QCD sum rules for D and B mesons in a strongly interacting medium

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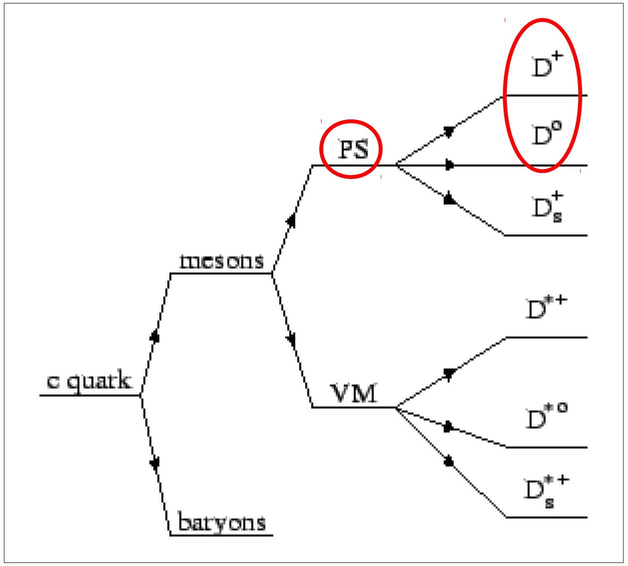
Motivation
D and B Mesons

quark contents:

$$D^+ \ (c\bar{d})\,, \quad B^+ \ (b\bar{d})$$
$$D^0 \ (c\bar{u})\,, \quad B^0 \ (b\bar{u})$$
$$D_s^+ \ (c\bar{s})\,, \quad B_s^+ \ (b\bar{s})$$



[modified figure from desy.de/~ameyer/hq/node38.html]

$$m_{D^+} = 1870 \text{ MeV}$$
$$m_{D^0} = 1865 \text{ MeV}$$
$$m_{D_s^+} = 1968 \text{ MeV}$$
$$m_{D^{*+}} = 2010 \text{ MeV}$$
$$m_{D^{*0}} = 2007 \text{ MeV}$$
$$m_{D_s^{*+}} = 2112 \text{ MeV}$$

Why D and B mesons ?

- input parameters for investigations of exotic charmed mesons (X, $Y_{(b)}$, $Z_{(b)}$, etc.)
- probes of hot and dense nuclear matter via medium modifications
  - good reconstruction at LHC, FAIR
  - recent interest [Blaschke et al., PRD 85 (2012)], [He et al., PRL 110 (2013)], [Tolos et al., PRD 88 (2013)], [Yasui et al., PRC 87 (2013)]

Mitglied der Helmholtz-Gemeinschaft
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Motivation

Chiral Symmetry Breaking / Restoration in Medium

medium modification of D and B mesons

changing order parameters of spontaneous chiral symmetry breaking

vacuum – (spontaneous) chiral symmetry breaking:

order parameter $\langle \bar{q} q \rangle \neq 0$

further chirally odd condensates

➔ mass splitting of chiral partner mesons

medium modifications:

non-zero temperature (T) and baryon density (n)

$$\langle \bar{q} q \rangle_{T,n} = \langle \bar{q} q \rangle \left(1 - \frac{T^2}{8 f_\pi^2} - \frac{\sigma_N \, n}{m_\pi^2 f_\pi^2} \right)$$

➔ signal of chiral symmetry restoration

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QCD Sum Rules

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➜ signal of chiral symmetry restoration

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QCD Sum Rules in Medium

causal current-current correlator $$\Pi(q) = \int d^4x\, e^{iqx} \langle \mathrm{T}\left[j(x) j^\dagger(0)\right] \rangle$$

quark d.o.f.

$u \quad d \quad s$

$c \quad b$

QCD

Sum ⟷ **Rule**

hadronic d.o.f.

$\pi \quad \rho \quad \omega \quad J/\psi \quad a_1 \, D \quad B$

$p \quad n \quad \Delta \quad \Lambda$

phenomenology

dispersion relation (from analyticity) $$\Pi(q_0, \vec{q}) = \frac{1}{\pi} \int_{-\infty}^{\infty} d\omega \frac{\mathrm{Im}\Pi(\omega, \vec{q})}{\omega - q_0}$$

$$\Pi_{\mathrm{OPE}}(q_0) = \int_{-\infty}^{\infty} d\omega \frac{\mathrm{discontinuity}\,(\omega)}{\omega - q_0}$$

Wick rotation of OPE side ensures $q^2 < 0$

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In-Medium OPE

$$\Pi_{\mathrm{OPE}}(q) = \sum_n C_n(q)\langle O_n\rangle$$

determined [Hilger et al., PRC 97 (2009)]

$$= C_0(q)\mathbb{1} + C_3(q)\langle\bar{q}q\rangle + C_4(q)\langle G^2\rangle + C_5(q)\langle\bar{q}Gq\rangle$$

$$+ C_{6,q}(q)\langle\bar{q}q\bar{q}q\rangle + C_{6,G}(q)\langle G^3\rangle + \ldots$$

determined for light mesons
[Thomas et al., PRL 95 (2005)]

determined for vacuum situations
[Nikolaev, Radyushkin, NPB 213 (1983)]

heavy-light mesons (qQ) [Hilger+Buchheim]

$m_Q\langle\bar{q}q\rangle_{T,n}$ ➔ sizable impact on medium modification of D and B mesons

light mesons (qq) [recollection]

$m_q\langle\bar{q}q\rangle_{T,n}$ ➔ suppressed, four-quark condensates are important instead

$\langle\bar{q}q\bar{q}q\rangle_{\mathrm{odd}}$ identified in ρ OPE [Hilger et al., PLB 709 (2012)]

