Re(\mathcal{A}(q^2)) = \mathcal{A}(0) + \frac{1}{\beta_l(q^2)}\left(\frac{\pi^2}{12} + \frac{1}{4} ln^2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right) + Li_2\left(\frac{1-\beta_l(q^2)}{1+\beta_l(q^2)}\right)\right)$$

Dissection of η→l$^+$l$^-$

PDG value dominated by the KTeV measurement

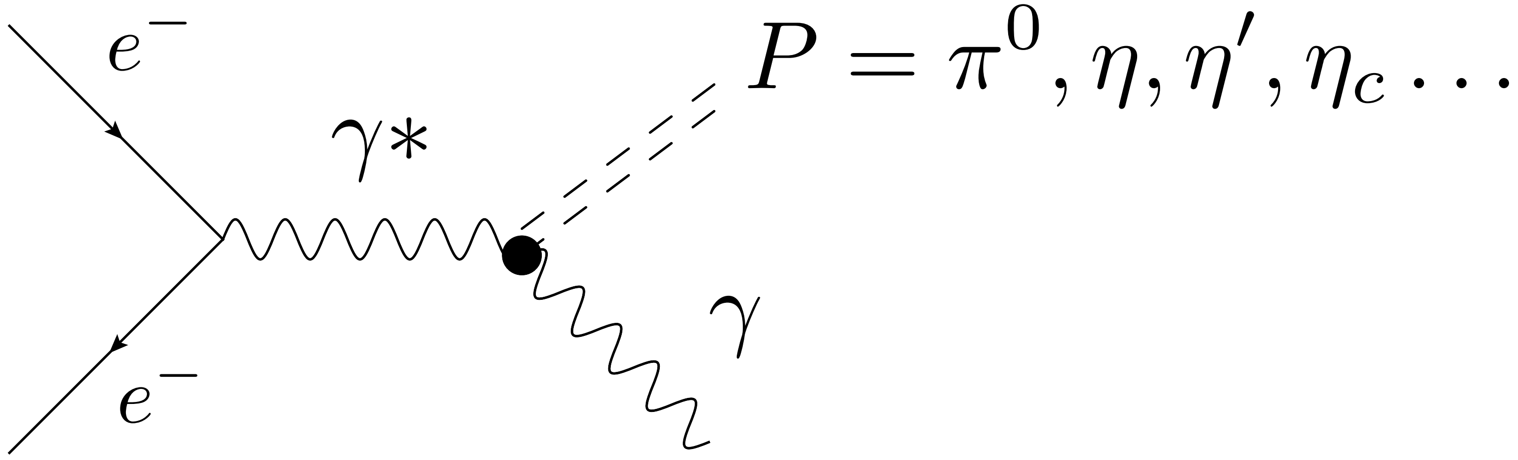
$$\frac{BR(P\to\bar{\ell}\ell)}{BR(P\to\gamma\gamma)} = 2\left(\frac{\alpha m_\ell}{\pi m_P}\right)^2 \beta_\ell(m_P^2)|\mathcal{A}(m_P^2)|^2 \quad \begin{aligned} &= 5.8(8)\cdot 10^{-6} \quad (\mu^+\mu^-) \\ &\leq 5.6\cdot 10^{-6} \quad (e^+e^-) \end{aligned}$$

