Hidden charm meson production in antiproton-induced reactions on nuclei

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Outline

- Motivation
- Glauber model (probabilistic)
- J/Ψ and Ψ´ production: influence of absorption
- Generalized eikonal approximation (quantum)
- Polarized χ_{c2} production: nondiagonal transitions
- Exotic XYZ meson production
- Summary and outlook

A.L., M. Bleicher, A. Gillitzer, M. Strikman, PRC 87, 054608 (2013);
A.L., M. Strikman, M. Bleicher, PRC 89, 014621 (2014);
and work in progress

Why to study charmonium-nucleon interactions ?

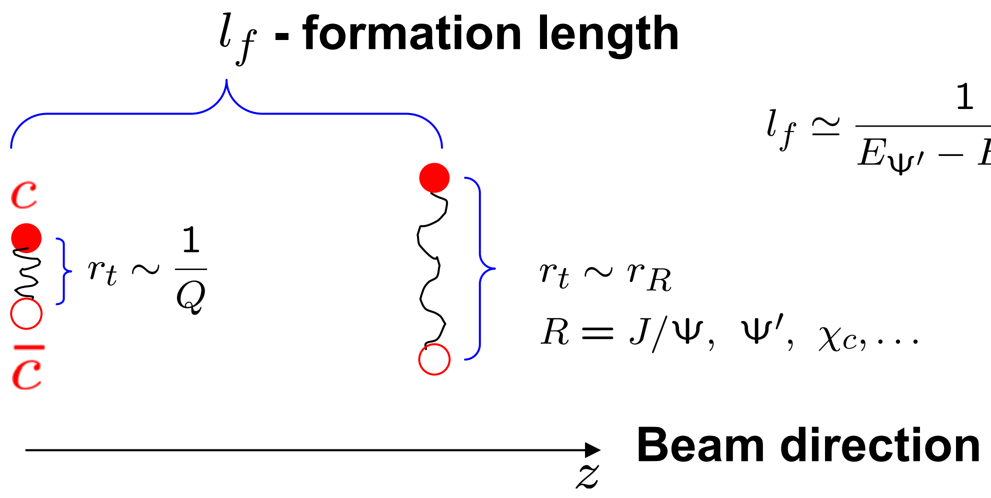
- Important for the interpretation of J/Ψ suppression in relativistic heavy-ion collisions and separation of the quark-gluon plasma signals from the cold nuclear matter effects.

- May constrain the QCD-inspired models of charmonia and of the charmonium-like XYZ mesons.

- Deepens our understanding of the nonperturbative vs perturbative QCD: factorization theorem, color dipole cross section, color transparency …

Color transparency

At high momentum transfer Q the small-size quark-antiquark configuration is created which expands to the normal meson size:



E.g., for J/Ψ:

$$l_f \simeq \frac{1}{E_{\Psi'} - E_{J/\Psi}} \simeq \frac{2p_{J/\Psi}}{m^2_{\Psi'} - m^2_{J/\Psi}} \simeq 0.1 \text{ fm } p_{J/\Psi} \text{ (GeV)}$$

G.R. Farrar, H. Liu,
L.L. Frankfurt, M.I. Strikman,
PRL 61, 686 (1988)

Color dipole – proton cross section (in the pQCD limit $r_t \to 0$) :

$$\sigma_{q\bar{q}} \propto r_t^2 \propto -Q^{-2} \sim m_R^{-2}$$

Within formation length charmonium-nucleon cross section is small.

internucleon spacing

At p_{lab} > 20 GeV the formation length is large: $l_f > d_{NN} \simeq 2$ fm

The information on the *genuine* J/Ψ N cross section from hadron- and photon-induced reactions on nuclei at high energies *is blured* by uncertain interactions within formation length .

Antiproton-nucleus reactions can be used to determine $\sigma_{J/\Psi N}$!

Formation reaction: $\bar{p}p \to J/\Psi$, $p_{J/\Psi} \simeq p_{lab} \simeq 4$ GeV/c ,

$l_f \simeq 0.4$ fm $< d_{NN}$.

***J/Ψ is formed before it collides with a nucleon*.**

- **Possible to study the *genuine* J/Ψ N interactions**

- **Difficulty - due to Fermi motion the J/Ψ production cross section on a nucleus is reduced:**

$$\frac{\sigma_{\bar{p}A \to J/\Psi(A-1)^*}}{Z\sigma_{\bar{p}p \to J/\Psi}} \sim 10^{-4}$$

G.R. Farrar et al., NPB 345, 125 (1990)

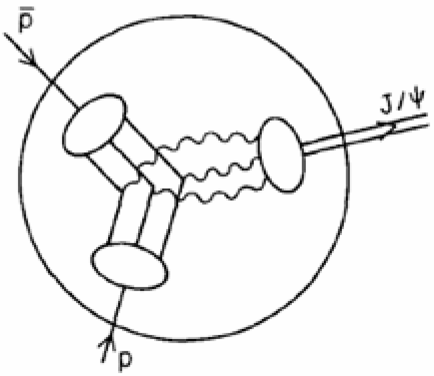


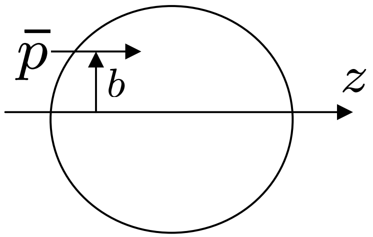
Fig. 2. The dominant mechanism for pp̄ exclusive annihilation into J/ψ.

Figure from S.J. Brodsky and A.H. Mueller, PLB 206, 685 (1988)

Other charmonia $(\Psi'(2S), \chi_c(1P), \ldots)$ can be also produced in $\bar{p}A$ reactions at threshold (p_{lab}=4-6 GeV/c). Their internal structure can be tested by interactions with target nucleons.

Possible at PANDA@FAIR: antiproton beam at p_{lab}~1.5-15 GeV/c, luminocity L~ $2 \cdot 10^{32}$ cm^{-2} s^{-1}=0.2 nb^{-1} s^{-1}, proton and nuclear targets.

How good are the antiproton-nucleus reactions to probe the charmonium-nucleon interactions ?



Charmonium $R = J/\Psi,\ \Psi',\ \chi_c, \ldots$ production cross section in the Glauber model:

$$\sigma_{\bar{p}A\to R(A-1)^*} = 2\pi \int\limits_0^\infty db\, b \int\limits_{-\infty}^{\infty} \frac{dz}{v_{\bar{p}}} \mathcal{P}_{\bar{p},\mathrm{surv}}(z,b)\Gamma_{\bar{p}\to R}(z,b)\mathcal{P}_{R,\mathrm{surv}}(z,b)\ ,$$

$$\Gamma_{\bar{p}\to R}(z,b) = \int \frac{2d^3p}{(2\pi)^3} v_{\bar{p}p}\sigma_{\bar{p}p\to R}(p,p_{\bar{p}}) f_p(z,b,\mathbf{p})\ , \quad f_p(z,b,\mathbf{p}) = \Theta(p_{F,p} - |\mathbf{p}|)\ ,$$

$$\mathcal{P}_{\bar{p},\mathrm{surv}}(z,b) = \mathrm{e}^{-\int\limits_{-\infty}^{z} dz'\rho(z',b)\sigma^{\mathrm{inel}}_{\bar{p}N}(p_{\mathrm{lab}})}\ , \quad \mathcal{P}_{R,\mathrm{surv}}(z,b) = \mathrm{e}^{-\int\limits_{z}^{\infty} dz'\rho(z',b)\sigma^{\mathrm{eff}}_{RN}(p_R,z'-z)}\ .$$

$$\sigma^{\mathrm{eff}}_{RN}(p_R,z) = \sigma_{RN}(p_R)\left(\left[\frac{z}{l_f} + \frac{<n^2k_t^2>}{m_R^2}\left(1 - \frac{z}{l_f}\right)\right]\Theta(l_f - z) + \Theta(z - l_f)\right)$$

- charmonium-nucleon effective interaction cross section in the color diffusion model G.R. Farrar, L.L. Frankfurt, M.I. Strikman, and H. Liu, NPB 345, 125 (1990)

$\sigma_{RN}(p_R)$ **– total fully-formed-charmonium-nucleon cross section**

$< k_t^2 >^{1/2} \simeq 0.35$ GeV/c **- r.m.s. transverse momentum of a quark in a hadron**

$n = 3$ **- number of intermediate gluons.**

Charmonium dissociation cross sections (expectations):

$$RN \to \Lambda_c \bar{D}(+\text{pions})\ , \quad R = J/\Psi, \chi_c, \Psi'$$

$\sigma_{J/\Psi N} = 6 - 7$ mb

- from J/Ψ transparency ratios for AA at $\sqrt{s} = 20$ GeV (except PbPb), $pA, \gamma A, \pi A$ and $\bar{p}A$ reactions on nuclei *without* including sidefeeding effects from χc and ψ' decays

C. Gerschel and J. Hüfner, Z. Phys. C 56, 171 (1992);
D. Kharzeev et al., Z. Phys. C 74, 307 (1997)

$\sigma_{J/\Psi N} = 3.62$ mb,
$\sigma_{\Psi' N} = 20.0$ mb,
$\sigma_{\chi_{c1}}(L_z = 0) = 6.82$ mb,
$\sigma_{\chi_{c1}}(L_z = \pm 1) = 15.9$ mb