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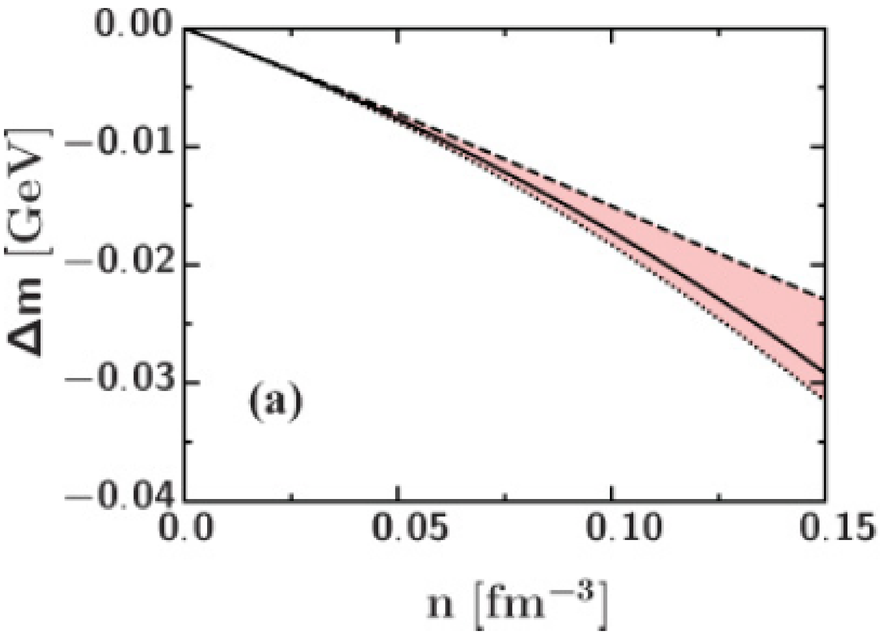
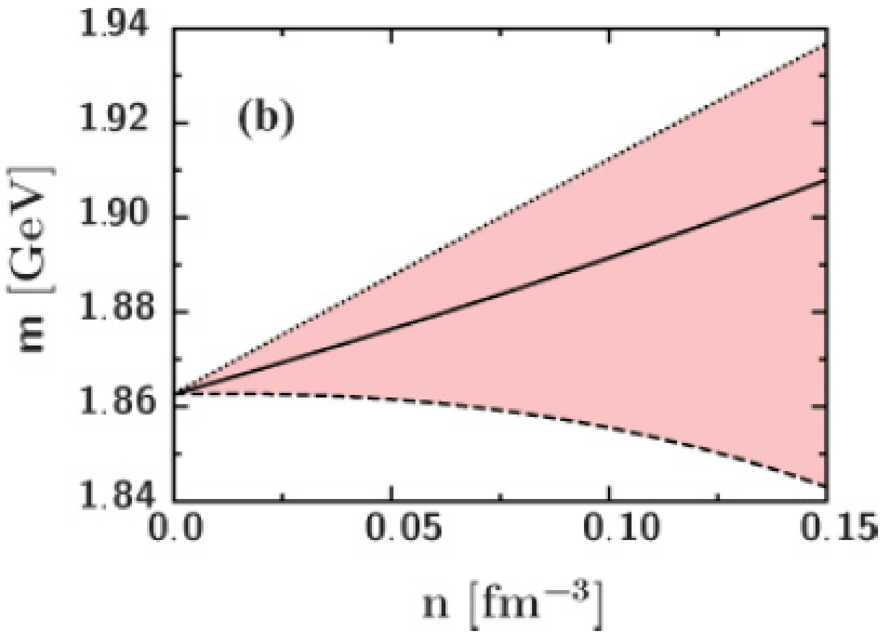
Numerical Evaluation: pseudo-scalar D meson

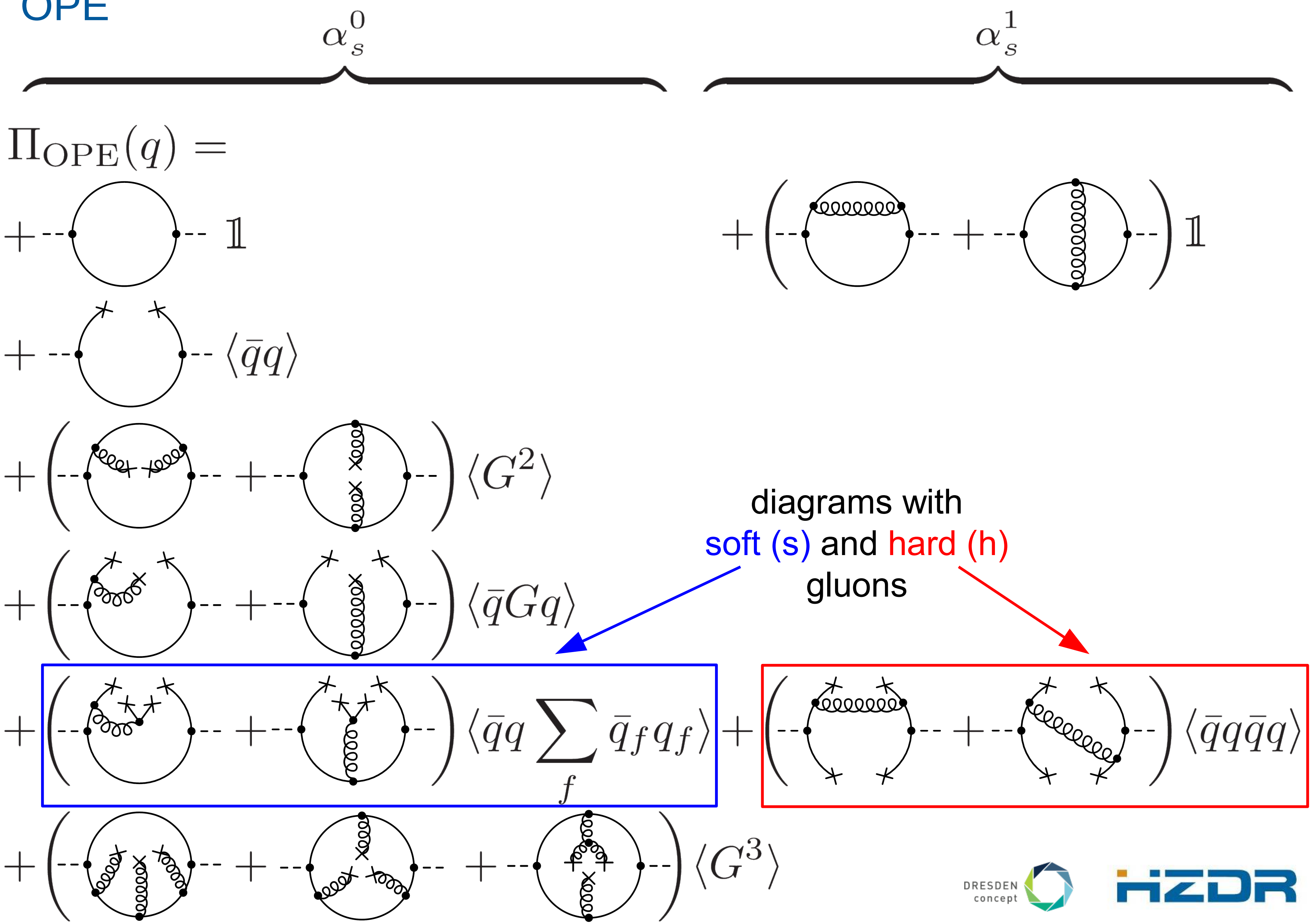
utilizing in-medium OPE up to mass dimension 5 [Hilger et al., PRC 79 (2009)]





