Testing the Standard Model at the precision frontier: The anomalous magnetic moment of the muon

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Johannes Gutenberg-University Mainz

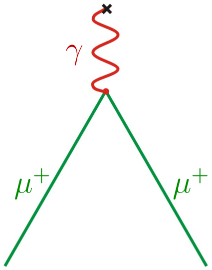
MESON, Cracow
May 29, 2014

The anomalous magnetic moment of the muon a_μ

gyromagnetic ratio: g

$$\vec{\mu} = g\frac{e\hbar}{2mc}\vec{S}$$

(1918) Dirac particles: $g = 2$



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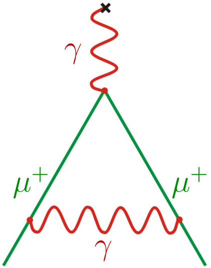
QED:



Schwinger: $a_\mu^{QED,LO} = \frac{\alpha}{2\pi}$

[Schwinger, PR **73**, 416 (1948)]

corresponds to an additional magnetic moment associated with the electron spin, of magnitude $\delta\mu/\mu = (\frac{1}{2}\pi)e^2/\hbar c$ $= 0.001162$. It is indeed gratifying that recently acquired experimental data confirm this prediction. Measurements

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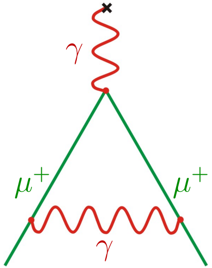

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$\frac{\alpha\pi}{2}$ ↑

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QED	11 658 471.895 ± 0.008
Schwinger	11 620 000

a_μ units in 10^{-10}

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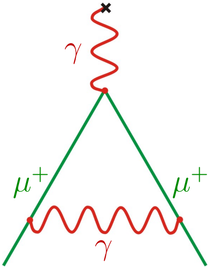
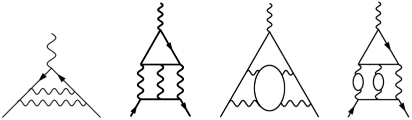


examples for higher order QED corrections:



[T.Kinoshita *et al.*, PR L **109**, 111808 (2012)]

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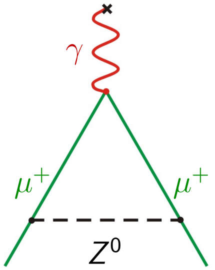

[A. Czarnecki *et al.*, PR D **67**, 073006 (2003)
Erratum-ibid. D**73**, 119901 (2006)]
[M. Knecht *et al.*, JHEP 0211, 003 (2002)]

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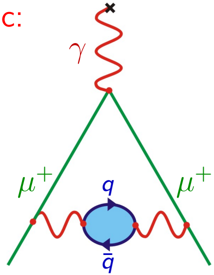
hadronic:



[M. Davier *et al.*, EPJ **C** 71, 1515 (2011)]

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BNL E821	11 659 208.9	± 6.4
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BNL−SM	28.7	± 8.0

a_μ units in 10^{-10}

experiment:

Brookhaven National Laboratory (BNL)
[G.W. Bennett *et al.*, PR D **73**, 072003 (2006)]

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$a_\mu^{\text{SM}} - a_\mu^{\text{exp}}$:
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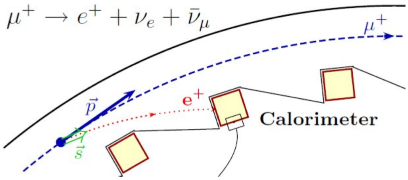
$(g-2)_\mu$ measurements

$(g-2)_\mu$ measurement: E821 at BNL



- $\pi^+ \to \mu^+ \nu$ violates P $\Rightarrow$ μ^+ longitudinally polarized
- μ^+ stored in a storage ring with constant $\vec{B}$
  - μ^+ rotating with cyclotron frequency: $\vec{\omega}_c$
  - μ^+-spin precessing frequency: $\vec{\omega}_s$
  - $\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right] = -a_\mu \frac{q}{m_\mu}\vec{B}$
- $\mu^+ \to e^+ \bar{\nu} \nu$ violates P
  $\Rightarrow$ e^+ direction remembers μ^+ polarization
  $\Rightarrow$ Fraction of detected e^+ above an E-threshold is modulated with $\vec{\omega}_a$

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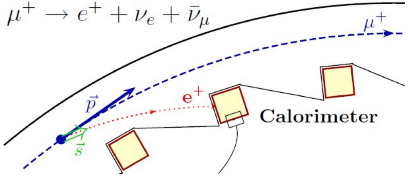

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magic energy: $E_\mu = 3.094\,\mathrm{GeV} \Rightarrow \gamma = 29.3$

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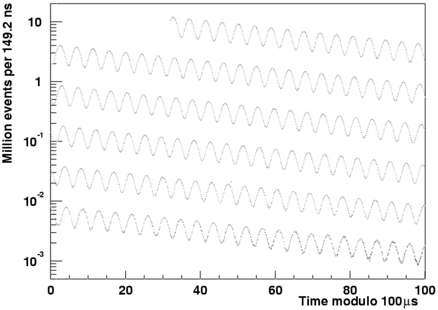
$(g-2)_\mu$ measurement: E821 at BNL

$\Rightarrow$ measure $\vec{\omega}_a$ and $\vec{B}$ [G.W. Bennett *et al.*, PR D **73**, 072003 (2006)]



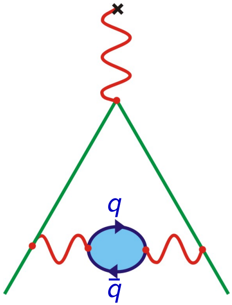
Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL	1997	μ^+	11 659 251(150)	13
BNL	1998	μ^+	11 659 191(59)	5
BNL	1999	μ^+	11 659 202(15)	1.3
BNL	2000	μ^+	11 659 204(9)	0.73
BNL	2001	μ^-	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

$(g-2)_\mu$ measurements: Future prospects

- Fermilab g-2 collaboration (E989) [Nucl.Phys.Proc.Suppl. 225-227 (2012), 277-281]
- J-PARC collaboration [Nucl.Phys.Proc.Suppl. 218 (2011) 242-246]

$\Rightarrow$ improve experimental precision by a factor of 4

Theoretical prediction of $(g-2)_\mu$



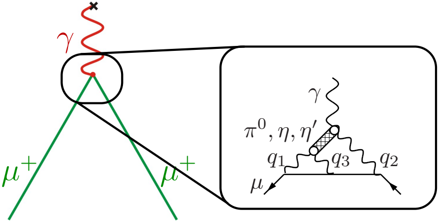
Hadronic contributions dominate the uncertainty!