- from QCD factorization theorem and nonrelativistic quarkonium model. Consistent with $\Psi'/J/\Psi$ ratio in pA collisions *with* sidefeeding effects from χc and Ψ' decays

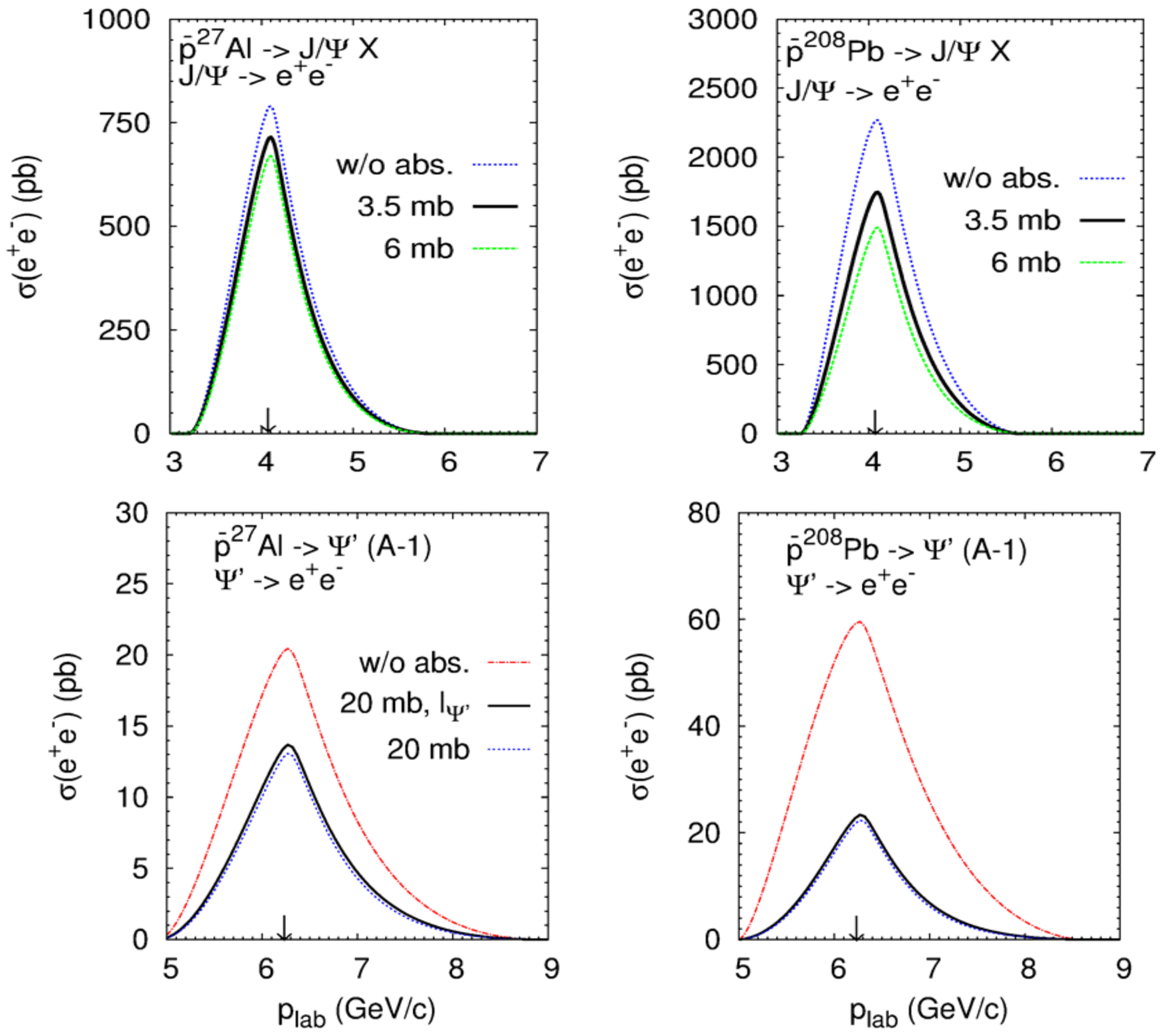
L. Gerland et al., PRL 81, 762 (1998)

$\sigma_{J/\Psi N} \simeq 5$ mb

- hadronic model

R. Molina, C.W. Xiao, E. Oset, PRC 86, 014604 (2012)

Influence of charmonium dissociation cross section σ_{RN} (R=J/Ψ,Ψ′) and charmonium formation length: $l_{\Psi'} \simeq 2l_{J/\Psi} \simeq 1$ fm

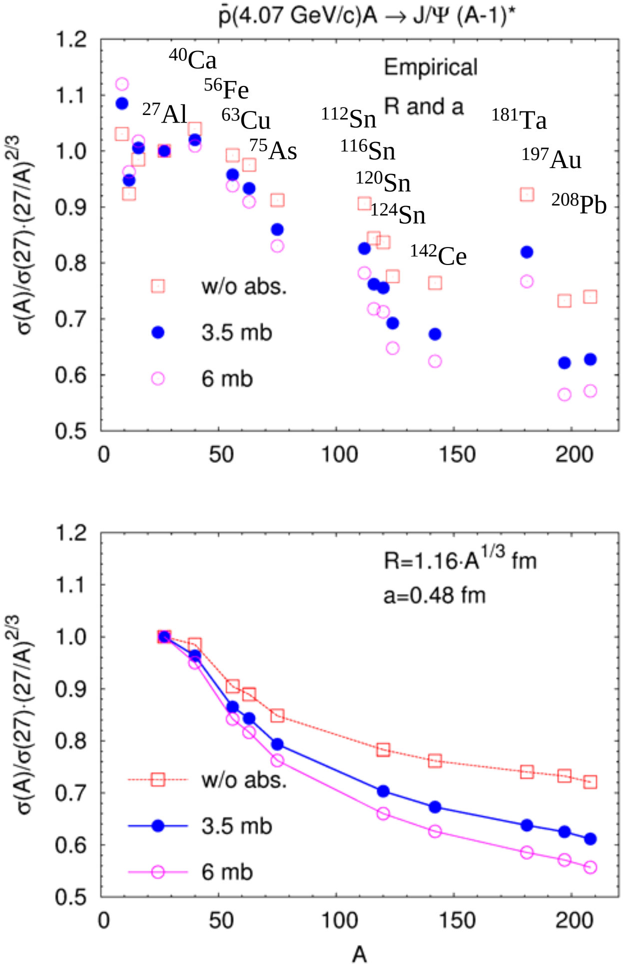


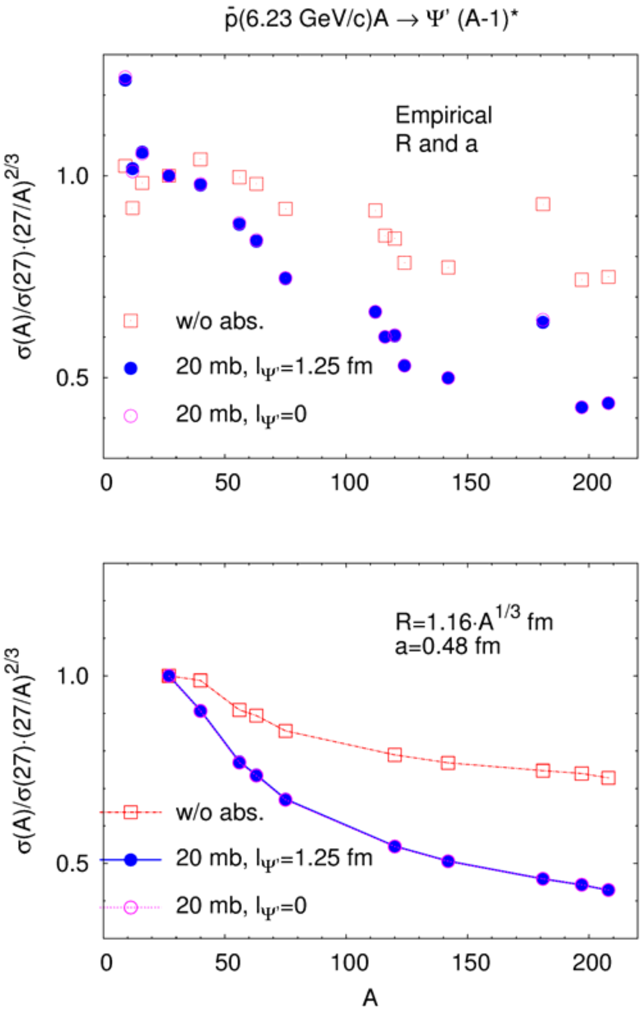
- For heavy nuclei - strong sensitivity to σ_{RN} .

- Almost no sensitivity to formation length.