Unitary Bound for the μμ case $= 4.37\cdot 10^{-6}$

SM calculations with $m_\eta^2/\Lambda^2\sim 0$ $= 4.99\cdot 10^{-6}$

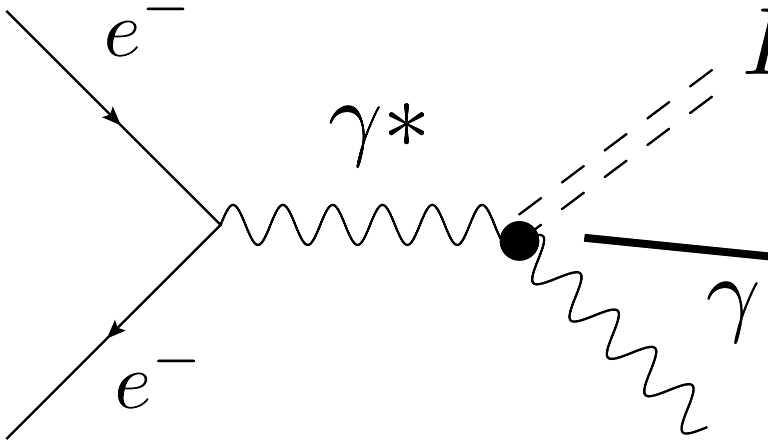
Our result from SL+TL (full result) $= 4.51(2)\cdot 10^{-6}$

Time-like TFF: prediction



- Asymptotic limits in time-like and space-like FFs are expected to be close, is important to measure this time-like FF because:
  - the charmonium region is between the perturbative and non-perturbative regimes of the π-, η-, and η'-TFF
  - background for charmonium decays

Time-like TFF: prediction



$P = \pi^0, \eta, \eta', \eta_c \ldots$

The vertex of interest is:

$$\Gamma_\mu = -ie^2 F_P(Q^2)\epsilon_{\mu\nu\rho\sigma}p^\nu \epsilon^\rho q^\sigma$$

Differential cross section:

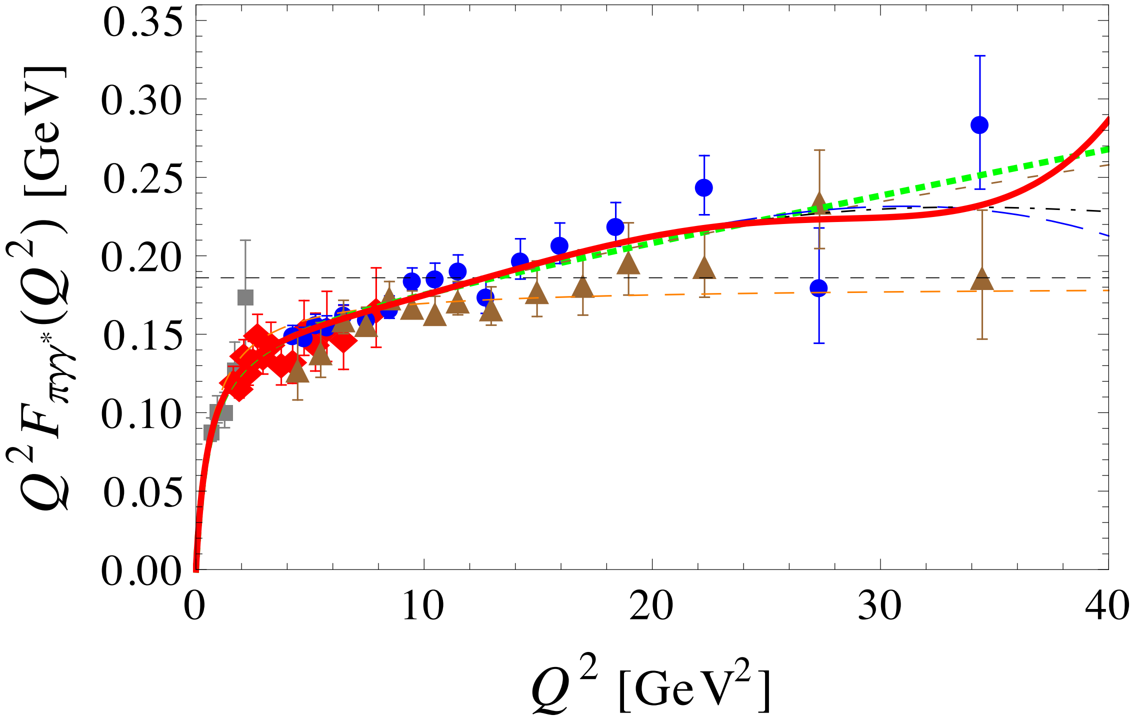
$$\frac{d\sigma(e^+e^- \to \gamma^* \to \gamma P)}{d(\cos\theta)} = \frac{\pi^2\alpha^3}{4}\big(F_{P\gamma^*\gamma}(s,0)\big)^2\Big(1 - \frac{M_P^2}{s}\Big)^3(1+\cos^2\theta)$$