- hadronic Light-by-Light
- hadronic Vacuum Polarization

Dominating hadronic contributions (1): $a_\mu^{\text{had,LbL}}$

total hadronic contribution: $a_\mu^{\text{had}} = (693.0 \pm 4.9) \cdot 10^{-10}$

Light-by-Light (LbL): $a_\mu^{\text{had,LbL}} = (10.5 \pm 2.6) \cdot 10^{-10}$



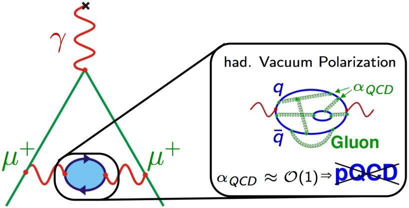
- pseudoscalar meson exchange contribution important (π^0, η, η')
- photon-meson transition form factors (TFF) need to be measured
- mesons in LbL term are virtual
  - measure real meson TFFs in experiments
  - models: relate real to virtual meson TFF

[J. Prades *et al.*, arXiv:0901.0306 (2009)]

Dominating hadronic contributions (2): $a_\mu^{\mathrm{had,VP}}$

total hadronic contribution: $a_\mu^{\mathrm{had}} = (693.0 \pm 4.9) \cdot 10^{-10}$

Hadronic Vacuum Polarization (VP): $a_\mu^{\mathrm{had,VP}} = (692.3 \pm 4.2) \cdot 10^{-10}$

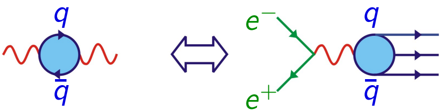


- largest hadronic contribution to a_μ
- largest absolute uncertainty of a_μ
- running of $\alpha_s \Rightarrow$ pQCD not applicable

[M. Davier *et al.*, EPJ **C** 71, 1515 (2011)]

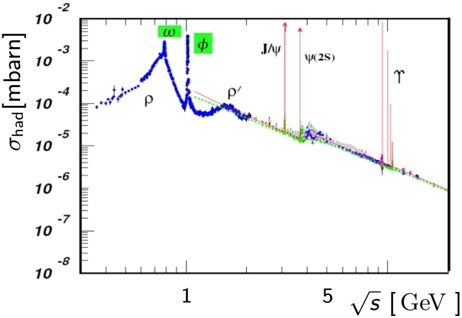
Dominating hadronic contributions (2): $a_\mu^{\text{had},VP}$

Optical theorem



Dispersion integral

$$a_{\mu,LO}^{\text{had}} = \frac{1}{4\pi^3} \int_{m_{\pi^0}^2}^{\infty} ds\, K(s)\, \sigma_{\text{had}}(s)$$



$$\sigma_{\text{had}}(s) \sim 1/s \quad \& \quad K(s) \sim 1/s$$

$$\Downarrow$$

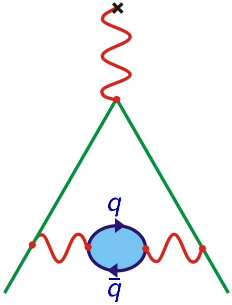
$$\sim 1/s^2$$

Low energy contributions important!

Needed:

hadronic cross section σ_{had}

Experimental input for a_μ^{Theory}



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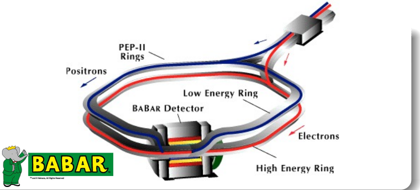
$\Rightarrow$ Need experimental input!

B-factories: *BABAR* and Belle – A success story

main purpose: B-physics $\rightarrow$ CP violation

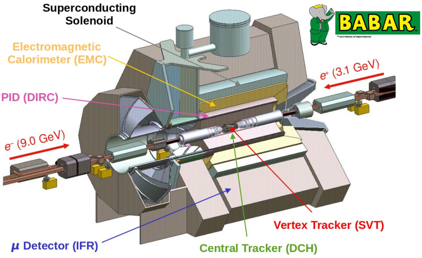
PEP-II@SLAC and KEKB@Tsukuba

- asymmetric e^+e^--colliders
- $\sqrt{s} = 10.58\,\mathrm{GeV} \Rightarrow \Upsilon(4S)$
  $\Rightarrow$ above $B\bar{B}$-threshold



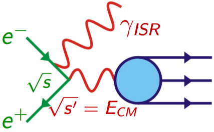
BABAR- and Belle-detector

- multi purpose detectors
- *BABAR* from 1999 – 2008:
  $\mathcal{L}_{int} \approx 0.5\,\mathrm{ab}^{-1} \approx 4.7 \cdot 10^8\ B\bar{B}$
- Belle from 1999 – 2010:
  $\mathcal{L}_{int} \approx 1\,\mathrm{ab}^{-1} \approx 7.7 \cdot 10^8\ B\bar{B}$



BUT, there is more...

Initial State Radiation (ISR)-Physics



- **high energy** $\gamma_{ISR} \Rightarrow$ lower cms energy
- produce vectors: $J^{PC} = 1^{--}$
- measure σ_{had}

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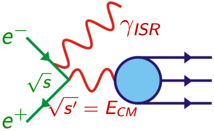
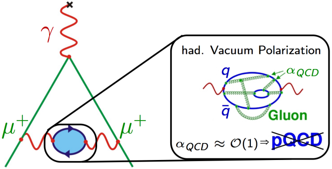


$\Rightarrow a_\mu^{had,VP}$:



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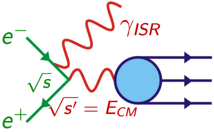

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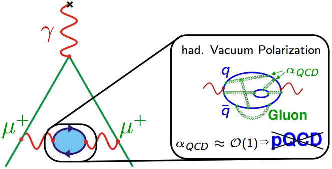
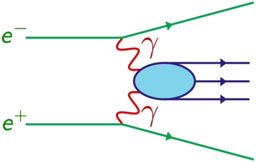


$\gamma\gamma$-Physics



- produce pseudoscalars $J^{PC} = 0^{-+}, \cdots$
- measure **meson photon transition FFs**

$\Rightarrow$ low-energy hadron Physics (u-d-s-quarks) at the B-Factories

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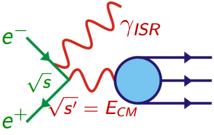