issue: associate production of $c\bar{c}$
vs. in-medium masses

OPE



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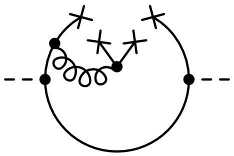
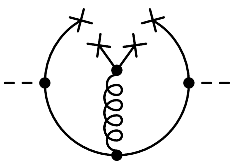
Wilson Coefficients

$$\Pi_{\mathrm{OPE}}(q) = \sum_k C_k(q) \langle \mathcal{O}_k \rangle$$

$$C_k(q) = \sum_l \mathcal{C}_{kl} L_{kl}$$



k	$\mathcal{C}^s_{kl}$ $l=1$	$\mathcal{C}^s_{kl}$ $l=2$	$\mathcal{C}^s_{kl}$ $l=3$	L^s_{kl} $l=1$	L^s_{kl} $l=2$	L^s_{kl} $l=3$
1	$-\frac{1}{3}\frac{1}{(q^2-m_Q^2)^2}\left(1-\frac{q^2}{q^2-m_Q^2}\right)^2$	$\frac{2}{9}\frac{1}{(q^2-m_Q^2)^3}$	$-\frac{8}{3}\frac{1}{(q^2-m_Q^2)^4}$	1	$q^2-4\frac{(vq)^2}{v^2}$	$\frac{3}{8}q^4-2\frac{q^2(vq)^2}{v^2}+\frac{(vq)^4}{v^4}$
2	0	$-\frac{4}{9}\frac{1}{(q^2-m_Q^2)^3}$	$\frac{8}{3}\frac{1}{(q^2-m_Q^2)^4}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$q^4-7\frac{q^2(vq)^2}{v^2}+6\frac{(vq)^4}{v^4}$
3	0	$-\frac{4}{3}\frac{m_Q}{(q^2-m_Q^2)^3}$	$\frac{8}{3}\frac{m_Q}{(q^2-m_Q^2)^4}$	—	$\frac{(vq)}{v^2}$	$\frac{(vq)}{v^2}\left(q^2-\frac{(vq)^2}{v^2}\right)$
4	0	$\frac{2}{9}\frac{1}{q^6}$	$-\frac{8}{3}\frac{1}{q^8}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$\frac{3}{8}q^4-2\frac{q^2(vq)^2}{v^2}+\frac{(vq)^4}{v^4}$
5	0	$-\frac{4}{9}\frac{1}{q^6}$	$\frac{8}{3}\frac{1}{q^8}$	—	$q^2-4\frac{(vq)^2}{v^2}$	$q^4-7\frac{q^2(vq)^2}{v^2}+6\frac{(vq)^4}{v^4}$



k	$\mathcal{C}^s_{kl}$ $l=1$	$\mathcal{C}^s_{kl}$ $l=2$	L^s_{kl} $l=1$	L^s_{kl} $l=2$
1	$\frac{1}{3}\frac{1}{(q^2-m_Q^2)^2}\left(2-\frac{q^2}{q^2-m_Q^2}\right)$	$-\frac{1}{9}\frac{1}{(q^2-m_Q^2)^3}$	1	$q^2-4\frac{(vq)^2}{v^2}$
2	0	$\frac{2}{9}\frac{1}{(q^2-m_Q^2)^3}$	—	$q^2-4\frac{(vq)^2}{v^2}$
3	0	$\frac{2}{3}\frac{m_Q}{(q^2-m_Q^2)^3}$	—	$\frac{(vq)}{v^2}$
4	$\frac{1}{3}\frac{1}{q^4}$	$-\frac{1}{9}\frac{1}{q^6}$	1	$q^2-4\frac{(vq)^2}{v^2}$
5	0	$\frac{2}{9}\frac{1}{q^6}$	—	$q^2-4\frac{(vq)^2}{v^2}$

Four-Quark Condensates

- no numerical values – factorization ansatz ? :
  - justified by large-N_c arguments for vacuum and finite densities only [Leupold, PLB 616 (2005)]
  - spoils behaviour under chiral transformations

- in qQ sector, e.g. D and B mesons:
  - 27 condensates in medium
  - heavy-light condensates
  - use and extend heavy-quark expansion method (HQE)

- medium behaviour ?

k	$\langle \mathcal{O}_k \rangle_{(\mathbb{1})}$	$\langle \mathcal{O}_k \rangle_{(t^A)}$
1	$\langle : \bar{q} q \bar{Q} Q : \rangle$	$\langle : \bar{q} t^A q \bar{Q} t^A Q : \rangle$
2	$\langle : \bar{q} \gamma_\nu q \bar{Q} \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_\nu t^A q \bar{Q} \gamma^\nu t^A Q : \rangle$
3	$\langle : \bar{q} \sigma_{\nu\rho} q \bar{Q} \sigma^{\nu\rho} Q : \rangle$	$\langle : \bar{q} \sigma_{\nu\rho} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle$
4	$\langle : \bar{q} \gamma_5 \gamma_\nu q \bar{Q} \gamma_5 \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_5 \gamma_\nu t^A q \bar{Q} \gamma_5 \gamma^\nu t^A Q : \rangle$
5	$\langle : \bar{q} \gamma_5 q \bar{Q} \gamma_5 Q : \rangle$	$\langle : \bar{q} \gamma_5 t^A q \bar{Q} \gamma_5 t^A Q : \rangle$
6	$\langle : \bar{q} \slashed{v} q \bar{Q} \slashed{v} Q : \rangle / v^2$	$\langle : \bar{q} \slashed{v} t^A q \bar{Q} \slashed{v} t^A Q : \rangle / v^2$
7	$\langle : \bar{q} \sigma^{\sigma\omega} q \bar{Q} \sigma^{\nu\rho} Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$	$\langle : \bar{q} \sigma^{\sigma\omega} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$
8	$\langle : \bar{q} \gamma_5 \slashed{v} q \bar{Q} \gamma_5 \slashed{v} Q : \rangle / v^2$	$\langle : \bar{q} \gamma_5 \slashed{v} t^A q \bar{Q} \gamma_5 \slashed{v} t^A Q : \rangle / v^2$
9	$\langle : \bar{q} \slashed{v} q \bar{Q} Q : \rangle$	$\langle : \bar{q} \slashed{v} t^A q \bar{Q} t^A Q : \rangle$
10	$\langle : \bar{q} q \bar{Q} \slashed{v} Q : \rangle$	$\langle : \bar{q} t^A q \bar{Q} \slashed{v} t^A Q : \rangle$
11	$\langle : \bar{q} \sigma^{\sigma\omega} q \bar{Q} \gamma_5 \gamma^\nu Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$	$\langle : \bar{q} \sigma^{\sigma\omega} t^A q \bar{Q} \gamma_5 \gamma^\nu t^A Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$
12	$\langle : \bar{q} \gamma_5 \gamma^\nu q \bar{Q} \sigma^{\sigma\omega} Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$	$\langle : \bar{q} \gamma_5 \gamma^\nu t^A q \bar{Q} \sigma^{\sigma\omega} t^A Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$

