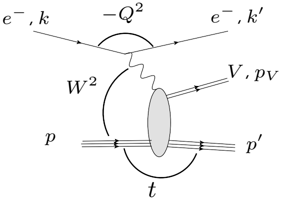
Constraints from leptoproduction of vector meson within different frameworks

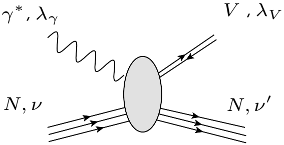
Adrien Besse

Irfu - SPhN

MESON 2014, Cracow



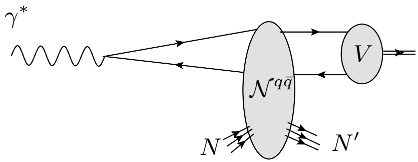
$$\to \mathcal{M}_{V,\{\lambda_V \nu';\lambda_\gamma,\nu\}} =$$



- $W^2 \gg |t|, Q^2 \gg \Lambda^2_{QCD}$, the Bjorken $x \sim \frac{Q^2}{W^2+Q^2}$
- Spin density matrix elements (SDME) linked to the helicity amplitudes : (Schilling Wolf, '73) & (Dielh, '07)
  - small x HERA (H1 and ZEUS)
  - mid-x region: COMPASS, HERMES, E665, NMC
  - valence region: CLAS

Theoretical descriptions

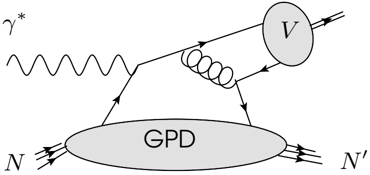
Color dipole picture (small$-x$)



Interaction via gluons exchange

Convenient to introduce saturation effects at small$-x$

Collinear fact. picture

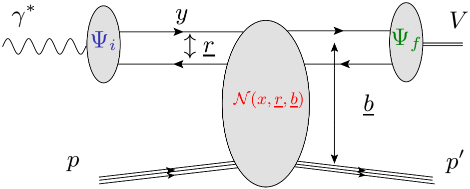


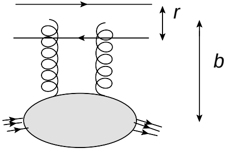
Interaction via gluon and quark exchange

Valid from small$-x$ to the valence region

Color dipole factorization scheme

- Impact parameter space representation of the amplitudes in the infinite momentum frame
  Nikolaev, Zakharov, '91, Mueller, '90



$\mathcal{N}(x, r, b) =$ 

- Initial Ψ_i and final Ψ_f states wave functions.
- Universal dipole/target scattering amplitude $\mathcal{N}(x, \underline{r}, \underline{b})$:
  (DIS structure functions, diffractive DIS, exclusive processes ...)

Dipole model picture

- Skewness effects can be taken into account in dipole cross-section model (Shuvaev, Golec-Biernat, Martin, Ryskin, '99)

$$\mathcal{N}(x,\underline{r}) \equiv \mathcal{N}(x,\xi,\underline{r}) \quad \text{such that} \quad \mathcal{N}(x,\xi=0,\underline{r}) = s\,\hat{\sigma}(x,\underline{r})$$

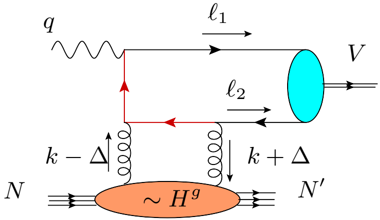
- Dipole models :
  - access to $\mathcal{I}m\,\mathcal{M}_V^g$
  - $\mathcal{R}e\,\mathcal{M}_V^g$ can be deduced from $\mathcal{I}m\,\mathcal{M}_V^g$ using dispersion relations

