Images of Dynamical Chiral Symmetry Breaking

Ian Cloët

Argonne National Laboratory

Collaborators

Lei Chang	–	Adelaide
Craig Roberts	–	ANL
Sebastian Schmidt	–	Jülich
Jorge Segovia	–	ANL
Peter Tandy	–	Kent State

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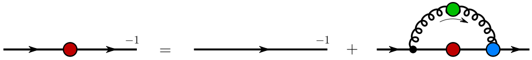
The Challenge of QCD

- QCD is the only known example in nature of a fundamental quantum field theory that is innately non-perturbative
  - *a priori* no idea what such a theory can produce
- Solving QCD will have profound implications for our understanding of the natural world
  - e.g. it will explain how massless gluons and light quarks bind together to form hadrons, and thereby explain the origin of $\sim$98% of the mass in the visible universe
  - *given QCDs complexity, the best promise for progress is a strong interplay between experiment and theory*
- QCD is characterized by two emergent phenomena:
  - confinement & dynamical chiral symmetry breaking (DCSB)
  - a world without DCSB would be profoundly different, e.g. $m_\pi \sim m_\rho$
- *Must discover the origin of confinement, its relationship to DCSB and understand how these phenomenon influence hadronic observables*

- The equations of motion of QCD $\Longleftrightarrow$ QCDs Dyson–Schwinger equations
  - an infinite tower of coupled integral equations
  - must implement a symmetry preserving truncation
- The most important DSE is QCDs gap equation $\Longrightarrow$ quark propagator

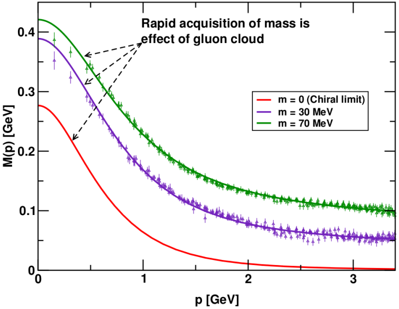


  - ingredients – dressed gluon propagator & dressed quark-gluon vertex

$$S(p) = \frac{Z(p^2)}{i\not{p} + M(p^2)}$$

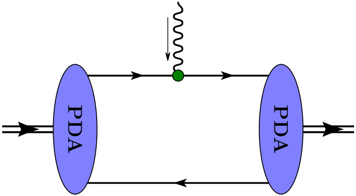
- $S(p)$ has correct perturbative limit
- mass function, $M(p^2)$, exhibits dynamical mass generation
- complex conjugate poles
  - no real mass shell $\Longrightarrow$ confinement

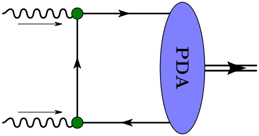
[M. S. Bhagwat *et al.*, Phys. Rev. **C 68**, 015203 (2003)]

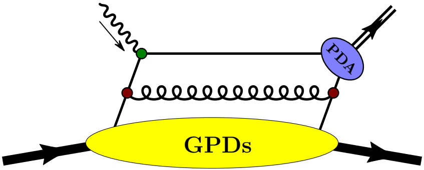


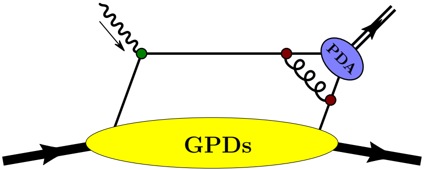
Pion's Parton Distribution Amplitude

- pion's PDA – $\varphi_\pi(x)$: *is a probability amplitude that describes the momentum distribution of a quark and antiquark in the bound-state's valence Fock state*
  - it's a function of the lightcone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2









- PDAs enter numerous hard exclusive scattering processes

Pion's Parton Distribution Amplitude

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  - it's a function of the lightcone momentum fraction $x = \frac{k^+}{p^+}$ and the scale Q^2
- The pion's PDA is defined by

$$f_\pi\,\varphi_\pi(x) = Z_2 \int \frac{d^4k}{(2\pi)^2}\,\delta\left(k^+ - x\,p^+\right)\,\mathrm{Tr}\left[\gamma^+\gamma_5\,S(k)\,\Gamma_\pi(k,p)\,S(k-p)\right]$$

