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Complete next-to-next-to-leading order calculation of NN→NNπ in chiral EFT

in collaboration with
V. Baru, E. Epelbaum, C. Hanhart, H. Krebs, A. E. Kudryavtsev, and F. Myhrer

Outline:

- Introduction: chiral EFT and NN→NNπ
- Why is pion production interesting?
- High accuracy pion production operator
- Importance of Delta(1232)-resonance
- Summary and outlook

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Method and goal

Approaches to QCD at low energies:

- Phenomenological models
- Lattice calculations
- Chiral effective field theory

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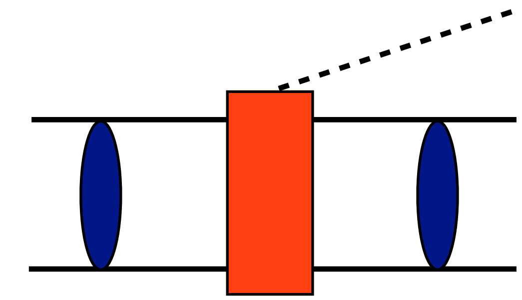
Chiral EFT - effective field theory of QCD below 1 GeV

- most general Lagrangian compatible with symmetries of QCD
- effective degrees of freedom: pions, nucleons, Delta(1232)-resonance
- systematic expansion in small momenta and small masses
- suited for investigations of $\pi\pi$, πN, NN interactions and nuclear forces
  Weinberg, Gasser, Leutwyler, Bernard, Meißner, Epelbaum, Glöckle, ...
- successful application to pion reactions on few-nucleon systems:
  $\pi A \to \pi A$ (A=2,3,4), $\pi d \to \gamma nn$, $\gamma d \to \pi NN$,...
  Weinberg, Beane, Hanhart, Meißner, Phillips, Baru, ...

Our goal is to study $NN \to NN\pi$ within chiral EFT

Specifics of pion production



NN interactions are non-perturbative [deuteron]

Hybrid chiral EFT method:

1. Calculate irreducible production operator perturbatively in chiral EFT
2. Convolute it with non-perturbative NN wave functions

   realistic phenomenological NN WF: CD-Bonn, CCF, AV18, ...

Large transferred momenta

- NN momenta in CMS are large enough to produce a pion

  $|\vec{p}| \sim \sqrt{m_\pi m_N} \sim 360\ \mathrm{MeV}$– new scale

  Special counting: Momentum Counting Scheme (MCS)

  expansion parameter $\chi_{\mathrm{MCS}} \sim \sqrt{\frac{m_\pi}{m_N}}$
- Explicit Delta(1232)-resonance $m_\Delta - m_N \sim 280\ \mathrm{MeV} \sim |\vec{p}|$

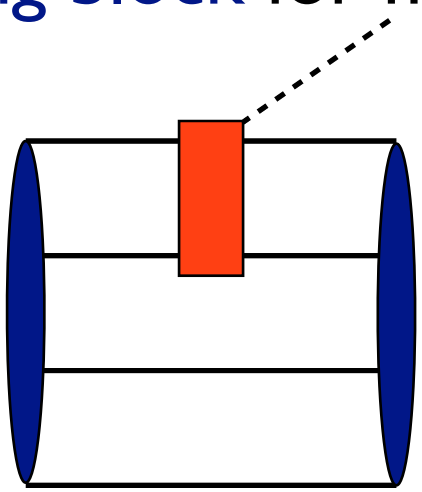
Why is pion production interesting?

- First inelastic process in nucleon-nucleon interactions
- Several channels:
  $pp \to pp\pi^0$ and $pp \to d\pi^+$ cross sections differ by an order of magnitude

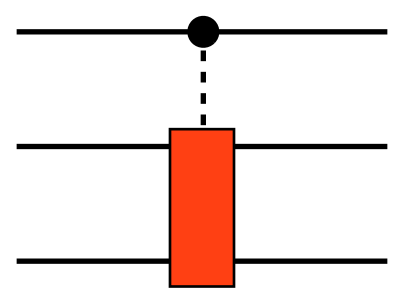
$\sigma_{\rm tot}(pp \to pp\pi^0) \simeq 3\,\mu b$ $\quad$ $\sigma_{\rm tot}(pp \to d\pi^+) \simeq 43\,\mu b$ $\quad$ $T_{\rm lab} = 293.5$ MeV

COSY-TOF (2003)

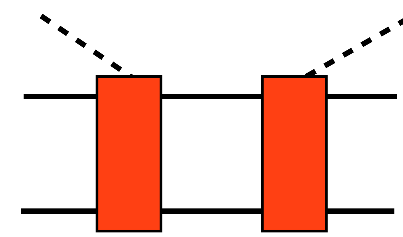
- Building block for more complicated processes:



CSB in $dd \to \alpha\pi^0$

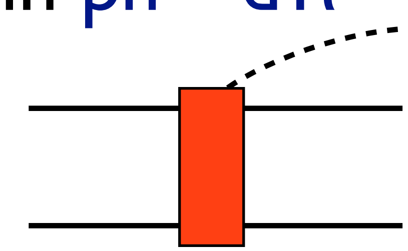


3N forces



pionic deuterium
$\pi d \to NN \to \pi d$

- Charge symmetry breaking in $pn \to d\pi^0$



Why is pion production interesting?