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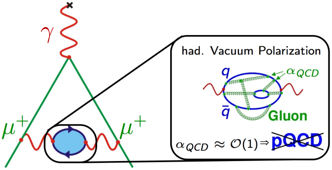

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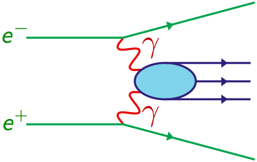
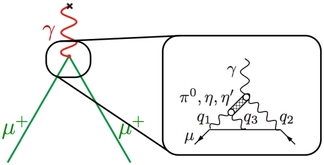


$\Rightarrow a_\mu^{had,LbL}$:



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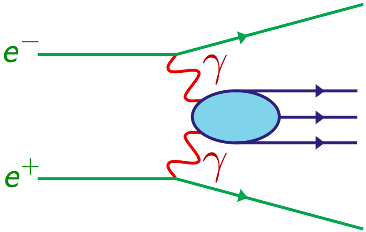
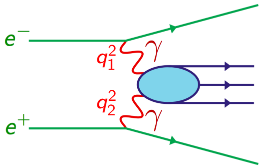
Meson photon transition form factors for $a_\mu^{had,LbL}$



Selection for meson-photon TFF at B-Factories



Amplitude for $\gamma\gamma^* \to$ P transition contains unknown function:
transition form factor (TFF): $F(q_1^2, q_2^2)$

$F(q_1^2, q_2^2)$ at B-factories in single tagged analyses:

- one $e^\pm$ along beamline $\Rightarrow q_2^2 \approx 0$
- other $e^\pm$ detected $\Rightarrow Q^2 = -q_1^2 > 4\,\mathrm{GeV}^2$
- pseudoscalar (P) meson fully reconstructed
  $\Rightarrow$ kinematic constraints to reject background

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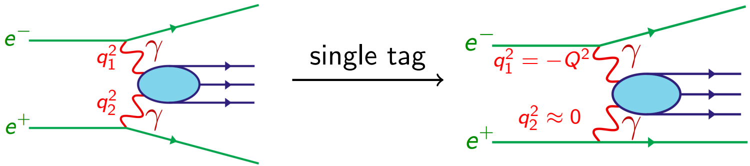

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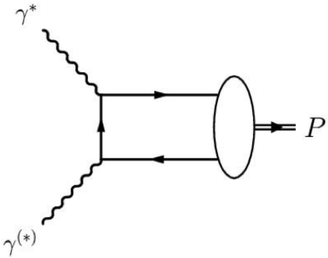
Why is the form factor interesting?



$$F(Q^2) = \int T(x, Q^2) \cdot \phi(x, Q^2) dx$$

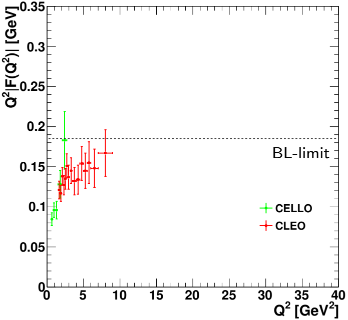
- ϕ: Meson distribution amplitude
- T: Hard scattering amplitude (pQCD)
- x: fraction of meson momentum carried by one of the quarks

Meson distribution amplitude ϕ

- important role in theoretical description of many QCD processes: $\gamma^* \rightarrow \pi^+\pi^-, \gamma\gamma \rightarrow \pi\pi, B \rightarrow \pi l \nu, \cdots$
- shape (x dependence) not known
- evolution with Q^2 predicted by QCD

$\Rightarrow$ test ϕ shape models with $F(Q^2)$ data!

π^0-photon transition form factor

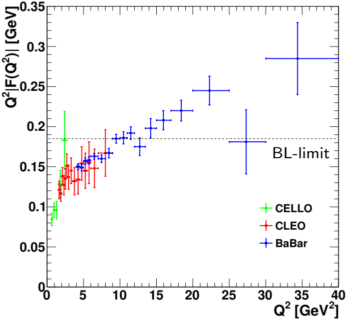


asymptotic limit from pQCD:

$$\lim_{Q^2 \to \infty} Q^2 F(Q^2) = \sqrt{2} f_\pi$$

[Brodsky, Lepage, Phys. Lett. B **87**, 359 (1979)]

π^0-photon transition form factor



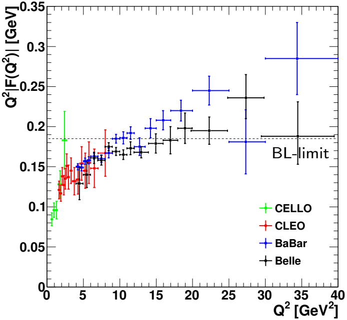
BABAR measurement:

- systematic uncertainty: 3-6%
- $4\,\mathrm{GeV}^2 < Q^2 < 9\,\mathrm{GeV}^2$: reasonable agreement with CLEO
- $Q^2 > 10\,\mathrm{GeV}^2$: pQCD: $\lim_{Q^2\to\infty} Q^2 F(Q^2) = \sqrt{2} f_\pi$ $\Rightarrow$ asymptotic limit exceeded!

$\Rightarrow$ triggered a lot of theoretical work

$>$ 200 citations since 2009

π^0-photon transition form factor



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- $Q^2 > 10\,\mathrm{GeV}^2$:
  pQCD: $\lim_{Q^2\to\infty} Q^2 F(Q^2) = \sqrt{2} f_\pi$
  $\Rightarrow$ asymptotic limit exceeded!