- $S(k)\,\Gamma_\pi(k,p)\,S(k-p)$ is the pion's Bethe-Salpeter wave function
  - in the non-relativistic limit it corresponds to the Schrodinger wave function
- $\varphi_\pi(x)$: is the axial-vector projection of the pion's Bethe-Salpeter wave function onto the light-front [pseudo-scalar projection also non-zero]
- Pion PDA is interesting because it is calculable in perturbative QCD and, e.g., in this regime governs the Q^2 dependence of the pion form factor

$$Q^2\,F_\pi(Q^2) \overset{Q^2\to\infty}{\longrightarrow} 16\,\pi\,f_\pi^2\,\alpha_s(Q^2) \qquad \Longleftrightarrow \qquad \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$$

QCD Evolution & Asymptotic PDA

- ERBL (Q^2) evolution for pion PDA [c.f. DGLAP equations for PDFs]

$$\mu \frac{d}{d\mu}\, \varphi(x,\mu) = \int_0^1 dy\, V(x,y)\, \varphi(y,\mu)$$

- This evolution equation has a solution of the form

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1 + \sum_{n=2,\,4,\ldots} a_n^{3/2}(Q^2)\, C_n^{3/2}(2x-1)\right]$$

  - $\alpha = 3/2$ because in $Q^2 \to \infty$ limit QCD is invariant under the collinear conformal group $SL(2;\mathbb{R})$
  - Gegenbauer-$\alpha = 3/2$ polynomials are irreducible representations $SL(2;\mathbb{R})$

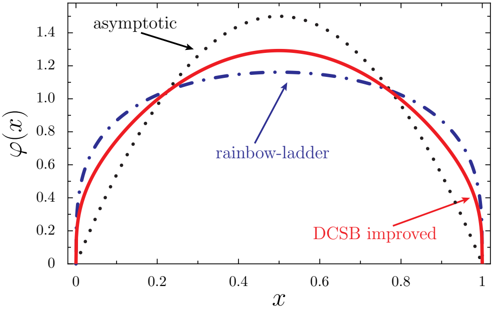
- The coefficients of the Gegenbauer polynomials, $a_n^{3/2}(Q^2)$, evolve logarithmically to zero as $Q^2 \to \infty$: $\varphi_\pi(x) \to \varphi_\pi^{\text{asy}}(x) = 6\,x\,(1-x)$

- At what scales is this a good approximation to the pion PDA

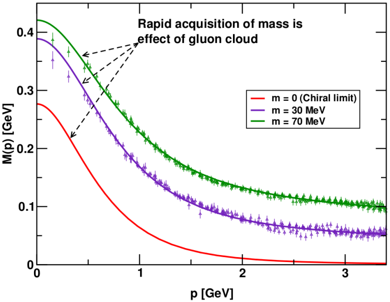
- E.g., AdS/QCD find $\varphi_\pi(x) \sim x^{1/2}\,(1-x)^{1/2}$ at $Q^2 = 1\,\text{GeV}^2$ expansion in terms of $C_n^{3/2}(2x-1)$ convergences slowly: $a_{32}^{3/2} / \, a_2^{3/2} \sim 10\,\%$

Pion PDA from the DSEs

[L. Chang, ICC, *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)]



[C.D. Roberts, Prog. Part. Nucl. Phys. 61 50 (2008)]



- Both DSE results, each using a different Bethe-Salpeter kernel, exhibit a pronounced broadening compared with the asymptotic pion PDA
  - scale of calculation is given by renormalization point $\zeta = 2\,\text{GeV}$
- Broading of the pion's PDA is directly linked to DCSB
- As we shall see the dilation of pion's PDA will influence the Q^2 evolution of the pion's electromagnetic form factor