Charge symmetry – invariance under interchange of u- and d-quarks

- Approximate symmetry of QCD
- Explicitly broken by quark mass difference and electromagnetic effects
- On the level of hadrons → invariance under interchange of p and n

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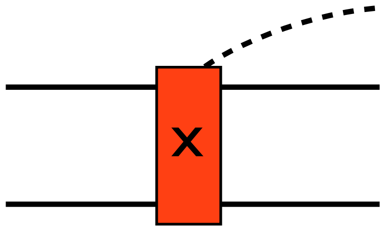

- Interchange of p and n changes differential cross section
- Forward backward-asymmetry $A_{fb} \propto \left(\frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right) / \frac{d\sigma}{d\Omega}(\theta)$
- Experiment: $A_{fb}=(17{,}2 \pm 8 \pm 5{,}5)\ 10^{-4}$ TRIUMF (2003)

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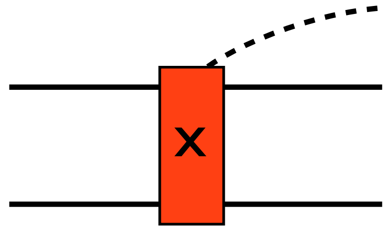

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- Theory $A_{fb} \propto \frac{\mathrm{Re}(M^{\mathrm{CSB}}_{\text{s-wave}} M^{\mathrm{CS}*}_{\text{p-wave}})}{|M^{\mathrm{CS}}_{\text{s-wave}}|^2} \propto \frac{(m_p - m_n)^{\mathrm{str}}}{|M^{\mathrm{CS}}_{\text{s-wave}}|^2}$

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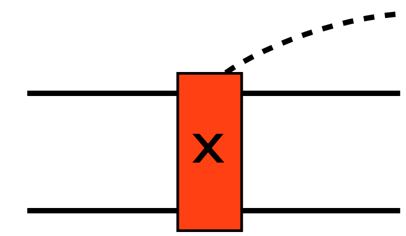

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- s-wave amplitude $M^{\mathrm{CS}}_{\text{s-wave}}$ is important prerequisite

  to extract $(m_p - m_n)^{\mathrm{str}}$ – strong part of p – n mass difference

s-wave pion production

Introduction

At threshold only s-wave gives non-zero contribution

General s-wave production amplitude at threshold

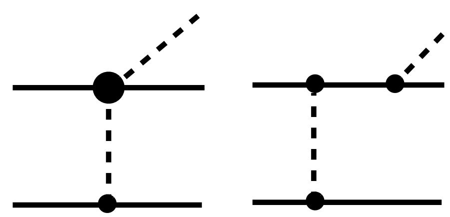
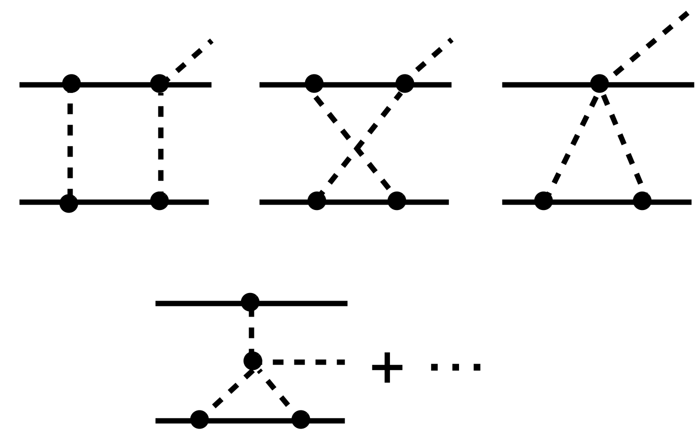
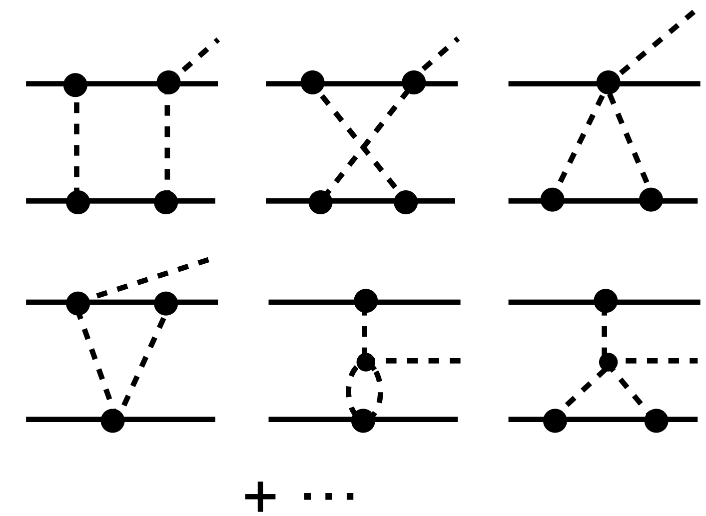
$$M_{\rm th}(NN \to NN\pi) = \text{A}\ (\vec{\sigma}_1 \times \vec{\sigma}_2) \cdot \vec{p}\ (\tau_1 + \tau_2) \cdot \phi^* + \text{B}\ (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{p}\ (\tau_1 \times \tau_2) \cdot \phi^*$$

