Exotic and conventional mesons from lattice QCD

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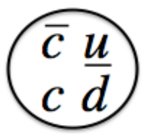
Outline

Spectrum from lattice QCD

- States well below strong decay threshold: $\underline{c}c$
- Excited states: single-hadron approximation
  $\underline{c}c$, D, D_s, isoscalar mesons $\underline{u}u$ & $\underline{d}d$ & $\underline{s}s$
- Near-threshold states: rigorous treatment
  - ★ first evidence for Z_c^+ from lattice QCD
  - ★ $D_{s0}^*(2317)$ near DK, $D_{s1}(2460)$ near D*K
  - ★ first evidence for X(3872) from lattice QCD



- Resonance above threshold: rigorous treatment
  ρ, K*, a_1, b_1, $D_0^*(2400)$, $D_1(2430)$

Lattice status:

Straightforward procedure,
many precision results

extensive results completed recently
highly excited states, many J^{PC}

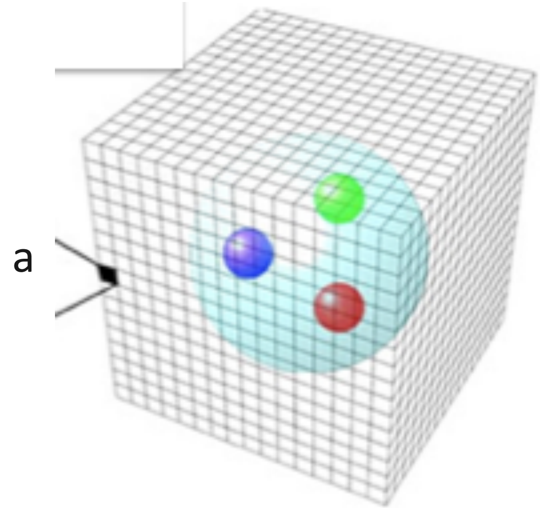
Only few pioneering results available

Only few resonances have been studied rigorously

QCD on lattice: ab initio non-perturbative metod

$$L_{QCD} = -\tfrac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_{q=u,d,s,c,b,t} \bar{q} i \gamma_\mu (\partial^\mu + i g_s G_a^\mu T^a) q - m_q \bar{q} q$$

input : g_s , m_q



Evaluation of Feynman path integrals in discretized space-time

$$S_{QCD} = \int d^4x \, L_{QCD}[G(x), q(x), \bar{q}(x)]$$

$$\langle C \rangle = \int DG \, Dq \, D\bar{q} \;\; C \, e^{-S_{QCD}}$$

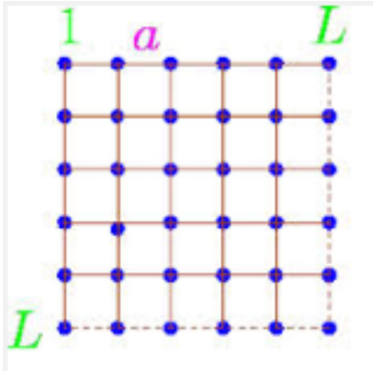
$$S = \int dt \, L[x(t)]$$

"Precision" spectrum:
States well below strong decay threshold

States well below threshold

Lattice QCD already determined masses of these states
very reliably and precisely O(10 MeV):

- m=E (for P=0)
- extrapolation : $a \to 0$, $L \to \infty$
- extrapolation or interpolation : $m_q \to m_q^{phy}$
- particular care needed for am_c and am_b discretization errors: several complementary methods give compatible results

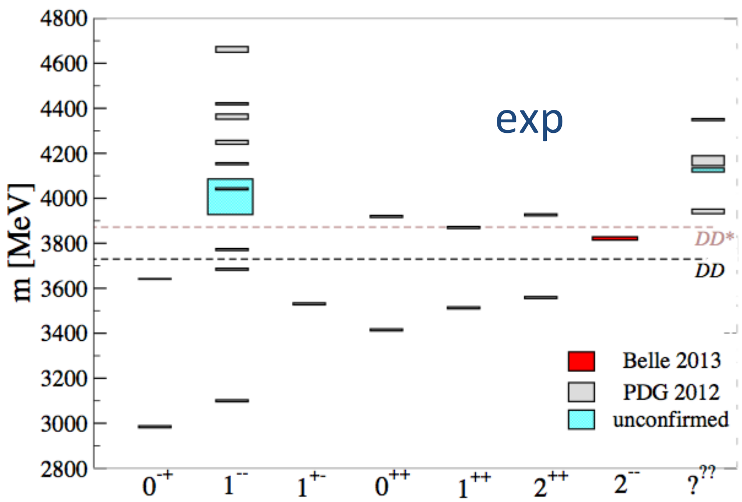


Many precision lattice results available for a number of years!

States well below threshold

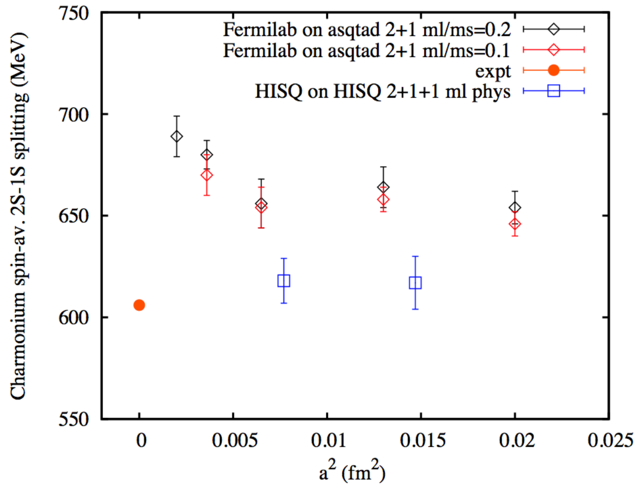
example: charmonium

Many results available,
only few examples shown



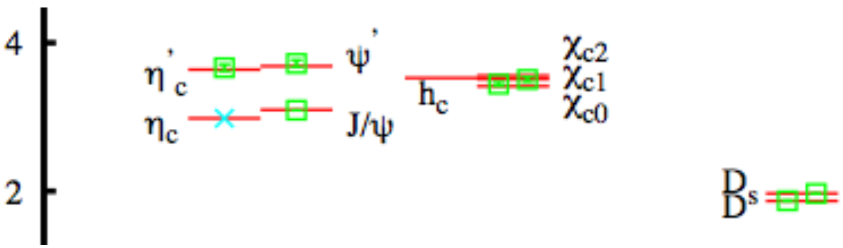
[HPQCD/MILC, 2014: Knecht, Galloway, Koponen, Davies and DeTar]

$$\tfrac{1}{4}M[\eta_c(2S)]+\tfrac{3}{4}M[\psi(2S)]$$
$$-\tfrac{1}{4}M[\eta_c(1S)]-\tfrac{3}{4}M[\psi(1S)]$$

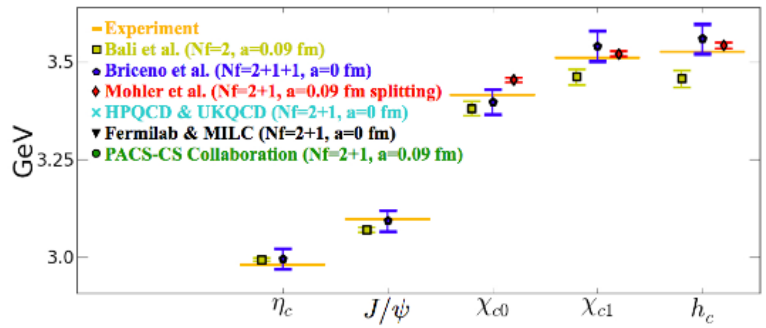


m [GeV]

[HPQCD: 1207.5149, PRD]



[Briceno, Lin, Bolton,1207.3536, PRD]



"Non-precision" spectrum: states near or above threshold

only one or two a, L, $m_{u/d}$

limits $a \rightarrow 0$, $L \rightarrow \infty$, $m_{u/d} \rightarrow m_{u/d}^{phy}$ usually not performed

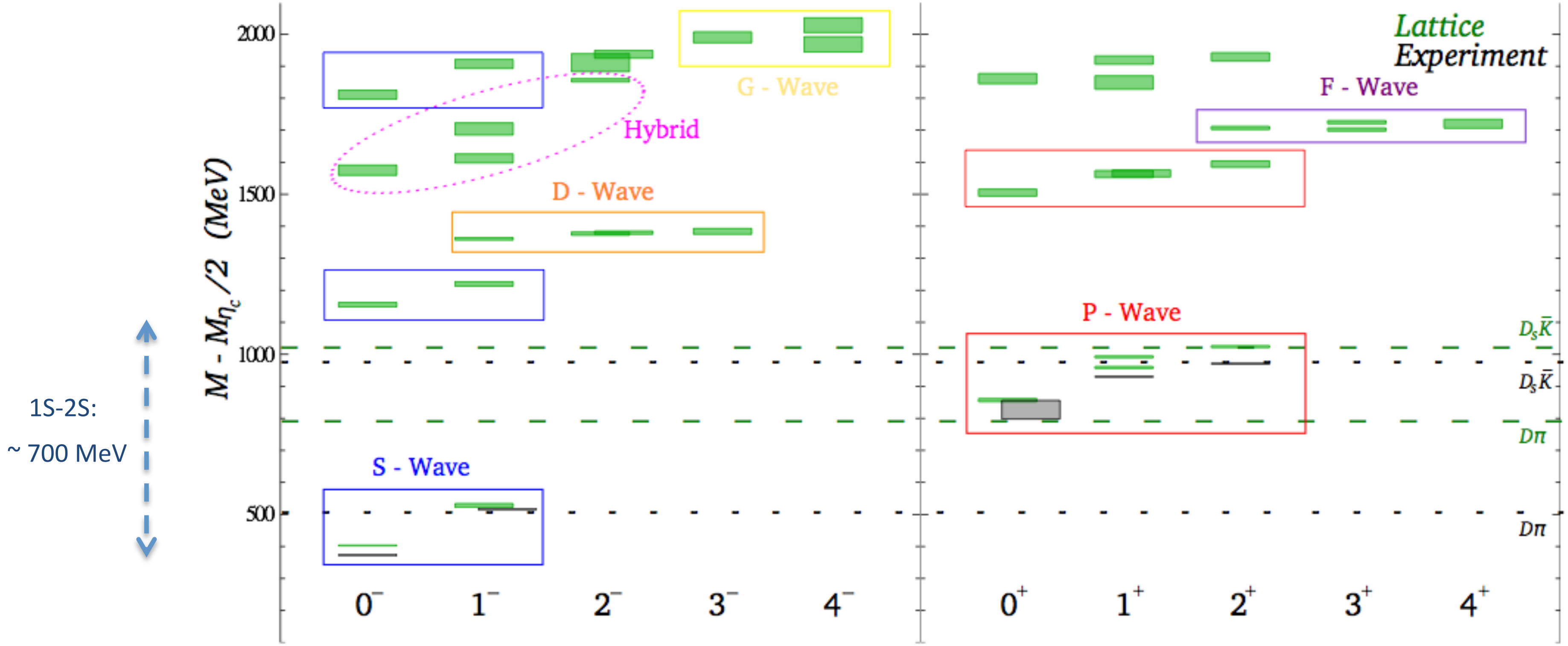
Excited states: single-hadron approximation

- only interpolating fields $\mathcal{O} \approx \bar{q}\, q$
- assumptions: all energy levels correspond to "one-particle" states

  m=E (for P=0)

  these are strong assumptions ...