Pion PDA from lattice QCD

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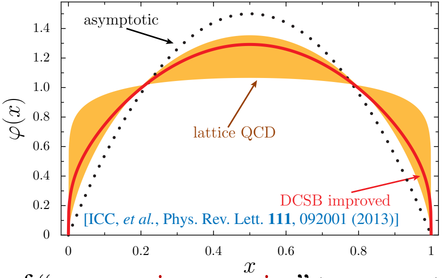
- Lattice QCD can only determine one non-trivial moment

$$\int_0^1 dx\,(2\,x-1)^2\varphi_\pi(x) = 0.27 \pm 0.04$$

[V. Braun *et al.*, Phys. Rev. D **74**, 074501 (2006)]

  - scale is $Q^2 = 4\,\text{GeV}^2$



- Standard practice to fit first coefficient of "*asymptotic expansion*" to moment

$$\varphi_\pi(x,Q^2) = 6\,x\,(1-x)\left[1+\sum\nolimits_{n=2,\,4,\ldots}\,a_n^{3/2}(Q^2)\,C_n^{3/2}(2x-1)\right]$$

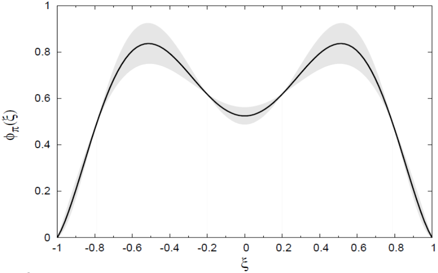
  - however this expansion is guaranteed to converge rapidly only when $Q^2 \to \infty$
  - this procedure results in a *double-humped* pion PDA

- Advocate using a *generalized expansion*

$$\varphi_\pi(x,Q^2) = N_\alpha\,x^{\alpha-1/2}(1-x)^{\alpha-1/2}\left[1+\sum\nolimits_{n=2,\,4,\ldots}\,a_n^{\alpha}(Q^2)\,C_n^{\alpha}(2x-1)\right]$$

- Find $\varphi_\pi \simeq x^\alpha(1-x)^\alpha,\ \alpha = 0.35^{+0.32}_{-0.24}$; good agreement with DSE: $\alpha \simeq 0.30$

Pion PDA from lattice QCD



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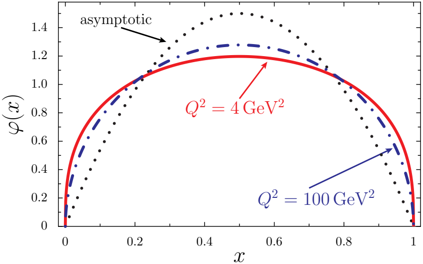
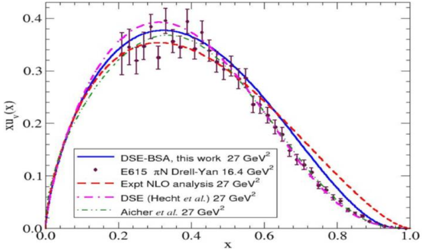
When is the Pion's PDA Asymptotic

[I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 092001 (2013)]

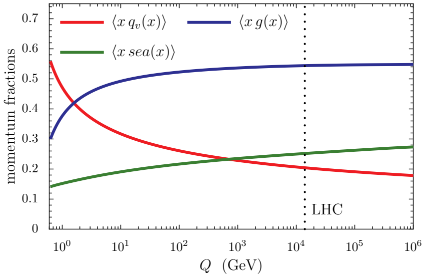


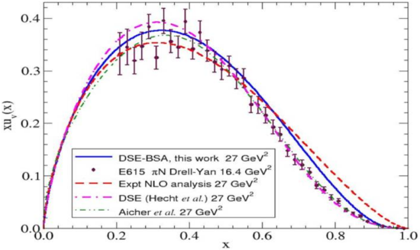
[T. Nguyen, *et al.*, Phys. Rev. C **83**, 062201 (2011)]