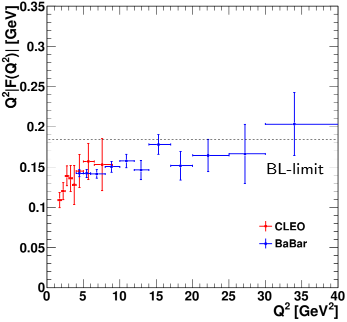
$\Rightarrow$ triggered a lot of theoretical work and a new measurement at Belle:

- disagreement with *BABAR* at large Q^2
- BUT: also exceeds asymptotic limit
- slope in better agreement with pQCD

$\Rightarrow$ need more data/experiments

η/η'-photon transition form factors

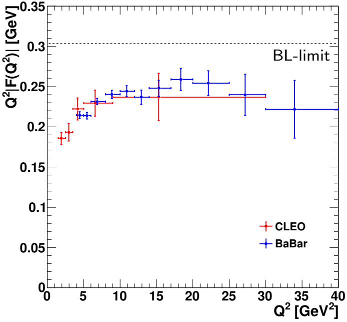
$\eta \to \pi^+\pi^-\pi^0, \pi^0 \to \gamma\gamma$



systematic uncertainty: 2.9%

η-FF approaches asymptotic limit

$\eta' \to \pi^+\pi^-\eta, \eta \to \gamma\gamma$

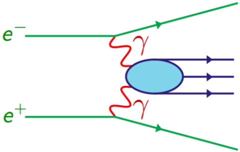


systematic uncertainty: 3.5%

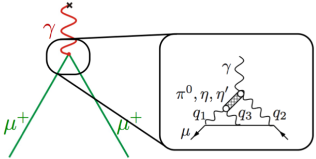
η'-FF below asymptotic expectation

Status of $g_\mu - 2$: had. LbL

$a_\mu^{\text{LBL}} = (10.5 \pm 2.6) \cdot 10^{-10}$

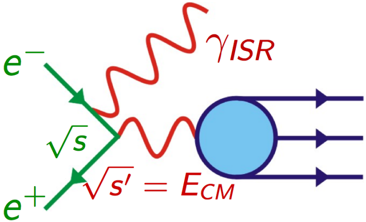


⇒ LBL:



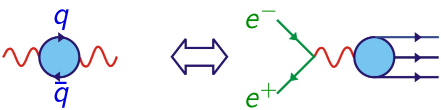
- evaluation not as straight forward as for VP:
  ⇒ models: relate measured $\pi^0/\eta/\eta'$ TFF to hadronic LbL term
- $Q^2 > 4\,\text{GeV}^2$: *B*-factories dominate the uncertainties:
  π^0-puzzle should be solved ⇒ Belle-2
- most relevant input for a_μ^{LBL}: low Q^2 region
  ⇒ BESIII, CMD3, SND, MAMI, ...
- alternative approach: a_μ^{LbL} from lattice QCD

Hadronic cross sections for $a_\mu^{had,VP}$



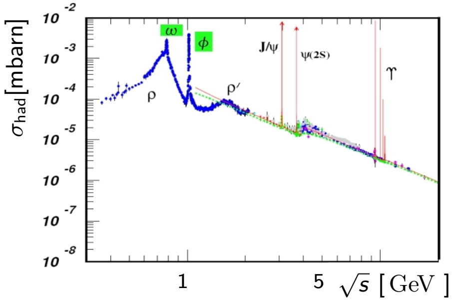
Experimental input for a_μ^{had}

Optical Theorem



Energy scan

- CMD & SND, VEPP-2M & VEPP-2000, Novosibirsk
- BES-I & II, BEPC, Beijing



Experimental input for a_μ^{had}

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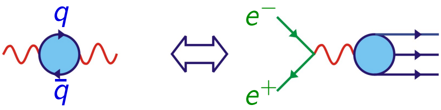
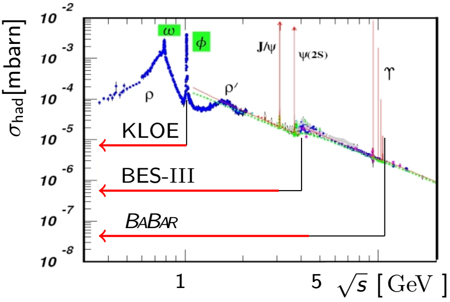
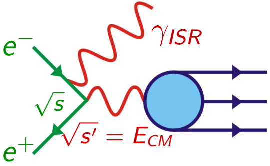




Energy scan

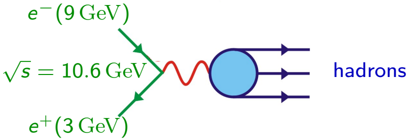
- CMD & SND@VEPP-2M & VEPP-2000 in Novosibirsk
- BES-III@BEPC-II in Beijing

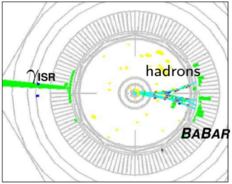
Initial State Radiation



- KLOE@DAϕNE in Frascati
- *BABAR*@PEP-II in Stanford
- BES-III@BEPC-II in Beijing

Initial State Radiation (ISR) events at *BABAR*

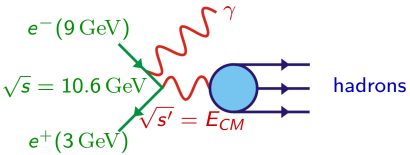


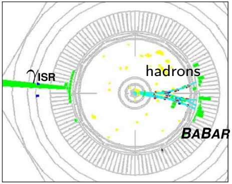


ISR selection

- Detected high energy photon: $E_\gamma > 3\,\mathrm{GeV}$
  $\rightarrow$ defines E_{CM} & provides strong background rejection
- Event topology: γ_{ISR} back-to-back to hadrons
  $\rightarrow$ high acceptance
- Kinematic fit including γ_{ISR}
  $\rightarrow$ very good energy resolution ($4 - 15\,\mathrm{MeV}$)
- Continuous measurement from threshold to $\sim 4.5\,\mathrm{GeV}$
  $\rightarrow$ provides common, consistent systematic uncertainties

Initial State Radiation (ISR) events at *BABAR*





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ISR analyses at BABAR

published

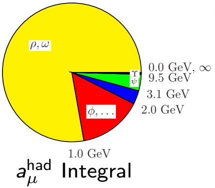
$e^+e^- \to \pi^+\pi^-$	PR D 86 (2012) 032013, PR L 103 (2009) 231801
$e^+e^- \to K^+K^-$	PR D 88, (2013) 032013
$e^+e^- \to \phi f_0(980)$	PR D 74 (2006) 091103, PR D 76 (2007) 012008
$e^+e^- \to \pi^+\pi^-\pi^0$	PR D 70 (2004) 072004
$e^+e^- \to K^+K^-\eta, K^+K^-\pi^0, K_S^0K^\pm\pi^\mp$	PR D 77 (2008) 092002, PR D 71 (2005) 052001
$e^+e^- \to 2(\pi^+\pi^-)$	PR D 85 (2012) 112009, PR D 76 (2007) 012008
$e^+e^- \to K^+K^-\pi^0\pi^0, K^+K^-\pi^+\pi^-, 2(K^+K^-)$	PR D 86 (2012) 012008, PR D 76 (2007) 012008
$e^+e^- \to K_S^0K_L^0, K_S^0K_L^0\pi^+\pi^-, K_S^0K_S^0\pi^+\pi^-, K_S^0K_S^0K^+K^-$	PR D 89 (2014) 092002
$e^+e^- \to 2(\pi^+\pi^-)\pi^0, 2(\pi^+\pi^-)\eta, K^+K^-\pi^+\pi^-\pi^0, K^+K^-\pi^+\pi^-\eta$	PR D 76 (2007) 092005
$e^+e^- \to 3(\pi^+\pi^-), 2(\pi^+\pi^-\pi^0), 2(\pi^+\pi^-)K^+K^-$	PR D 73 (2006) 052003
$e^+e^- \to p\bar{p}$ (small $\sqrt{s}$)	PR D 87 (2013) 092005, PR D 73 (2006) 012005
$e^+e^- \to p\bar{p}$ (large $\sqrt{s}$)	PR D 88 (2013) 072009
$e^+e^- \to \Lambda\bar{\Lambda}, \Lambda\bar{\Sigma}^0, \Sigma^0\bar{\Sigma}^0$	PR D 76 (2007) 092006
$e^+e^- \to c\bar{c} \to \ldots$	... ...

