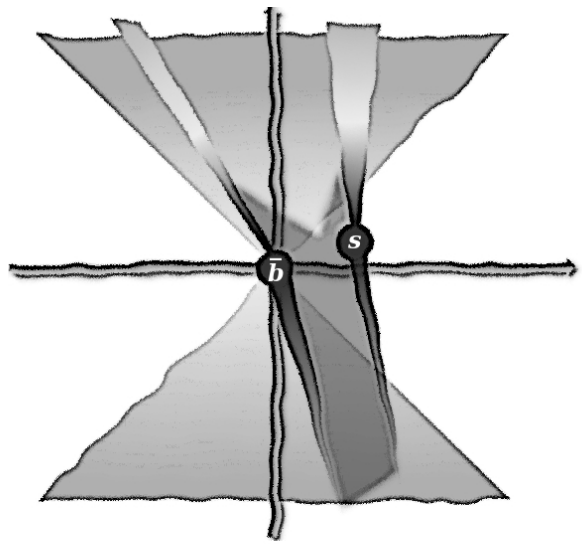
Description of hadrons with Covariant Quark Model



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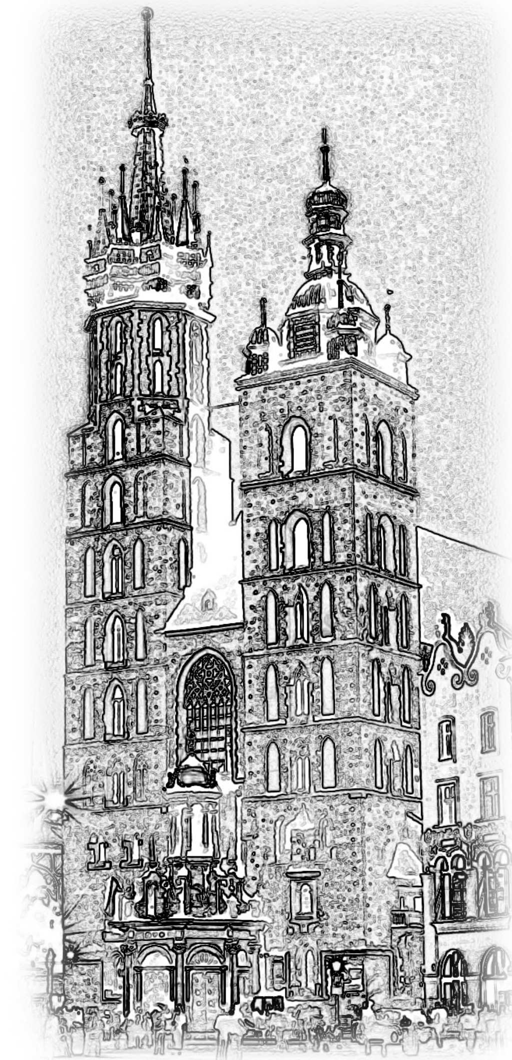

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Overview

- Motivation
- Main characteristics of the model
- Overview of results
- Summary and outlook

Motivation

- **Domain with large experimental activity:**
  - ***Heavy quark factories***
    - Ongoing experiments: LHCb, BaBar, Belle, BESII
    - Near future: Belle II (SuperKEKB)
  - ***Unceasing appearance of new and unique data***
    - Rare B decays
    - Multiquark state claims
    - Precision measurements of decay widths and branching fractions

- **Theory only slowly catching up**
  - ***Low energy domain – a challenge***
    - High-quantity high-precision data vs. theory outcomes
    - Lattice QCD, ChPT: still limited results ⇒ *complementary model-dependent approach needed!*
  - ***Covariant quark model***
    - Successful tool for a unified description of the multiquark states (mesons, baryons, tetraquarks, …)
    - Effective quantum field approach to hadronic interactions (hadrons interacting with constituent quarks, gluons are absent)
    - Few free parameters: constituent quarks masses, infrared cutoff and parameters describing hadron effective size.