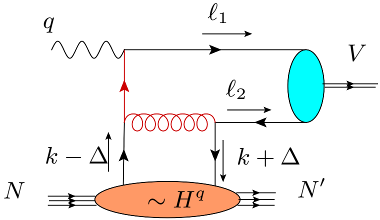
- In the limit $\underline{\Delta} = 0$, i.e. $|t| = |t|_{min}$,

$$\mathcal{I}m\,\mathcal{M}_{\lambda_V\lambda_\gamma}(Q^2,x) \propto i\int dy \int d\underline{r}\,\hat{\Psi}^*_{\lambda_V}(y,\underline{r})\,\hat{\Psi}_{\lambda_\gamma}(y,\underline{r})\,\mathcal{N}(x,\xi,\underline{r})$$

Description of exclusive processes within
Collinear factorization approach

- Description of DVMP, DVCS, TCS, ... in the Bjorken limit
- Collinear factorization proven for LT amplitude $\mathcal{M}_{V,\{0+;0+\}}$
  (Collins, Frankfurt, Strikman, '97, Radyushkin, '97)
- Set of GPDs, $H(x,\xi,t)$, $E(x,\xi,t)$, $\tilde{H}(x,\xi,t)$, $\tilde{E}(x,\xi,t)$
- Quark and Gluon contributions:

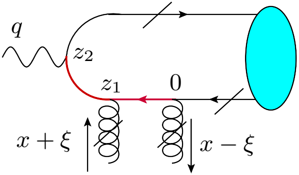




Gluon contribution in MPA

$$\ell_q = y p_1 + \ell_\perp + \frac{\ell^2}{2sy} p_2$$

$$\mathcal{M}_V^g = \int dx \int dy \int \frac{d^2 \underline{\ell}}{(2\pi)^2}$$



$$\langle V(p_1)|\, \bar{\psi}_i(z_2)\psi_j(0)\,|0\rangle \rightarrow \not{p}_{1,ij}\Psi_V(y, \ell_\perp)$$

$$\langle N(p')|\, \not{A}(z_1)\not{A}^*(0)\,|N(p)\rangle \rightarrow \sum_\lambda \not{\epsilon}_\perp^{(\lambda)} \not{\epsilon}_\perp^{(\lambda)*} \frac{H^g(x,\xi,t)}{x^2-(\xi-i\epsilon)^2}$$

- Neglect then $\frac{\ell_\perp^2}{y\bar{y}Q^2}$ terms in numerator in the MPA spirit
- Fourier transform in transverse space $\rightarrow$ impact parameter space
- Sudakov form factor (Sterman, Li ,'92)
  (Resums soft gluon emmisions from the quark-antiquark dipole)

Model dependences

- Models from (Kroll, Goloskokov, '08) :
  - GPDs with evolution approximated by the DGLAP evolution
  - Wavefunction models (Gaussian ansatz)

$$\hat{\Psi}_V(y,\underline{r}) \propto \text{ Leading twist DA } \times \exp\left(-\frac{\underline{r}^2}{4a_V^2}y\bar{y}\right)$$

  - Sudakov form factor (Dahm, Jakob, Kroll, '95)

- Kroll&Goloskokov GPD model based on double distribution ansatz (Musatov, Radyushkin, '00)

$$H(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta+\xi\alpha-x)\, f(\beta,\alpha,t')$$

- MPA result for the helicity amplitude $\gamma_L^* \, N(p) \to V_L \, N(p')$ (A.B. in preparation):

$$\mathcal{M}_{V,\{0+,0+\}} = \frac{s}{2\sqrt{2\pi}} \int_0^1 dy \int d^2\underline{r} \sum_f C_V^f \left(\hat{\Psi}_V(y,-\underline{r}) \hat{\Psi}^f_{\gamma_L^*}(y,\underline{r}) \right)$$

$$\times \left(\frac{\pi \, \alpha_s(\mu^2)}{N_c} \left(\frac{4}{y\bar{y}(2\xi s)} \right) \left\{ \int_0^1 dx \frac{2\xi H^g(x,\xi,t) + 2\,x\,C_F\,\xi H^f_{\text{singlet}}(x,\xi,t)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)} \right\} \right)$$