Transparency ratio $\frac{\sigma_{\bar{p}A\to R(A-1)^*}}{\sigma_{\bar{p}^{27}\mathrm{Al}\to R^{26}\mathrm{Mg}^*}}\left(\frac{27}{A}\right)^{2/3}$ $(R = J/\Psi, \Psi')$ at the on-shell peak

Possible uncertainties in the in-medium production width $\Gamma_{\bar{p}\to R}$ cancel-out



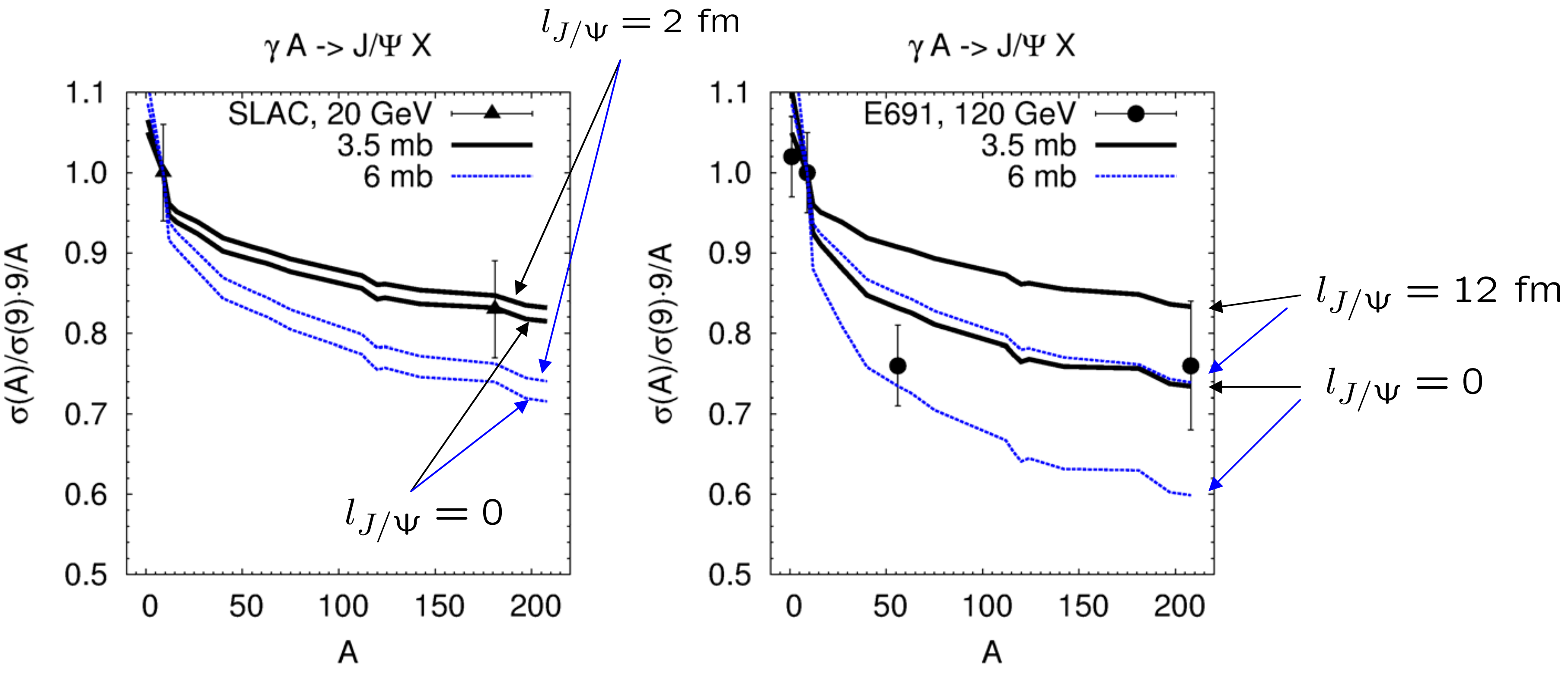


- ***Local variations of A-dependence due to details of nuclear density profiles.***
- ***Careful selection of the target nuclei needed.***

Influence of J/Ψ formation length on transparency ratio in γ-induced reactions:

$$S_A = \frac{\sigma_{\gamma A \to J/\Psi X}}{A\sigma_{\gamma p \to J/\Psi X}} = \frac{2\pi}{A} \int\limits_0^\infty db\, b \int\limits_{-\infty}^{\infty} dz \rho(z,b) P_{J/\Psi,\mathrm{surv}}(z - l_c, b)$$

$l_c = 2E_\gamma / m^2_{J/\Psi}$ **- coherence length**



$l_{J/\Psi} = 2$ fm

$l_{J/\Psi} = 0$

$l_{J/\Psi} = 12$ fm

$l_{J/\Psi} = 0$

- Difficult (at 120 GeV impossible) to determine $\sigma_{J/\Psi N}$ due to formation length effects.

χ_{cJ} $(J = 0, 1, 2)$ ***production:***

- Mass splitting between different χ_{cJ} states is small ~ 140 MeV

- Nondiagonal transitions $\chi_{cJ_1}N \to \chi_{cJ}N$ are easily possible

-In the simplest quark model with central (e.g Cornell) potential *the physical* χ_{cJ} state with helicity ν can be decomposed in the basis of $c\bar{c}$ states with fixed orbital and spin angular momentum projections on the charmonium momentum axis:

$$|J\nu\rangle = \sum_{L_z, S_z} |1L_z; 1S_z\rangle\langle 1L_z; 1S_z|J\nu\rangle \ .$$

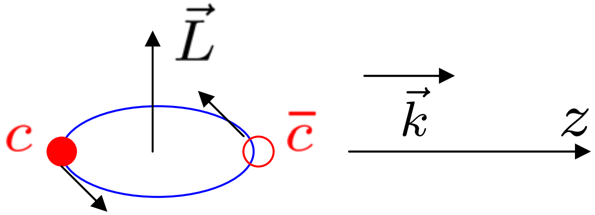
- **For the basis states $|1L_z;1S_z\rangle$ the interaction cross section with a nucleon depends on L_z (QCD factorization theorem and nonrelativistic quarkonium model, L. Gerland et al, PRL 81, 762 (1998)):**

$$\sigma_{L_z} = \int \sigma_{q\bar{q}}(r_t) R^2(r) |Y_{1L_z}(\widehat{\vec{r}})|^2 d^3r$$

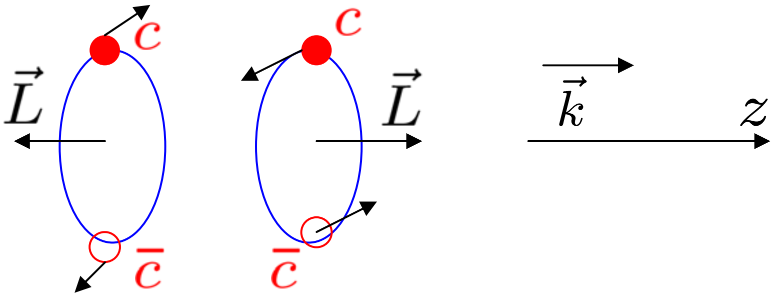
color dipole cross section (nonperturbative evaluation)

$$\boxed{\sigma_{L_z=0} = 6.8 \text{ mb}, \ \sigma_{L_z=\pm 1} = 15.9 \text{ mb}}$$

$L_z = 0$



$L_z = \pm 1$



pQCD estimate:

$$\sigma_{\bar{q}q}(r_t \to 0) \propto r_t^2$$

$$\frac{\sigma_1}{\sigma_0} \simeq \frac{< r_t^2 >_{L_z=\pm 1}}{< r_t^2 >_{L_z=0}} = 2$$

Longitudinally polarized $c\bar{c}$ pair has a larger transverse size and, hence, a larger interaction cross section with a nucleon.

Diagonal (elastic) or nondiagonal $\chi_c N$ scattering: $\chi_{J_1\nu}N \to \chi_{J\nu}N$

Assume that the interaction with a nucleon does not change the spin and internal angular momentum of $c\bar{c}$ pair:

Invariant matrix element: $M_{J\nu;J_1\nu}(\mathrm{q}_t) = \mathrm{e}^{-B_{\chi N}\mathrm{q}_t^2/2} \sum\limits_{L_z,S_z} \langle J\nu|1L_z;1S_z\rangle M_{L_z}(0)\langle 1L_z;1S_z|J_1\nu\rangle$

$B_{\chi N} \simeq 3\ \mathrm{GeV}^{-2}$ **- two-gluon exchange (L. Gerland et al, PLB 619, 95 (2005))**

Optical theorem: $M_{L_z}(0) = 2ip_{\mathrm{lab}}m_N\sigma_{L_z}(1-i\rho_{\chi N})\ ,\ \rho_{\chi N} = \dfrac{\mathrm{Re}M_{L_z}(0)}{\mathrm{Im}M_{L_z}(0)} \simeq 0.15 - 0.30$

(soft Pomeron exchange – pQCD limits)

The amplitudes of nondiagnal transitions are proportional to σ_1-σ_0 :