D spectrum: single hadron approximation



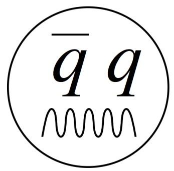
[G. Moir et al, HSC (Hadron Spectrum Coll.): 1301.7670, JHEP]

- $m_\pi \approx 400$ MeV, L≈2.9 fm, Nf=2+1
- reliable J^P determination; many excited states
- identification with $n\ ^{2S+1}L_J$ multiplets using $\langle O|n\rangle$
- green: lat, black: exp

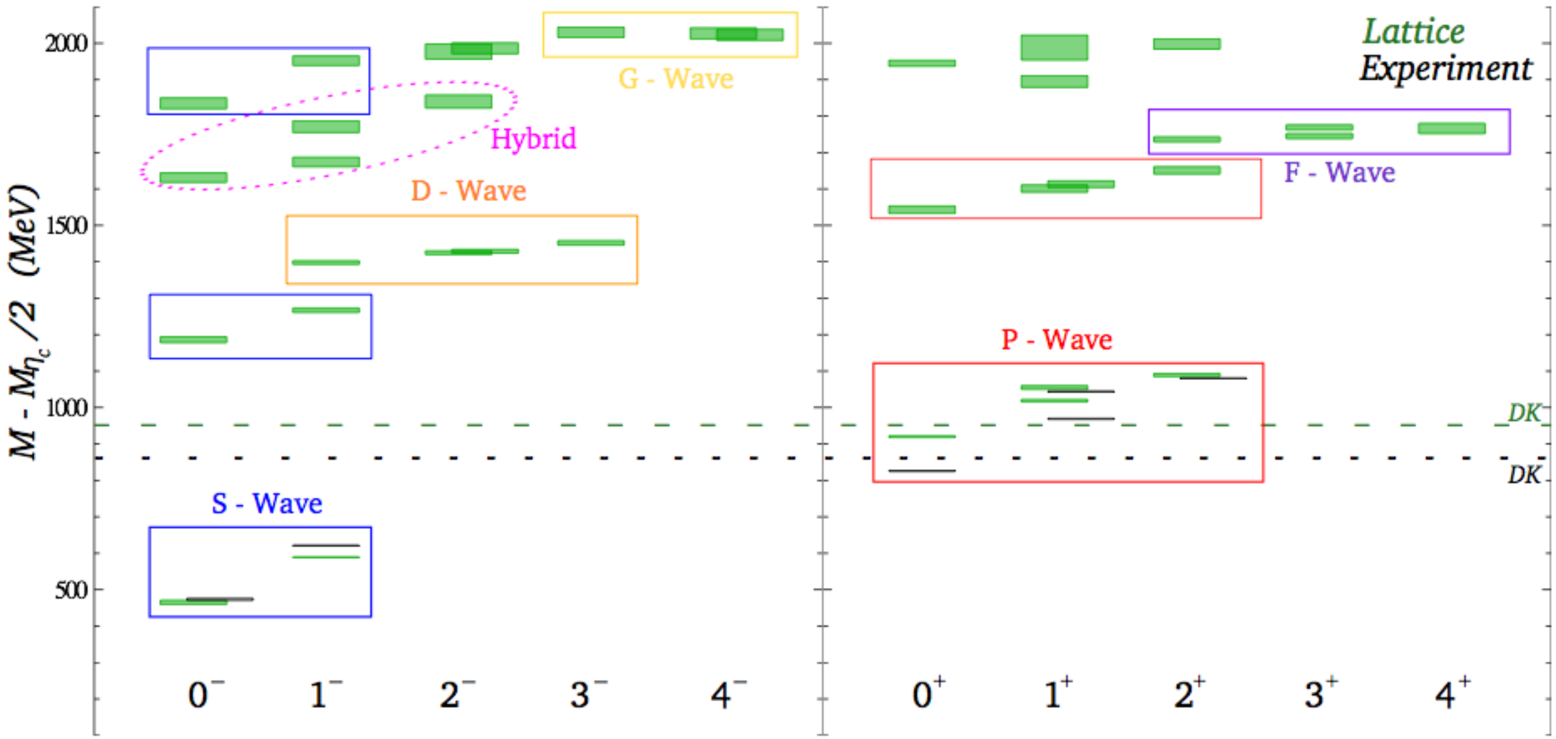
Hybrids:

large overlap with $O = \bar{q}\, F_{ij}\, q$

gluonic tensor $F_{ij}=[D_i\, , D_j\,]$



D_s spectrum: single hadron approximation



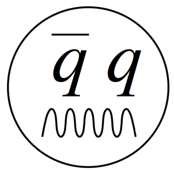
[G. Moir et al., HSC : 1301.7670, JHEP]

- $m_\pi \approx 400$ MeV, L≈2.9 fm, Nf=2+1
- reliable J^{PC} determination
- identification with n $^{2S+1}L_J$ multiplets using <*O*|n>
- green: lat, black: exp

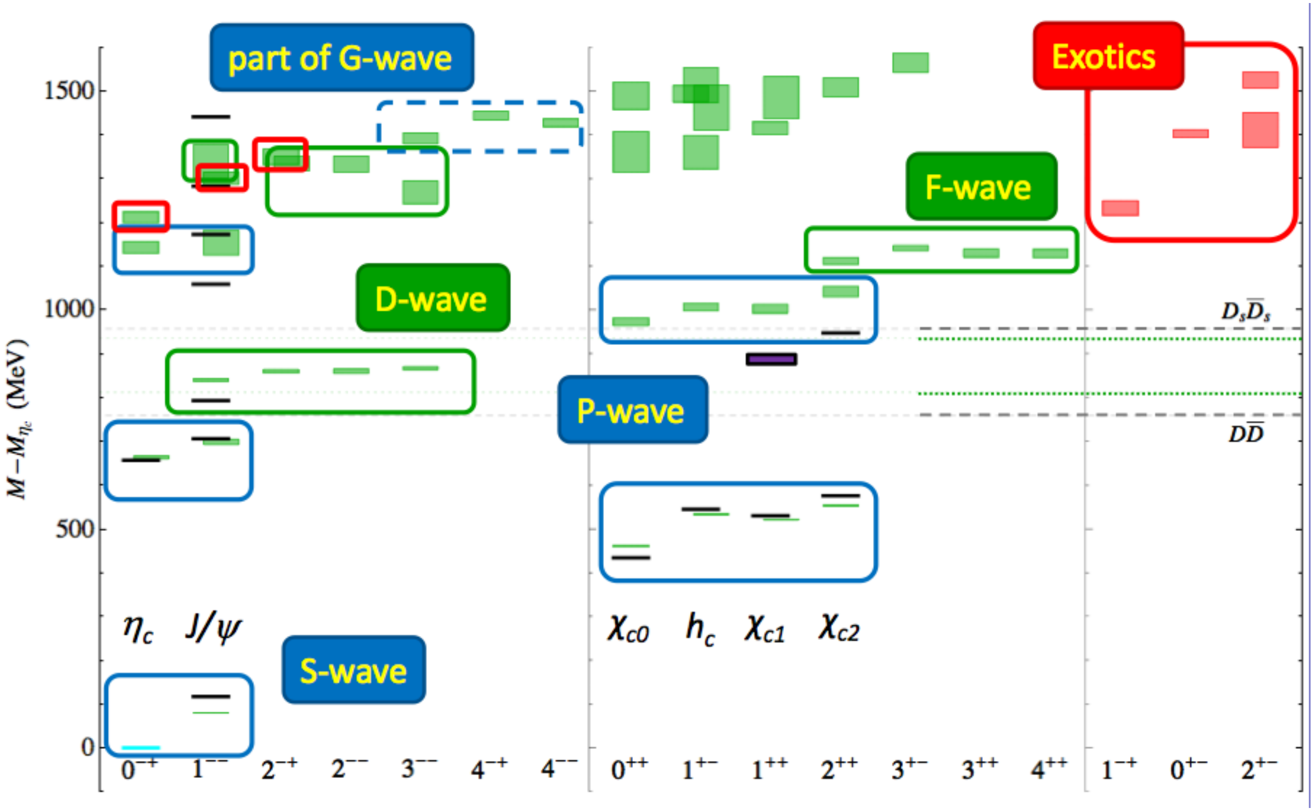
Hybrids:

large overlap with *O*= $\bar{q}\, F_{ij}\, q$

gluonic tensor $F_{ij}=[D_i\ ,\ D_j\ ]$



$\underline{c}c$ spectrum: single hadron approximation



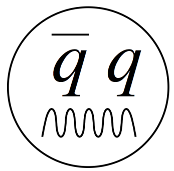
[HSC , L. Liu et al: 1204.5425, JHEP]

- $m_\pi \approx 400$ MeV, $L \approx 2.9$ fm, Nf=2+1
- reliable J^{PC} determination
- identification with $n\ ^{2S+1}L_J$ multiplets using <O|n>
- green: lat, black: exp

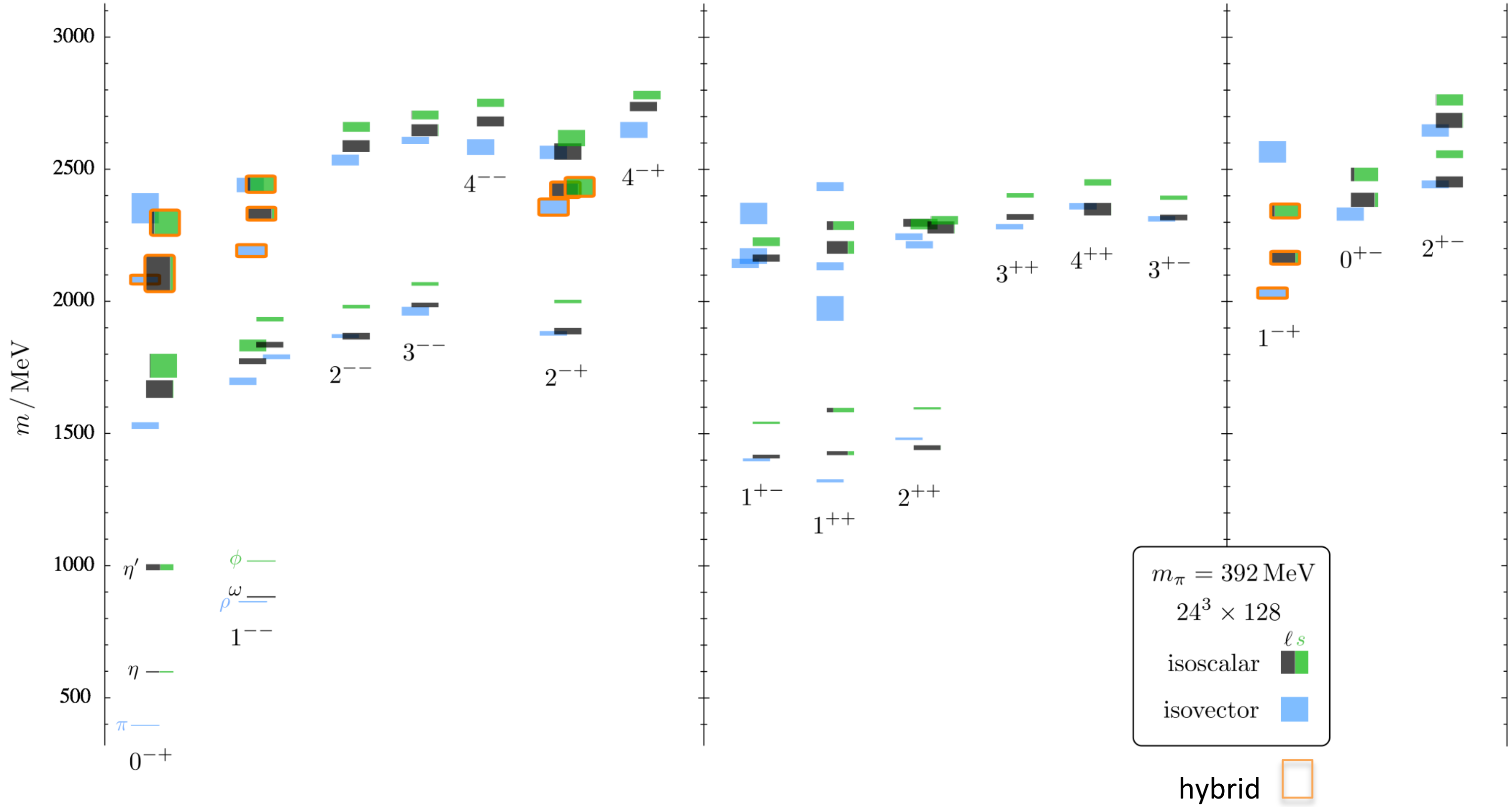
Hybrids:

some of them have exotic J^{PC}

large overlap with $O = \underline{q}\ F_{ij}\ q$



Isoscalar mesons: single hadron approximation

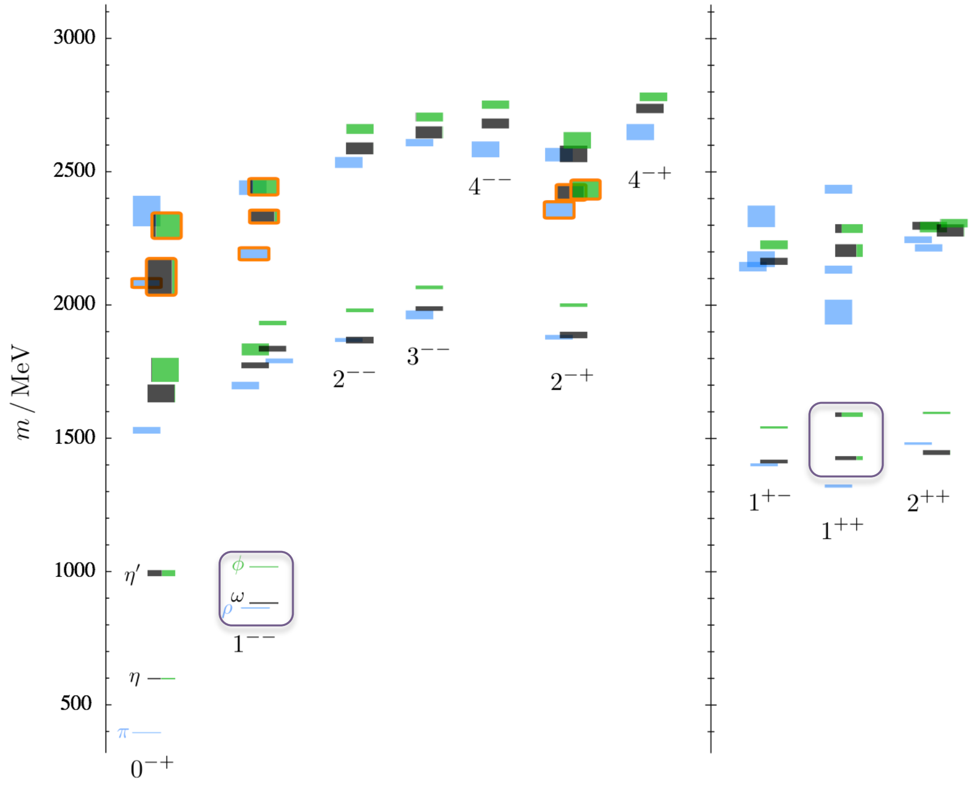


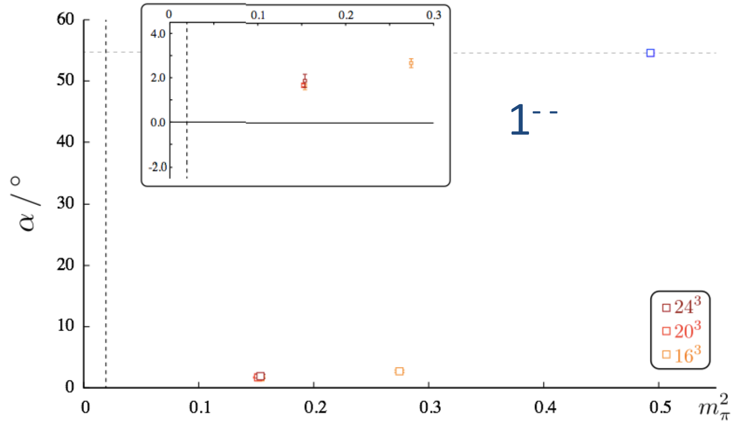
$$\begin{pmatrix} |\mathfrak{a}\rangle \\ |\mathfrak{b}\rangle \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} |\ell\rangle \\ |s\rangle \end{pmatrix}$$

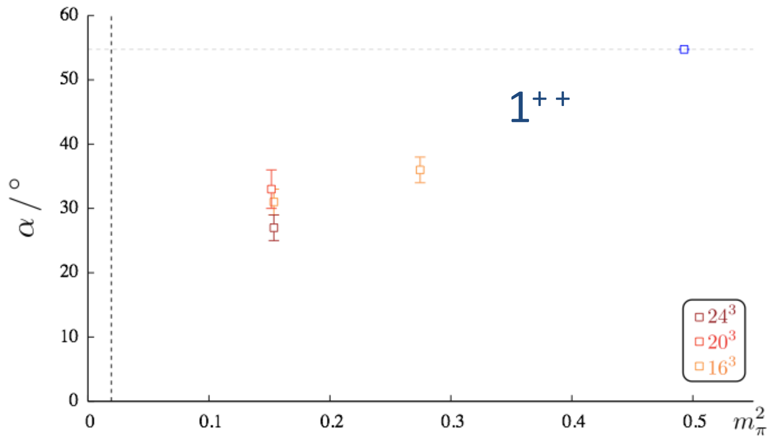
$$|\ell\rangle \equiv \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle\right)$$
$$|s\rangle \equiv |s\bar{s}\rangle$$

[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

Isoscalar mesons: mixing angle







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$$|s\rangle \equiv |s\bar{s}\rangle$$

[HSC : Dudek, Edward, Guo, Thomas: 1309.2608, PRD]

States near threshold: rigorous treatment

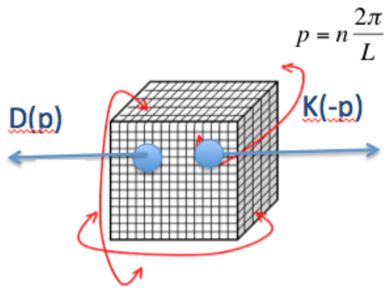
note: most of interesting states are found near threshold:

$D_{s0}^{*}(2317)$, $X(3872)$, Z_c^+, Z_b^+

Discrete energy spectrum from correlators

$$\langle C\rangle \propto \int DG\,Dq\,D\bar{q}\;C(q,\bar{q},G)\,e^{i S_{QCD}/\hbar},\quad S_{QCD}=\int d^4x\,L_{QCD}$$

Example: meson channel with given J^{PC}



$$\mathcal{O}=\bar{q}\Gamma q,\quad \bar{q}\Gamma' q,\quad (\bar{q}\Gamma_1 q)(\bar{q}\Gamma_2 q),\ldots$$

$$C_{ij}(t)=\langle 0|\,\mathcal{O}_i(t)\;\mathcal{O}_j^{+}(0)\,|0\rangle$$

$$=\sum_n \langle 0|\mathcal{O}_i|n\rangle\; e^{-E_n t}\,\langle n|\mathcal{O}_j^{+}|0\rangle=\sum_n Z_i^n\,Z_j^{n*}\,e^{-E_n t}\qquad Z_i^n=\langle 0|\mathcal{O}_i|n\rangle$$

All physical states with given J^{PC} appear as energy levels E_n in principle : single particle, two-particle,...

channel :	"eigenstates"
$J^{PC}=0^{++},\ \bar{s}c:$	$D_{s0}^*(2317),\ DK$
$J^{PC}=1^{+-},\ \bar{c}c\bar{d}u:$	$Z_c^+,\quad J/\psi\,\pi^+,\ldots$
$J^{PC}=1^{--},\ \bar{s}u:$	$K^*(892),\quad K\pi$

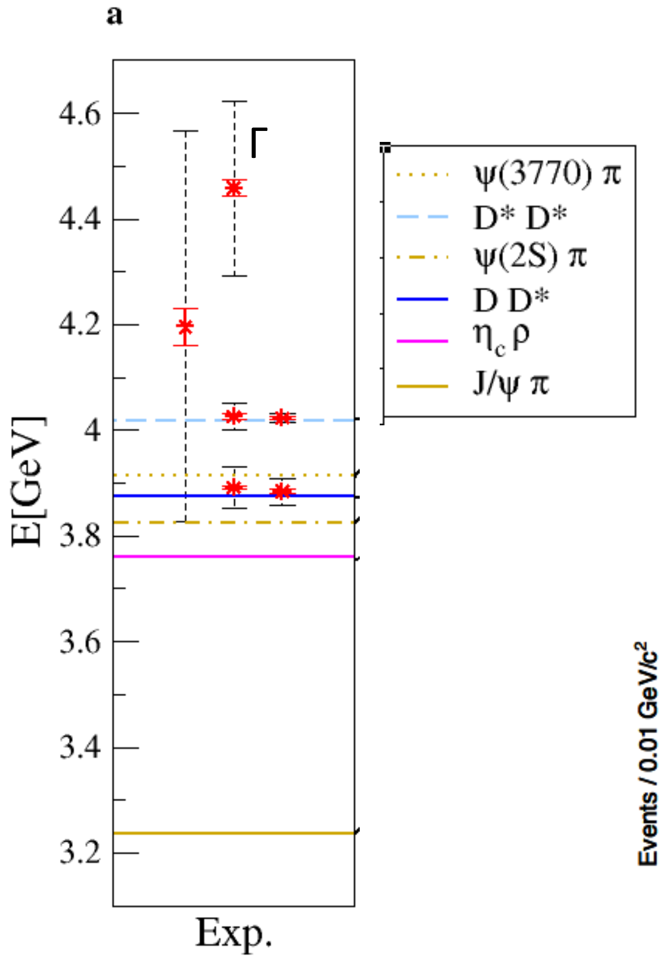
In experiment: these correspond to two-meson decay products with continous spectrum.

On lattice: these are discrete due to finite box and periodic BC.

Charged charmonium Z_c^+ : experimental status

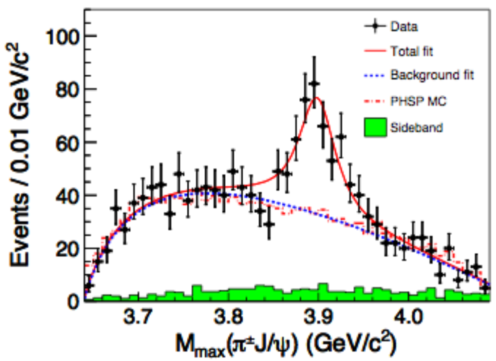
$\bar{c}\,u$
$c\,\bar{d}$

candidates with
preferred
$I^G=1^+,\ J^{PC}=1^{+-}$



[review: Brambilla et al., 1404.3723]

particle	C	J^P	decay	year	coll
$Z^+(4430)$	-	1+	$\psi(2S)\ \pi^+$	2008	Belle, ~~BABAR~~, LHCb
$Z_c^+(3900)$	-	?	$J/\psi\ \ \pi^+$	2013	BESIII, Belle, CLEOc
$Z_c^+(3885)$	-	1+	$(DD^*)^+$	2013	BESIII
$Z_c^+(4020)$	-	?	$h_c(1P)\ \pi^+$	2013	BESIII
$Z_c^+(4025)$	-	?	$(D^*\ D^*)^+$	2013	BES III
$Z^+(4200)$	-	1+	$J/\psi\ \ \pi^+$	2014	Belle
$Z^+(4050)$	+	?	$\chi_{c1}\ \ \pi^+$	2008	Belle
$Z^+(4250)$	+	?	$\chi_{c1}\ \ \pi^+$	2008	Belle



[BESIII, 2013, 1303.5949, PRL]