Results in the two different approaches in the limit $t \sim 0$

- MPA result for the helicity amplitude $\gamma_L^* \, N(p) \to V_L \, N(p')$ (A.B. in preparation):

$$\mathcal{I}m \, \mathcal{M}^g_{V,\{0+,0+\}} = \frac{s}{2\sqrt{2\pi}} \int_0^1 dy \int d^2\underline{r} \sum_f C_V^f \left(\hat{\Psi}_V(y,-\underline{r}) \hat{\Psi}^f_{\gamma_L^*}(y,\underline{r}) \right) \times \left(- \frac{\pi^2}{N_c} \frac{4}{y\bar{y}Q^2} \alpha_s H^g(\xi,\xi,0) \right)$$

- Dipole model result for the helicity amplitude $\gamma_L^* \, N(p) \to V_L \, N(p')$:

$$\mathcal{I}m \, \mathcal{M}^g_{V,\{0+,0+\}} = \frac{s}{2\sqrt{2\pi}} \int dy \int d^2\underline{r} \sum_f C_V^f \left(\hat{\Psi}_V(y,-\underline{r}) \hat{\Psi}^f_{\gamma_L^*}(y,\underline{r}) \right) \times \left(- \frac{\mathcal{N}(x,\xi,\underline{r})}{s} \right)$$

- At small$-x$: $2\xi s \approx Q^2$

Interpretation

- Forward dipole cross-section (Frankfurt, Radyushkin, Strikman, '97) :

$$\frac{\mathcal{N}(x,0,\underline{r})}{s} = \hat{\sigma}(x,\underline{r}) = \frac{\pi^2\,\alpha_s}{N_c}\,\underline{r}^2\,xg(x) \qquad \text{(color transparency)}$$

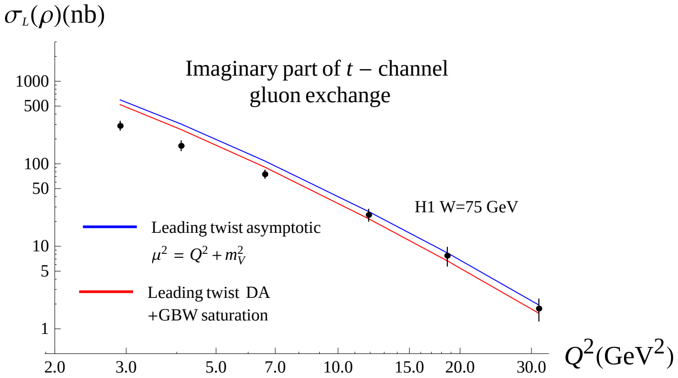
- Comparing the results for DVMP, (Forward limit of gluon GPD : $H^g(x,\xi \to 0,0) = x\,g(x)$):

$$\frac{\mathcal{N}(x,\xi,\underline{r})}{s} \leftrightarrow \frac{\pi^2\,\alpha_s}{N_c}\left(\frac{4}{y\bar{y}Q^2}\right) H^g(\xi,\xi,0) = \frac{\pi^2\,\alpha_S}{N_c}\,(\underline{r}_0^2)\,H^g(\xi,\xi,0)$$

$$\text{with } \underline{r}_0^2 = \frac{4}{y\bar{y}Q^2}$$

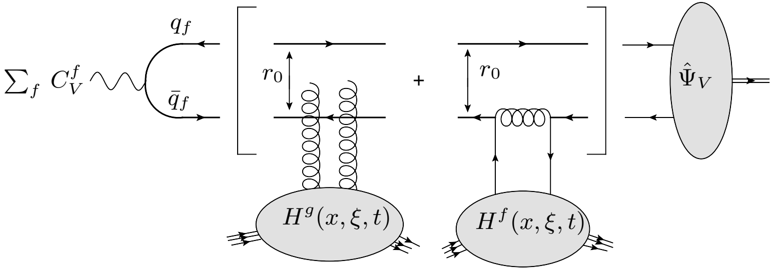
- $\sqrt{\langle r_0^2\rangle} \geq 2\,R_0(x)$ (Saturation radius) for $Q^2 \sim 5$ GeV^2 for $W = 75$ GeV