ongoing analyses

$e^+e^- \to \pi^+\pi^-\pi^0\pi^0, \pi^+\pi^-\pi^0\pi^0\pi^0, K_S^0K^\pm\pi^\mp\pi^0/\eta$

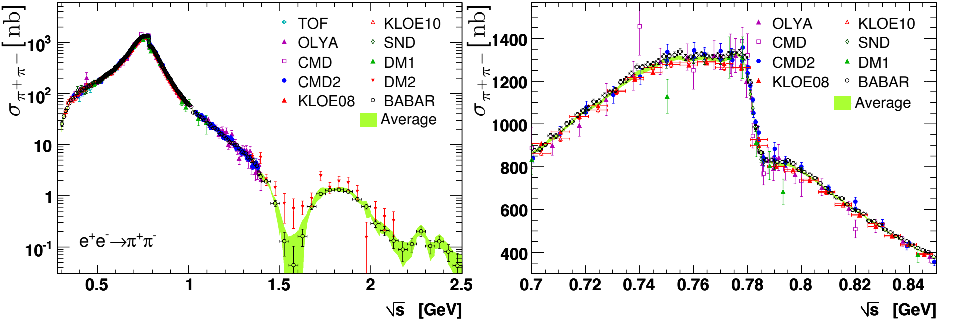
Contributions of exclusive final states to $g_\mu - 2$

Contributions of different energy regions to the dispersion integral



$\rightarrow E < 1\,\mathrm{GeV}$ region dominates
$\rightarrow \pi^+\pi^-$ channel needed!

$\pi^+\pi^-$ cross section

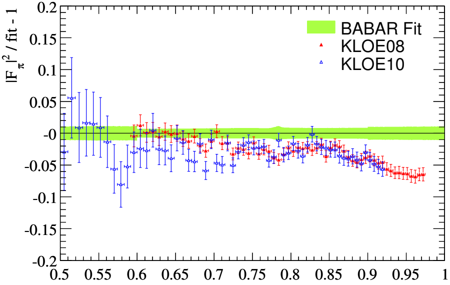


- ρ peak
- $\rho - \omega$ interference
- Dip at 1.6 GeV: excited ρ states
- Dip at 2.2 GeV
- Contribution to a_μ^{had}: 75%!

Systematic Uncertainties

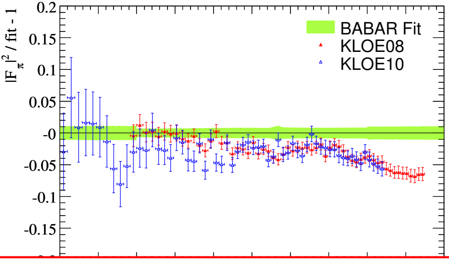
BABAR: 0.5%
CMD2: 0.8%
SND: 1.5%
KLOE: 0.8%

$\pi^+\pi^-$ cross section



- KLOE and *BABAR* dominate the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference, especially above ρ peak
- Difference $\to$ relatively large uncertainty for a_μ^{had}

$\pi^+\pi^-$ cross section

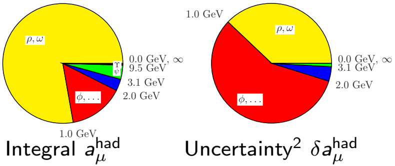


Need to solve this discrepancy!

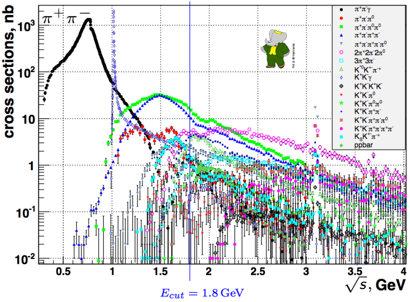
- KLOE and *BABAR* dominate the world average
- Uncertainty of both measurements smaller than 1%
- Systematic difference, especially above ρ peak
- Difference $\to$ relatively large uncertainty for a_μ^{had}

Contributions of exclusive final states

Contributions of different energy regions to the dispersion integral



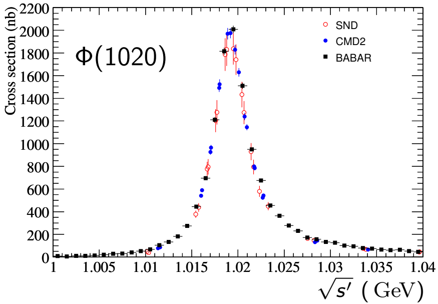
⇒ Precise measurements $1\,\mathrm{GeV} < E < 2\,\mathrm{GeV}$ needed!

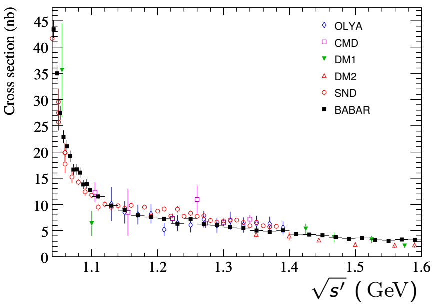


⇒ Other channels important!