- Under leading order Q^2 evolution the pion PDA remains broad to well above $Q^2 > 100\,\mathrm{GeV}^2$, compared with $\varphi_\pi^{\mathrm{asy}}(x) = 6\,x\,(1-x)$
- *Consequently, the asymptotic form of the pion PDA is a poor approximation at all energy scales that are either currently accessible or foreseeable in experiments on pion elastic and transition form factors*
- Importantly, $\varphi_\pi^{\mathrm{asy}}(x)$ is only guaranteed be an accurate approximation to $\varphi_\pi(x)$ when pion valence quark PDF satisfies: $q_v^\pi(x) \sim \delta(x)$
- This is far from valid at forseeable energy scales

When is the Pion's Valence PDF Asymptotic





- LO QCD evolution of momentum fraction carried by valence quarks

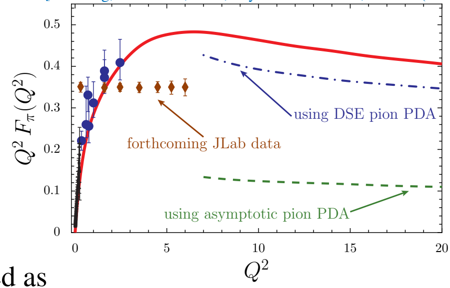
$$\langle x\, q_v(x)\rangle (Q^2) = \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{qq}^{(0)2}/(2\beta_0)} \langle x\, q_v(x)\rangle (Q_0^2) \quad \text{where} \quad \frac{\gamma_{qq}^{(0)2}}{2\beta_0} > 0$$

  - therefore, as $Q^2 \to \infty$ we have $\langle x\, q_v(x)\rangle \to 0$ implies $q_v(x) = \delta(x)$

- At LHC energies valence quarks still carry 20% of pion momentum
  - the gluon distribution saturates at $\langle x\, g(x)\rangle \sim 55\%$

- *Asymptotia is a long way away!*

[L. Chang, I. C. Cloët, *et al.*, Phys. Rev. Lett. **111**, 141802 (2013)]



- Extended the pre-experiment DSE prediction to $Q^2 > 4\,\text{GeV}^2$
- Predict max at $Q^2 \approx 6\,\text{GeV}^2$; within domain accessible at JLab12
- Magnitude directly related to DCSB
- The QCD prediction can be expressed as

$$Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda^2_{\text{QCD}}}{\sim} 16\,\pi\, f_\pi^2\, \alpha_s(Q^2)\, w_\pi^2; \qquad w_\pi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_\pi(x)$$

- Using $\varphi_\pi^{\text{asy}}(x)$ significantly underestimates experiment
- Within DSEs there is consistency between the direct pion form factor calculation and that obtained using the DSE pion PDA
  - 15% disagreement explained by higher order/higher-twist corrections
- *We predict that QCD power law behaviour sets in at* $Q^2 \sim 8\,\text{GeV}^2$

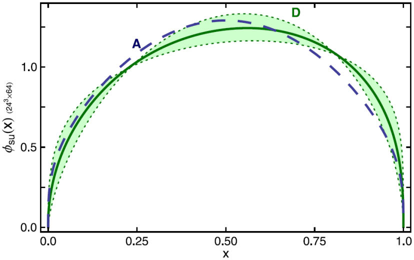
Kaon PDAs from Lattice QCD

[R. Arthur, P. A. Boyle, *et al.*, Phys. Rev. D **83**, 074505 (2011)]

Meson PDA moments obtained using numerical simulations of lattice-regularised QCD with $N_f = 2+1$ domain-wall fermions and nonperturbative renormalisation of lattice operators [30]: linear extrapolation to physical pion mass, $\overline{\mathrm{MS}}$-scheme at $\zeta = 2$ GeV, two lattice volumes. The first error is statistical, the second represents an estimate of systematic errors, including those from the s-quark mass, discretisation and renormalisation.