Comparison of predictions



- Leading twist result from collinear factorization (Blue)
- Leading twist DA + dipole cross-section with saturation (Red)

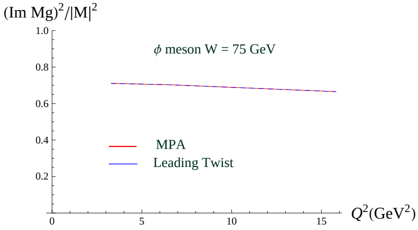
Contribution from gluons and quarks



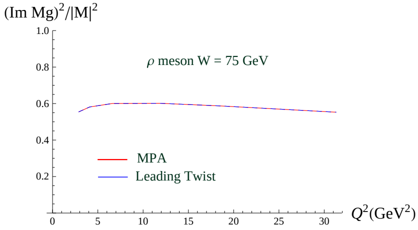
$$
-\sum_f e_f C_V^f \, \mathcal{N}(x,\underline{r}) \longleftrightarrow \sum_f e_f C_V^f \, \frac{\pi\, \alpha_s(\mu^2)}{N_c} \left(\frac{4}{y\bar{y}Q^2}\right)
$$

$$
\times \left\{ \int_0^1 dx \frac{2\xi H^g(x,\xi,t) + 2\,x\,C_F\,\xi H^f_{\text{singlet}}(x,\xi,t)}{(x-\xi+i\epsilon)(x+\xi-i\epsilon)} \right\}
$$

Contributions of other amplitudes



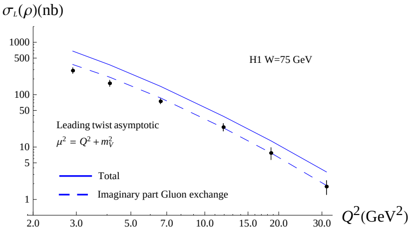
$$(\mathcal{I}m\,\mathcal{M}_V^g)^2 \,/\, |\mathcal{M}_V|^2 \sim 70\%$$

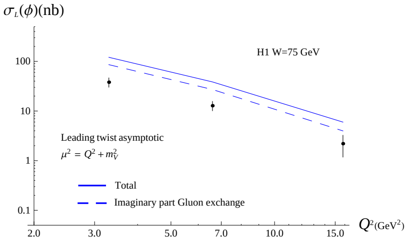


$$(\mathcal{I}m\,\mathcal{M}_V^g)^2 \,/\, |\mathcal{M}_V|^2 \sim 60\%$$

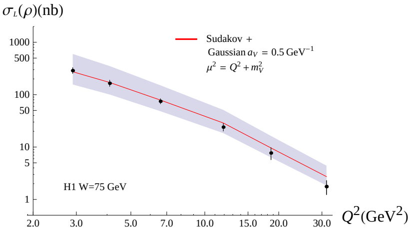
- Sea quark contribution (via interference term) not negligeable in MPA approach with GK GPDs based on (CTEQ6M, '02) fits ($10^{-4} < x < 0.5$ and $4 < Q^2 < 40$ GeV^2)

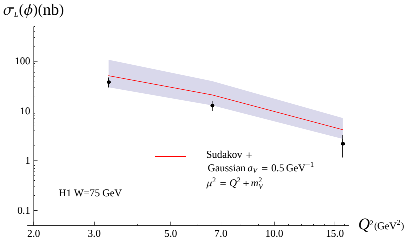
Results
Comparison Gluon vs Total contributions





Results
Sudakov + meson wavefunction ansatz





Dipole model from GPDs?