Integrating with respect to cosθ

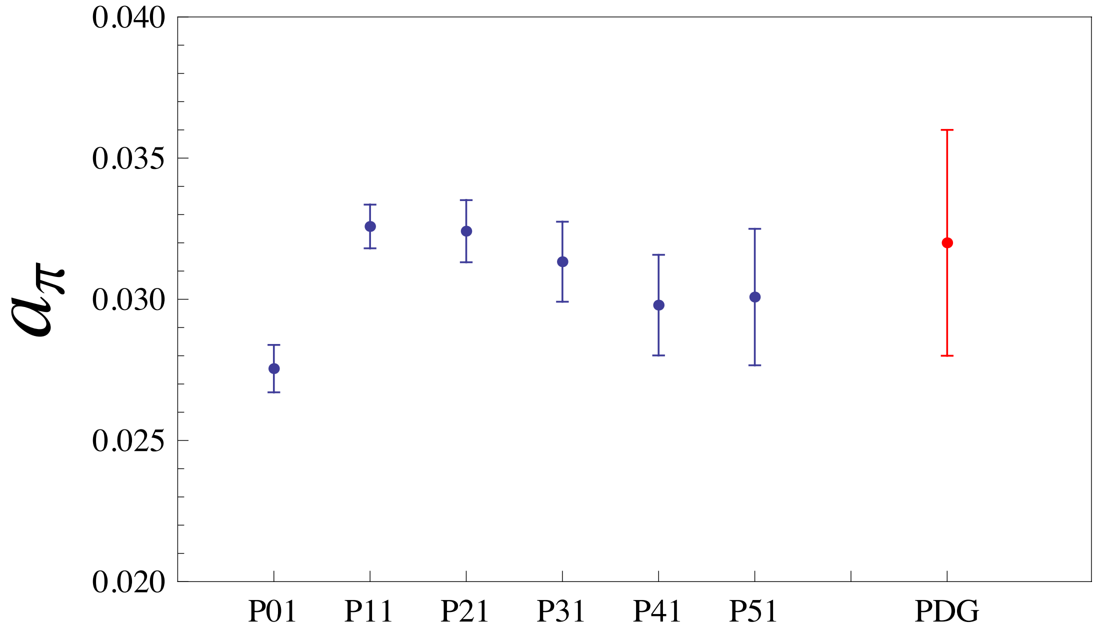
$$\boxed{\sigma(e^+e^- \to \gamma^* \to \gamma P) = \frac{2\pi^2\alpha^3}{3}\big(F_{P\gamma^*\gamma}(s,0)\big)^2\Big(1 - \frac{M_P^2}{s}\Big)^3}$$