Meson	$\langle (x-\bar{x})^n \rangle$	$16^3 \times 32$	$24^3 \times 64$
π	$n=2$	0.25(1)(2)	0.28(1)(2)
$\rho_\parallel$	$n=2$	0.25(2)(2)	0.27(1)(2)
ϕ	$n=2$	0.25(2)(2)	0.25(2)(1)
K	$n=1$	0.035(2)(2)	0.036(1)(2)
$K^*_\parallel$	$n=1$	0.037(1)(2)	0.043(2)(3)
K	$n=2$	0.25(1)(2)	0.26(1)(2)
$K^*_\parallel$	$n=2$	0.25(1)(2)	0.25(2)(2)

[J. Segovia, L. Chang, ICC, *et al.*, Phys. Lett. B **731**, 13 (2014)]



- For the kaon lattice can determine two non-trivial moments; Generalization:

$$\varphi_K \simeq x^\alpha (1-x)^\beta; \quad \begin{array}{lll} 16^3 \times 32 & \alpha = 0.56^{+0.21}_{-0.18} & \beta = 0.45^{+0.19}_{-0.16} \\ 24^3 \times 64 & \alpha = 0.48^{+0.19}_{-0.16} & \beta = 0.38^{+0.17}_{-0.15} \end{array}$$

- For kaon $\langle x - \bar{x} \rangle \neq 0$ $\;[\bar{x} = 1 - x]$ skews PDA
  - skewness is a measure of $SU(3)$ flavour breaking
  - peak of kaon PDA shifted by 10% from $x = 1/2$; $SU(3)$ flavour breaking $\sim 10\%$
  - DCSB masks flavour breaking; naive expectation $m_s/m_q \simeq 25$

Kaon/Pion form factor ratio from Lattice QCD

- QCD prediction:

[J. Segovia, L. Chang, ICC, *et al.*, Phys. Lett. B **731**, 13 (2014)]

$Q^2 F_{PS}(Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{\sim} 16\,\pi\, f^2_{PS}\, \alpha_s(Q^2)\, w^2_{PS};$

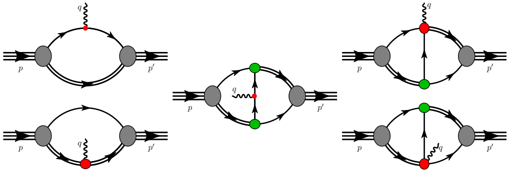
$w_{PS} = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\, \varphi_{PS}(x)$

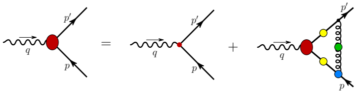
$Q^2 = 4\,\mathrm{GeV}^2$	$16^3 \times 32$	$24^3 \times 64$
$F_K(Q^2)/F_\pi(Q^2)$	$1.21^{+1.22}_{-0.62}$	$0.74^{+1.21}_{-0.51}$

- From QCD relation can make lattice-based estimate for $F_K(Q^2)/F_\pi(Q^2)$
- What should we expect for this ratio?
  - at $Q^2 = 0$ charge conservation implies: $F_K(0)/F_\pi(0) = 1$
  - in conformal limit, $Q^2 = \infty$, must have: $F_K(\infty)/F_\pi(\infty) = f_K^2/f_\pi^2 \simeq \sqrt{2}$
  - expect $F_K(Q^2)/F_\pi(Q^2)$ to grow monotonically toward conformal limit; to do otherwise would require a new dynamically generated mass scale
  - expectation is supported by DSE predictions: $F_K/F_\pi = 1.13$ at $Q^2 = 4\,\mathrm{GeV}^2$
- Central value obtained from $16^3 \times 32$ lattice is consistent with expectations and DSE prediction (albeit with large errors)
- $24^3 \times 64$ lattice result suggests this larger lattice produces a pion PDA which is too broad