- K^+K^-
- $K_S^0 K_L^0$
- $\pi^+\pi^-\pi^+\pi^-$
- $\pi^+\pi^-\pi^0$
- $\pi^+\pi^-\pi^0\pi^0$

Cross section $\sigma(e^+e^- \to K^+K^-)$





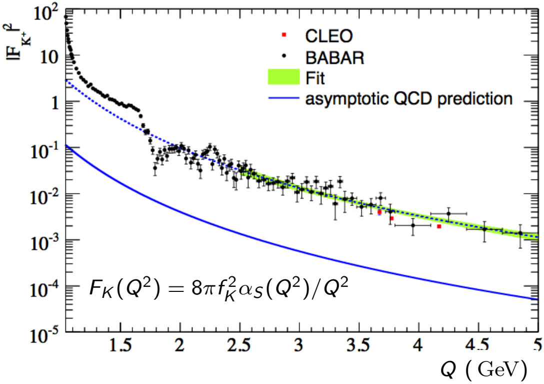
- uncertainties near ϕ peak:
  BABAR: 0.8%
  CMD2: 2.2%
- extracted m_ϕ agree within calibration uncertainties
- normalization difference:
  $\approx$ 1 s.d. to SND
  $\approx$ 2 s.d. to CMD2

$\Rightarrow$ some tension between the results

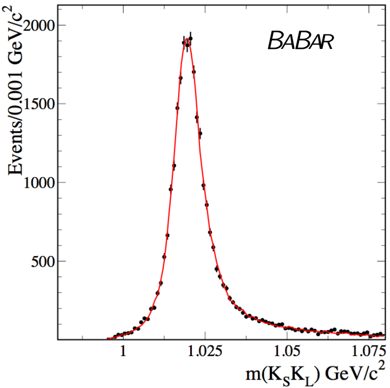
Charged kaon form factor at large Q^2

Predictions based on QCD in asymptotic regime (Chernyak, Brodsky-Lepage, Farrar-Jackson)

- Power law: $F_K \propto \alpha_S(Q^2)Q^{-n}$ with $n = 2$
  $\rightarrow$in good agreement with the data (2.5-5 GeV $n = 2.10 \pm 0.23$)
- HOWEVER: data on $|F_K|^2$ factor ≈ 20 above prediction!
- No trend in data up to 25 GeV^2 for approaching the asymp. QCD prediction

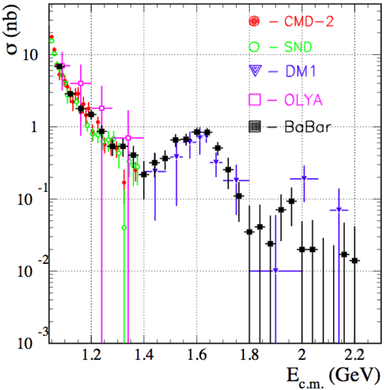


Cross section $\sigma(e^+e^- \to K_S^0 K_L^0)$



$\Gamma_\phi^{ee} \mathcal{B}_{K_S^0 K_L^0} = 0.4200 \pm 0.0033 \pm 0.0122 \pm 0.0019$

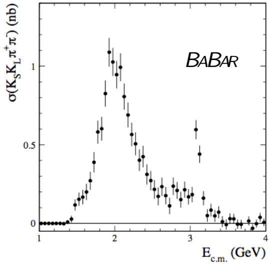
- within 1σ wrt CMD-2
- syst uncertainty: 2.9%
- dominated by trigger



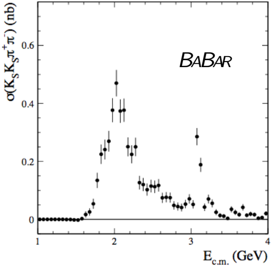
systematic uncertainty:

- $\sim 10\%$ for $\sigma > 0.5\,\mathrm{nb}$
- $\sim 30\%$ for $\sigma < 0.5\,\mathrm{nb}$
- dominated by bkg-subtraction

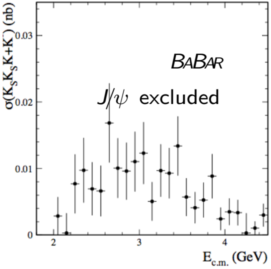
Cross Section of $e^+e^- \to K_S^0K_L^0\pi^+\pi^-$, $K_S^0K_S^0\pi^+\pi^-$, and $K_S^0K_S^0K^+K^-$



$\mathcal{B}_{J/\psi \to K_S^0K_L^0\pi^+\pi^-} = (3.7 \pm 0.6 \pm 0.4) \cdot 10^{-3}$



$\mathcal{B}_{J/\psi \to K_S^0K_S^0\pi^+\pi^-} = (1.68 \pm 0.16 \pm 0.08) \cdot 10^{-3}$



$\mathcal{B}_{J/\psi \to K_S^0K_S^0K^+K^-} = (0.42 \pm 0.08 \pm 0.02) \cdot 10^{-3}$

Systematic uncertainties:

- $\sim 10\%$ in peak region
- $\sim 30\%$ at $1.5/3\,\mathrm{GeV}$
- $\sim 100\%$ above $3\,\mathrm{GeV}$

- $\sim 5\%$ in peak region
- $\sim 20\%$ at $1.5/3\,\mathrm{GeV}$
- $\sim 50 - 70\%$ above $3\,\mathrm{GeV}$

- $\sim 5\%$
- stat. error dominates

1^{st} measurement:

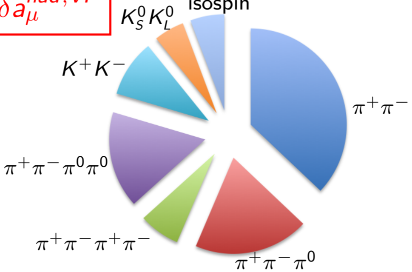
previously, isospin relations needed to estimate $(g-2)_\mu$ contribution

Impact on $g_\mu - 2$: had. VP

$a_\mu^{\text{VP,LO}} = (692.3 \pm 4.2) \cdot 10^{-10}$

$a_\mu^{had,VP}$

$\delta a_\mu^{had,VP}$



[data from Davier, EPJ **C** 71, 1515 (2011)]

channels estimated with isospin rels

largest contributions: $K\bar{K}\pi$ and $K\bar{K}\pi\pi$

$K_S^0 K_L^0$

BABAR not evaluated, yet

K^+K^-

BABAR reduces $\delta a_\mu^{had}(K^+K^-)$ by factor 2.7

$\pi^+\pi^-\pi^+\pi^-$

BABAR reduces $\delta a_\mu^{had}(\pi^+\pi^-\pi^+\pi^-)$ by 40%

$\pi^+\pi^-, \pi^+\pi^-\pi^0, \pi^+\pi^-\pi^0\pi^0$

wait for *BABAR*, BESIII, and CMD3 results

Summary & outlook

hadronic cross sections for $a_\mu^{had,VP}$

- dominate the uncertainty of a_μ^{had} and a_μ^{theory}
- $e^+e^- \to \pi^+\pi^-\pi^+\pi^-, K^+K^-, K_S^0 K_L^0$: well under control
- $e^+e^- \to \pi^+\pi^-\pi^0, \pi^+\pi^-\pi^0\pi^0, \pi^+\pi^- \Rightarrow$ BESIII, CMD3, *BABAR*

transition form factors for $a_\mu^{had,LbL}$

- *BABAR*-Belle discrepancy for π^0 at high Q^2
- η and η' compatible with pQCD prediction
- $a_\mu^{had,LbL}$: low Q^2 measurements needed $\Rightarrow$ BESIII, CMD3, ...