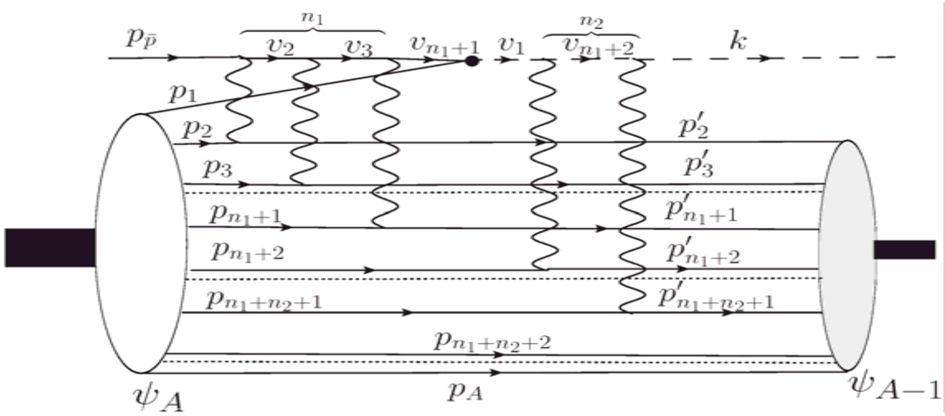
$$\begin{aligned} M_{20;00}(0) &= 2ip_{\mathrm{lab}}m_N\frac{\sqrt{2}}{3}(\sigma_1-\sigma_0)(1-i\rho_{\chi N})\ , \\ M_{2,\pm1;1,\pm1}(0) &= \pm 2ip_{\mathrm{lab}}m_N\frac{1}{2}(\sigma_1-\sigma_0)(1-i\rho_{\chi N})\ . \end{aligned}$$

Multiple scattering diagrams

— keep only diagrams with elastic rescattering: inelastic diffractive cross sections are small (e.g. $\sigma(\bar{p}p \to \bar{N}^* p + c.c.) \simeq 0.1$ mb at $p_{\text{lab}} \simeq 6$ GeV/c)

Diagonal: **number of involved nucleons**

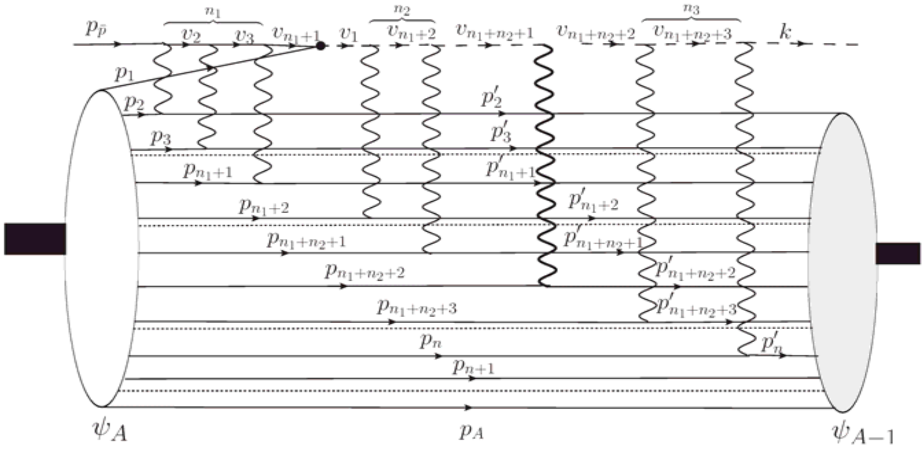
$$M^J(1,2,\dots,n)\ ,\ n = n_1 + n_2 + 1$$



Nondiagonal, i.e. with transition

$\chi_{J_1} N_{n_1+n_2+2} \to \chi_J N'_{n_1+n_2+2}$:

$$M^{J_1 J}(1,2,\dots,n)\ ,\ n = n_1 + n_2 + n_3 + 2$$



Generalized eikonal approximation (GEA): **L. Frankfurt, M. Sargsian, M. Strikman, PRC 56, 1124 (1997); M. Sargsian, Int. J. Mod. Phys. E 10, 405 (2001).**

— neglect energy transfer in rescatterings (soft rescatterings on nonrelativistic nucleons);

— eikonal form of propagators (nonrelativistic initial and final nucleons);

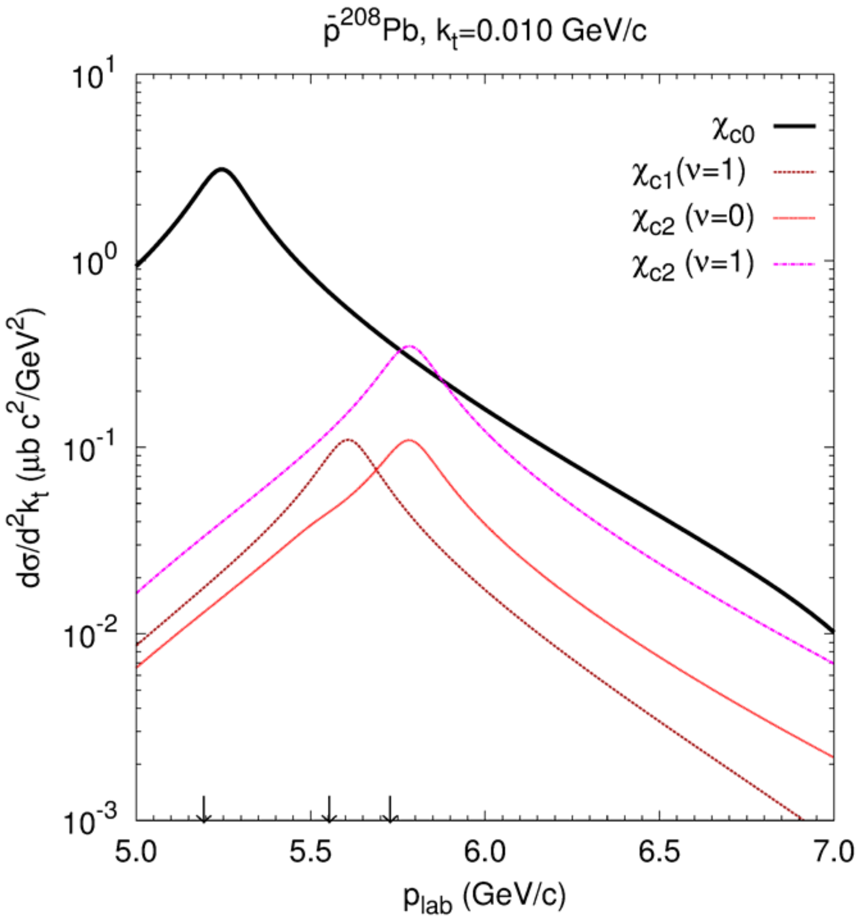
— keep only transverse momentum transfer dependence in elementary amplitudes (soft scatterings at high energies);

— quasifree kinematics of the produced charmonium: $|p_{\mathrm{lab}} - k^z| \ll p_{\mathrm{lab}}$ **;**

— systematic expansion of $|M|^2$ in the number of rescatterings.

Differential cross sections:

$$\frac{d\sigma_{\bar{p}A\to\chi_{J\nu}(A-1)^*}}{d^2k_t} = \frac{|M|^2}{16\pi^2 p_{\mathrm{lab}}^2}$$



Strong overlap in p_{lab} for the different χ_c flavors. Interference is possible.

On-shell production in $\bar{p}p \to \chi_c$:

$p_{\mathrm{lab}} = 5.194,\ 5.553$ and 5.727 GeV/c for χ_{c0}, χ_{c1} and χ_{c2}, resp.