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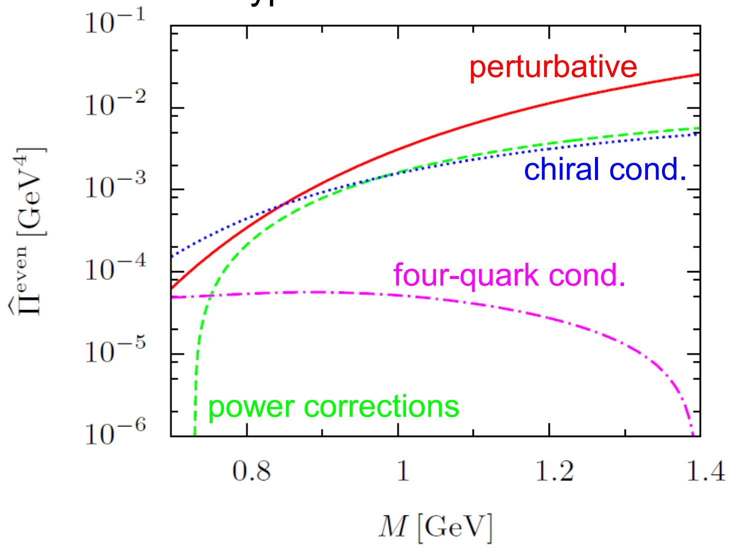
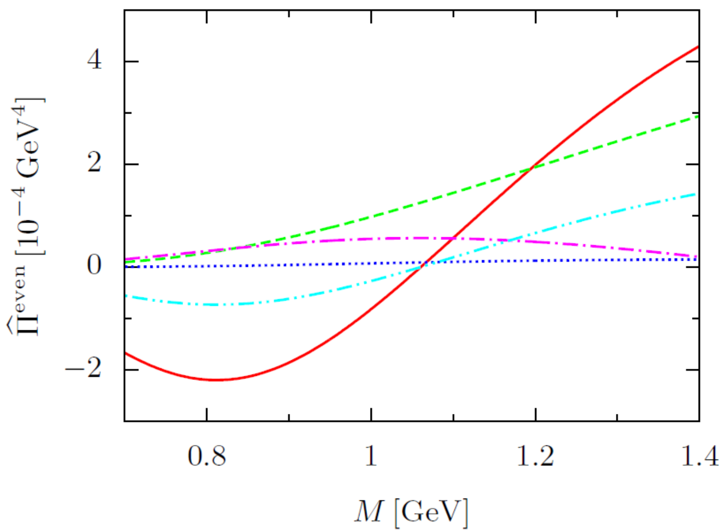
Numerical Evaluation: pseudo-scalar D meson

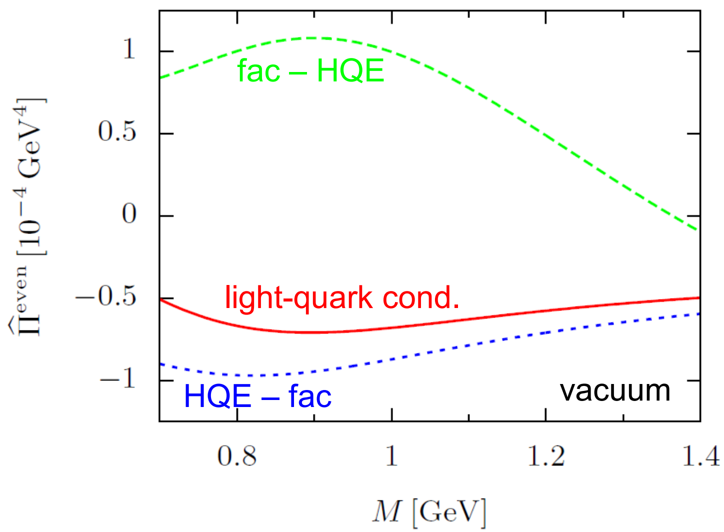
scale of typical OPE contributions

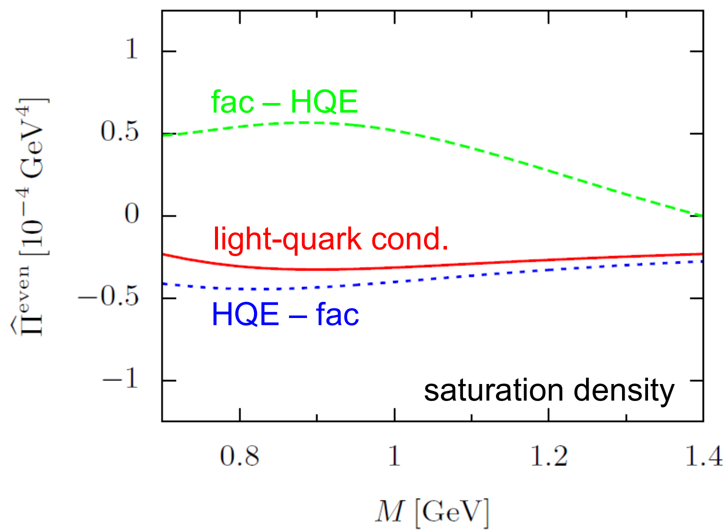


OPE contributions of mass dimension 4 and 5 condensates



OPE contributions of four-quark condensates in three different approaches





Summary

- medium modifications of D and B mesons can be traced back to changing condensates within the QCD sum rule approach
- chiral condensate dominates D meson sum rules
- four-quark condensates contributions are important for ρ and ω, but subleading for D mesons