CQM Lagrangian

- **Non-local Lagrangian (density):**

$$\mathrm{L}_{\mathrm{int}} = g_H \cdot H(x) \cdot J_H(x)$$

$$\mathrm{J}_M(x) = \int dx_1 \int dx_2\, F_M(x, x_1, x_2) \cdot \bar{q}^a_{f_1}(x_1)\, \Gamma_M\, q^a_{f_2}(x_2)$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3\, F_B(x, x_1, x_2, x_3) \times \Gamma_1\, q^{a_1}_{f_1}(x_1) \left(q^{a_2}_{f_2}(x_2) C\, \Gamma_2\, q^{a_3}_{f_3}(x_3) \right) \cdot \varepsilon^{a_1 a_2 a_3}$$

$$J_T(x) = \int dx_1 \ldots \int dx_4\, F_T(x, x_1, \ldots, x_4)$$
$$\times \left(q^{a_1}_{f_1}(x_1)\, C\Gamma_1\, q^{a_2}_{f_2}(x_2) \right) \cdot \left(\bar{q}^{a_3}_{f_3}(x_3)\, \Gamma_2 C\, \bar{q}^{a_4}_{f_4}(x_4) \right) \cdot \varepsilon^{a_1 a_2 c} \varepsilon^{a_3 a_4 c}$$

- **Translational invariance and calculational convenience:**

$$F_H(x, x_1, \ldots, x_n) = \delta \left(x - \sum_{i=1}^{n} w_i x_i \right) \Phi_H \Big(\sum_{i<j} ((x_i - x_j)^2 \Big)$$

$$w_i = m_i / \sum_{j=1}^{n} m_j$$

$$\bar{\Phi}_H(-\,k^2) = \exp\left(k^2 / \Lambda_H^2\right)$$

- **CQM Lagrangian:**
  - Full Lorentz invariance
  - Standard QFT diagram techniques

Parameters and compositeness condition

- **Free parameters:**
  - Constituent quark masses [4], hadron-size related parameters [N] and universal cut-off [1]: N+5
  - Fitting basic observables (leptonic decay constants, EM decay widths) ⇒num. values: $m_{u,d} = 0.235$, $m_s = 0.424$, $m_c = 2.16$, $m_b = 5.09$, $\lambda_{cut\text{-}off} = 0.181$, $\Lambda_\pi = 0.87$, $\Lambda_K =$... in GeV

- **Compositeness condition:**
  - $\mathscr{L}_{int}$: quarks and hadrons elementary ↔ *nature*: hadrons compound of quarks
  - Compositeness condition: renormalization constant $Z_H^{1/2}$ can be interpreted as the matrix element between a physical state and the corresponding bare state.
  - $Z_H^{1/2} = \langle H_{bare} | H_{dressed} \rangle = 0$ ⇒Physical state does not contain bare state and is properly described as a bound state.

    [A. Salam, Nuovo Cim. 25, 224 (1962), S. Weinberg, Phys. Rev. 130, 776 (1963)]
  - Couplings eliminated as free variables ($\prod_H$ – meson mass operator) :

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H \left(m_H^2\right) = 0$$

Calculation methods

- **Feynman diagram:**
  - General form:
    $$\Pi(p_1,\ldots,p_j) = \int [d^4k]^\ell \prod_{i_1=1}^{m} \Phi_{i_1+n}\left(-K^2_{i_1+n}\right) \prod_{i_3=1}^{n} S_{i_3}(\tilde{k}_{i_3} + \tilde{p}_{i_3})$$
    $$\mathrm{K}^2_{i_1+n} = \sum_{i_2} (\tilde{k}^{(i_2)}_{i_1+n} + \tilde{p}^{(i_2)}_{i_1+n})^2$$
    - j external momenta
    - n quark propagators
    - l loop integrations
    - m vertices
    - $\tilde{k}_i$: linear combination of loop momenta k_i
    - $\tilde{p}_i$: linear combination of external momenta p_i