Nucleon Electromagnetic Form Factors

- Elastic form factors provide information on the *momentum space* distribution of charge and magnetization within the nucleon
- Accurate form factor measurements are creating a paradigm shift in our understanding of hadron structure; e.g.
  - proton radius puzzle, $\mu_p \, G_{Ep}/G_{Mp}$ ratio and a possible zero in G_{Ep}
  - flavour decomposition and diquark correlations
  - tests perturbation QCD scaling predictions
- In the DSEs the nucleon current is given by:





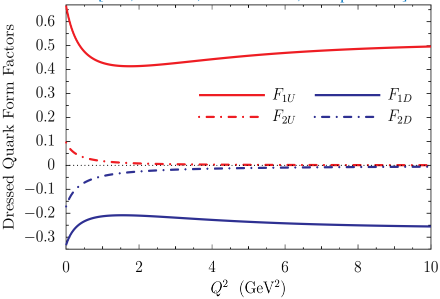
$$\Gamma^\mu = \Gamma^\mu_L + \Gamma^\mu_T; \quad q_\mu \, \Gamma^\mu_T = 0$$

$$q_\mu \, \Gamma^\mu_L = \hat{Q} \left[S^{-1}(p') - S^{-1}(p) \right]$$

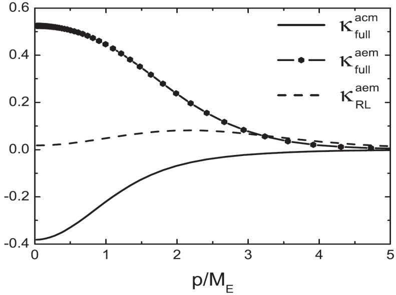
- Feedback with experiment can constrain DSE quark–gluon vertex
- Knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs
  - also gives the β-function which may shed light on confinement

Dressed Quarks are not Point Particles

[ICC, W. Bentz, A. W. Thomas, to be published]



[L. Chang, Y. -X. Liu, C. D. Roberts, PRL **106**, 072001 (2011)]

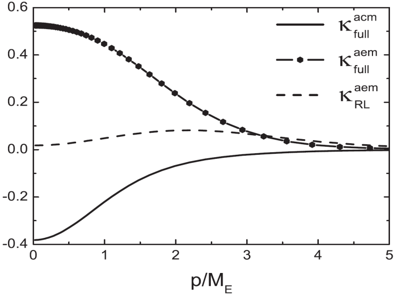


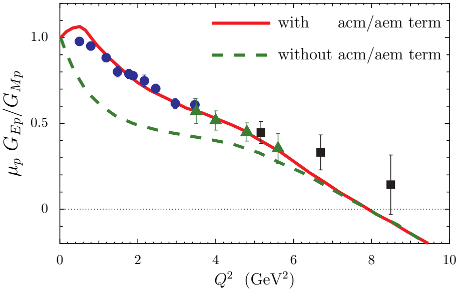
- EM properties of a spin-$\frac{1}{2}$ point particle are characterized by two quantities:
  - charge: e & magnetic moment: μ
- Strong gluon cloud dressing produces – from a massless quark – a dressed quark with $M \sim 400\,\text{MeV}$
  - expect gluon dressing to produce non-trival EM structure for a dressed quark
  - analogous to pion dressing on nucleon giving large anomalous magnetic moment
- A large quark anomalous chromomagnetic moment in the quark-gluon vertex – *produces a large quark anomalous electromagnetic moment*

Proton G_E/G_M Ratio

[L. Chang, Y. -X. Liu, C. D. Roberts, Phys. Rev. Lett. **106**, 072001 (2011)]

[ICC, C. D. Roberts, PPNP, *in press* (2014)]