$Z_c^+(3900)\ \rightarrow\ \ J/\Psi\ \ \pi^+$

$c\bar{c}\ \ \bar{d}u$

$\bar{c}\,u$
$c\,\bar{d}$

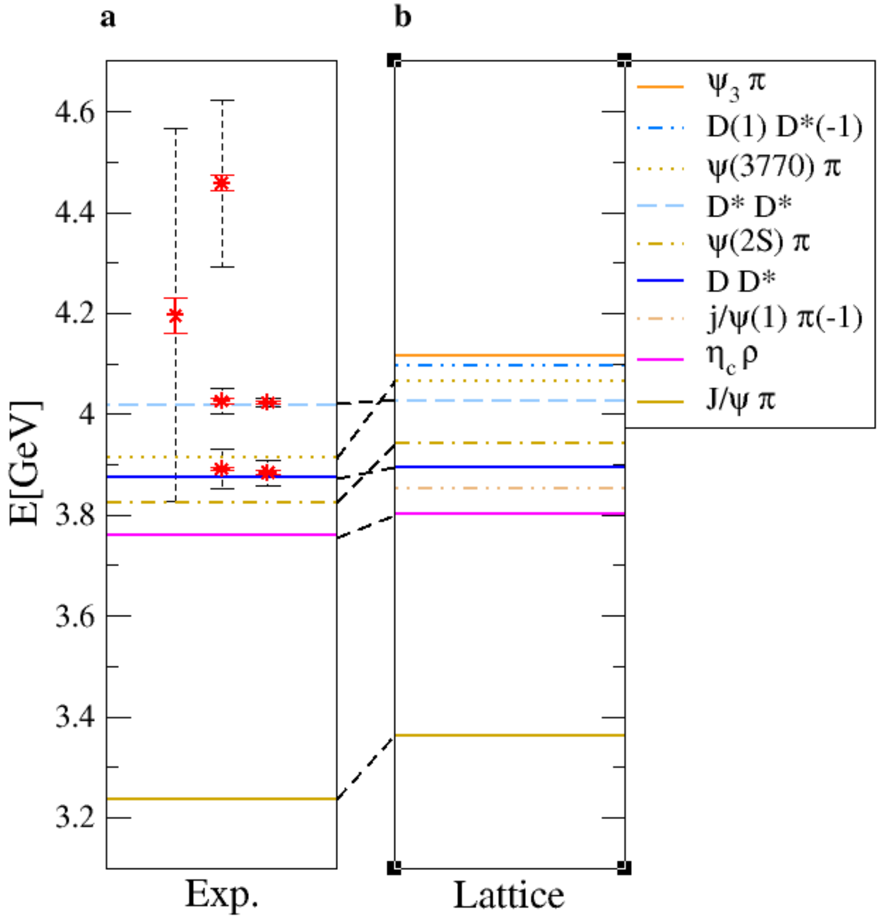
Previous searches for Zc+ from lattice

- Search in $J^{PC}=1^{+-}$ channel for m < 4 GeV:
  no Z_c^+ candidate found
  [S.P. & L. Leskovec, 1308.2097, PLB]

- Search for resonance in D$\underline{D}$* scattering with $J^{PC}=1^{+-}$ near threshold E ~ 3.9 GeV
  no Z_c^+ candidate found
  [Y. Chen et al, 1403.1318, PRD]

First evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$

$\begin{matrix} \bar{c} & u \\ c & \bar{d} \end{matrix}$



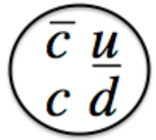
Lattice:

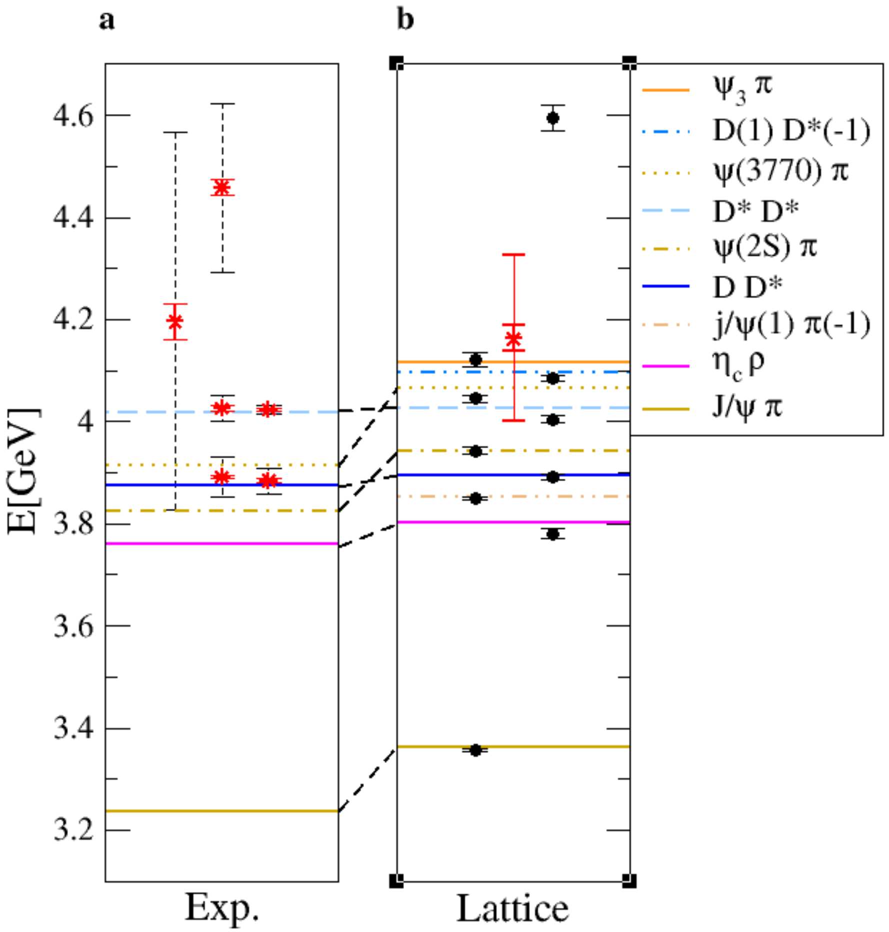
Energies of all two-particle states with E<4.3 GeV

[S.P., Lang, Leskovec, Mohler, 1405.7623]

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

First evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$

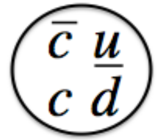


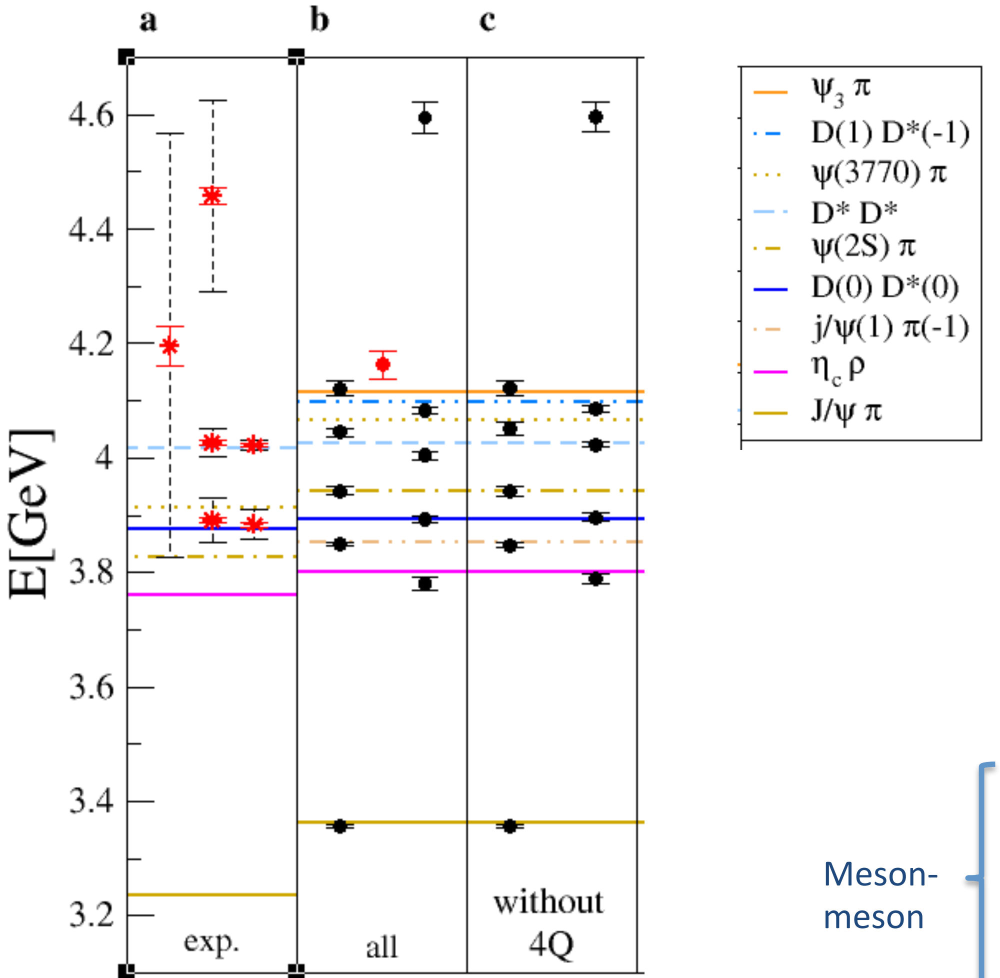


[S.P., Lang, Leskovec, Mohler, 1405.7623]

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

First evidence for Z_c^+ from lattice: $I^G=1^+$, $J^{PC}=1^{+-}$





Exp. Lattice

[S.P., Lang, Leskovec, Mohler, 1405.7623]

Meson-meson

$$O_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)$$

$$O^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,$$

$$O_1^{D(0)D^*(0)} = \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}$$

and 11 others ..

Diquark-antidiquark

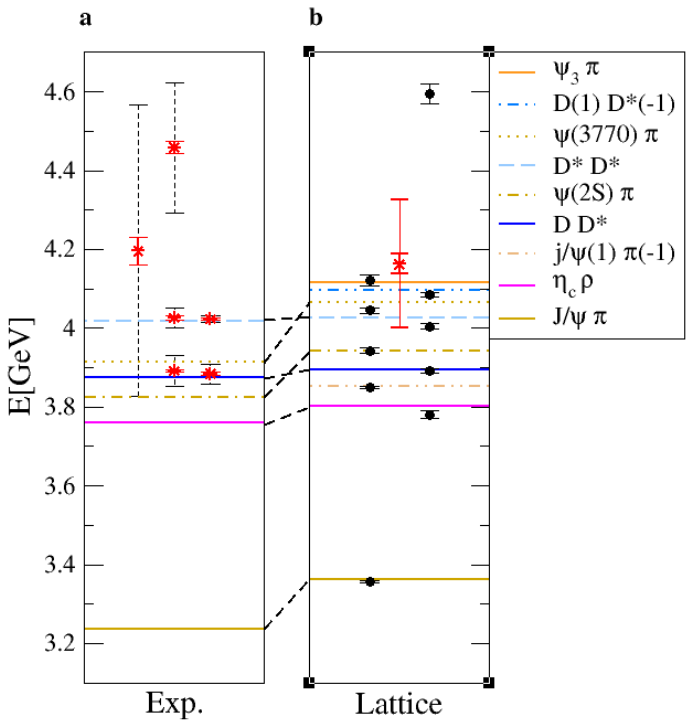
$$O_1^{4q} \approx [\bar{c}\ C\gamma_5\ \bar{d}]_{3_c} [c\ \gamma_i C\ u]_{\bar{3}c}$$

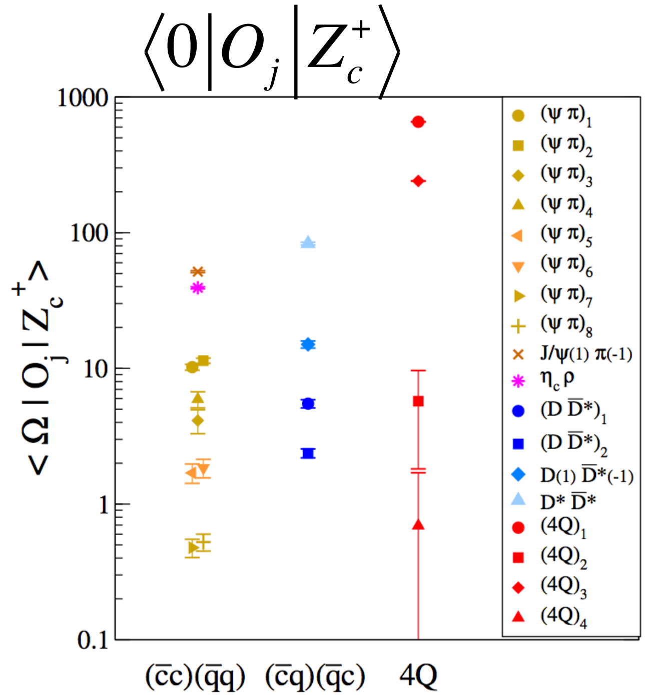
$$O_2^{4q} \approx [\bar{c}\ C\ \bar{d}]_{3_c} [c\ \gamma_i \gamma_5 C\ u]_{\bar{3}c}$$

and 2 others ..