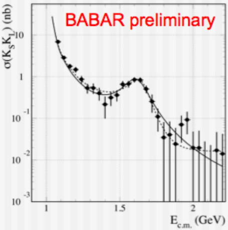
direct measurement of a_μ^{exp}

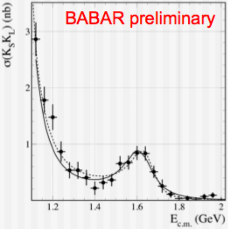
- new experiments at Fermilab and J-PARC
  $\Rightarrow$ reduce experimental uncertainty by factor ≈ 4!

backup slides

$\phi(1680) \to K_S^0 K_L^0$

Is it $\phi(1680)$?





$$\sigma(s) = \frac{P(s)}{s^{5/2}} \left| \frac{A_{\phi(1020)}}{\sqrt{P(m_\phi)}} + \frac{A_X}{\sqrt{P(m_X)}} \cdot e^{i\varphi} + A_{bkg} \right|^2$$

$$P(s) = \left((s/2)^2 - m_{K^0}^2 \right)^{3/2}$$

$$A(s) = \frac{\Gamma(m^2) \cdot m^3 \sqrt{\sigma_0 \cdot m}}{s - m^2 + i\sqrt{s}\Gamma(s)}$$

$$\Gamma(s) = \Gamma \cdot \sum_f B_f \cdot \frac{P_f(s)}{P_f(m_f^2)}$$

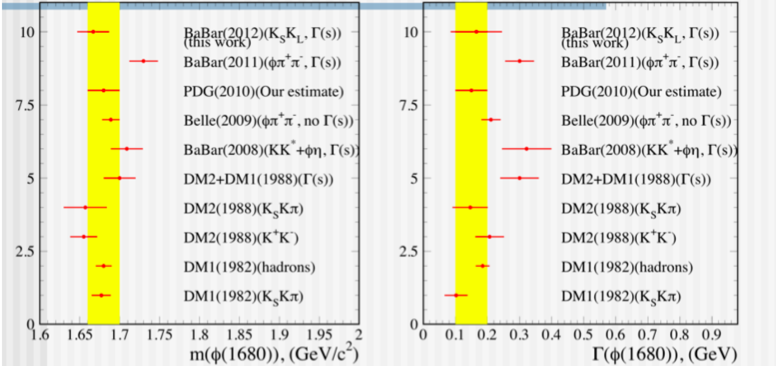
$$A_{\phi(1020)} = A_\phi + A_\omega - A_\rho, \qquad f = K^* K, \phi\eta, \phi\pi\pi, K_S K_L$$

$\sigma_0 = 0.46 \pm 0.10 \pm 0.04$ nb
m = 1674 ± 12 ± 6 MeV/c^2
$\Gamma_0 = 165 \pm 38 \pm 70$ MeV
$\varphi = 3.01 \pm 0.38$ – fixed to π
$\sigma_{bkg} = 0.36 \pm 0.18$ nb
$\Gamma_{ee} \cdot B_{KSKL} = 14.3 \pm 2.4 \pm 1.5 \pm 6.0$ eV

Simultaneous $K_S K_L$ and K^+K^- (and $\pi\pi$) fit is needed to separate I=0,1 states and ω(1420, 1650), ρ(1450,1700) contribution ..

$\phi(1680)$ observations in other channels

What we know about φ(1680)

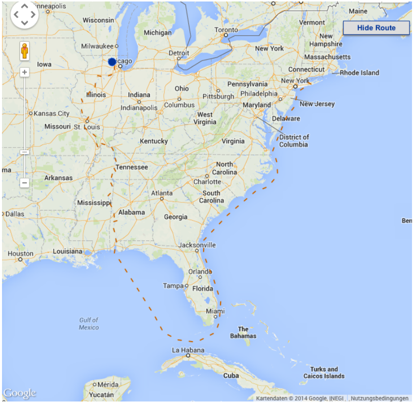


Energy dependence significantly increase width.

BaBar has measured φ(1680) parameters in major decay modes:

φ(1680) ➔ $K_SK\pi$, $KK\pi^0$ (K*K), φη, φππ, K_SK_L (preliminary) - still no info in PDG

$(g-2)_\mu$ measurements: Future prospects



- Fermilab g-2 collaboration (E989)
  - Nucl.Phys.Proc.Suppl. 225-227 (2012), 277-281
  - aiming for precision of 0.14 ppm
  - magnet moved from Brookhaven in July 2013
- J-PARC collaboration
  - Nucl.Phys.Proc.Suppl. 218 (2011) 242-246
  - aiming for precision of 0.1 ppm
  - use ultra-cold muon beam to limit transverse momentum
  - eliminate electric focusing field $\Rightarrow$ no magic energy

$\Rightarrow$ improve precision by a factor of 4!