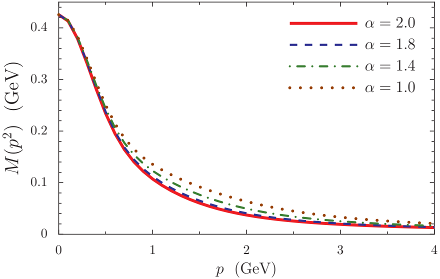


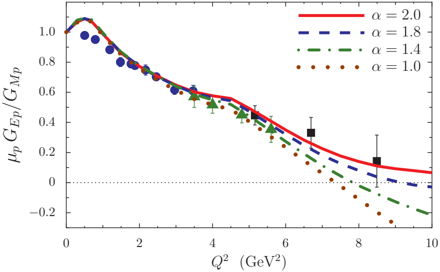


- Latest results include effect from anomalous chromomagnetic moment term in the quark–gluon vertex
  - generates large anomalous electromagnetic term in quark–photon vertex
- Quark anomalous magnetic moment required for good agreement with data
  - important for low to moderate Q^2
- For massless quarks anomalous chromomagnetic moment is only possible via DSCB

Proton G_E form factor and DCSB

[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]





- Find that slight changes in $M(p)$ on the domain $1 \lesssim p \lesssim 3\,$GeV have a striking effect on the G_E/G_M proton form factor ratio
  - position of zero, or lack thereof, in G_E is extremely sensitive to underlying quark-gluon dynamics
- Zero in $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$ largely determined by evolution of $Q^2\,F_2$
  - F_2 is sensitive to DCSB through the dynamically generated quark anomalous electromagnetic moment – *vanishes in perturbative limit*
  - the quicker the perturbative regime is reached the quicker $F_2 \to 0$

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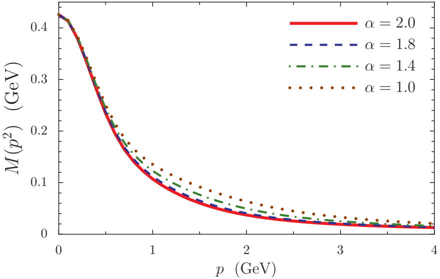
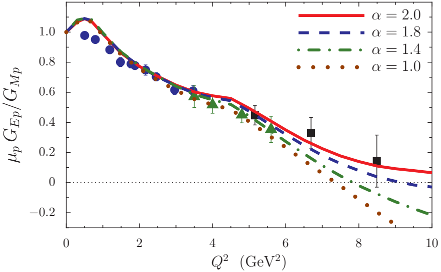
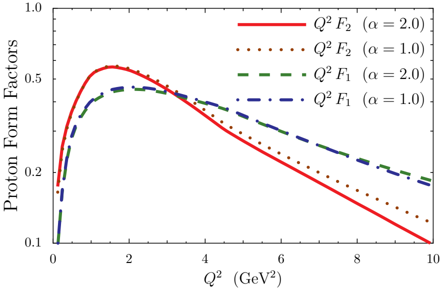
[I. C. Cloët, C. D. Roberts and A. W. Thomas, Phys. Rev. Lett. **111**, 101803 (2013)]







- Recall: $G_E = F_1 - \frac{Q^2}{4\,M_N^2}\,F_2$
- Only G_E is senitive to these small changes in the mass function
- *Accurate determination of zero crossing would put important contraints on quark-gluon dynamics within DSE framework*

Conclusion

- QCD and therefore Hadron Physics is unique:
  - must confront a fundamental theory in which the elementary degrees-of-freedom are intangible (confined) and only composites (hadrons) reach detectors
- QCD will only be solved by deploying a diverse array of experimental and theoretical methods:
  - must define and solve the problems of confinement and its relationship with DCSB
- These are two of the most important challenges in fundamental Science
- Both DSEs and lattice QCD agree that the pion PDA is significantly broader than the asymptotic result
  - using LO evolution find dilation remains significant for $Q^2 > 100\,\text{GeV}^2$
  - asymptotic form of pion PDA only guaranteed to be valid when $q_v^\pi(x) \propto \delta(x)$
- Feedback with EM form factor measurements can constrain QCD's quark–gluon vertex within the DSE framework
  - knowledge of quark–gluon vertex provides $\alpha_s(Q^2)$ within DSEs $\Leftrightarrow$ confinement