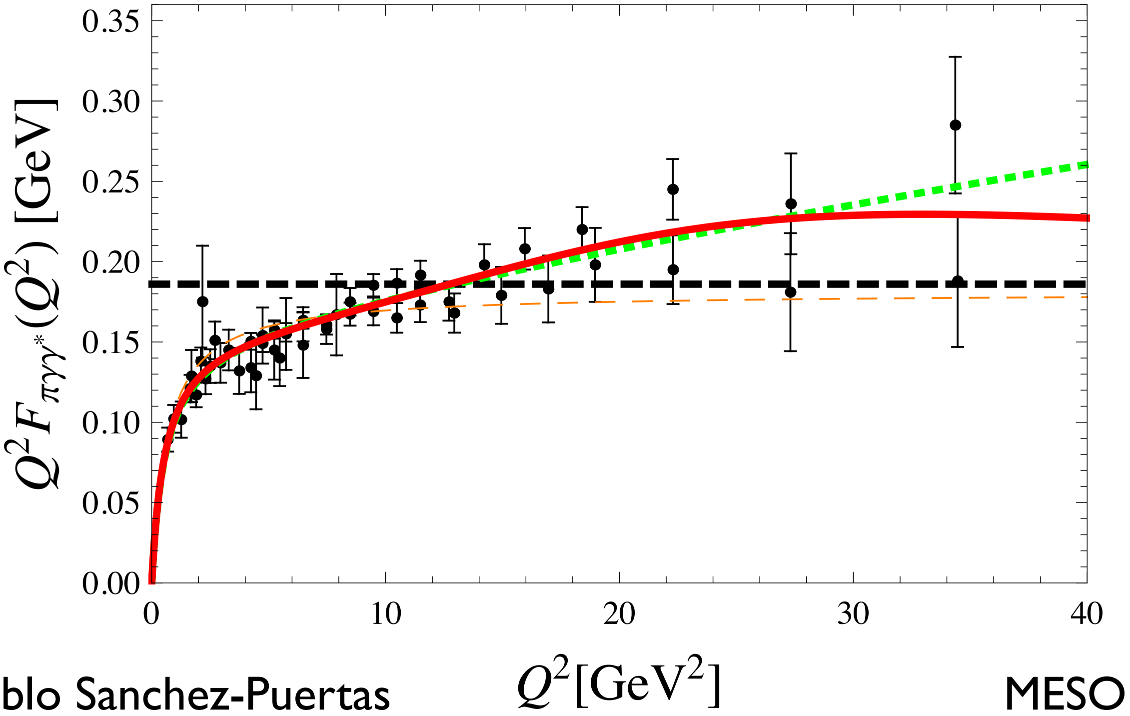
π⁰-TFF

Fit to Space-like data: CELLO'91, CLEO'98, BABAR'09 and Belle'12



$P_1^N(Q^2)$ up to N=5 [P.M, '12]