Amplitudes A and B contribute to different reaction channels

- A contributes to $pp \to pp\pi^0$
- B contributes to $pp \to d\pi^+$

Goal: derive pion production operators A and B within chiral EFT

s-wave pion production operators

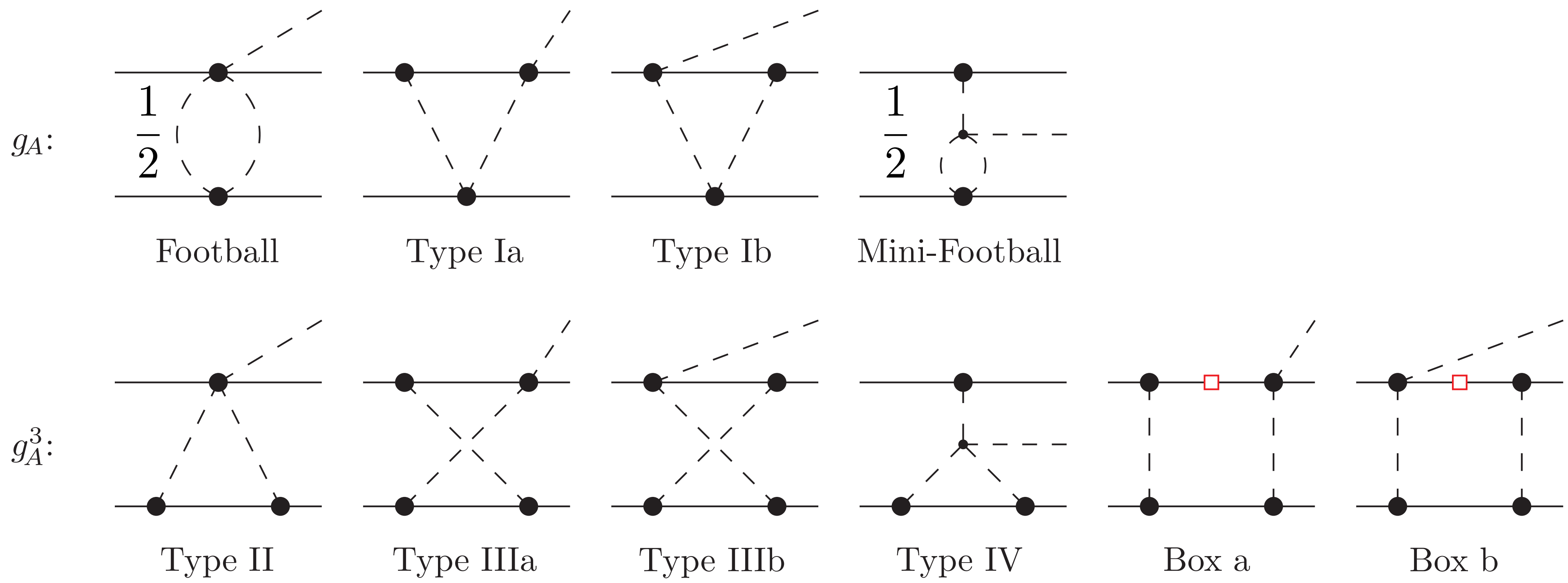
$\chi_{\rm MCS} \sim \sqrt{\frac{m_\pi}{m_N}}$	LO	NLO	NNLO
			
$pp \to d\pi^+$	big contribution Koltun et al. (1966)	0 Lensky et al. (2006)	small, correction to LO this work
$pp \to pp\pi^0$	almost negligible Cohen et al. (1996), Park et al. (1996)	0 Hanhart and Kaiser (2005)	small, but main contribution (!) this work

For $pp \to pp\pi^0$ LO rescattering contribution is forbidden, NLO is zero

$\Rightarrow$ effects of NNLO loops are very important

NNLO loop-diagrams

- Topologies of NNLO diagrams:

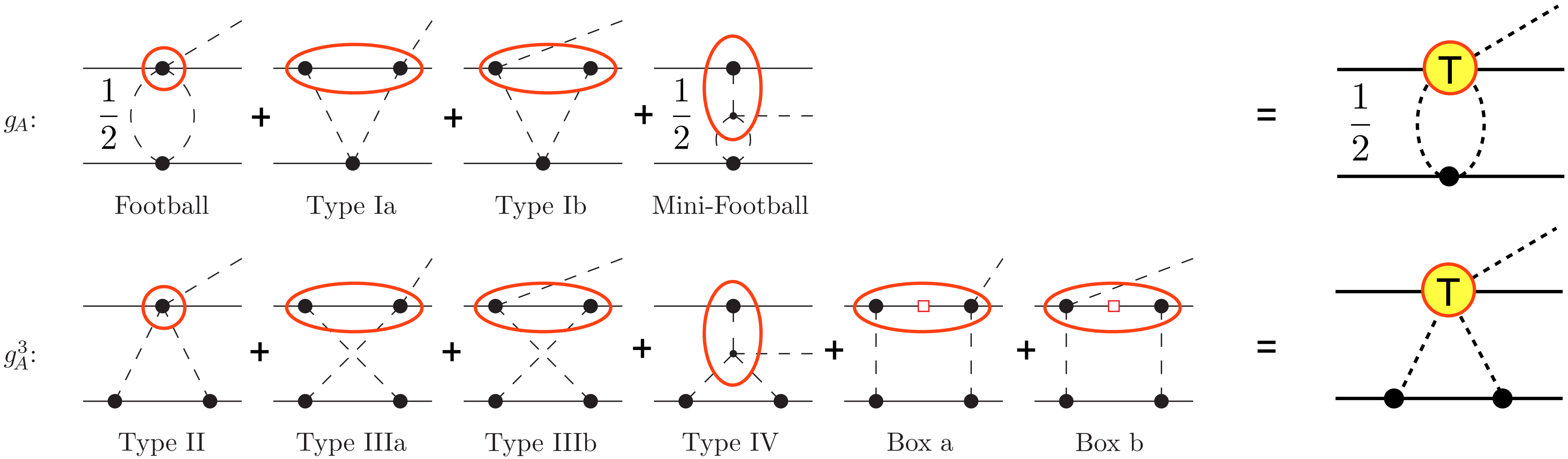


- $1/m_N$ correction should be included in every vertex
  → Lots of terms and lengthy calculation

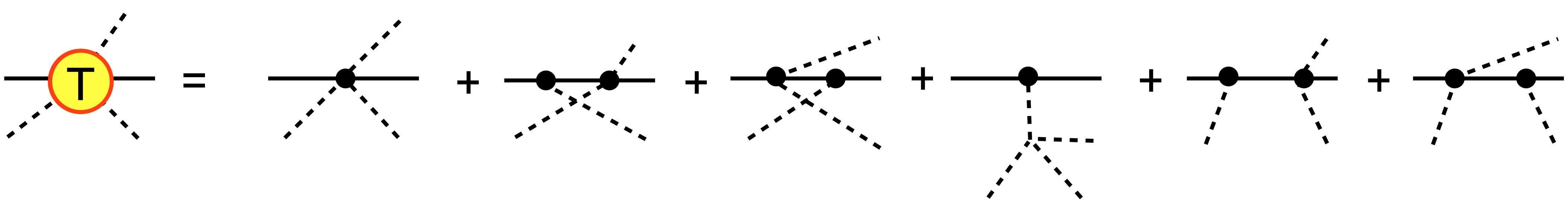
- Efficient method: collect $\pi N \to \pi\pi N$ subgraphs

NNLO loop-diagrams: calculation method

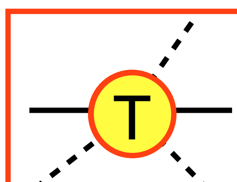
Collecting subgraphs – efficient way to calculate NNLO loop diagrams



Operator T is the sum of all πN→ππN subgraphs



We found that for s-wave pion production at threshold:



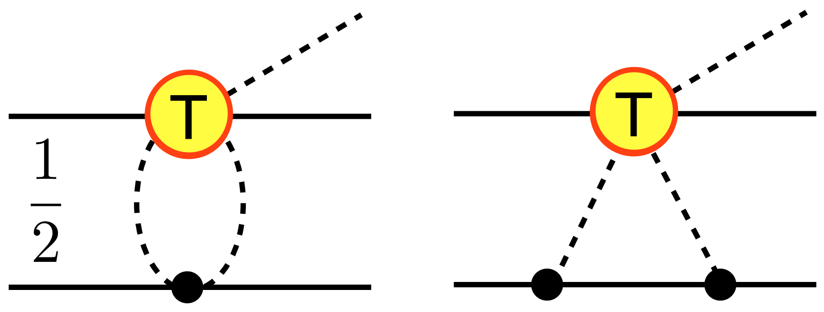
= 0 + [few NNLO terms] + [p-wave terms]