→ spectral properties obtained in [Hilger et al., PRC 79 (2009)] hold

Outlook

Chirally Odd Condensates in Weinberg Sum Rules

Example: ρ meson

$$\varphi = \begin{pmatrix} u \\ d \end{pmatrix}$$

chiral transformations

$$\varphi_{\mathrm{L,R}} \longrightarrow e^{-i\Theta^a_{\mathrm{L,R}}\tau_a}\varphi_{\mathrm{L,R}}\,, \qquad \tau_a = \sigma_a/2$$

Pauli metrices

$$\langle \bar{q}q\bar{q}q\rangle_{\mathrm{odd}} = \langle (\bar{\varphi}_{\mathrm{R}}\gamma_\mu\lambda^A\tau_3\varphi_{\mathrm{R}})(\bar{\varphi}_{\mathrm{L}}\gamma^\mu\lambda^A\tau_3\varphi_{\mathrm{L}})\rangle$$

[Hilger et al., PLB 709 (2012)]

Gell-Mann metrices

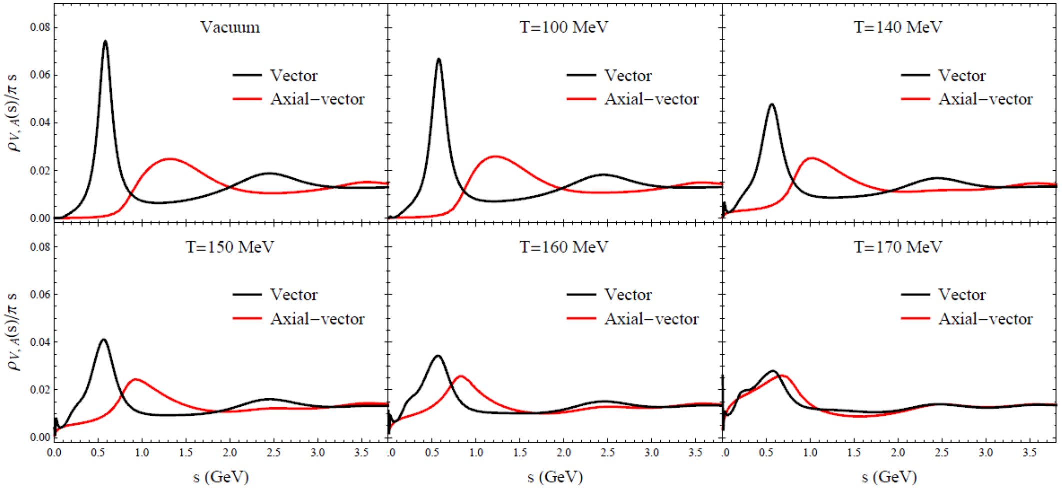
Weinberg Sum Rules (WSR) of ρ and a_1 mesons – degeneracy of spectral functions

$$\int ds\, \frac{1}{s}\Delta\rho(s) = f_\pi^2$$

$$\int ds\, \Delta\rho(s) = -2m_q\langle\bar{q}q\rangle$$

$$\int ds\, s\Delta\rho(s) = -2\pi\alpha_s\langle\bar{q}q\bar{q}q\rangle_{\mathrm{odd}}$$

with $\Delta\rho = \rho_{\mathrm{V}} - \rho_{\mathrm{A}}$



fixed by microscopic calculation in had. eff. field theory

ansatz with parameters selected by minimization of inequalities in the WSR

[Hohler, Rapp, PLB 731 (2014)]

Back up

QCD Sum Rules in Vacuum

causal current-current correlator $\Pi(q) = \int d^4x \, e^{iqx} \langle \mathrm{T} \left[j(x) j^\dagger(0) \right] \rangle$

quark d.o.f.

$u \quad d \quad s$

$c \quad b$

QCD

Sum

Rule

hadronic d.o.f.

$\pi \quad \rho \quad \omega \quad J/\psi \quad a_1 \quad D \quad B$

$p \quad n \quad \Delta \quad \Lambda$

phenomenology

dispersion relation (from analyticity of $\Pi(q)$) $\Pi(q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{s - q^2}$

$$\Pi_{\mathrm{OPE}}(q^2) = \int_0^\infty ds \frac{\text{spectral density}\,(s)}{s - q^2}$$

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OPE: loop expansion

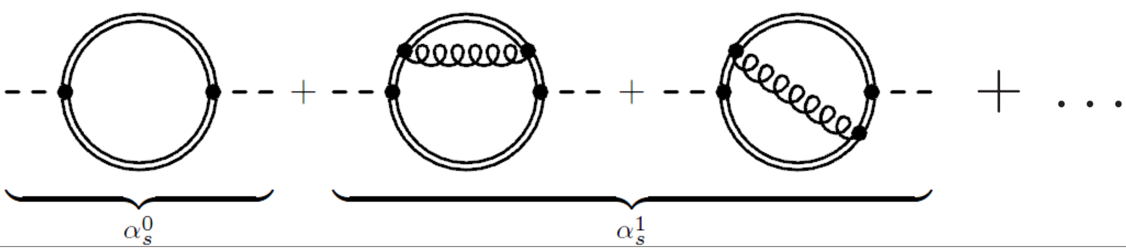
$$\Pi_{\mathrm{OPE}}(q) = \int d^4x\, e^{ipx} \langle \mathrm{T}\Big[j(x) j^\dagger(0) \Big(\mathbb{1} + \frac{(i)^2}{2!} \int d^4y_1 d^4y_2 \mathcal{L}_{\mathrm{int}}(y_1) \mathcal{L}_{\mathrm{int}}(y_2) + \dots \Big) \Big] \rangle$$

$$= \Pi_{\alpha_s^0}(q) + \Pi_{\alpha_s^1} + \dots$$

with reduced interaction Lagrangian:

$$\mathcal{L}_{\mathrm{int}}(y) = g\bar{q}(y)\gamma^\mu t^A q(y) G_\mu^A(y) + (q \longrightarrow Q)$$

$\Pi_{\mathrm{OPE}}(q) =$



OPE: QCD quark propagator

background field method in Fock-Schwinger gauge

$$S(p) = \sum_{i=0}^{\infty} S^{(i)}(p) \qquad S^{(i)}(p) = -S^{(i-1)}(p)\gamma^{\mu}\tilde{A}_{\mu}S^{(0)}(p)\,,\ i \geq 1$$