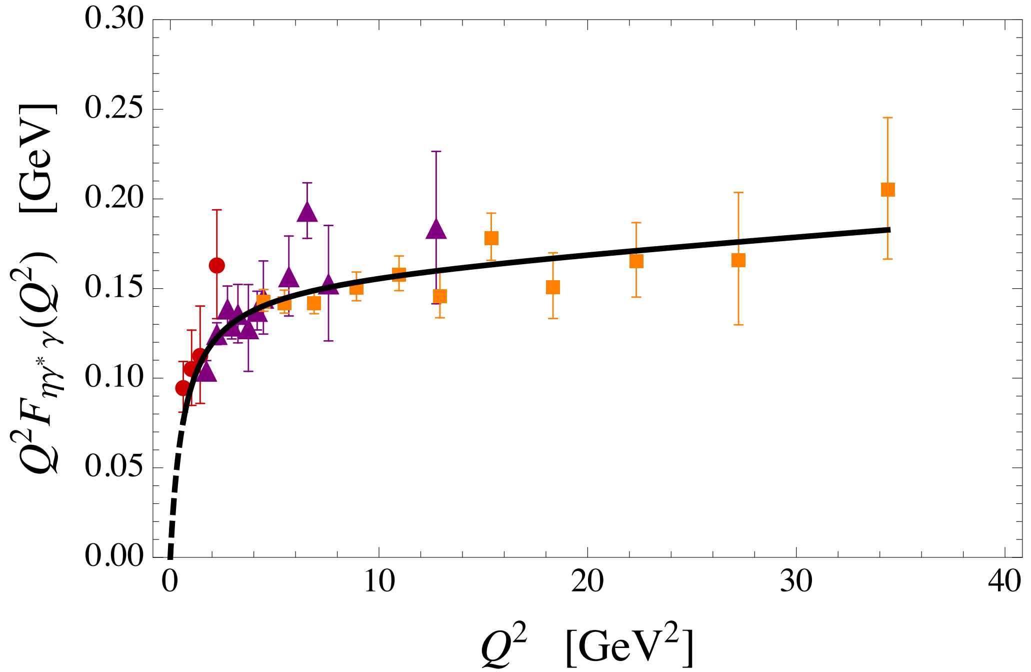


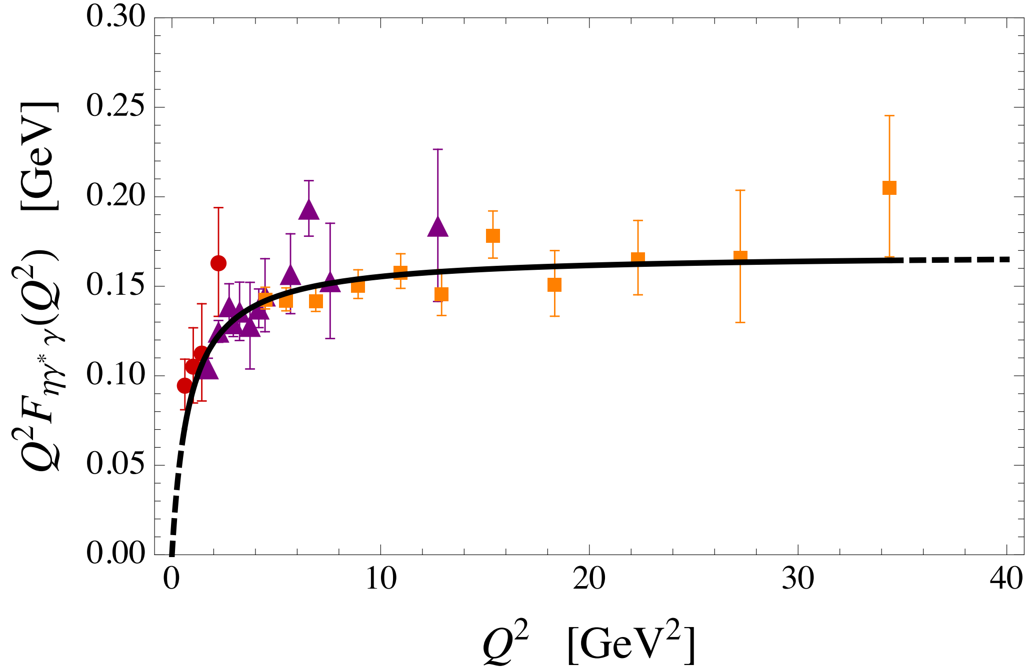


$P_N^N(Q^2)$ up to N=3