- **Calculation steps:**
  - Schwinger representation of quark propagators:
    $$\tilde{\mathrm{S}}_\mathrm{q}(\mathrm{k}) = \left(\mathrm{m} + \hat{\mathrm{k}}\right) \int_0^\infty \mathrm{d}\alpha\ \mathrm{e}^{[-\alpha(\mathrm{m}^2 - \mathrm{k}^2)]}$$
  - Loop momenta integration:
    $$\int \mathrm{d}^4\mathrm{k}\ \mathrm{P}(\mathrm{k})\mathrm{e}^{2\mathrm{kr}} = \int \mathrm{d}^4\mathrm{k}\ \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\mathrm{e}^{2\mathrm{kr}} = \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\int \mathrm{d}^4\mathrm{k}\ \mathrm{e}^{2\mathrm{kr}}$$
  - Simplification in trace evaluation:
    $$\int_0^\infty \mathrm{d}^\mathrm{n}\alpha\ \mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}}\right)\mathrm{e}^{-\frac{\mathrm{r}^2}{\mathrm{a}}} = \int_0^\infty \mathrm{d}^\mathrm{n}\alpha\ \mathrm{e}^{-\frac{\mathrm{r}^2}{\mathrm{a}}}\mathrm{P}\left(\frac{1}{2}\frac{\partial}{\partial \mathrm{r}} - \frac{\mathrm{r}}{\mathrm{a}}\right),\ \ \mathrm{r} = \mathrm{r}(\alpha_\mathrm{i}),\ \ \mathrm{a} = \mathrm{a}(\Lambda_\mathrm{H}, \alpha_\mathrm{i})$$

Infrared confinement

- **Introduction of infrared cut-off**
  - Unity in form of δ-function introduced ⇒ single cut-off parameter

$$1 = \int_0^\infty \mathrm{dt}\ \delta(\mathrm{t} - \sum_{\mathrm{i}=1}^{\mathrm{n}} \alpha_{\mathrm{i}})$$

$$\Pi = \int_0^\infty \mathrm{d^n}\alpha\ \mathrm{F}(\alpha_1, \cdots, \alpha_{\mathrm{n}}) = \int_0^{\infty \to \frac{1}{\lambda^2}} \mathrm{dt}\ \mathrm{t}^{\mathrm{n}-1} \int_0^1 \mathrm{d^n}\alpha\ \delta\Big(1 - \sum_{\mathrm{i}=1}^{\mathrm{n}} \alpha_{\mathrm{i}}\Big) \mathrm{F}(\mathrm{t}\alpha_1, \ldots, \mathrm{t}\alpha_{\mathrm{n}})$$

  - Universal value $\lambda_{\text{cut-off}} = 0.181$ established for all processes.
  - Π becomes a smooth function, thresholds in the quark loop diagrams and corresponding branch points are removed
  - Numerical integration

- **Example:** scalar one-loop two-point function, case $p^2 = 4m^2$

$$\Pi_2(p^2) = \int \frac{d^4 k_E}{\pi^2} \frac{e^{-s\,k_E^2}}{[m^2 + (k_E + \frac{1}{2}p_E)^2][m^2 + (k_E - \frac{1}{2}p_E)^2]}$$

$e^{-s\,k_E^2} \longrightarrow$ product of non-local Gaussian vertex form factors

$p_E, k_E \longrightarrow$ Euclidean momenta

$$\Pi_2(p^2) = \int_0^{\infty \leftrightarrow \frac{1}{\lambda^2}} dt \frac{t}{(s+t)^2} \int_0^1 d\alpha\ \exp\Big\{ -\ t\,[m^2 - \alpha(1-\alpha)p^2] + \frac{st}{s+t}\Big(\alpha - \frac{1}{2}\Big)^2 p^2 \Big\}$$

Form factors and weak decays

- **Observables (usually) expressed via form factors**

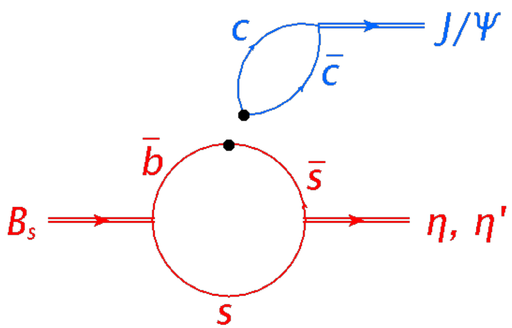
$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\,|\bar{q}_2 O^\mu q_1| P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = F_+(q^2)\,P^\mu + F_-(q^2)\,q^\mu$$

$$\left\langle P'_{[\bar{q}_3,q_2]}(p_2)\,|\bar{q}_2(\sigma^{\mu\nu}q_\nu)\,q_1| P'_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \frac{i}{m_1+m_2}\left(q^2P^\mu - q\cdot Pq^\mu\right)F_T(q^2)$$

$$\left\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\,|\bar{q}_2 O^\mu q_1| P_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \frac{\epsilon^\dagger_\nu}{m_1+m_2}\Big[-g^{\mu\nu}P\cdot q A_0(q^2) + P^\mu P^\nu A_+(q^2) + q^\mu P^\nu A_-(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta V(q^2)\Big]$$