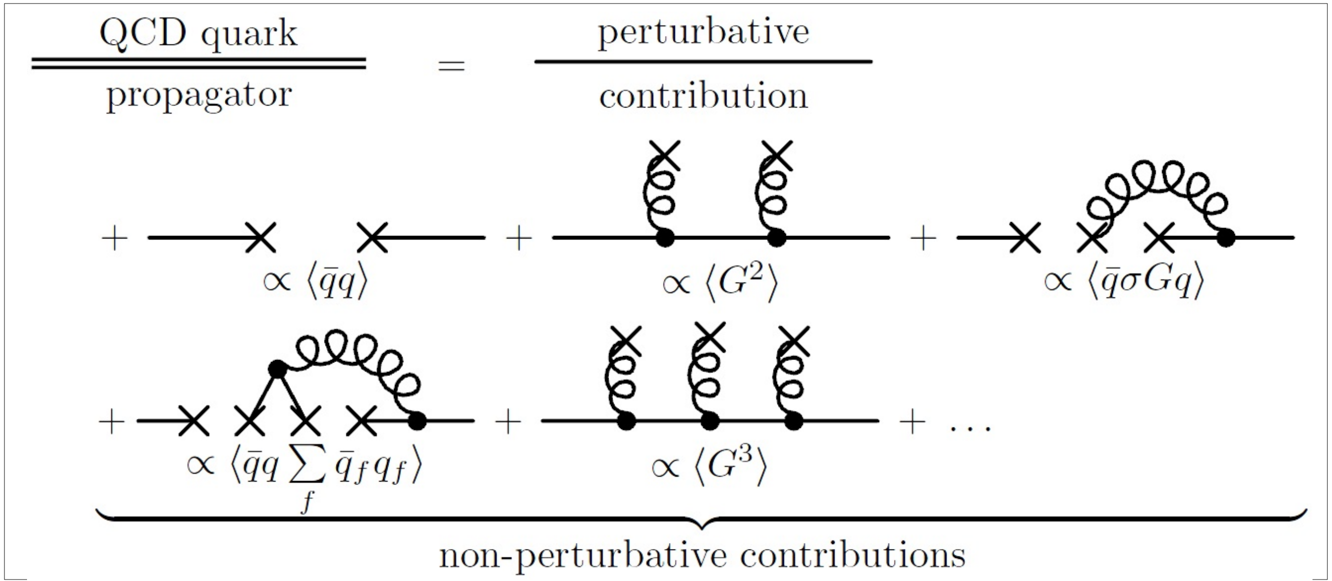
free quark propagator

with derivative operator

$$\tilde{A}_{\mu} = -\sum_{j=0}^{\infty} g \frac{(-i)^j}{j!(j+2)} D_{\vec{\alpha}_j} G_{\mu\nu} \partial^{\nu} \partial^{\vec{\alpha}_j}$$

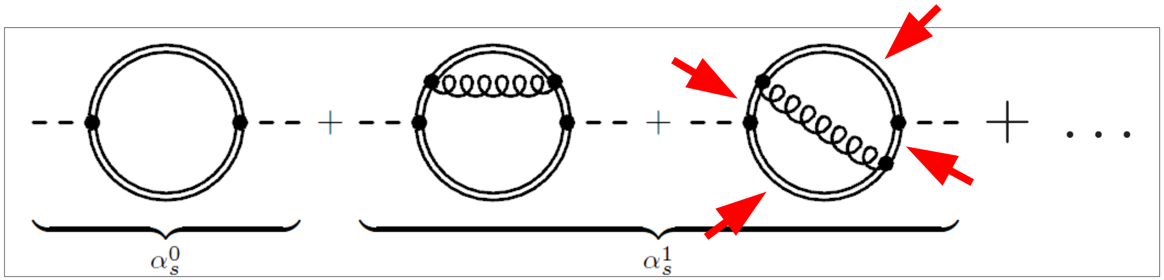
and 2 Wick uncontracted non-local quark operators

construction of condensates

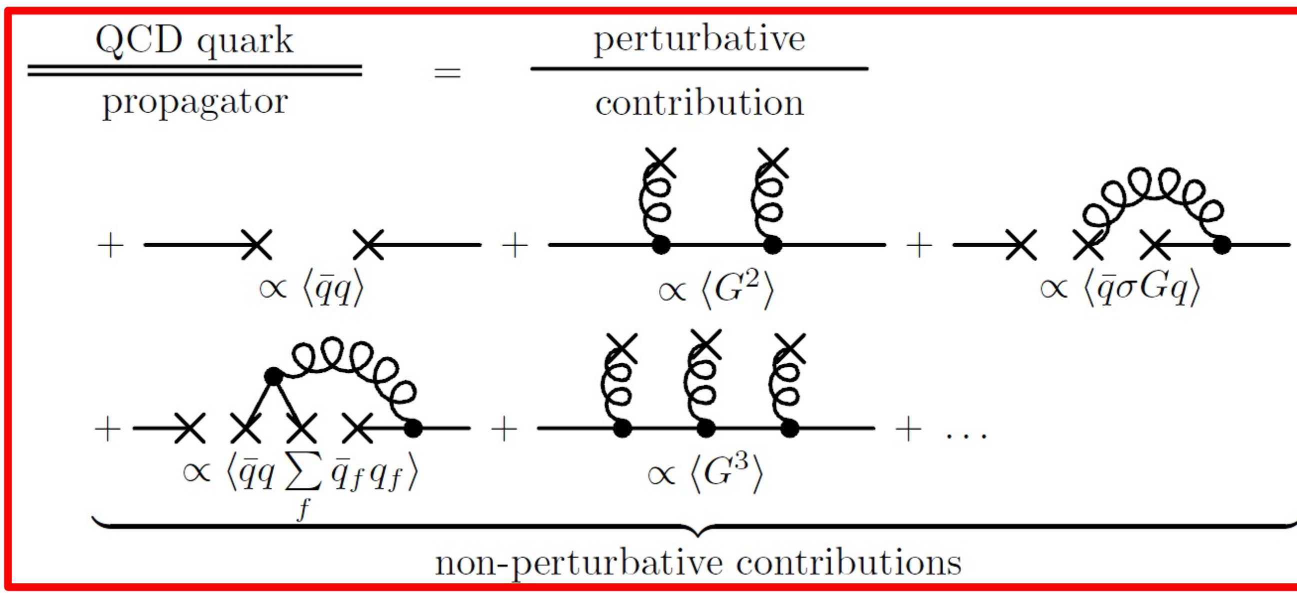


OPE

$$\Pi_{\mathrm{OPE}}(q) =$$

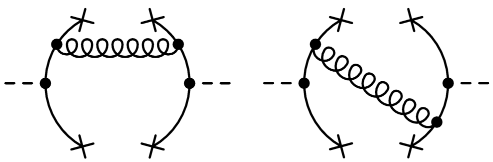


$$\otimes$$



Four-Quark Condensates

$$\Pi_{\mathrm{OPE}}(q) = \sum_k C_k(q) \langle \mathcal{O}_k \rangle$$



$$\langle \mathcal{O}_k \rangle^h = 4 \langle \mathcal{O}_k \rangle^h_{(\mathbb{1})} - 3 \langle \mathcal{O}_k \rangle^h_{(t^A)}$$