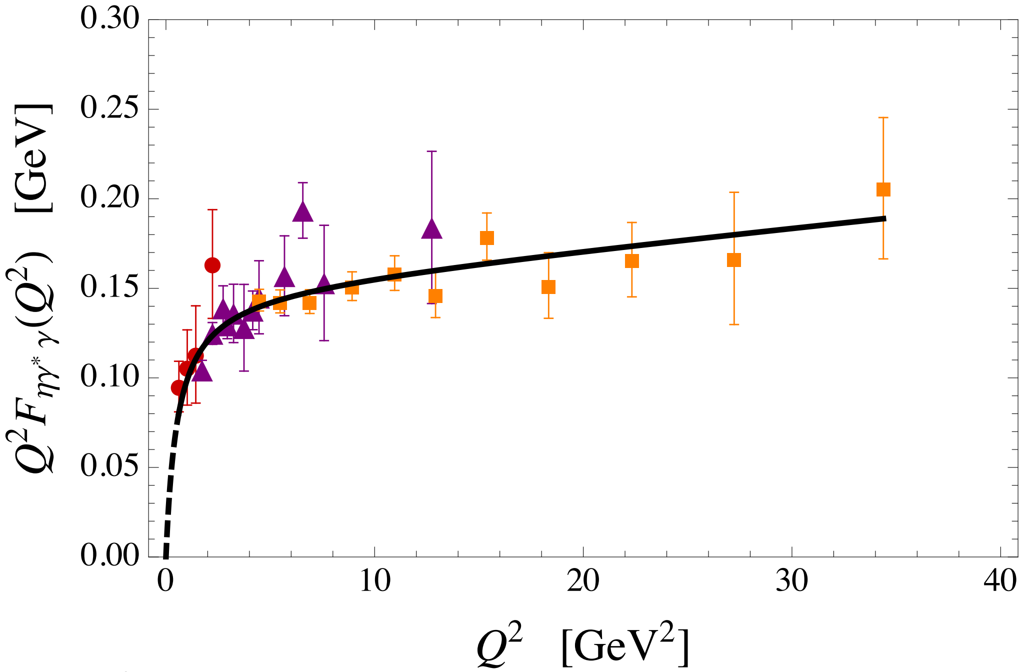
η-TFF

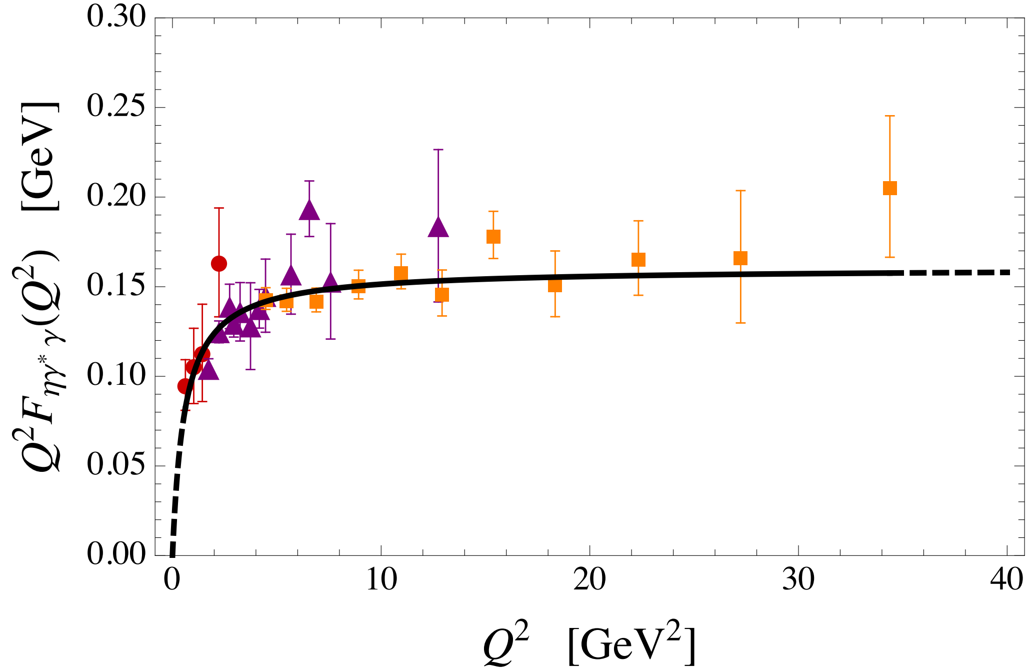
$\Gamma_{\eta\to\gamma\gamma}$ not included