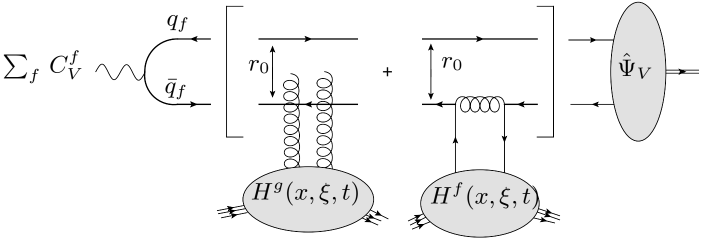
Contribution from gluons and quarks

- At small-x: Glauber-Mueller dipole model :

$$\hat{\sigma}(x,\underline{r}) = \frac{\pi^2\,\alpha_s}{N_c}\,\underline{r}^2\,xg(x) \to \sigma_0\left(1 - e^{-\frac{\pi^2\,\alpha_s}{N_c\,\sigma_0}\,\underline{r}^2\,xg(x)}\right)$$

- For exclusive process, (Martin, Ryskin, Teubner '99) skewness effect $\Rightarrow R_g$:

$$\hat{\sigma}(x,\underline{r}) = \sigma_0\left(1 - \exp\left\{-\frac{\pi^2\,\alpha_s}{N_c\,\sigma_0}\,\underline{r}^2\,R_g\,xg(x)\right\}\right)$$



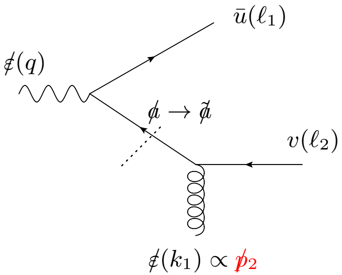
- Summary:
  - Within MPA, $\mathcal{M}_{V,\{0,+;0+\}}$ factorization of the overlap of the wavefunctions
  - Allows to compare results within dipole picture and collinear factorization framework
  - Role of the quark $t-$channel exchange within MPA can be important even at small$-x$
- Perspectives :
  - The universality of the relation between the dipole scattering amplitude and GPDs has to be checked
  - Other helicity amplitudes $\Rightarrow$ sensitivity to other GPDs

Acknowledgement

Thanks to my collaborators for encouraging discussions on this work

P. Kroll
C. Lorcé
C. Mezrag
H. Moutarde
Al. Mueller
S. Munier
B. Pire
F. Sabatié
L. Szymanowski
S. Wallon

Factorization of the wavefunctions and models



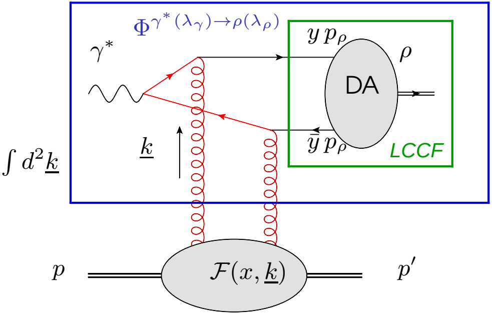
- Eikonal approximation leads to (Anikin, Wüsthof, '99)

$$\not a \, \not p_2 \to \not\tilde{a} \, \not p_2 \propto v(\tilde{a})\bar{v}(\tilde{a}) \, \not p_2 \,, \text{ such that } \tilde{a}^2 = 0$$

- The wavefunction factorizes

$$\Psi_{\gamma^*}(y, \underline{\ell}) \propto \frac{\bar{u}(\ell_1)\not\epsilon(q)v(\tilde{a})}{a^2 + i\epsilon}\Big|_{\ell_1 = yp_1 + \ell_\perp + \frac{\underline{\ell}^2}{2ys}p_2}$$

Factorization of helicity amplitudes



- Twist 2:
  - $\gamma_L^* \to \rho_L$ $(\equiv T_{00})$
  - $\gamma_T^* \to \rho_L$ $(\equiv T_{01})$

  Ginzburg, Panfil, Serbo, '85
- Twist 3, in the limit $t \sim 0$:
  - $\gamma_T^* \to \rho_T$ $(\equiv T_{11})$