k	$\langle \mathcal{O}_k \rangle^h_{(\mathbb{1})}$	$\langle \mathcal{O}_k \rangle^h_{(t^A)}$
1	$\langle : \bar{q} q \bar{Q} Q : \rangle$	$\langle : \bar{q} t^A q \bar{Q} t^A Q : \rangle$
2	$\langle : \bar{q} \gamma_\nu q \bar{Q} \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_\nu t^A q \bar{Q} \gamma^\nu t^A Q : \rangle$
3	$\langle : \bar{q} \sigma_{\nu\rho} q \bar{Q} \sigma^{\nu\rho} Q : \rangle$	$\langle : \bar{q} \sigma_{\nu\rho} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle$
4	$\langle : \bar{q} \gamma_5 \gamma_\nu q \bar{Q} \gamma_5 \gamma^\nu Q : \rangle$	$\langle : \bar{q} \gamma_5 \gamma_\nu t^A q \bar{Q} \gamma_5 \gamma^\nu t^A Q : \rangle$
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6	$\langle : \bar{q} \not{v} q \bar{Q} \not{v} Q : \rangle / v^2$	$\langle : \bar{q} \not{v} t^A q \bar{Q} \not{v} t^A Q : \rangle / v^2$
7	$\langle : \bar{q} \sigma^{\sigma\omega} q \bar{Q} \sigma^{\nu\rho} Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$	$\langle : \bar{q} \sigma^{\sigma\omega} t^A q \bar{Q} \sigma^{\nu\rho} t^A Q : \rangle g_{\nu\omega} v_\sigma v_\rho / v^2$
8	$\langle : \bar{q} \gamma_5 \not{v} q \bar{Q} \gamma_5 \not{v} Q : \rangle / v^2$	$\langle : \bar{q} \gamma_5 \not{v} t^A q \bar{Q} \gamma_5 \not{v} t^A Q : \rangle / v^2$
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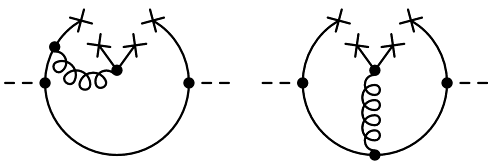
12	$\langle : \bar{q} \gamma_5 \gamma^\nu q \bar{Q} \sigma^{\sigma\omega} Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$	$\langle : \bar{q} \gamma_5 \gamma^\nu t^A q \bar{Q} \sigma^{\sigma\omega} t^A Q : \rangle \varepsilon_{\alpha\nu\sigma\omega} v^\alpha$



k	$\langle \mathcal{O}_k \rangle^s$
1	$\langle : \bar{q} \gamma^\nu t^A q \sum_f \bar{q}_f \gamma_\nu t^A q_f : \rangle$
2	$\langle : \bar{q} \not{v} t^A q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle / v^2$
3	$\langle : \bar{q} t^A q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle$
4	$\langle : \bar{Q} \gamma^\nu t^A Q \sum_f \bar{q}_f \gamma_\nu t^A q_f : \rangle$
5	$\langle : \bar{Q} \not{v} t^A Q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle / v^2$
6	$\langle : \bar{Q} t^A Q \sum_f \bar{q}_f \not{v} t^A q_f : \rangle$

medium

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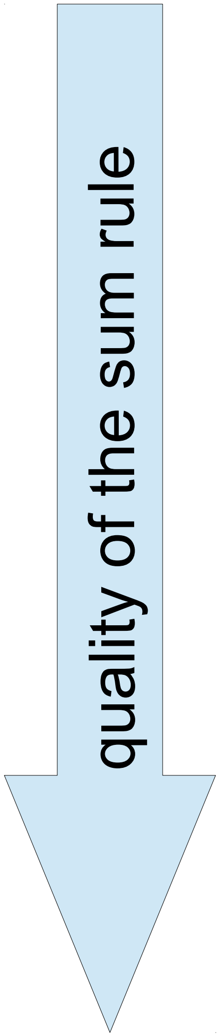
How to handle heavy quarks in four-quark condensates?

4 Approaches:



- neglecting condensates containing heavy quarks, factorization
- factorization and subsequent heavy-quark expansion
- heavy-quark expansion and subsequent factorization
- lattice calculations

DRESDEN concept

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Factorization of Four-Quark Condensats

colorless hadronic states and the QCD vacuum

vacuum

$$\langle \bar{q}\Gamma_1 t^A q \bar{q}\Gamma_2 t^A q \rangle = \sum_n c_n(\Gamma_1, \Gamma_2, t^A) \langle \bar{q}q|n\rangle\langle n|\bar{q}q\rangle$$
$$\approx c_0(\Gamma_1, \Gamma_2, t^A)\langle \bar{q}q\rangle^2$$

medium

reduction of light four-quark condensates:

$$\langle \bar{q}\Gamma_1 t^A q \bar{q}\Gamma_2 t^A q \rangle = a\langle \bar{q}q\rangle^2 + b\langle \bar{q}q\rangle\langle \bar{q}\not{v}q\rangle + c\langle \bar{q}\not{v}q\rangle^2$$

reduction of heavy-light four-quark condensates:

$$\langle \bar{q}\Gamma_1 t^A q \bar{Q}\Gamma_2 t^A Q \rangle = A\langle \bar{q}q\rangle\langle \bar{Q}Q\rangle + B\langle \bar{q}\not{v}q\rangle\langle \bar{Q}Q\rangle + C\langle \bar{q}q\rangle\langle \bar{Q}\not{v}Q\rangle + D\langle \bar{q}\not{v}q\rangle\langle \bar{Q}\not{v}Q\rangle$$