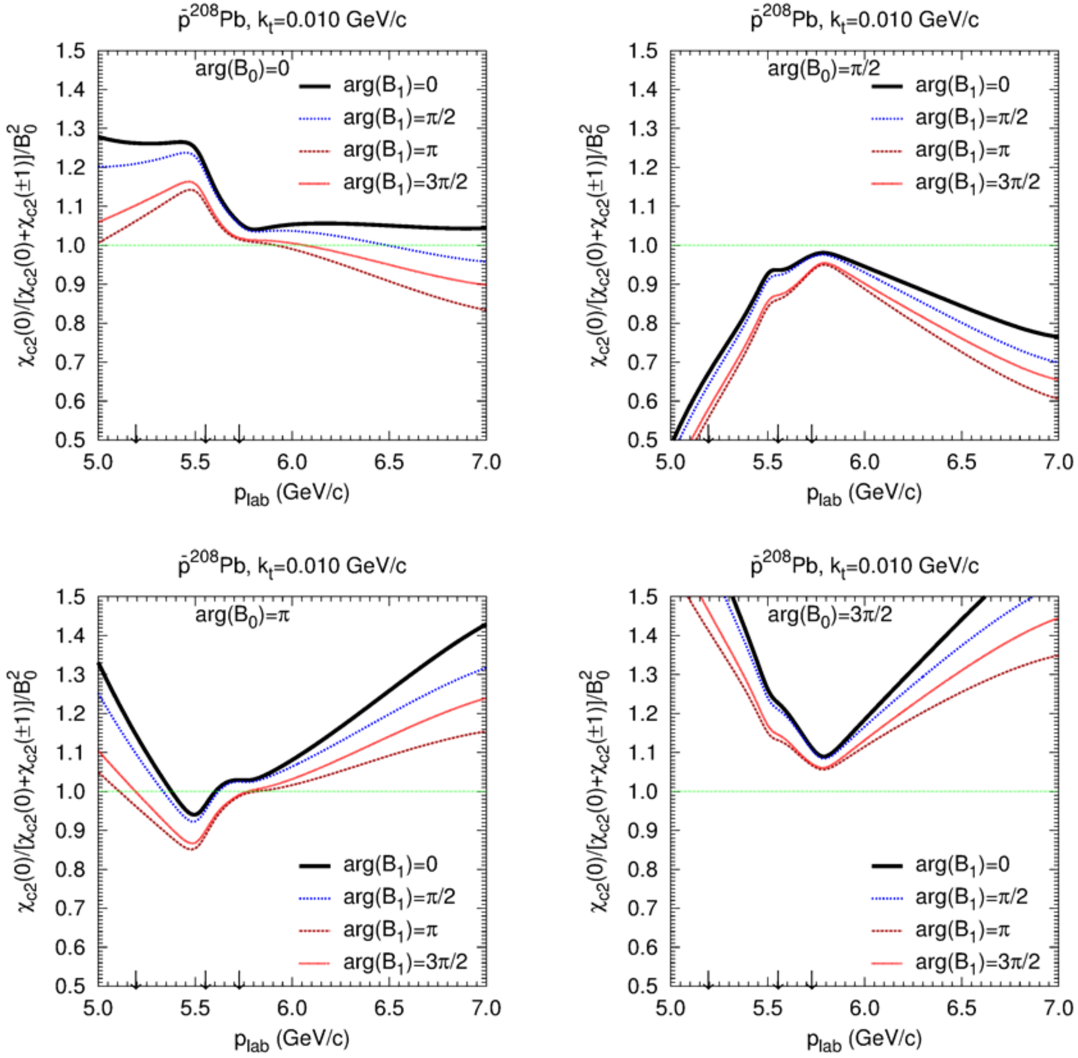
Helicity ratio

$$\mathcal{R} = \frac{\chi_{20}}{(\chi_{20} + 2\chi_{21})|B_0|^2}$$

$\mathcal{R} = 1$ for $\bar{p}p$

$B_0,\ B_1$ **- helicity amplitudes**

for $\bar{p}p \to \chi_{c2}(\nu = 0, \pm 1)$



$$|B_0|^2 + 2|B_1|^2 = 1$$

$$|B_0|^2 = 0.13 \pm 0.08$$

- from angular distributions for $\bar{p}p \to \chi_{c2} \to J/\Psi\gamma \to e^+e^-\gamma$

M. Ambrogiani et al. (E835), PRD 65, 052002 (2002)

The deviation of $\mathcal{R}$ from 1 is due to the interference of the direct $\bar{p}p \to \chi_{20}$ and the two-step $\bar{p}p \to \chi_{00},\ \chi_{00}N \to \chi_{20}N$ amplitudes **and proportional to $\sigma_1 - \sigma_0$.**

XYZ production

Noncharmonium mesons containing a $c\bar{c}$ pair :

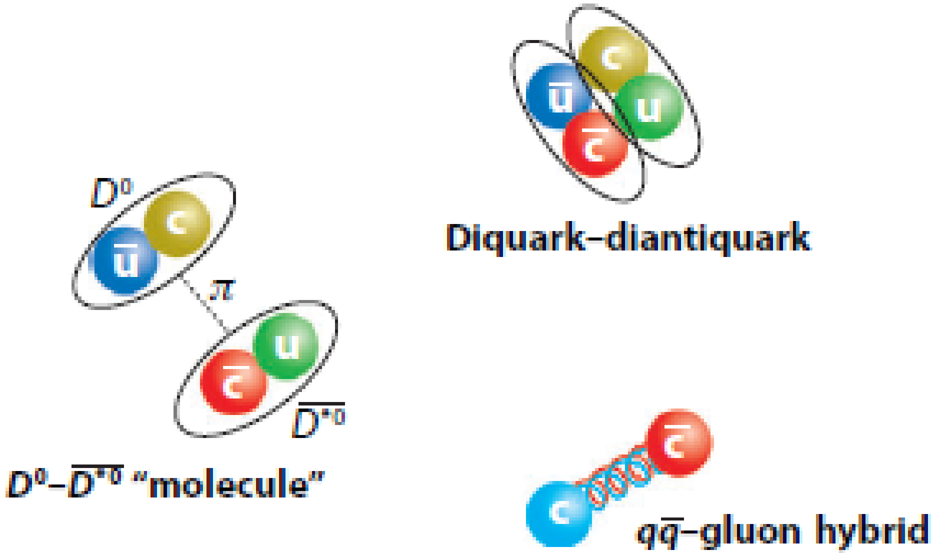
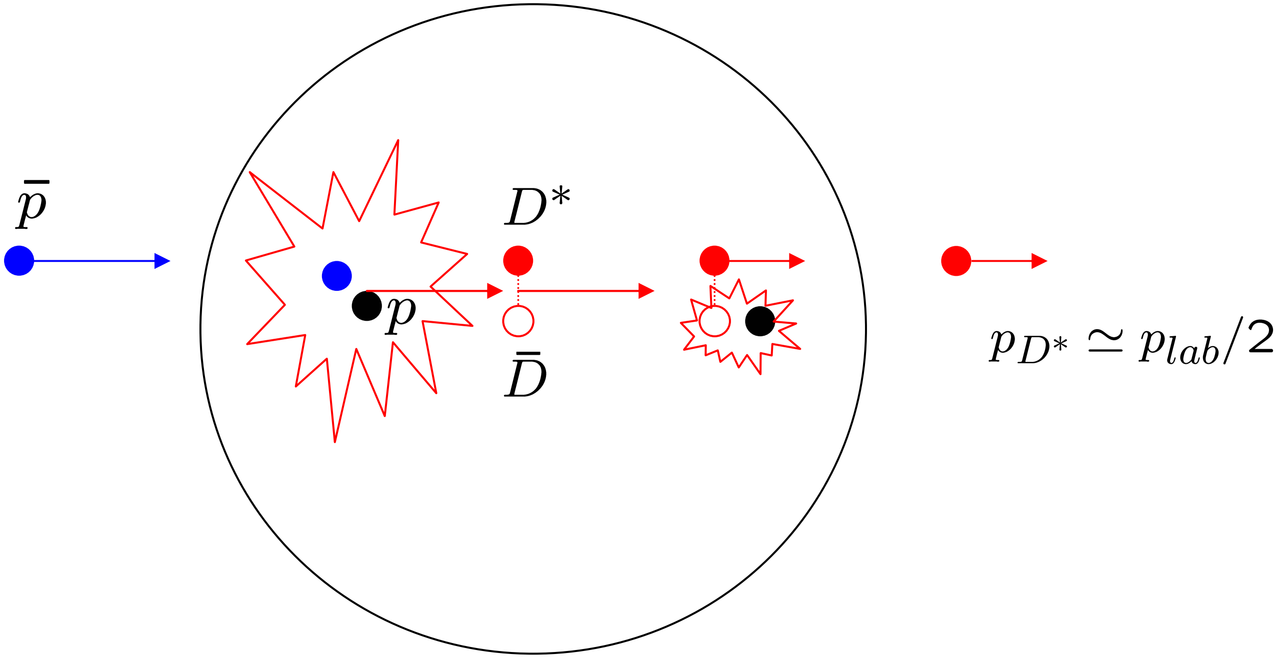


Figure from S. Godfrey and S.L. Olsen, Annu. Rev. Nucl. Part. Sci. 58, 51 (2008)

Noncharmonium candidates: X(3872), X(3915), X(3940), G(3900), Y(4008)

N. Brambilla et al., EPJ C 71, 1534 (2011)

Use nucleus to test the possible molecular structure of X(3872):



$$r_{r.m.s.}(\bar{D}D^*) \sim \hbar/\sqrt{2\mu\epsilon_B} \sim 1 - 10 \text{ fm} \qquad \mu = \frac{m_{\bar{D}} m_{D^*}}{m_{\bar{D}} + m_{D^*}}$$

$D^- D^{*+}$ $\qquad$ $\bar{D}^0 D^{*0}$

$\epsilon_B \simeq 7$ MeV $\qquad$ $\epsilon_B \simeq 0.4$ MeV

Expected elementary cross sections:

$$\frac{\sigma^{\text{tot}}_{Dp}(p_{\text{lab}}/2)}{\sigma^{\text{tot}}_{\pi^+p}(p_{\text{lab}}/2)} \sim \left(\frac{r_{Bohr}(D)}{r_{Bohr}(\pi)}\right)^2 \sim \frac{1}{2}$$

$$\bar{p}p \to X(3872), \qquad p_{\text{lab}} \simeq 7 \text{ GeV/c}$$

$$\sigma^{\text{tot}}_{\pi^+p} \simeq 29 \text{ mb}$$

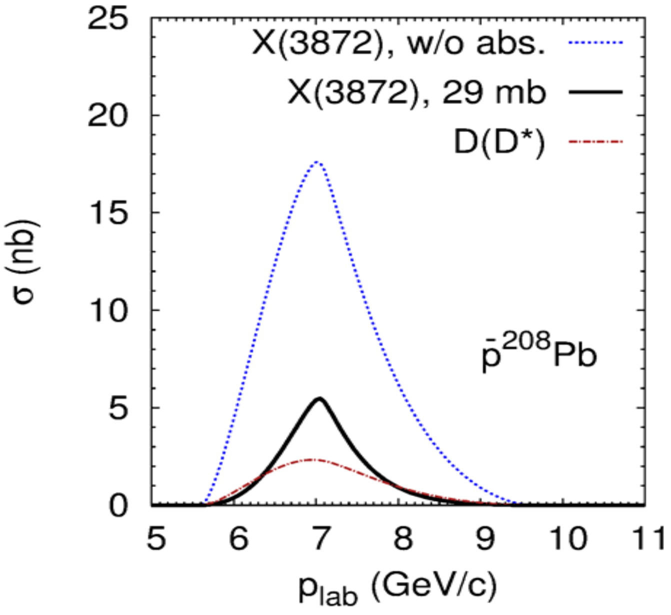
→ $\sigma^{\text{tot}}_{Dp} \simeq \sigma^{\text{tot}}_{D^*p} \simeq 14.5 \text{ mb}, \quad \sigma^{\text{tot}}_{Xp} \simeq \sigma^{\text{tot}}_{Dp} + \sigma^{\text{tot}}_{D^*p} \simeq 29 \text{ mb}$