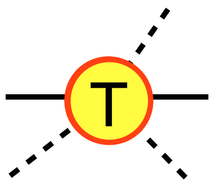
all NLO, and most of NNLO terms cancel

vanish at threshold

NNLO loop-diagrams: results

- Put [few NNLO terms] into common structures





= 0 + [few NNLO terms]

→ Get full NNLO loop operator
→ Very compact expression

- NNLO loop production operator (simplified in Dim.Reg.):

$$iM^{\text{Sym.}}_{nucl} = \frac{g_A^3}{f_\pi^5} v\cdot q\, \tau_+^a \left(i\varepsilon^{\alpha\mu\nu\beta} v_\alpha k_{1\mu} S_{1\nu} S_{2\beta}\right)(-2 I_{\pi\pi}) \longrightarrow \text{A}$$

$$+\frac{g_A^3}{f_\pi^5} v\cdot q\, \tau_\times^a (S_1+S_2)\cdot k_1 \left(-\frac{19}{24} I_{\pi\pi} + \frac{5}{9}\frac{1}{(4\pi)^2}\right)$$

$$+\frac{g_A}{f_\pi^5} v\cdot q\, \tau_\times^a (S_1+S_2)\cdot k_1 \left(\frac{1}{6} I_{\pi\pi} - \frac{1}{18}\frac{1}{(4\pi)^2}\right)$$

→ B

with only one basic integral: $I_{\pi\pi} = \frac{\mu^\epsilon}{i}\int \frac{d^{4-\epsilon}l}{(2\pi)^{4-\epsilon}} \frac{1}{(l^2-m_\pi^2+i0)((l+k_1)^2-m_\pi^2+i0)}$