First evidence for Z_c^+ from lat: $I^G=1^+$, $J^{PC}=1^{+-}$

$\bar{c}\, u$
$c\, \bar{d}$





$$O_1^{4q} \approx [\bar{c}\, C\gamma_5\, \bar{d}]_{3_c} [c\, \gamma_i C\, u]_{\bar{3}c}$$

$$O_2^{4q} \approx [\bar{c}\, C\, \bar{d}]_{3_c} [c\, \gamma_i \gamma_5 C\, u]_{\bar{3}c}$$

Nearby experimental candidates:

Zc+(4020), Γ=7.9 ± 3.7 MeV BESIII 2013

Zc+(4025), Γ=24.8 ± 9.5 MeV BESIII 2013

Zc+(4200), Γ=370 ± 110 MeV Belle, Moriond 2014

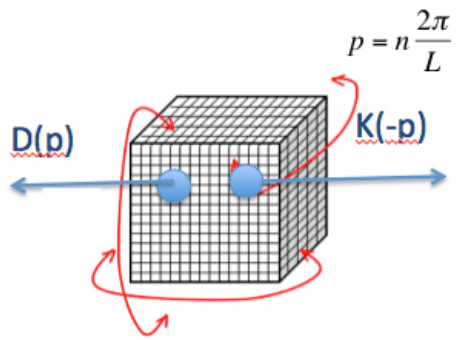
Lattice (m_π =266 MeV, Nf=2) :

$m(Z_c^+)$ = 4.16 GeV

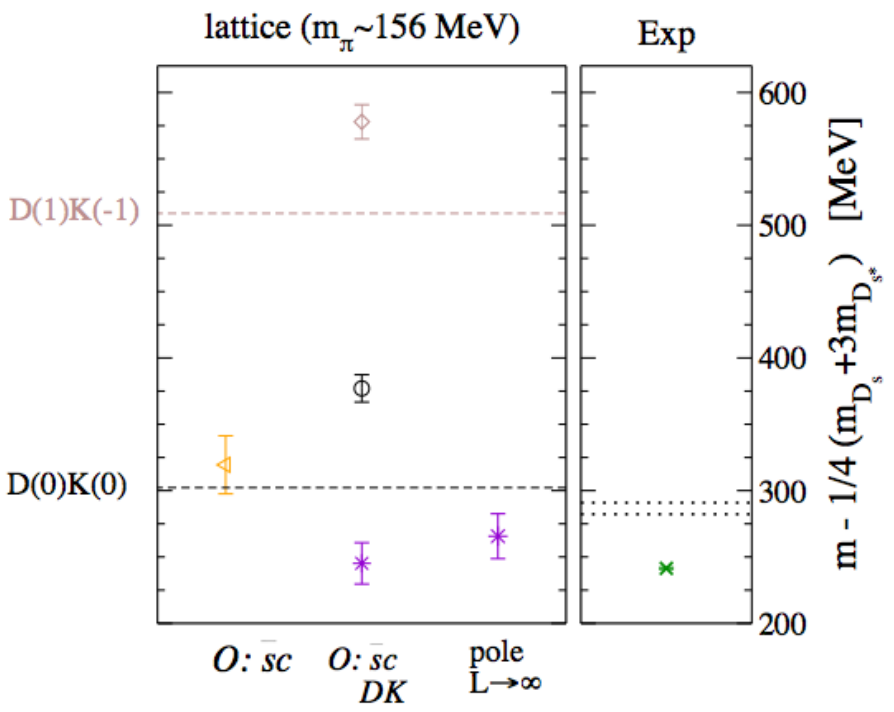
± 0.163 GeV ± O(Γ)

[S.P., Lang, Leskovec, Mohler, 1405.7623]

$D_{s0}^{*}(2317)$ and DK scattering : $J^P=0^+$



$$\mathcal{O}: \ \bar{s}\, c, \quad DK \approx [\bar{\mathrm{d}}\gamma_5 c]\ [\bar{\mathrm{s}}\gamma_5 d]$$



Rigorous relation [M. Luscher , 1991]:

$E \rightarrow \delta(E)$ phase shift for DK scattering in s-wave

$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2$$

pole position and mass of $D_{s0}^{*}(2317)$

$$S \propto \frac{1}{\cot\delta - i} = \infty \qquad \cot\delta(p_{BS}) = i$$

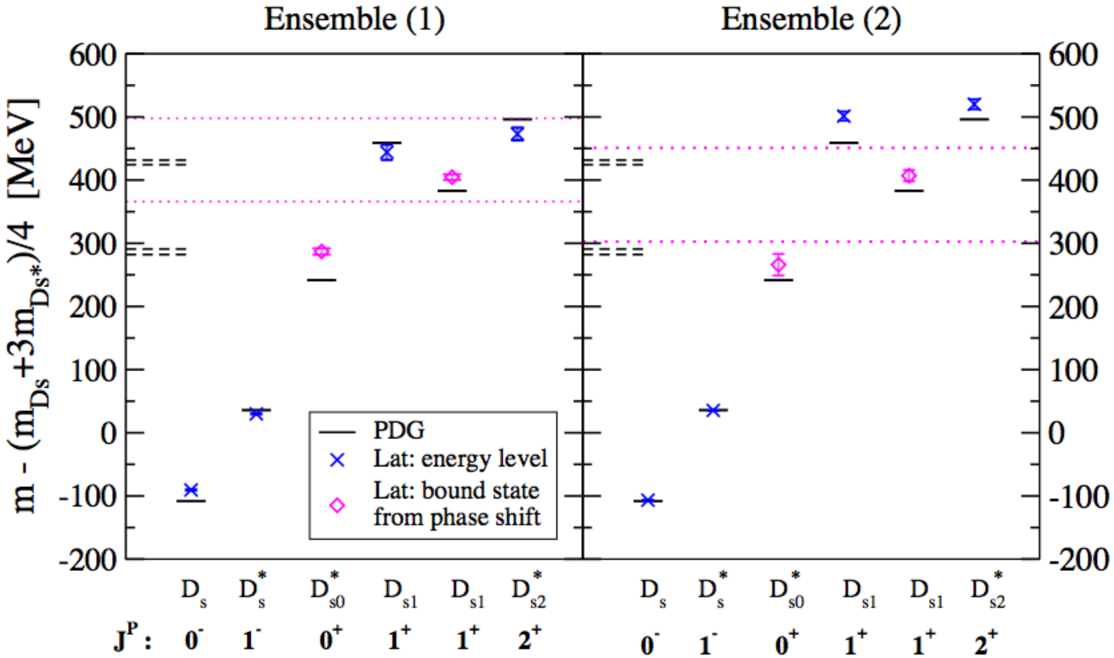
$$m_{D_{s0}}^{lat, L\to\infty} = E_D(p_{BS}) + E_K(p_{BS})$$

[D. Mohler, C. Lang, L. Leskovec, S.P. , R. Woloshyn: PRL 2013]

m_π≈156 MeV, L≈2.9 fm, Nf=2+1

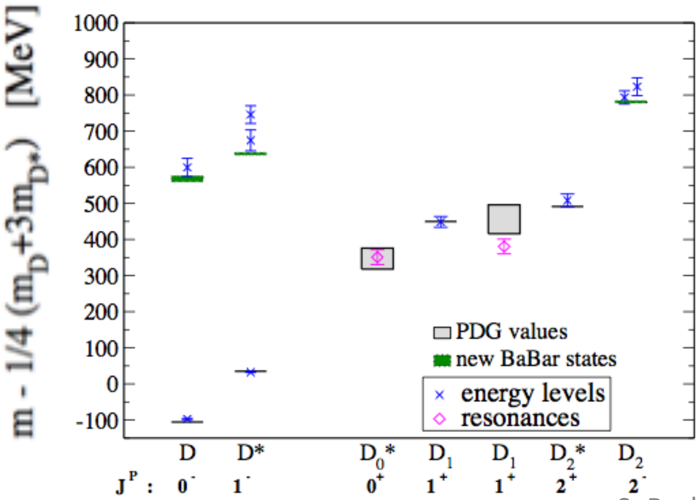
Charmed meson spectrum: rigorous treatment

$m_\pi \approx 266$ MeV, Nf=2 $m_\pi \approx 156$ MeV, Nf=2+1



Ds mesons (near-threshold)

[C. Lang, L. Leskovec, D. Mohler, S.P. , R. Woloshyn: PRL 2013, 1403.8103]



D mesons (resonances)

[D. Mohler, S.P. , R. Woloshyn: PRD 2013]

Charmed scalar meson "puzzle" revisited

- why do these scalar partners have mass so close ?

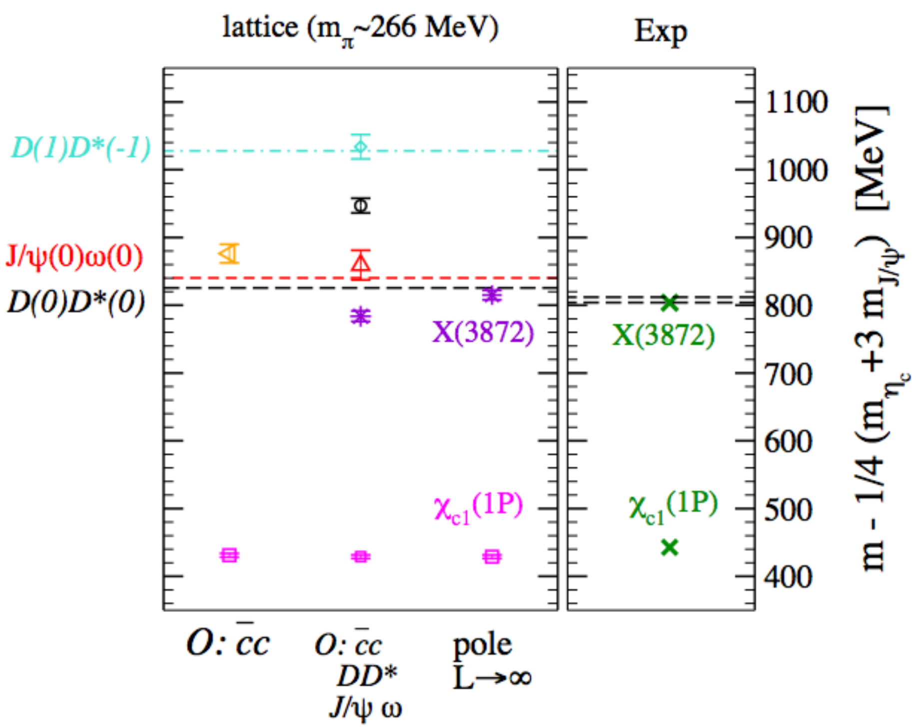
$D_0^*(2400): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 267 \text{ MeV} \quad \bar{c}u \text{ or } \bar{c}u\bar{s}s\,?$

$D_{s0}(2317): \quad M \approx 2318 \text{ MeV} \quad \Gamma \approx 0 \text{ MeV} \quad \bar{c}s \text{ or } \bar{c}s[\bar{u}u + \bar{d}d]\;?$