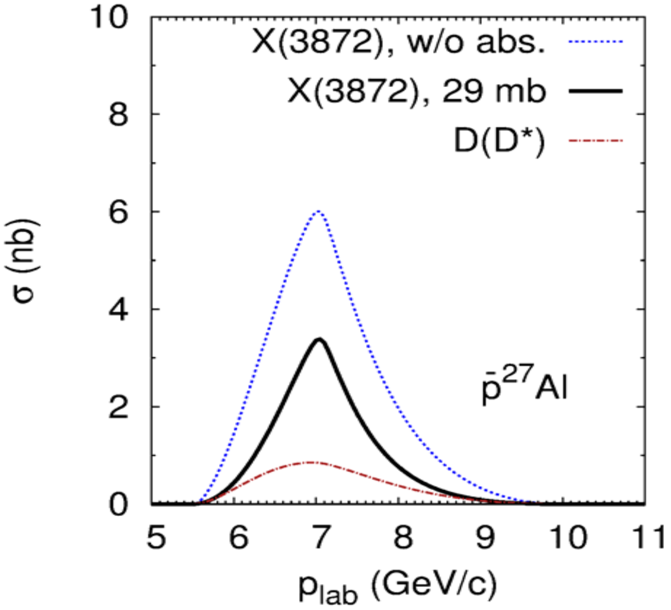
X(3872) and D (D*) production cross sections on nuclei

Input:

$$\Gamma_{X(3872)} = 2.3 \text{ MeV}$$

$$\frac{\Gamma_{X(3872)\to\bar{p}p}}{\Gamma^{\text{tot}}_{X(3872)}} = 1 \times 10^{-4}$$





- Strong absorption of X(3872)

- Molecular structure of X(3872) enhances D (D*) production

Summary

— Strong sensitivity of J/Ψ(Ψ´) production in antiproton-induced reactions to the genuine J/ΨN (Ψ´N) dissociation cross section

— For the quantitative determination of J/ΨN (Ψ´N) cross sections the density profiles are important

— Polarization effects in χ_{c2} production on nuclei due to σ_{L_z}

— Possible molecular structure of X(3872) manifests itself in the enhanced production of D(D*)

Further steps

— Differential cross sections of X(3872) and D(D*) production, shadowing effects

— Deuteron target

Thank you for your attention !

Backup

Partial width:

$$p_{\rm lab} = m_R\sqrt{m_R^2/4m_N^2 - 1} \quad (\text{for } \bar{p}p \to R_{\rm on-shell}):$$

$$\Gamma_{\bar{p}\to R} = \int \frac{2d^3p}{(2\pi)^3} v_{\bar{p}p}\sigma_{\bar{p}p\to R}(\sqrt{s}) f_p(\mathbf{p}) \simeq \frac{3m_R^2\Gamma_{R\to\bar{p}p}p_{F,p}^2}{8p_{\rm lab}E_{\bar{p}}E_p q_R} \propto \rho_p^{2/3}$$

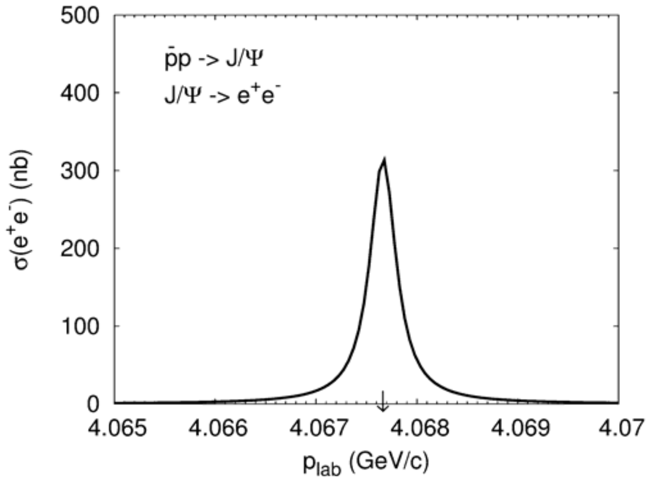
$$E_{\bar{p}} = \sqrt{p_{\rm lab}^2 + m_N^2},\ q_R = \sqrt{m_R^2/4 - m_N^2},\ f_p(\mathbf{p}) = \Theta(p_{F,p} - |\mathbf{p}|).$$

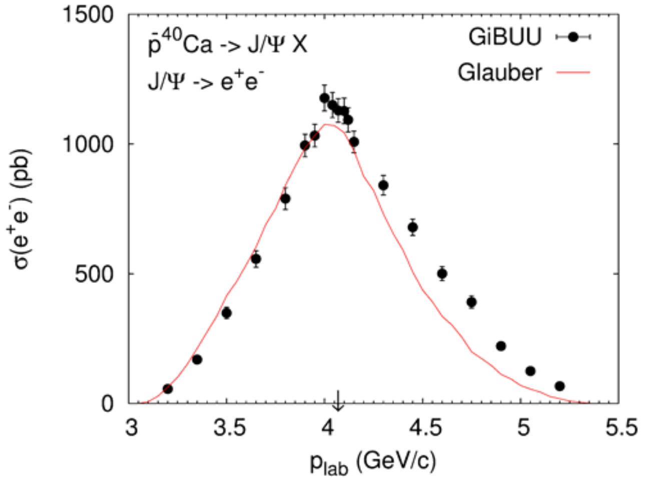
$$\frac{\sigma_{\bar{p}A\to R(A-1)*}}{Z\sigma_{\bar{p}p\to R}} \sim \frac{\Gamma_{\bar{p}\to R}}{v_{\bar{p}}\sigma_{\bar{p}p\to R}(m_R)\rho_p} = \frac{3\pi m_R m_N \Gamma_R}{4(m_R^2 - 2m_N^2)v_{\bar{p}}p_{F,p}} \sim 10^{-4}$$

(for $m_{J/\Psi} = 3.097$ GeV, $\Gamma_{J/\Psi} = 93$ keV,
$p_{F,p} \simeq 0.3$ GeV/c, $p_{\rm lab} = 4.07$ GeV/c)

Strong reduction of charmonium production due to Fermi motion

Fermi motion by Monte-Carlo:





Due to Fermi motion cross section drops by a factor of ~10^{-3} at the peak

Good agreement between GiBUU and Glauber calculations

GiBUU model review: **O. Buss et al., Phys. Rep. 512, 1 (2012)**

Effective charmonium-nucleon cross section:

$$\sigma^{\text{eff}}_{RN}(p_R, z) = \sigma_{RN}(p_R)\left(\left[\left(\frac{z}{l_R}\right)^{\tau} + \frac{< n^2 k_t^2 >}{m_R^2}\left(1 - \left(\frac{z}{l_R}\right)^{\tau}\right)\right]\Theta(l_R - z) + \Theta(z - l_R)\right) \ ,$$

$$\tau = 1 \ .$$

G.R. Farrar, L.L. Frankfurt, M.I. Strikman, and H. Liu, NPB 345, 125 (1990)

$< k_t^2 >^{1/2} \simeq 0.35 \text{ GeV/c}$ **— average quark transverse momentum in a hadron**

$n = 3$ **— number of intermediate gluons**

$$\left.\begin{aligned} l_{J/\Psi} &\simeq \frac{2p_{J/\Psi}}{m^2_{\Psi'} - m^2_{J/\Psi}} \simeq 3\text{fm}\frac{p_{J/\Psi}}{30\text{GeV}} \ , \\ l_{\Psi'} &\simeq 6\text{fm}\frac{p_{\Psi'}}{30\text{GeV}} \ , \\ l_{\chi_c} &\simeq 3\text{fm}\frac{p_{\chi_c}}{30\text{GeV}} \ . \end{aligned}\right\}$$

— formation lengths

L. Gerland et al,
PRL 81, 762 (1998)

Density profiles

For light nuclei (A ≤ 20) — harmonic oscillator model:

$$\rho_q(r) = \rho_q^0 \left[1 + a_q \left(\frac{r}{R_q}\right)^2\right] \exp\{-(r/R_q)^2\} \, , \quad q = p, n \, .$$

For heavy nuclei (A > 20) — two-parameter Fermi distribution:

$$\rho_q(r) = \frac{\rho_q^0}{\exp\left(\frac{r-R_q}{a_q}\right) + 1} \, , \;\; q = p, n$$

Charge density parameters: C. De Jager et al.,
Atom. Data Nucl. Data Tabl. 14, 479 (1974).