Inclusion of Delta(1232) explicitly

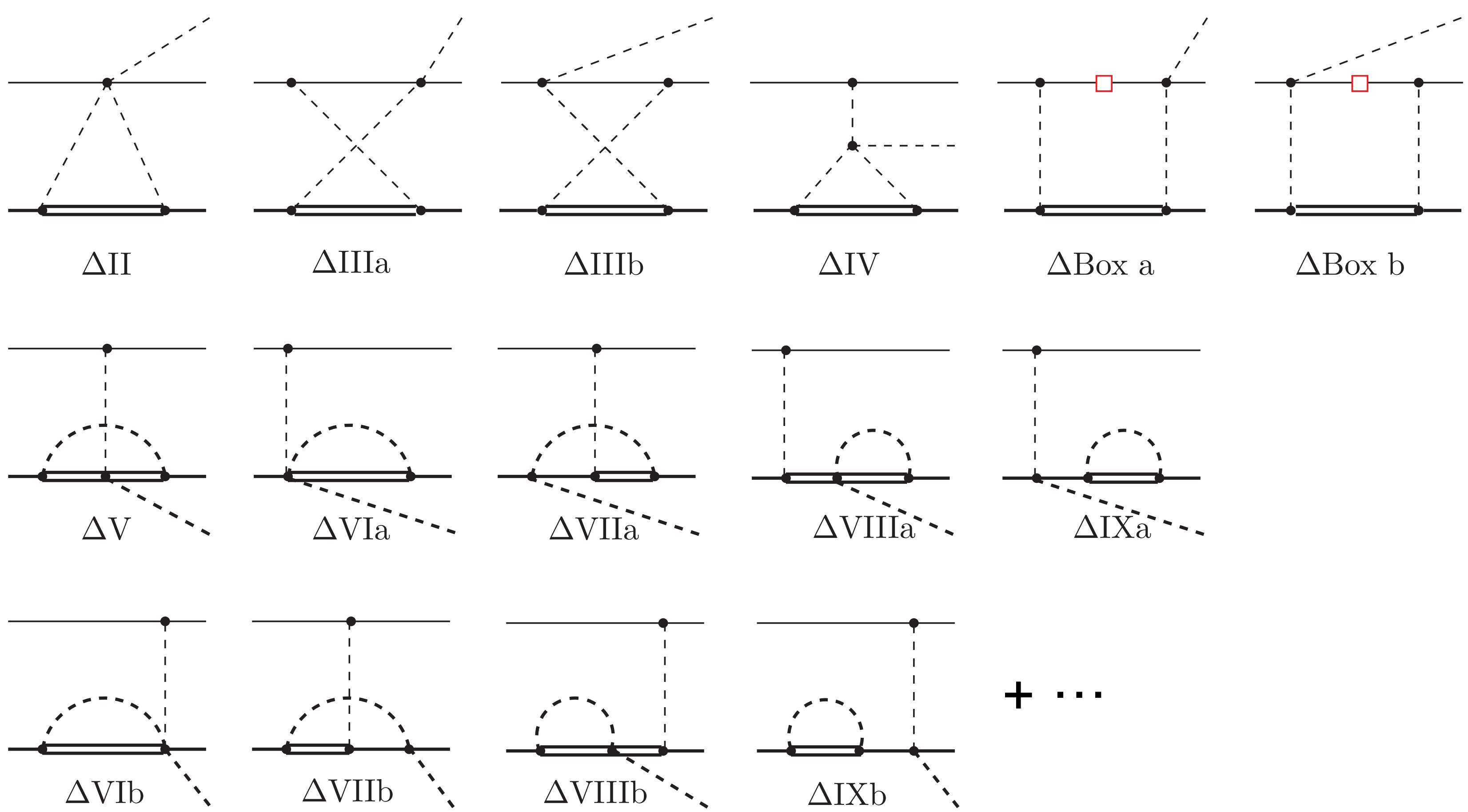
Motivation

For NN → NNπ Delta-resonance contribution is important

- Typical momenta in NN → NNπ is about p≃360 MeV
- Delta-nucleon mass difference m_Δ-m_N≃280 MeV → same order as p

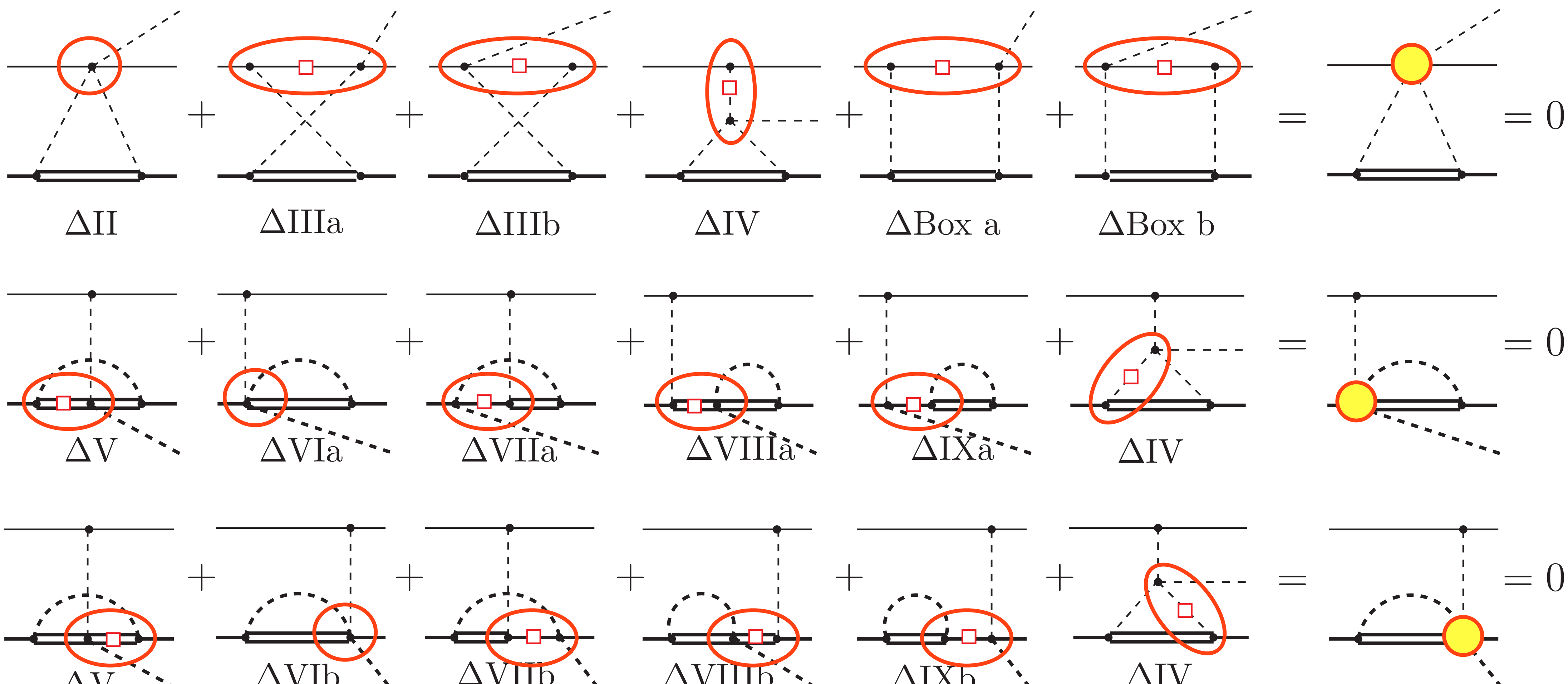
→ Delta(1232) should be included as a dynamic degree of freedom