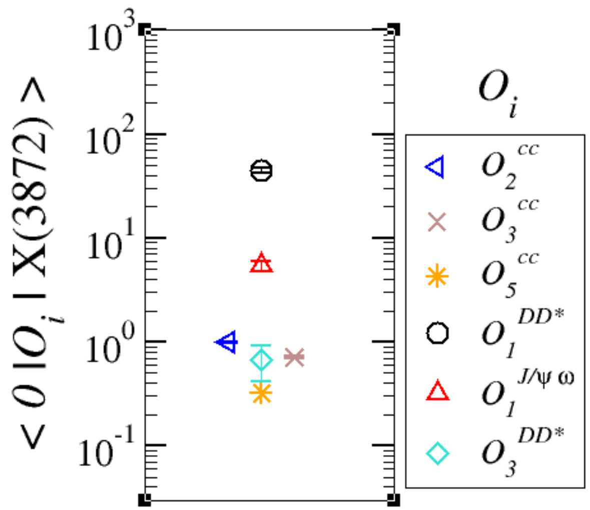
1) is D_0^* mass pushed up : valence ss pair ?? ✖
2) is D_{s0}^* mass pushed down : effect of DK threshold ?? ✔

First evidence for X(3872) from lattice : $J^{PC}=1^{++}$, I=0

$\mathcal{O}: \bar{c}\,c,\quad DD^{*},\quad J/\psi\omega$



$$\langle 0|O_j|X(3872)\rangle$$



Overlaps normalized to $\langle 0|O_1^{cc}\,|X(3872)\rangle$

X(3872)	m - $(m_{D0}+m_{D0^*})$
lat	- 11 ± 7 MeV
exp	- 0.14 ± 0.22 MeV

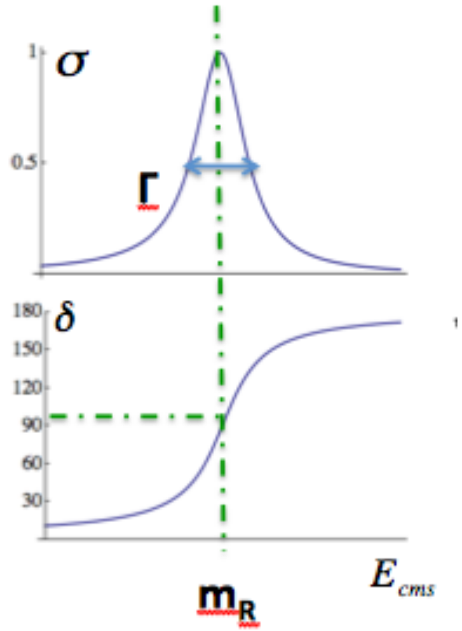
S. P. and L. Leskovec : 1307.5172, PRL

$m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

lat: simulations on larger L required

exp: Tomaradze et al., 1212.4191

States above threshold - resonances: rigorous treatment



Almost all mesons are resonances (decay strongly)

$\bar{u}u$	$\bar{s}u$	$\bar{c}u$
$\pi^\pm$	$K^\pm$	$D^\pm$
π^0	K^0	D^0
η	$K_S{}^0$	$D^*(2007)^0$
$f_0(500)$ or σ was $f_0(60$	$K_L{}^0$	$D^*(2010)^\pm$
$\rho(770)$	$K_0^*(800)$ or κ	$D_0^*(2400)^0$
$\omega(782)$	$K^*(892)$	$D_0^*(2400)^\pm$
$\eta'(958)$	$K_1(1270)$	$D_1(2420)^0$
$f_0(980)$	$K_1(1400)$	$D_1(2420)^\pm$
$a_0(980)$	$K^*(1410)$	$D_1(2430)^0$
$\phi(1020)$	$K_0^*(1430)$	$D_2^*(2460)^0$
$h_1(1170)$	$K_2^*(1430)$	$D_2^*(2460)^\pm$
$b_1(1235)$	$K(1460)$	$D(2550)^0$
$a_1(1260)$	$K_2(1580)$	$D(2600)$
$f_2(1270)$	$K(1630)$	$D^*(2640)^\pm$
$f_1(1285)$	$K_1(1650)$	$D(2750)$
$\eta(1295)$	$K^*(1680)$	
$\pi(1300)$	$K_2(1770)$	
$a_2(1320)$	$K_3^*(1780)$	
$f_0(1370)$	$K_2(1820)$	
$h_1(1380)$	$K(1830)$	
$\pi_1(1400)$		
$\eta(1405)$		
$f_1(1420)$		
$\omega(1420)$		
$f_2(1430)$		
$a_0(1450)$		
$\rho(1450)$		

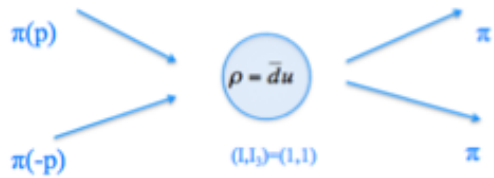
- stable on strong decay: ab-initio OK
- others decay strongly; hadronic resonances

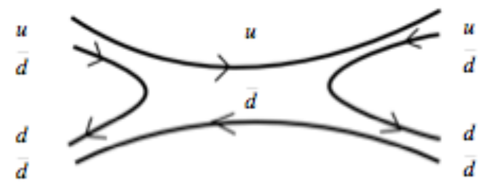
only resonance simulated properly by several collab. (first simulation: CP-PACS 2007)

SP and coll. (Lang, Mohler, Leskovec, Woloshyn) (2011- 2014)

- Most resonances simulated only using single-hadron app.
- Only few treated rigorously

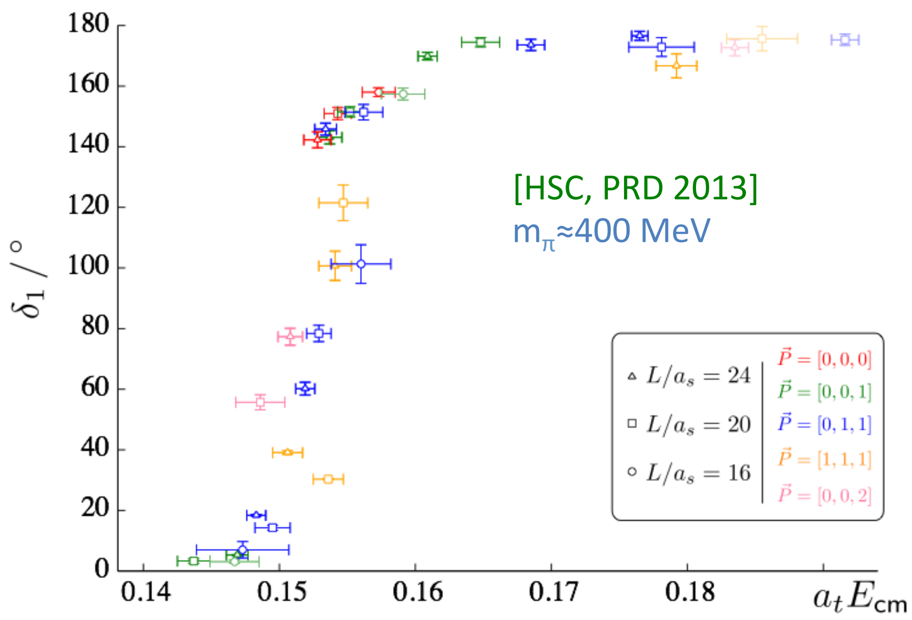
ρ resonance



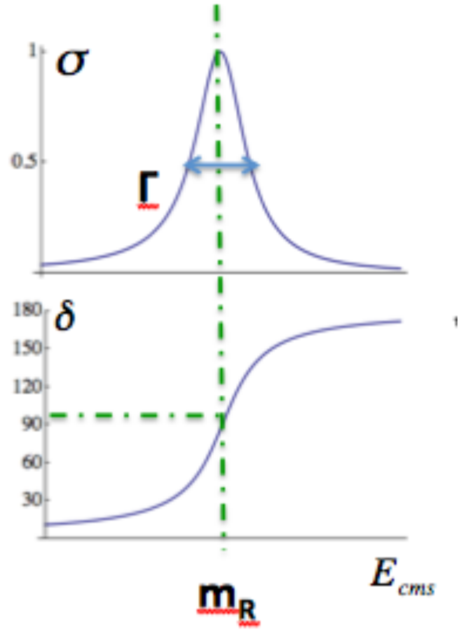


P≠0: $s=E^2-P^2$, Luscher-type relation $s \to \delta(s)$

$m_1=m_2$ [Rummukainen, Gotlieb 1995]



[HSC, PRD 2013]
m_π≈400 MeV

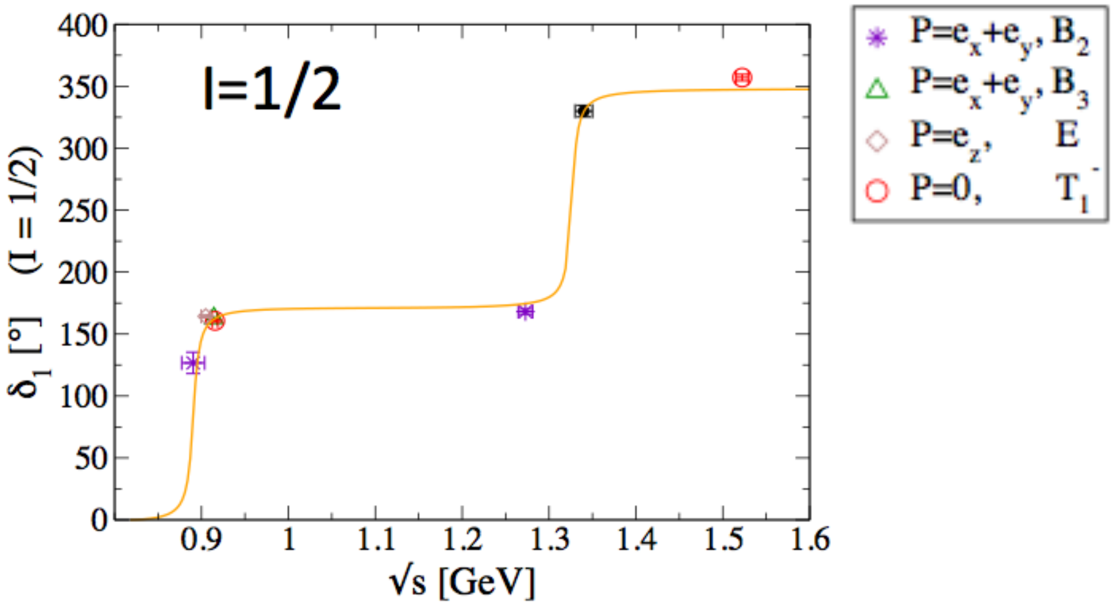


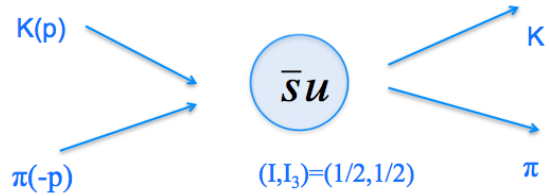
Simulation also by CP-PACS, PACS-CS, QCDSF, ETMC

[Lang, Mohler, S.P. ,Vidmar, PRD 2011] m_π≈266 MeV

K*(892) resonance: first lat determination of the width

Kπ , I=1/2: p-wave phase shift





$$\text{BW}: \quad \delta = \text{acot}\frac{m_R^2 - E_{cms}^2}{m_R\,\Gamma}$$

$$\Gamma[K^* \to K\pi] = \frac{g^2}{6\pi}\frac{p^{*3}}{s}$$

	$m_{K^*(892)}$ [MeV]	$g_{K^*(892)}$ [no unit]
lat	891 ± 14	5.7 ± 1.6
exp	891.66 ± 0.26	5.72 ± 0.06

[S.P. ,Lang, Leskovec, Mohler, 1307.0736, PRD 2013]

$m_\pi \approx 266$ MeV

D-meson resonance masses and widths

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

g is compared to exp instead of Γ (Γ depends on phase sp. and m_π)