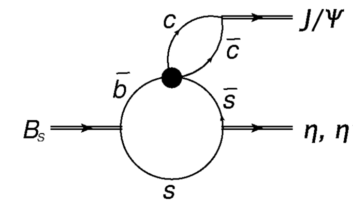
$$\left\langle V_{[\bar{q}_3,q_2]}(p_2,\epsilon_2)\,|\bar{q}_2\left[\sigma^{\mu\nu}q_\nu\left(1+\gamma^5\right)\right]q_1| P_{[\bar{q}_3,q_1]}(p_1)\right\rangle = \epsilon^\dagger_\nu\Big[-\left(g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)P\cdot q\; a_0(q^2) + \left(P^\mu P^\nu - q^\mu P^\nu\frac{p\cdot q}{q^2}\right)a_+(q^2) + i\varepsilon^{\mu\nu\alpha\beta}P_\alpha q_\beta\; g(q^2)\Big]$$

- **Flavor transitions**
  - Effective theory (Wilson coefficients) used to describe quark flavor transition
  - Factorization: convolution of form factor and expression proportional to decay constant



Mesons: $B_s \to J/\psi + \eta^{(')}$ decay

- **$B_s \to J/\psi + \eta$ and $B_s \to J/\psi + \eta'$:**
  - Measured by Belle in 2012 [PRL 108, 181808 (2012)]
  - Light-strange quark mixing



$$\mathrm{B_S^0}:\ \mathrm{s\bar{b}} \qquad \eta:\ \frac{1}{\sqrt{2}}\sin\delta(\mathrm{u\bar{u}}+\mathrm{d\bar{d}})-\cos\delta(\mathrm{s\bar{s}}) \qquad \eta':\ \frac{1}{\sqrt{2}}\cos\delta(\mathrm{u\bar{u}}+\mathrm{d\bar{d}})+\sin\delta(\mathrm{s\bar{s}})$$

$$\mathcal{L}_\eta(\mathrm{x}) = \mathrm{g}_\eta \eta(\mathrm{x}) \iint \mathrm{dx_1 dx_2}\,\delta\left(\mathrm{x}-\frac{1}{2}\mathrm{x_1}-\frac{1}{2}\mathrm{x_2}\right)\phi_\eta\left[(\mathrm{x_1}-\mathrm{x_2})^2\right]$$
$$\times\left\{\frac{1}{\sqrt{2}}\cos(\delta)\left[\bar{\mathrm{u}}(\mathrm{x_1})\,\mathrm{i}\gamma^5\mathrm{u}(\mathrm{x_2})+\bar{\mathrm{d}}(\mathrm{x_1})\,\mathrm{i}\gamma^5\mathrm{d}(\mathrm{x_2})\right]-\sin(\delta)\left[\bar{\mathrm{s}}(\mathrm{x_1})\,\mathrm{i}\gamma^5\mathrm{s}(\mathrm{x_2})\right]\right\}$$

$$\mathcal{L}_{\mathrm{eff}} = \frac{\mathrm{G_F}}{\sqrt{2}}\mathrm{V_{cb}V_{cs}^*}\sum_{\mathrm{i}}\mathrm{C_iQ_i} \qquad \mathrm{Q_1} = (\bar{\mathrm{c}}_{\mathrm{a_1}}\mathrm{b}_{\mathrm{a_2}})_{\mathrm{V-A}}(\bar{\mathrm{s}}_{\mathrm{a_2}}\mathrm{c}_{\mathrm{a_1}})_{\mathrm{V-A}} \qquad \mathrm{Q_2} = \ldots$$

$$(\bar{\psi}\psi)_{\mathrm{V-A}} = \bar{\psi}\mathrm{O}^\mu\psi,\ \mathrm{O}^\mu = \gamma^\mu(1-\gamma^5) \qquad (\bar{\psi}\psi)_{\mathrm{V+A}} = \bar{\psi}\mathrm{O}_+^\mu\psi,\ \mathrm{O}_+^\mu = \gamma^\mu(1+\gamma^5)$$

  - Model over-constrained:

$$\eta\to\gamma\gamma \quad \eta'\to\gamma\gamma \quad \rho\to\eta\gamma \quad \varphi\to\eta\gamma \quad \varphi\to\eta'\gamma$$
$$\mathrm{B_d}\to \mathrm{J}/\psi\ \eta \quad \mathrm{B_d}\to \mathrm{J}/\psi\ \eta' \quad \omega\to\eta\gamma \quad \eta'\to\omega\gamma$$