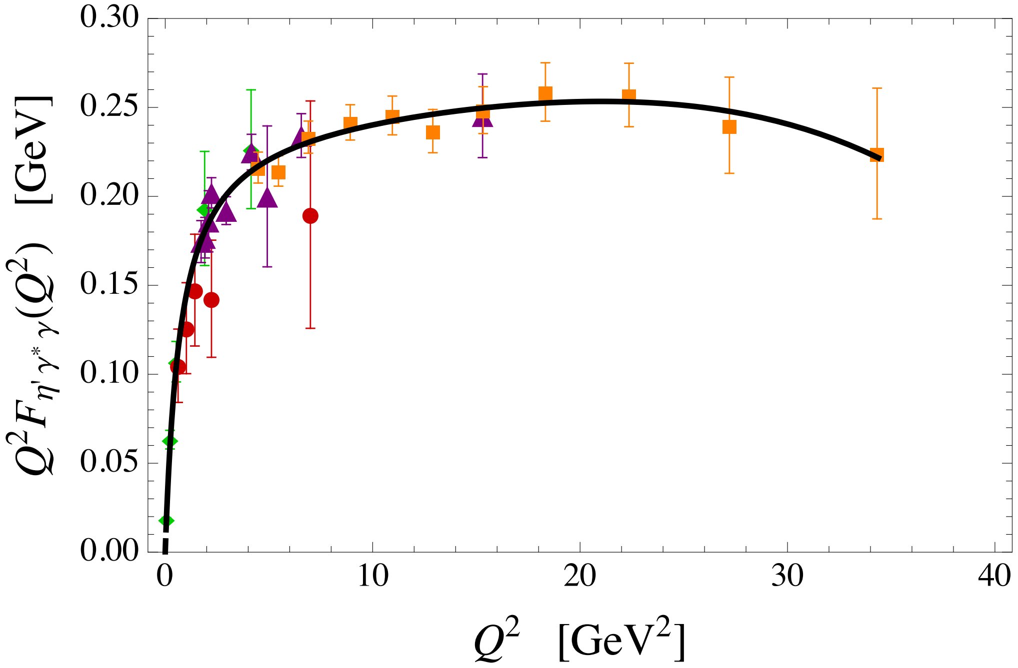
$\Gamma_{\eta\to\gamma\gamma}$ included

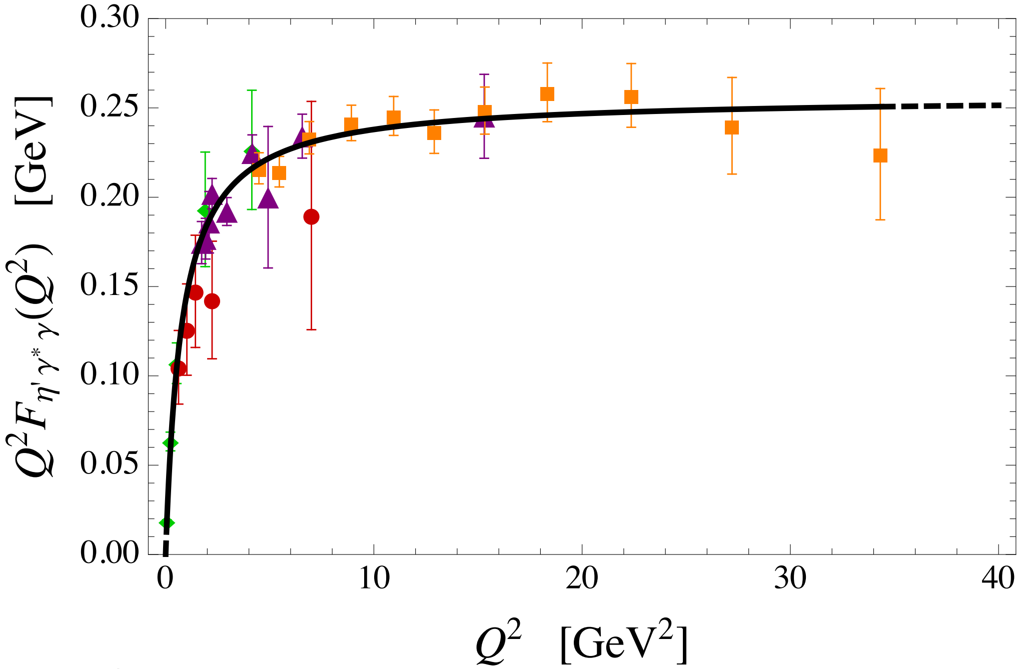




η'-TFF

$\Gamma_{\eta'\to\gamma\gamma}$ not included





$\Gamma_{\eta'\to\gamma\gamma}$ included