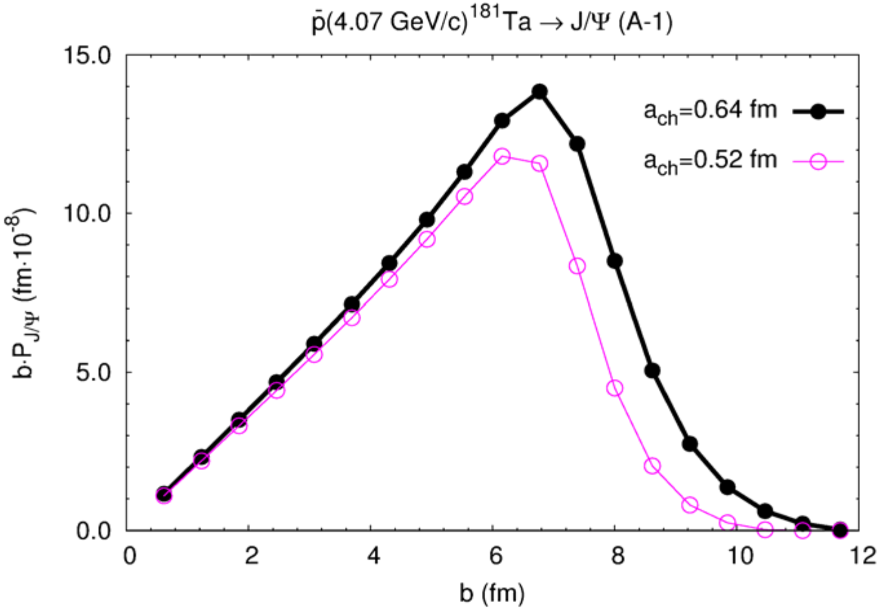
Neutron density parameters: J. Nieves et al., NPA 554, 509 (1993);
V. Koptev et al., Yad. Fiz. 31, 1501 (1980);
R. Schmidt et al., PRC 67, 044308 (2003).

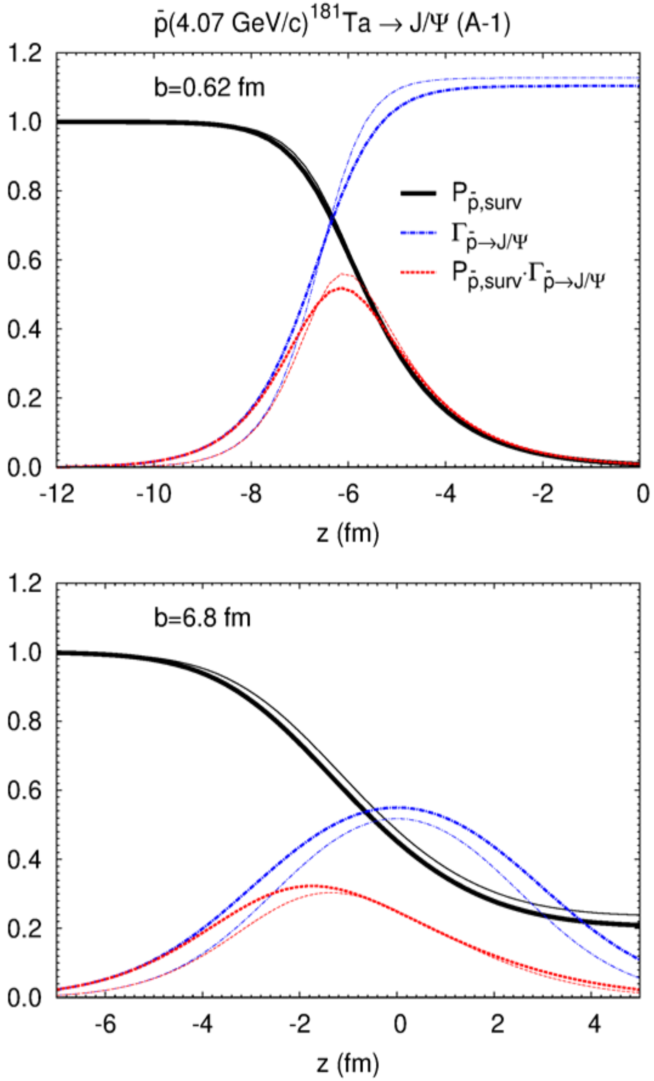
Probability of J/Ψ production:

$$\Gamma_{\bar{p}\to J/\Psi} \propto \rho_p^{2/3} \text{ (in } 10^{-8} \text{ c/fm)}$$

$$P_{\mathrm{J}/\Psi}(b) = v_{\bar{p}}^{-1} \int\limits_{-\infty}^{\infty} dz \mathcal{P}_{\bar{p},\mathrm{surv}}(z,b) \Gamma_{\bar{p}\to \mathrm{J}/\Psi}(z,b)$$

Thick (thin) lines: a_{ch}=0.64 (0.52) fm





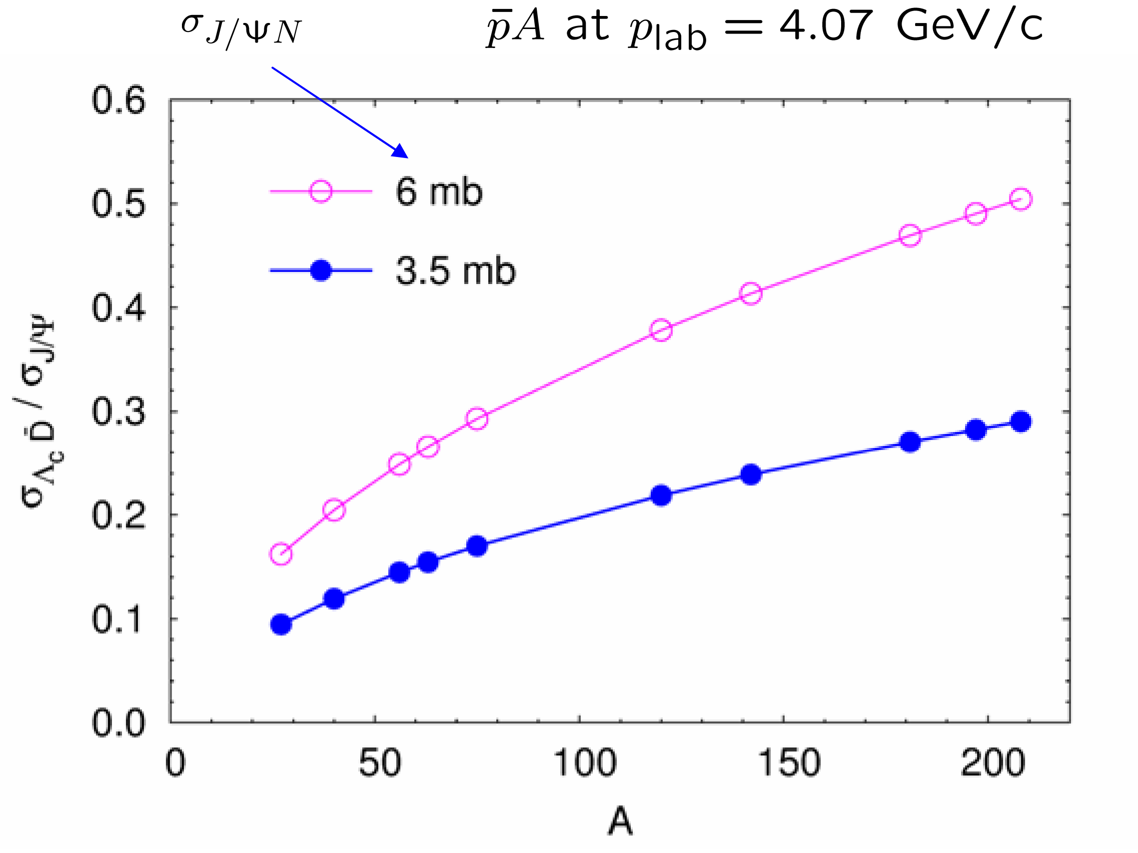
- Charmonium production is localized in the diffuse surface zone.

-Diffuseness parameter of the charge distribution influences sensitively.

The centre of the nucleus at b=0, z=0.