  Anikin, Ivanov, Pire, Szymanowski, Wallon, '10

- Link with the dipole model (A.B., Szymanowski, Wallon, '13) and implementation of the saturation effects using dipole cross-section models fitted on DIS

$$\mathcal{I}m\, \mathcal{M}^g_{V,\{0+,0+\}} \propto \int dy \int d^2\underline{r} \sum_f C_V^f \left(\varphi_1(y)\, \hat{\Psi}^f_{\gamma_L^*}(y,\underline{r})\right) \mathcal{N}(x,\xi,\underline{r})$$

with $\varphi_1(y)$ the Leading twist DA

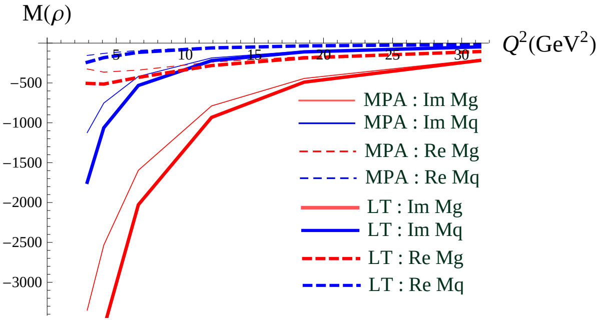
Models of dipole cross-section

- Small$-x$ evolution
  - Initial condition for the dipole cross-section at a given rapidity from DIS structure functions
  - Evolution with rc-BK equation (Balitsky, '07)

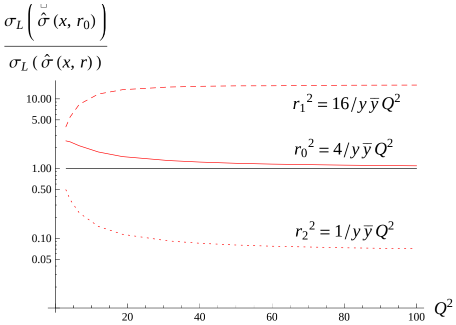
- DGLAP evolution (Bartels, Golec-Biernat, Kowalski, '02)

$$\hat{\sigma}(x, \underline{r}) = \sigma_0 \left(1 - \exp\left\{-\frac{\pi^2 \underline{r}^2 \alpha_s(\mu^2(\underline{r}^2)) \, xg(x, \mu^2(\underline{r}^2))}{3\sigma_0}\right\}\right)$$

Contributions of other amplitudes

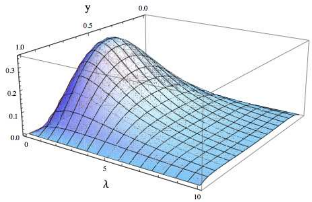


- $\sigma_L \propto \left| \int dy \int d^2\underline{r}\, (\Psi_V^* \Psi_\gamma)\, (y, \underline{r})\, \hat{\sigma}(x, r) \right|^2$
- Expect to find same numerical values at large Q^2 using :
  - a dipole cross-section model (here GBW model, (Golec-Biernat, Wüsthof, '98)): $\hat{\sigma}(x, r) = \sigma_0(1 - e^{-r^2/(4R_0(x)^2)})$
  - a dipole cross-section model with $\underline{r}^2 \to \underline{r}_0^2 = \frac{4}{y\bar{y}Q^2}$



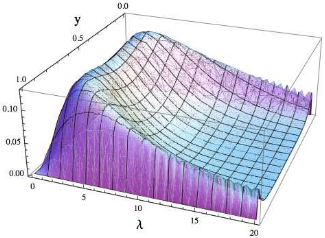
Transversely polarized cross-section

- Helicity amplitude $\mathcal{M}_{V,\{++,++\}}$ within MPA .. under study
- The overlap of wavefunctions appearing within k_T-factorization $\Psi_V^*(y,\underline{r})\Psi_{\gamma^*}(y,\underline{r})$:



$\lambda_\gamma = \lambda_V = 0$

Leading twist DA



$\lambda_\gamma = \lambda_V = \pm$

Twist 3 combination of DAs

(in Wandzura-Wilczek approx.)

(A.B., Szymanowski, Wallon, '13)

- Expected to be sensitive to the Sudakov form factor suppression