Additional NNLO loop-diagrams with delta:



Explicit Delta: groups and cancellations

- Same calculation method as for pure nucleon case: selecting groups
- Cancellation patterns for s-wave pion production:



- Finite remainder survives like in NN case

Explicit inclusion of Delta in NN → NNπ

Result:

NNLO loop-corrections to NN → NNπ operator due to explicit Δ(1232)

$$
\begin{aligned}
iM^{\rm NNLO}_{\Delta\text{-loops}} \;=\; & \frac{g_A g^2_{\pi N\Delta}}{f^5_\pi} v\cdot q\,\tau^a_+ \left(i\varepsilon^{\alpha\mu\nu\beta} v_\alpha k_{1\mu} S_{1\nu} S_{2\beta}\right) \\
& \times\left\{\frac{2}{9}\left(I_{\pi\pi} + \frac{1}{2}\frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2}\right) + \frac{1}{18} k_1^2 J_{\pi\pi N\Delta}\right\} \\
+\; & \frac{g_A g^2_{\pi N\Delta}}{f^5_\pi} v\cdot q\,\tau_\times (S_1 + S_2)\cdot k_1 \\
& \times\left\{\frac{5}{9}\left(I_{\pi\pi} + \frac{1}{2}\frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2}\right) + \frac{1}{18} k_1^2 J_{\pi\pi N\Delta}\right. \\
& \left. + \frac{8}{9}\frac{\delta^2}{k_1^2}\left(I_{\pi\pi} + \frac{1}{2}\frac{J_{\pi\Delta}}{\Delta} + \Delta J_{\pi\pi\Delta} + \frac{2}{(4\pi)^2}\right) - \frac{2}{27}\left(I_{\pi\pi} + \frac{1}{2}\frac{J_{\pi\Delta}}{\Delta} + \frac{1}{3}\frac{2}{(4\pi)^2}\right)\right\}.
\end{aligned}
$$

→ A (first curly-bracket term)

→ B (second curly-bracket term, both lines)

Correct analytic behavior:

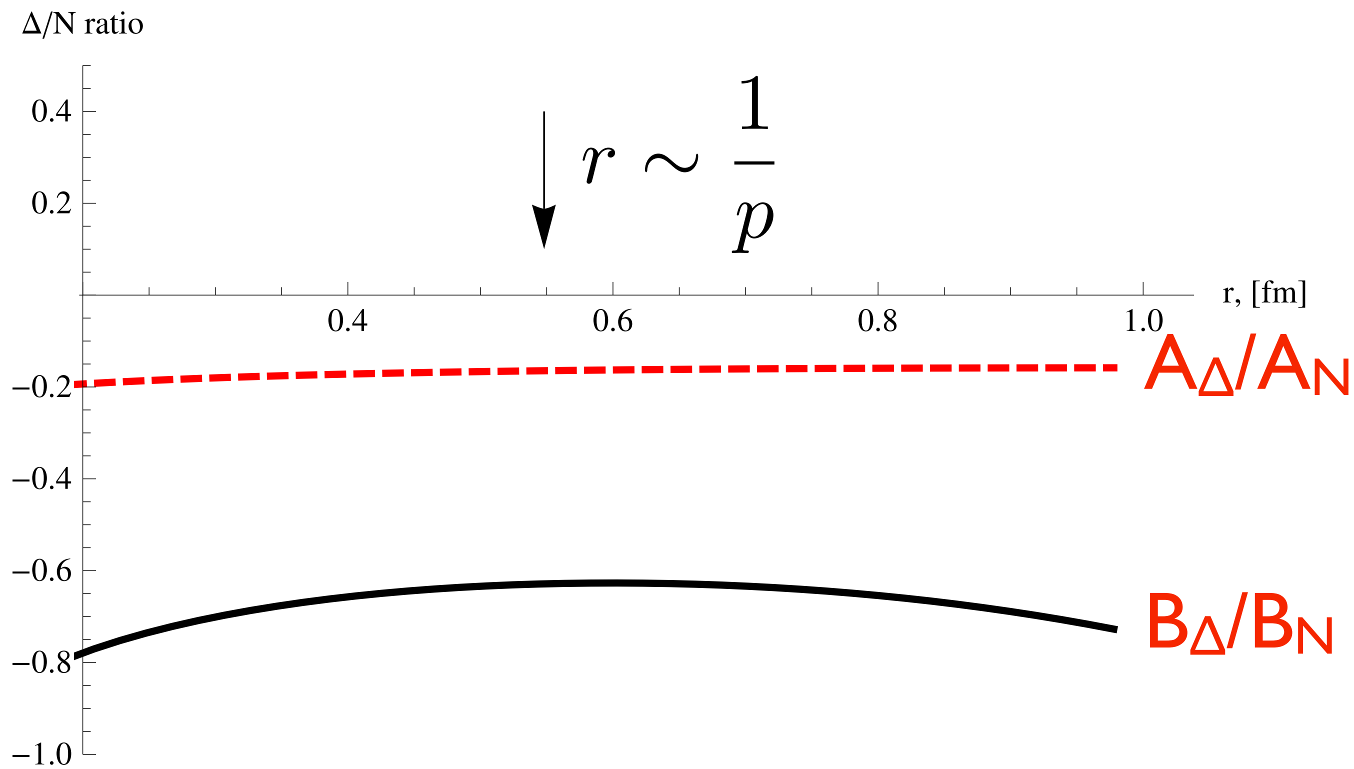
If $m_\Delta \to \infty$ then the contribution of Delta vanishes (decoupling of Delta)