Mesons – overview of results

- $B_s \to J/\psi + \eta^{(\prime)}$ results:

$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta) = 4.67$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta) = 5.10 \pm 1.12$$
$$\mathcal{B}_{\mathrm{CQM}}\,(\mathrm{J}/\psi\ \eta') = 4.04$$
$$\mathcal{B}_{\mathrm{Belle}}\,(\mathrm{J}/\psi\ \eta') = 3.71 \pm 0.95$$

$$R = \frac{\Gamma(J/\psi + \eta')}{\Gamma(J/\psi + \eta)} = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & \text{Belle} \\ 0.90 \pm 0.09^{+0.06}_{-0.02} & \text{LHCb} \end{cases}$$

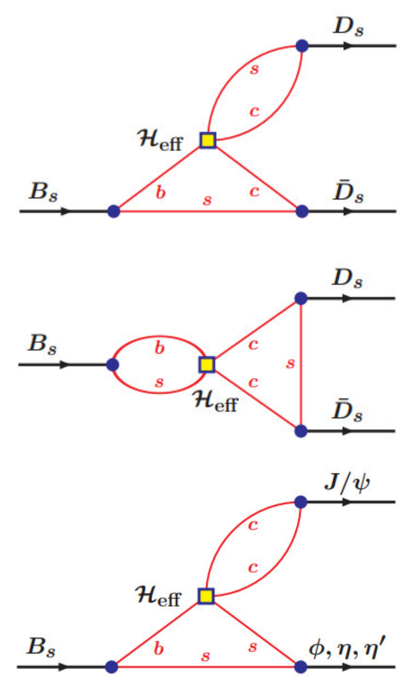
$$R^{\text{theor}} = \underbrace{\frac{|\mathbf{q}_{\eta'}|^3}{|\mathbf{q}_{\eta}|^3} \tan^2\delta}_{\approx 1.04} \times \underbrace{\left(\frac{F_+^{B_s\eta'}}{F_+^{B_s\eta}}\right)^2}_{\approx 0.83} \approx 0.86.$$

[S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and A. Liptaj Phys. Rev. D 87, 074201 (2013)]

- Other meson related results
  - Branching ratios (%) of the B_s nonleptonic decays

Process	CQM	PDG
$B_s \to D_s^- D_s^+$	1.65	$1.04^{+0.29}_{-0.26}$
$B_s \to D_s^- D_s^{*\,+} + D_s^{*\,-} D_s^+$	2.40	2.8 ± 1.0
$B_s \to D_s^{*\,-} D_s^{*\,+}$	3.18	3.1 ± 1.4
$B_s \to J/\psi\phi$	0.16	0.14 ± 0.05

[M. A. Ivanov, et. al., Phys. Rev., D85:034004, 2012.]



Baryons – overview of results

- **Nucleon as three quark state**
  - Three-quark current

$$J_{3q}^{(p)}(x_1,x_2,x_3) = \Gamma^A \gamma^5\, d^{a_1}(x_1) \times$$
$$\times [\epsilon^{a_1 a_2 a_3}\, u^{a_2}(x_2)\, C\, \Gamma_A\, u^{a_3}(x_3)]$$
$$\Gamma^A \otimes \Gamma_A = \gamma^\alpha \otimes \gamma_\alpha \quad \text{(vector)}$$
$$\Gamma^A \otimes \Gamma_A = \frac{1}{2}\sigma^{\alpha\beta} \otimes \sigma_{\alpha\beta} \quad \text{(tensor)}$$
$$J_N = x J_N^T + (1-x) J_N^V\,, \quad N = p,$$

  - Parameters: x = 0.8, Λ_N = 0.5 GeV

Quantity	CQM	PDG
μ_p (in n.m.)	2.96	2.793
μ_n (in n.m.)	-1.83	-1.913
r_E^p (fm)	0.805	0.8768 ± 0.0069
$< r_E^2 >^n$ (fm^2)	-0.121	-0.1161 ± 0.0022
r_M^p (fm)	0.688	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.685	$0.862^{+0.009}_{-0.008}$