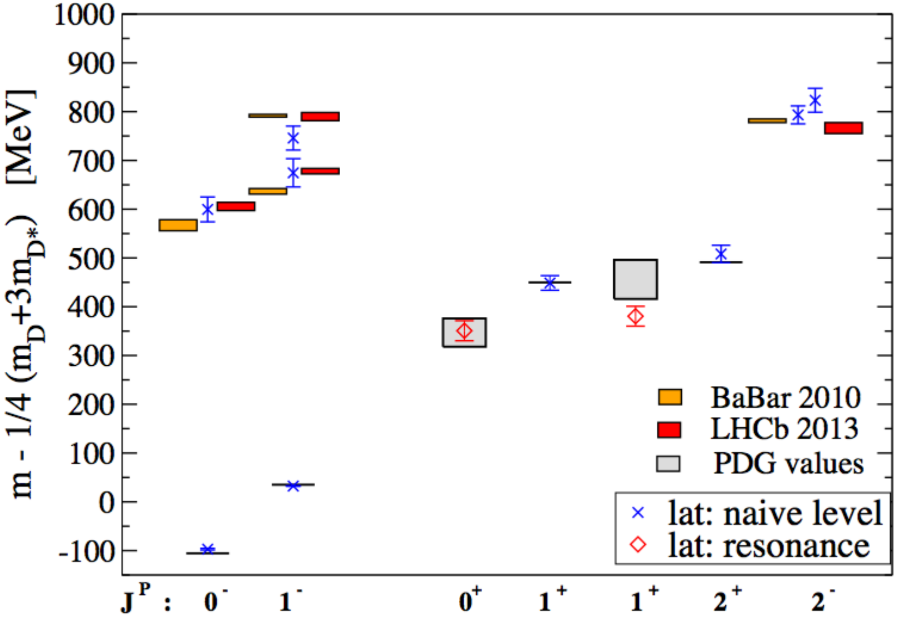
(analysis of spectrum in this case is based on an assumption given in paper below)

$J^P=0^+$: D π

$D_0^*(2400)$	m - 1/4(mD+3 mD*)	g
lat	351 ± 21 MeV	2.55 ± 0.21 GeV
exp	347 ± 29 MeV	1.92 ± 0.14 GeV

$J^P=1^+$: D* π

$D_1(2430)$	m - 1/4(mD+3 mD*)	g
lat	381 ± 20 MeV	2.01 ± 0.15 GeV
exp	456 ± 40 MeV	2.50 ± 0.40 GeV



first lattice result for strong decay width of a hadron containing charm quark

[D. Mohler, S.P. , R. Woloshyn: 1208.4059, PRD]

- $m_\pi \approx 266$ MeV, L≈2 fm, Nf=2

Lightest axial resonances a_1(1260) and b_1(1235)

Simulating scattering:

ρπ in 1++ channel to extract a1

ωπ in 1+- channel to extract b1

$$\Gamma(E) \equiv g^2 \frac{p}{E^2}$$

resonance	$a_1(1260)$			$b_1(1235)$	
quantity	$m_{a_1}^{\rm res}$ [GeV]	$g_{a_1\rho\pi}$ [GeV]	$a_{l=0}^{\rho\pi}$ [fm]	$m_{b_1}^{\rm res}$ [GeV]	$g_{b_1\omega\pi}$ [GeV]
lat	$1.435(53)(^{+0}_{-109})$	1.71(39)	0.62(28)	$1.414(36)(^{+0}_{-83})$	input
exp	1.230(40)	1.35(30)	-	1.2295(32)	0.787(25)

[Lang, Leskovec, Mohler, S.P. , 1401.2088, JHEP]

Conclusions

Lattice QCD is the only non-perturbative theoretical tool that depends on the same parameters as QCD Lagrangian.

Available already for some time

- Precise result for states well below threshold

Important developments during past two years

a) Extensive results for excited multiplets within single-hadron approximation

b) Effect of threshold on near-threshold states: crucial for (c) and (d)

c) First evidence for an exotic Z_c^+ with two valence quarks and antiquarks

d) First evidence for X(3872)

e) First determination of Γ for strange and charmed meson resonances

Thanks to my collaborators

Christian B. Lang (Graz)
Daniel Mohler (Fermilab)
Luka Leskovec (Ljubljana)
Richard Woloshyn (Triumf)

Backup slides

[review: Brambilla et al., 1404.3723]

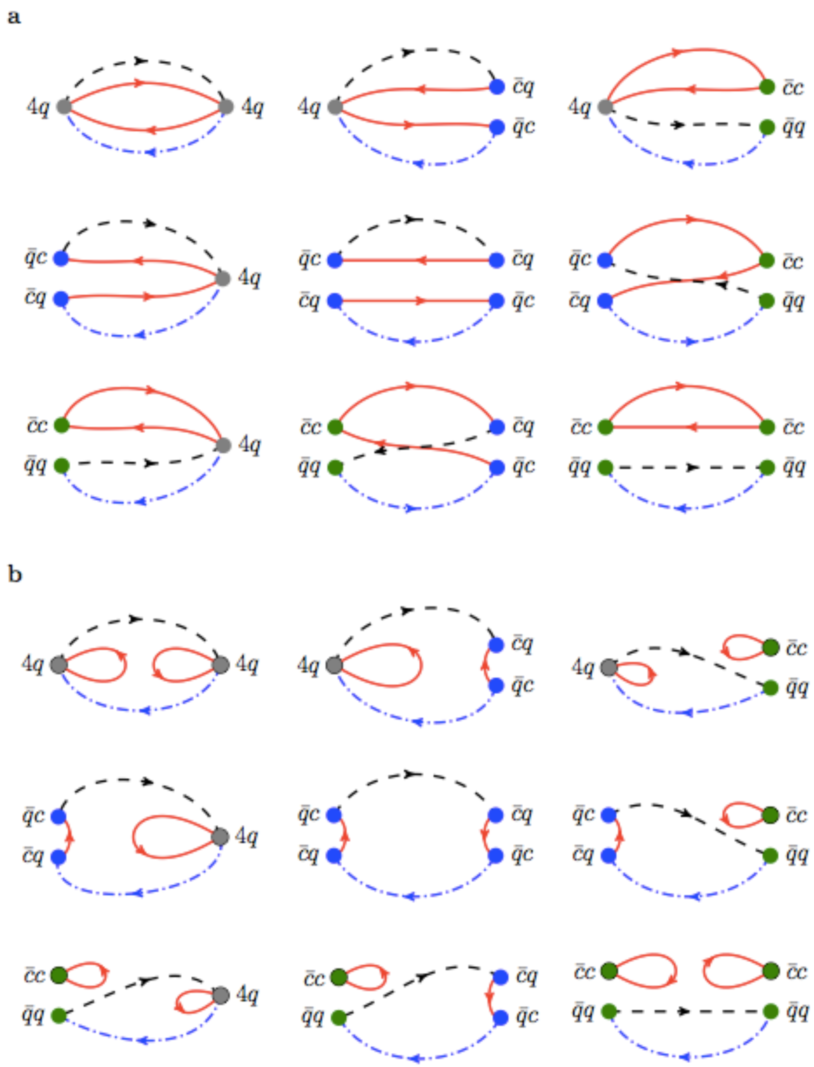
TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.68 ± 0.17	< 1.2	1^{++}	$B \to K(\pi^+\pi^- J/\psi)$	Belle [772, 992] (>10), BaBar [993] (8.6)	2003	Ok
				$p\bar{p} \to (\pi^+\pi^- J/\psi) \ldots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \to (\pi^+\pi^- J/\psi) \ldots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] (4.3), BaBar [1000] (4.0)	2005	Ok
				$B \to K(\gamma J/\psi)$	Belle [1001] (5.5), BaBar [1002] (3.5) LHCb [1003] (> 10)	2005	Ok
				$B \to K(\gamma \psi(2S))$	BaBar [1002] (3.6), Belle [1001] (0.2) LHCb [1003] (4.4)	2008	NC!
				$B \to K(D\bar{D}^*)$	Belle [1004] (6.4), BaBar [1005] (4.9)	2006	Ok
$Z_c(3885)^+$	3883.9 ± 4.5	25 ± 12	1^{+-}	$Y(4260) \to \pi^-(D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	3891.2 ± 3.3	40 ± 8	$?^{?-}$	$Y(4260) \to \pi^-(\pi^+ J/\psi)$	BES III [1007] (8), Belle [1008] (5.2) T. Xiao *et al.* [CLEO data] [1009] (>5)	2013	Ok
$Z_c(4020)^+$	4022.9 ± 2.8	7.9 ± 3.7	$?^{?-}$	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	4026.3 ± 4.5	24.8 ± 9.5	$?^{?-}$	$Y(4260) \to \pi^-(D^*\bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	10607.2 ± 2.0	18.4 ± 2.4	1^{+-}	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle [1012–1014] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B\bar{B}^*)^+$	Belle [1015] (8)	2012	NC!
$Z_b(10650)^+$	10652.2 ± 1.5	11.5 ± 2.2	1^{+-}	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle [1012, 1013] (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^-(B^*\bar{B}^*)^+$	Belle [1015] (6.8)	2012	NC!

[review: Brambilla et al., 1404.3723]

State	M, MeV	Γ, MeV	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$Y(3915)$	3918.4 ± 1.9	20 ± 5	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	3927.2 ± 2.6	24 ± 6	2^{++}	$e^+e^- \to e^+e^-(D\bar{D})$	Belle [1054] (5.3), BaBar [1055] (5.8)	2005	Ok
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi\,(D\bar{D}^*)$	Belle [1048, 1049] (6)	2005	NC!
$Y(4008)$	3891 ± 42	255 ± 42	1^{--}	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	4039 ± 1	80 ± 10	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.0)	2013	NC!
$Z(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (1.1)	2008	NC!
$Y(4140)$	4145.8 ± 2.6	18 ± 8	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (>5) D0 [1064] (3.1)	2009	NC!
$\psi(4160)$	4153 ± 3	103 ± 8	1^{--}	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle [1057] (6.5)	2013	NC!
$X(4160)$	4156^{+29}_{-25}	139^{+113}_{-65}	$?^{?+}$	$e^+e^- \to J/\psi\,(D^*\bar{D}^*)$	Belle [1049] (5.5)	2007	NC!
$Z(4200)^+$	4196^{+35}_{-30}	370^{+99}_{-110}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (7.2)	2014	NC!
$Z(4250)^+$	4248^{+185}_{-45}	177^{+321}_{-72}	$?^{?+}$	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle [1058] (5.0), BaBar [1059] (2.0)	2008	NC!
$Y(4260)$	4250 ± 9	108 ± 12	1^{--}	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11) Belle [1008, 1056] (15), BES III [1007] (np)	2005	Ok
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar [1067] (np), Belle [1008] (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III [1007] (8), Belle [1008] (5.2)	2013	Ok
				$e^+e^- \to (\gamma\, X(3872))$	BES III [1070] (5.3)	2013	NC!
$Y(4274)$	4293 ± 20	35 ± 16	$?^{?+}$	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (>3), D0 [1064] (np)	2011	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	13^{+18}_{-10}	$0/2^{?+}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [1071] (3.2)	2009	NC!
$Y(4360)$	4354 ± 11	78 ± 16	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (8), BaBar [1073] (np)	2007	Ok
$Z(4430)^+$	4458 ± 15	166^{+37}_{-32}	1^{+-}	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle [1074, 1075] (6.4), BaBar [1076] (2.4) LHCb [1077] (13.9)	2007	Ok
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle [1065] (4.0)	2014	NC!
$X(4630)$	4634^{+9}_{-11}	92^{+41}_{-32}	1^{--}	$e^+e^- \to (\Lambda_c^+\bar{\Lambda}_c^-)$	Belle [1078] (8.2)	2007	NC!
$Y(4660)$	4665 ± 10	53 ± 14	1^{--}	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle [1072] (5.8), BaBar [1073] (5)	2007	Ok
$\Upsilon(10860)$	10876 ± 11	55 ± 28	1^{--}	$e^+e^- \to (B_{(s)}^{(*)}\bar{B}_{(s)}^{(*)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S,2S,3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \to (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta\Upsilon(1S,2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	10888.4 ± 3.0	$30.7^{+8.9}_{-7.7}$	1^{--}	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!