[http://muon-g-2.fnal.gov/bigmove/gallery.html]

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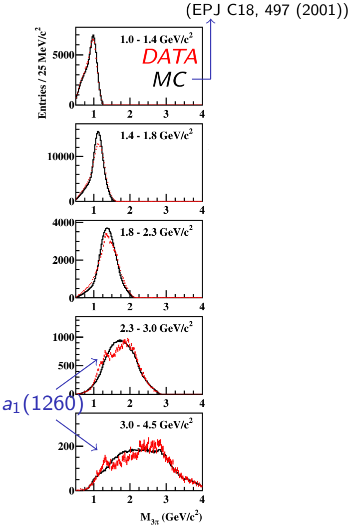
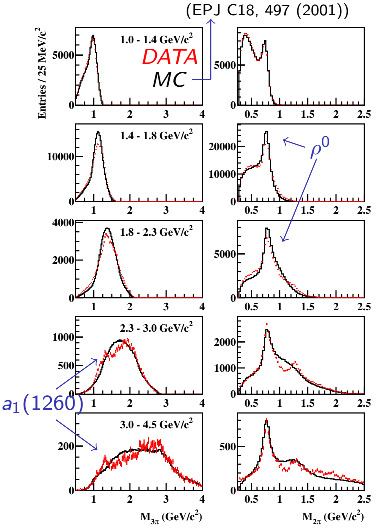
Internal structure in various E_{CM} energy slices



First column (4 entries/event):

$a_1(1260)$

Internal structure in various E_{CM} energy slices



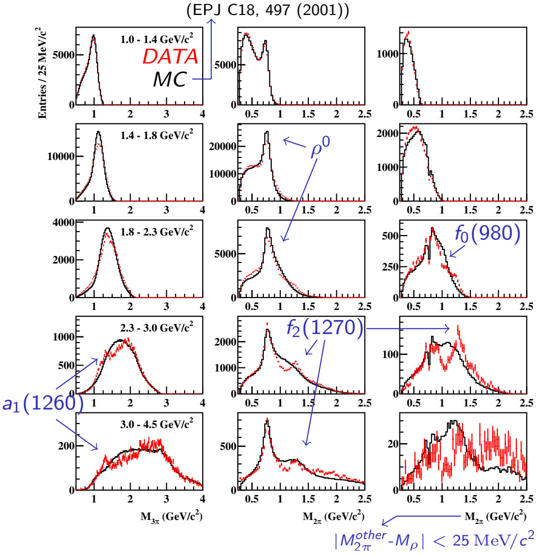
First column (4 entries/event):

$a_1(1260)$

Second column (4 entries/event):

strong ρ^0 contribution
e.g. for $M_{4\pi} > 1.4\,\mathrm{GeV}/c^2$:
$1/4th$ of entries in ρ^0 peak
$\rho^0\rho^0$ is forbidden
$\rightarrow \rho^0$ in each event!

Internal structure in various E_{CM} energy slices



First column (4 entries/event):

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$\rho^0\rho^0$ is forbidden
$\rightarrow \rho^0$ in each event!

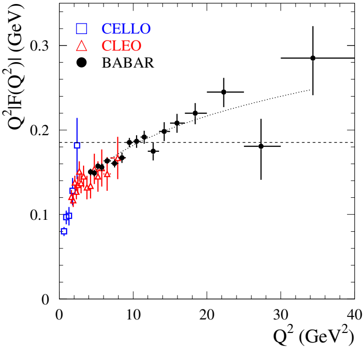
Third column (1 entry/event):

2π lie within ρ^0 mass
$\rightarrow$ other $\pi^+\pi^-$'s mass plotted

$f_2(1270)$, $a_1(1260)$, $f_0(980)\ldots$?

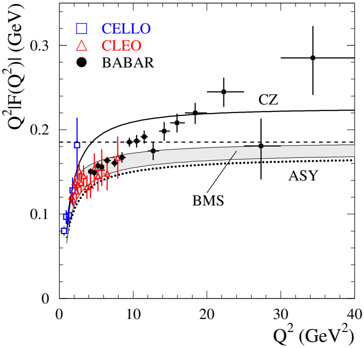
$\rightarrow$ Partial Wave Analysis needed

π^0-photon transition form factors



- systematic uncertainty:
  efficiency (trigger): 2.5%
  bkg ($e^+e^- \to e^+e^-\pi^0\pi^0$): 0.3 – 6.0%
  model uncertainty: 1.5%
- $4\,\mathrm{GeV}^2 < Q^2 < 9\,\mathrm{GeV}^2$:
  reasonable agreement with CLEO
- $Q^2 > 10\,\mathrm{GeV}^2$:
  pQCD: $\lim_{Q^2\to\infty} Q^2F(Q^2) = \sqrt{2}f_\pi$
  $\Rightarrow$ asymptotic limit exceeded!

π^0-photon transition form factors



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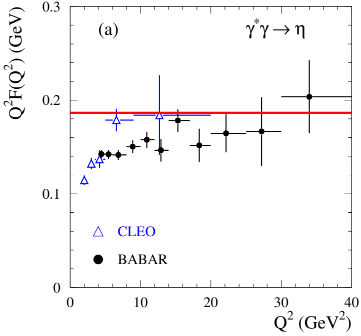
  $\Rightarrow$ asymptotic limit exceeded!

$\Rightarrow$ triggered a lot of theoretical work & a new measurement at Belle

$>$ 200 citations since 2009

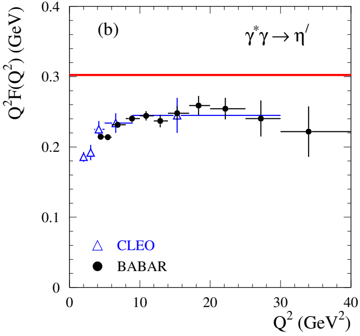
η/η'-photon transition form factors

$\eta \to \pi^+\pi^-\pi^0$



systematic uncertainty: 2.9%
dominated by model unc. & π^0 rec.

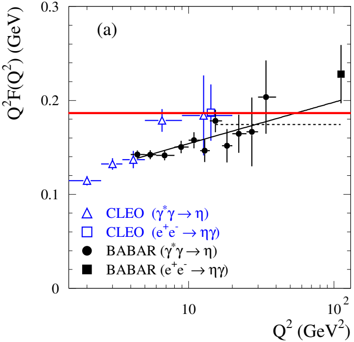
$\eta' \to \pi^+\pi^-\eta, \eta \to \gamma\gamma$



systematic uncertainty: 3.5%
dominated by model unc. & η rec.

η/η'-photon transition form factors

$\eta \to \pi^+\pi^-\pi^0$

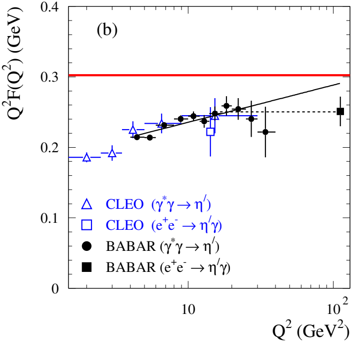


systematic uncertainty: 2.9%

dominated by model unc. & π^0 rec.

η-FF exceeds asymptotic limit

$\eta' \to \pi^+\pi^-\eta, \eta \to \gamma\gamma$



systematic uncertainty: 3.5%

dominated by model unc. & η rec.

η'-FF below asymptotic expectation