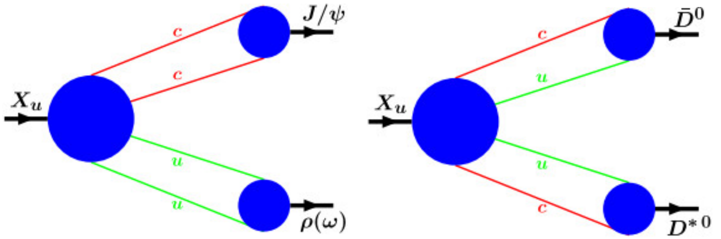
[T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij and P. Santorelli, Phys. Rev. D 87, 074031 (2013)]

- **Rare baryon decays $\Lambda_b \to \Lambda \ell^+ \ell^-$**
  - Existing ***CDF (1.73±0.69)×10⁻⁶*** and ***LHCb (0.96±0.25)×10⁻⁶*** measurements of branching fractions.
  - CQM: $B(\Lambda_b \to \Lambda \mu^+ \mu^-) = 1.0\times 10^{-6}$
  - Further results (Λ_s and Λ_c decays, Λ_b decay angular distributions and asymmetries) in the published paper:

[T. Gutsche, M. A. Ivanov, J. G. Körner, V. E. Lyubovitskij, P. Santorelli, Phys. Rev. D 87 074031 (2013)]

X(3872) tetraquark candidate

- **Experiment**
  - First observation by Belle (2003) in the decay $B^+ \to K^+(\pi^+\pi^- J/\psi)$,
    Resonance near $D^0 D^{*0}$ mass threshold: $M = 3871.68 \pm 0.17$ MeV, $\Gamma < 1.2$ MeV, $J^{PC} = 1^{++}$
  - Confirmed by BaBar, Belle [$X^{3872} \to J/\psi\gamma$] and CDF
- **Nature of the particle**
  - Pure charmonium state disfavored [LHCb: $B^+ \to X(3872)K^+ \to J/\psi\pi^+\pi^- K^+$; $J/\psi \to \mu^+\mu^-$]
  - Tetraquak hypothesis: $D^0 + D^{*0}$ molecule, bound state with small binding energy [diquark and antidiquark]
- **Description within CQM** (strong and radiative decays)
  - Coupling constant: compositeness condition
  - Off-mass-shell decay: narrow-width approximation
  - Results:



$$\frac{\Gamma(X_l \to \gamma J/\psi)}{\Gamma(X_l \to J/\psi + \pi\pi)}\bigg|_{\text{CQM}} = 0.15 \pm 0.03$$

$$\frac{\Gamma(X \to D^0\bar{D}^0\pi^0)}{\Gamma(X \to J/\psi\pi^+\pi^-)} = \begin{cases} 4.5 \pm 0.2 & \text{CQM} \\ 10.5 \pm 4.7 & \text{exp.} \end{cases}$$

$$\frac{\Gamma(X \to \gamma J/\psi)}{\Gamma(X \to 2\pi)} = \begin{cases} 0.14 \pm 0.05 & \text{Belle} \\ 0.22 \pm 0.06 & \text{BaBar} \end{cases}$$

[S. Dubnicka, A. Z. Dubnickova, M. A. Ivanov and J. G. Körner, Phys. Rev. D 81, 114007 (2010)]

[M. A. Ivanov *et. al.*, Phys. Rev. D 84, 014006 (2011)]

Summary and outlook

- **Summary**
  - ***Predictions of the CQ model for various processes***
    - Light and heavy hadrons
    - Mesons, baryons, tetraquarks
    - Strong and weak decays
  - ***Validity and suitability of the CQM consists in***
    - Nice agreement with experimental data
    - Wide application spectra
    - High activity in this physics area
- **Outlook**
  - Hadron physics at heavy quark factories - rich source for applications of the CQ model:

$$B \to K l^+ l^- \qquad \mathrm{B}^0_{(\mathrm{s})} \to \mu^+ \mu^-$$

$$\mathrm{B_s} \to \mathrm{J}/\Psi\, \mathrm{f_0}(980) \qquad \mathrm{B}^0_\mathrm{s} \to \mathrm{J}/\Psi \mathrm{K}^0_\mathrm{S}$$

$$\mathrm{B}^0_\mathrm{s} \to \pi^+ \pi^- \qquad \mathrm{B}^0 \to \mathrm{K}^+ \mathrm{K}^-$$