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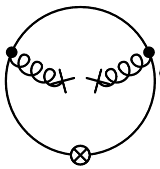
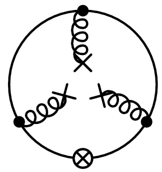
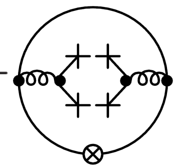
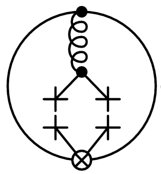
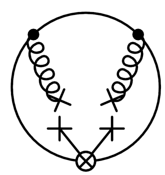
Heavy-Quark Expansion (HQE)

heavy two-quark condensate: [Generalis, Broadhurst, PLB139 (1984)]

$\langle \bar{Q} Q \rangle =$  $\langle G^2 \rangle +$  $\langle G^3 \rangle +$  $\langle \sum_f \bar{q}_f q_f \sum_{f'} \bar{q}_{f'} q_{f'} \rangle + \ldots$

leading order: $$\langle \bar{Q} Q \rangle = -\frac{g^2}{48\pi^2 m_Q} \langle G^2 \rangle + \mathcal{O}(1/m_Q^3)$$

heavy-light four-quark condensate:

$\langle \bar{q} A q \bar{Q} B Q \rangle =$  $\langle \bar{q} q \sum_f \bar{q}_f q_f \rangle +$  $\langle \bar{q} q G^2 \rangle + \ldots$

leading order:

$$\langle \bar{q} \gamma_\nu t^A q \sum_f \bar{q}_f \gamma^\nu t^A q_f \rangle \,, \langle \bar{q} \not{v} t^A q \sum_f \bar{q}_f \not{v} t^A q_f \rangle / v^2 \,, \langle \bar{q} t^A q \sum_f \bar{q}_f \not{v} t^A q_f \rangle$$

with HQE coefficiens of order $1/m_Q^0$

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Chirally Odd Condensates

besides $\langle \bar{q}q \rangle$ further candidates for order parameters

➔ condensates not invariant under chiral transformations

chirally odd condensates, e.g. chiral condensate

$\varphi \ldots N_f$ vector

$$\varphi = \varphi_{\mathrm{L}} + \varphi_{\mathrm{R}} \,, \qquad \varphi_{\mathrm{L,R}} = P_{\mathrm{L,R}} \varphi = \frac{1}{2}(1 \mp \gamma_5)\varphi$$

chiral transformations

$$\varphi_{\mathrm{L,R}} \longrightarrow e^{-i\Theta^a_{\mathrm{L,R}}\tau_a} \varphi_{\mathrm{L,R}} \,,$$

τ_a generators of $\mathrm{SU}(N_f)$

recover vector (V) and axial-vector (A) transformations for $\Theta^a_{\mathrm{L}} = -\Theta^a_{\mathrm{R}} = \Theta^a$

V: $\varphi \longrightarrow e^{-i\Theta^a\tau_a}\varphi$ A: $\varphi \longrightarrow e^{-i\Theta^a\tau_a\gamma_5}\varphi$

test chiral condensate

$$\langle \bar{\varphi}\varphi \rangle = \langle \bar{\varphi}_{\mathrm{L}}\varphi_{\mathrm{R}} \rangle + \langle \bar{\varphi}_{\mathrm{R}}\varphi_{\mathrm{L}} \rangle \longrightarrow \langle \bar{\varphi}_{\mathrm{L}} e^{-i(\Theta^a_{\mathrm{R}} - \Theta^a_{\mathrm{L}})\tau_a} \varphi_{\mathrm{R}} \rangle + (\mathrm{L} \longleftrightarrow \mathrm{R})$$

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Translation to D and B Mesons

[Hilger et al., PRC 84 (2011)]

$$\varphi = \begin{pmatrix} u \\ d \\ c, b \end{pmatrix}$$

chiral transformations restricted to light part

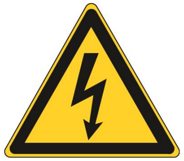
Gell-Mann metrices

$$\varphi_{\mathrm{L,R}} \longrightarrow e^{-i\Theta^a_{\mathrm{L,R}}\tau_a}\varphi_{\mathrm{L,R}}\,, \qquad \tau_a = \lambda_a/2$$

with special choice $\Theta_{\mathrm{L,R}} = (\Theta^1_{\mathrm{L,R}}, \Theta^2_{\mathrm{L,R}}, \Theta^3_{\mathrm{L,R}}, 0, \ldots, 0)$

➔ leaves Lagrangian $\bar{\varphi} M \varphi\,,\, M = \mathrm{diag}(0, 0, m_{c,b})$ invariant

mixing of chiral partner meson currents under corresponding axial transformations



observed splitting of chiral partner spectra

$$\frac{m_P}{m_S} \sim \frac{1800}{2300} \qquad \frac{m_V}{m_A} \sim \frac{2000}{2400}$$

➔ spontaneous symmetry breaking, driven by order parameters,

e.g. $\langle \bar{q}q \rangle$, $\langle \bar{q}q\bar{q}q \rangle_{\mathrm{odd}}$

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Extension of Weinberg Sum Rules for D and B mesons

Weinberg-Type Sum Rules for qQ Mesons:

- set up to condensates of mass dimension 5 for P/S and V/A
  [Hilger et al., PRC 84 (2011)]

- our **goal**: extension to mass dimension 6

➜ identified for P/S by its transformation properties

$$\langle \bar{q}q\bar{q}q \rangle_{\mathrm{odd}} = \langle (\bar{\varphi}_{\mathrm{R}} \lambda^A \varphi_{\mathrm{L}})(\bar{\varphi}_{\mathrm{L}} \not{v} \lambda^A \varphi_{\mathrm{L}}) \rangle + \langle (\bar{\varphi}_{\mathrm{L}} \lambda^A \varphi_{\mathrm{R}})(\bar{\varphi}_{\mathrm{L}} \not{v} \lambda^A \varphi_{\mathrm{L}}) \rangle + (\mathrm{L} \longleftrightarrow \mathrm{R})$$

➜ quantifies difference of chiral partner spectra

- chiral restoration scenario in a strongly interacting medium

$\langle \bar{q}q \rangle \longrightarrow 0$, $\langle \bar{q}q\bar{q}q \rangle_{\mathrm{odd}} \longrightarrow 0$, etc.

➜ drive degeneracy of chiral partner spectra, cf. [Hohler, Rapp, PLB 731 (2014)]

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