Appelquist, Carazzone (1975)

No additional unknown LECs

Comparison of Delta- and nucleon-loops

Ratio of long-range loop-contributions: nucleon / Delta



A: $pp \to pp\pi^0$

B: $pp \to d\pi^+$

- Explicitly proves MCS counting estimation: delta~p

- Sum of Delta and nucleon-loop contributions:

In A: net NNLO effect ~ A_N – of natural size in MCS

In B: net NNLO effect is smaller than MCS expectations due to cancellations

→ Both facts are consistent with indications from data:

For B there is already a good description of data at NLO

For A we probe NNLO contributions directly (LO + NLO ≈ 0)

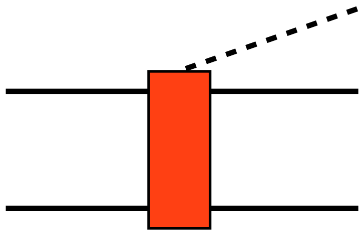
Summary and outlook

The reaction $NN \rightarrow NN\pi$ in Chiral EFT

- Tool to study charge symmetry breaking ($pn \rightarrow d\pi^0$)
- Building block for more complicated reactions ($dd \rightarrow \alpha\pi^0$, 3NF,...)
- Cross section puzzle (different cross sections in different channels)

Current results:

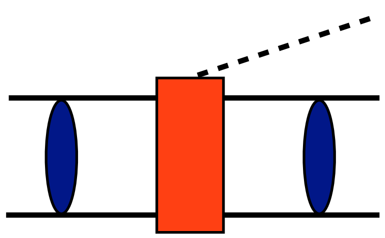
- s-wave pion production operator at threshold up to N^2LO MCS (6%) including explicit Delta(1232)



- Loop-diagrams with Delta are needed for quantitative understanding of both $pp \rightarrow pp\pi^0$ and $pp \rightarrow d\pi^+$

Next step

- Convolution with nucleon-nucleon wave functions and calculation of the observables



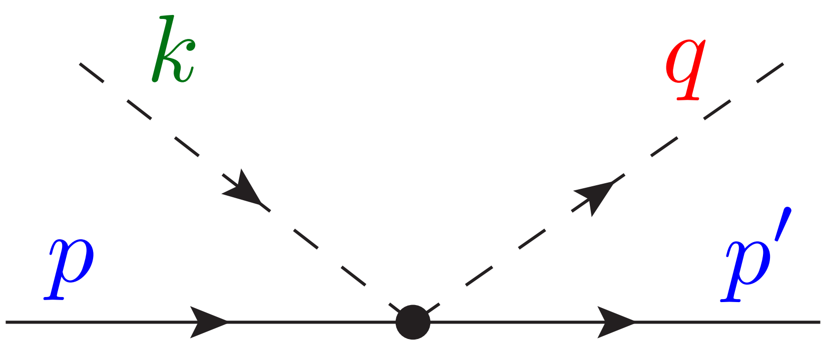
spares

πN→πN vertex and nucleon propagator

Leading order πN→πN vertex (Weinberg-Tomozawa vertex)

$$V_{\pi N \to \pi N} = \frac{1}{4f_\pi^2}\epsilon_{abc}\tau^c(\not{k} + \not{q})$$

$$k = q - p + p'$$



can be identically rewritten as:

$$\begin{aligned} V_{\pi N \to \pi N} &= \frac{1}{4f_\pi^2}\epsilon_{abc}\tau^c(2\not{q} - \not{p} + \not{p}') \\ &= \frac{1}{4f_\pi^2}\epsilon_{abc}\tau^c(\underbrace{2\not{q}} - \underbrace{(\not{p} - m_N)} + \underbrace{(\not{p}' - m_N)}) \end{aligned}$$

Remaining 2q term

incoming nucleon propagator

outgoing nucleon propagator

Parts of πN→πN vertex can cancel nucleon propagators