Wick contractions for Zc+

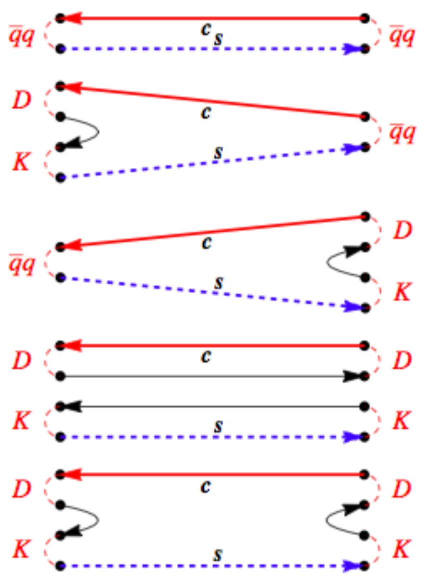


$D_{s0}^{*}(2317)$ with $J^P=0^+$

$$\mathcal{O} = \bar{s}\, c$$

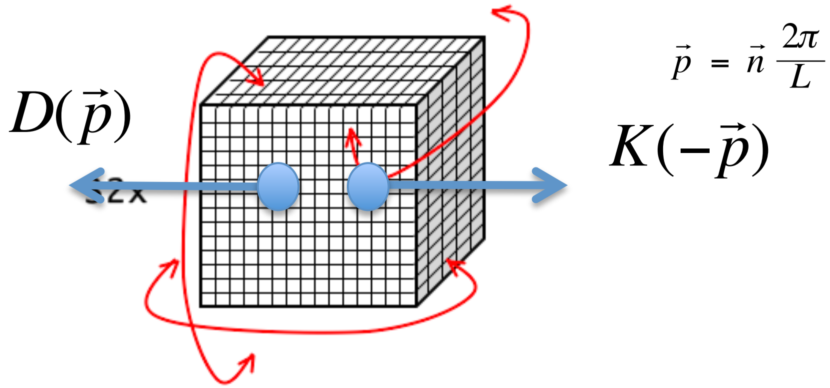
$$\mathcal{O} = DK \approx [\bar{d}\gamma_5 c]\,[\bar{s}\gamma_5 d]$$

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\, \mathcal{O}_j^{+}(0)|0\rangle$$



We **use distillation method**

[Peardon et al. 2009] to calculate C_{ij}



Extract E_n from $C_{ij}(t)$: variational method

$$C_{ij}(t) = \sum_n A_n^{ij}\, e^{-E_n t}$$

due to strong int.

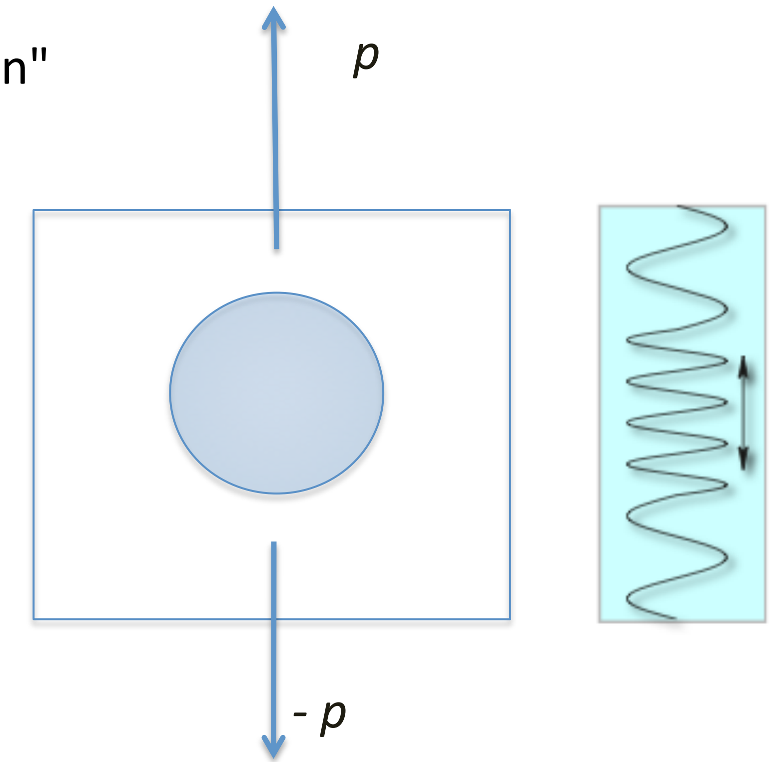
$$\vec{p} = \vec{n}\,\frac{2\pi}{L} \qquad E(L) = \sqrt{m_D^2 + \vec{p}^{\,2}} + \sqrt{m_K^2 + (-\vec{p})^2} + \Delta E$$

Energy levels that appear in addition to these discrete two-particle states correspond to bound states or resonances

Meson-meson scattering: extracting δ(p) from E_n at $P=p_1+p_2=0$ [M. Luscher,1991]

- extract $E_n(L)$
- E_n renders p in the region "outside-interaction"

$$E=\sqrt{s}=\sqrt{m_1^2+p^2}+\sqrt{m_2^2+p^2}$$

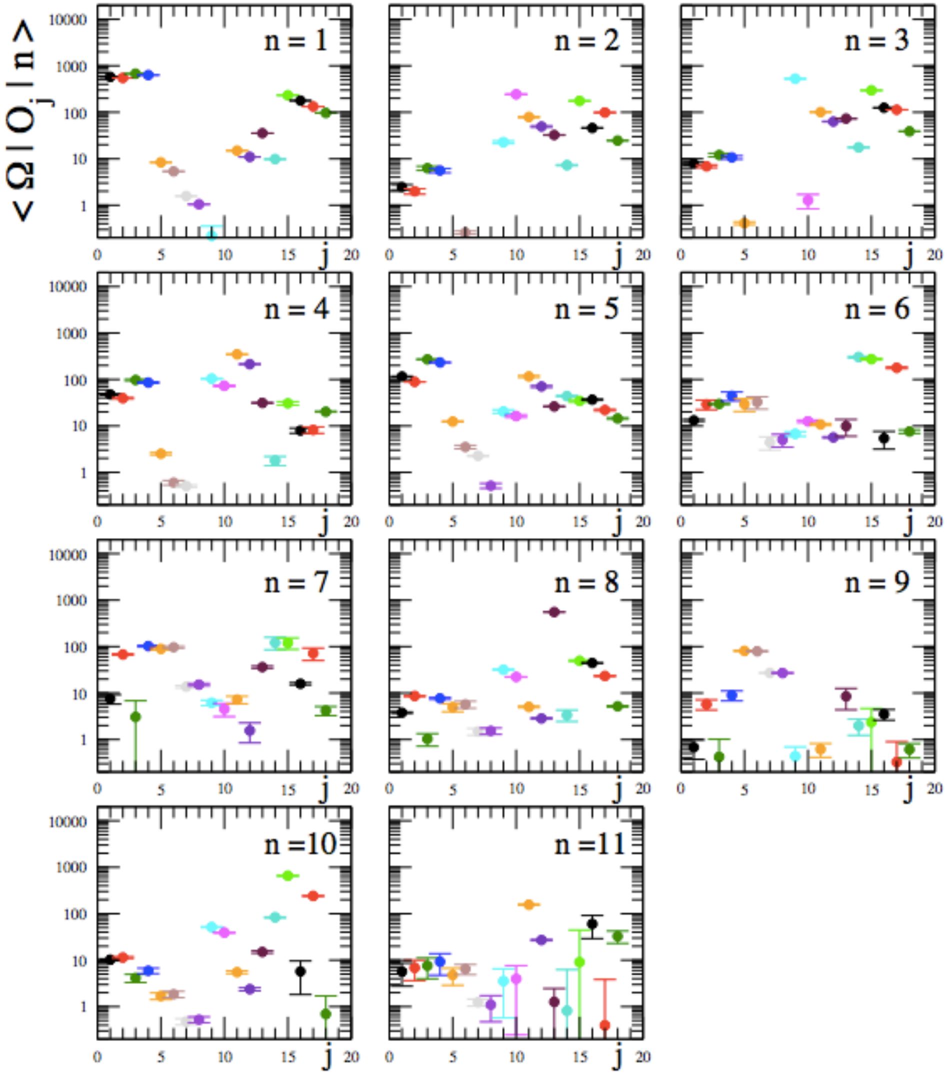


- p contains info on δ(p) for periodic BC

$$\det[\delta_{ll'}\delta_{mm'}-\tan\delta_l(p)\,M_{ll',mm'}(q^2)]=0$$

$$\tan\delta(p)=\frac{\pi^{3/2}q}{Z_{00}(1;q^2)} \qquad Z_{00}(1;q^2)\equiv\sum_{\vec{n}\in N^3}\frac{1}{\vec{n}^2-q^2} \qquad q\equiv\frac{L}{2\pi}p$$

Overlaps of all states in Zc+ channel



Interpolators in Zc channel

$$\mathcal{O}_1 = \mathcal{O}_1^{\psi(0)\pi(0)} = \bar{c}\gamma_i c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_2 = \mathcal{O}_2^{\psi(0)\pi(0)} = \bar{c}\gamma_i\gamma_t c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_3 = \mathcal{O}_3^{\psi(0)\pi(0)} = \bar{c}\overleftarrow{\nabla}_j\gamma_i\overrightarrow{\nabla}_j c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_4 = \mathcal{O}_4^{\psi(0)\pi(0)} = \bar{c}\overleftarrow{\nabla}_j\gamma_i\gamma_t\overrightarrow{\nabla}_j c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_5 = \mathcal{O}_5^{\psi(0)\pi(0)} = |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_6 = \mathcal{O}_6^{\psi(0)\pi(0)} = |\epsilon_{ijk}||\epsilon_{klm}|\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c(0)\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_7 = \mathcal{O}_7^{\psi(0)\pi(0)} = R_{ijk}Q_{klm}\ \bar{c}\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c\ \bar{d}\gamma_5 u(0)\,,$$
$$\mathcal{O}_8 = \mathcal{O}_8^{\psi(0)\pi(0)} = R_{ijk}Q_{klm}\ \bar{c}\gamma_t\gamma_j\overleftarrow{\nabla}_l\overrightarrow{\nabla}_m c\ \bar{d}\gamma_5 u(0)\,.$$
$$\mathcal{O}_9 = \mathcal{O}^{\psi(1)\pi(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_i c(e_k)\ \bar{d}\gamma_5 u(-e_k)\,,$$
$$\mathcal{O}_{10} = \mathcal{O}^{\eta_c(0)\rho(0)} = \bar{c}\gamma_5 c(0)\ \bar{d}\gamma_i u(0)\,,$$
$$\mathcal{O}_{11} = \mathcal{O}_1^{D(0)D^*(0)} = \bar{c}\gamma_5 u(0)\ \bar{d}\gamma_i c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$
$$\mathcal{O}_{12} = \mathcal{O}_2^{D(0)D^*(0)} = \bar{c}\gamma_5\gamma_t u(0)\ \bar{d}\gamma_i\gamma_t c(0) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$
$$\mathcal{O}_{13} = \mathcal{O}^{D(1)D^*(-1)} = \sum_{e_k=\pm e_{x,y,z}} \bar{c}\gamma_5 u(e_k)\ \bar{d}\gamma_i c(-e_k) + \{\gamma_5 \leftrightarrow \gamma_i\}\,,$$
$$\mathcal{O}_{14} = \mathcal{O}^{D^*(0)D^*(0)} = \epsilon_{ijk}\ \bar{c}\gamma_j u(0)\ \bar{d}\gamma_k c(0)\,,$$
$$\mathcal{O}_{15} = \mathcal{O}_1^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\gamma_5\bar{d}_c\ c_{b'}\gamma_i C u_{c'} - \bar{c}_b C\gamma_i\bar{d}_c\ c_{b'}\gamma_5 C u_{c'})$$
$$\mathcal{O}_{16} = \mathcal{O}_2^{4q} = N_L^3\ \epsilon_{abc}\epsilon_{ab'c'}(\bar{c}_b C\bar{d}_c\ c_{b'}\gamma_i\gamma_5 C u_{c'} - \bar{c}_b C\gamma_i\gamma_5\bar{d}_c\ c_{b'} C u_{c'})$$
$$\mathcal{O}_{17} = \mathcal{O}_3^{4q} = \mathcal{O}_1^{4q}(N_v{=}32)\,,$$
$$\mathcal{O}_{18} = \mathcal{O}_4^{4q} = \mathcal{O}_2^{4q}(N_v{=}32)\,.$$