The only channel of $\Lambda_c\bar{D}$-pair production at $p_{lab} < 5.194$ GeV/c (χ_{c0} production threshold in $\bar{p}p$ collisions) is $J/\Psi N \to \Lambda_c \bar{D}$

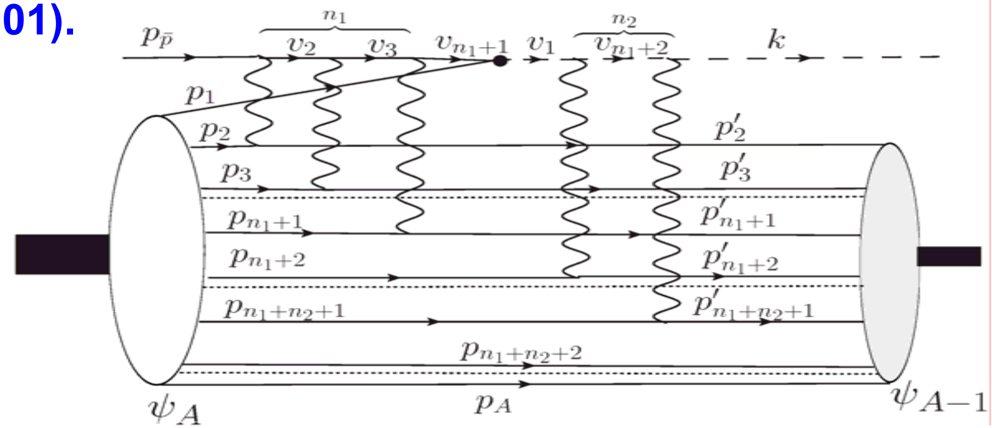
$$\Longrightarrow \quad \sigma_{\Lambda_c\bar{D}} = \sigma^{w/o\,J/\Psi abs.}_{\bar{p}A\to J/\Psi(A-1)^*} - \sigma_{\bar{p}A\to J/\Psi(A-1)^*}$$



Generalized eikonal approximation (GEA)

L. Frankfurt, M. Sargsian, M. Strikman, PRC 56, 1124 (1997);
M. Sargsian, Int. J. Mod. Phys. E 10, 405 (2001).

Multiple scattering diagram:



$n = n_1 + n_2 + 1$

- number of involved nucleons

— neglect energy transfer in rescatterings (soft rescatterings on nonrelativistic nucleons):

$$S^J(1,2,\dots,n) = \frac{i(2\pi)\delta(E_{\bar p} + E_1 - \omega)}{(2E_{\bar p}V2\omega V)^{1/2}} M^J(1,2,\dots,n)\ ,$$

$$\begin{aligned}
M^J(1,2,\dots,n) \ &= \ \frac{1}{\sqrt{2E_1}(2m)^{n-1}(2\pi)^{6(n-1)}} \int d^3x'_2 \cdots d^3x'_n \int d^3x_1 \cdots d^3x_A \\
&\times \psi^*_{A-1}(\mathbf{x}'_2,\dots,\mathbf{x}'_n,\mathbf{x}_{n+1},\dots,\mathbf{x}_A)\psi_A(\mathbf{x}_1,\dots,\mathbf{x}_A) \\
&\times \int d^3p'_2 \cdots d^3p'_n \int d^3p_1 \cdots d^3p_n \delta^{(3)}(\mathbf{p}_{\bar p} + \mathbf{p}_1 + \mathbf{q}_2 + \cdots + \mathbf{q}_n - \mathbf{k}) \\
&\times \mathrm{e}^{i\mathbf{p}'_2\mathbf{x}'_2 + \cdots + i\mathbf{p}'_n\mathbf{x}'_n} \frac{M_{JN}(q_n)M_{JN}(q_{n-1})\cdots M_{JN}(q_{n_1+2})M_{J;\bar p p}(p_1)}{D_J(v_{n-1})\cdots D_J(v_{n_1+2})D_J(v_1)} \\
&\times \frac{M_{\bar p N}(q_{n_1+1})\cdots M_{\bar p N}(q_2)}{D_{\bar p}(v_{n_1+1})\cdots D_{\bar p}(v_2)} \mathrm{e}^{-i\mathbf{p}_1\mathbf{x}_1 - \cdots - i\mathbf{p}_n\mathbf{x}_n}\ ,
\end{aligned}$$

$$q_i = p_i - p'_i\ , \qquad i = 2,\dots,n\ .$$

Inverse propagators of $\bar{p}$ and χ_J (for nonrelativistic initial and final nucleons):

$$
\begin{aligned}
-D_{\bar{p}}(v_i) &= (p_{\bar{p}} + \sum_{j=2}^{i} q_j)^2 - m^2 + i\varepsilon \simeq 2p_{\mathrm{lab}}(-l_i + i\varepsilon) \ , \quad p_{\bar{p}} = (E_{\bar{p}}, 0, 0, p_{\mathrm{lab}}) \\
& l_i = \sum_{j=2}^{i} q_j^z \ , \quad i = 2, \ldots, n_1 + 1 \ ; \\
-D_J(v_1) &= (p_{\bar{p}} + p_1 + \sum_{j=2}^{n_1+1} q_j)^2 - m_J^2 + i\varepsilon \simeq 2p_{\mathrm{lab}}(\Delta_J^0 - l_1 + i\varepsilon) \ , \\
& l_1 = p_1^z + \sum_{j=2}^{n_1+1} q_j^z \ ; \\
-D_J(v_i) &= (p_{\bar{p}} + p_1 + \sum_{j=2}^{i} q_j)^2 - m_J^2 + i\varepsilon \simeq 2p_{\mathrm{lab}}(\Delta_J^0 - l_i + i\varepsilon) \ , \\
& l_i = p_1^z + \sum_{j=2}^{i} q_j^z \ , \quad i = n_1 + 2, \ldots, n - 1 \ .
\end{aligned}
$$

$$
\Delta_J^0 = \frac{m^2 + E_1^2 + 2E_{\bar{p}}E_1 - m_J^2}{2p_{\mathrm{lab}}}
$$

Longitudinal momentum transfer in case of on-shell χ_J production

— keep only transverse momentum transfer dependence in elementary amplitudes (soft scatterings at high energies), i.e. $M_{\bar{p}N}(q_i) \to M_{\bar{p}N}(\mathbf{t}_i), \quad \mathbf{t}_i = \mathbf{q}_{it}$ **etc.;**

— coordinate representation of propagators: $\dfrac{1}{\Delta_J^0 - p_1^z + i\varepsilon} = -i \int dz^0 \Theta(z^0) \mathrm{e}^{i(\Delta_J^0 - p_1^z) z^0}$

→ **Gribov-Glauber-type expression:**

$$
\begin{aligned}
M^J(1,2,\ldots,n) \; = \; & \frac{i^{n-1}}{(2E_1)^{1/2}(2\pi)^{2(n-1)}(4mp_{\text{lab}})^{n-1}} \\
& \times \int d^3x_1 \cdots d^3x_A \psi^*_{A-1}(\mathbf{x}_2,\ldots,\mathbf{x}_A)\psi_A(\mathbf{x}_1,\ldots,\mathbf{x}_A) \\
& \times \Theta(z_3 - z_2) \cdots \Theta(z_{n_1+1} - z_{n_1}) \Theta(z_1 - z_{n_1+1}) \Theta(z_{n_1+2} - z_1) \\
& \times \Theta(z_{n_1+3} - z_{n_1+2}) \cdots \Theta(z_n - z_{n-1}) \\
& \times \exp\{i(p_{\text{lab}} - k^z + \Delta_J^0) z_n - i\Delta_J^0 z_1 - i\mathbf{k}_t \mathbf{b}_1\} \int d^2t_2 \cdots d^2t_n \\
& \times \exp\{-i\mathbf{t}_2(\mathbf{b}_2 - \mathbf{b}_1) - \cdots - i\mathbf{t}_n(\mathbf{b}_n - \mathbf{b}_1)\} M_{\bar{p}N}(\mathbf{t}_2) \cdots M_{\bar{p}N}(\mathbf{t}_{n_1+1}) \\
& \times M_{J;\bar{p}N}(\mathbf{k}_t - \mathbf{t}_2 - \cdots - \mathbf{t}_n) M_{JN}(\mathbf{t}_{n_1+2}) \cdots M_{JN}(\mathbf{t}_n) \; .
\end{aligned}
$$

Quasifree production: $|p_{\text{lab}} - k^z| \ll p_{\text{lab}} \; , \quad k^z - p_{\text{lab}} \simeq \Delta_J^0 - \cancel{(\Delta_J^0)^2/2p_{\text{lab}}}$

Sum over different orders of scatterings:

$$\Theta(z_3 - z_2) \cdots \Theta(z_{n_1+1} - z_{n_1}) \Theta(z_1 - z_{n_1+1}) \Theta(z_{n_1+2} - z_1) \Theta(z_{n_1+3} - z_{n_1+2}) \cdots \Theta(z_n - z_{n-1})$$

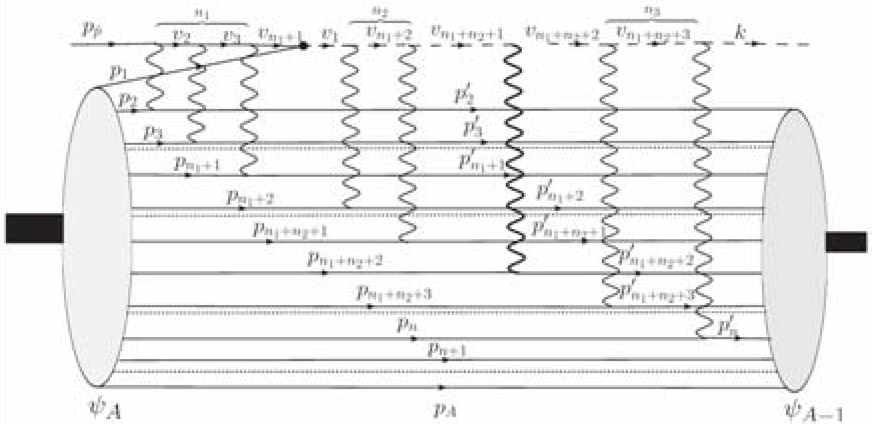
↓

$$\Theta(z_1 - z_2) \cdots \Theta(z_1 - z_{n_1+1}) \Theta(z_{n_1+2} - z_1) \cdots \Theta(z_n - z_1)$$

Diagram with one nondiagonal transition $\chi_{J_1} N_{n_1+n_2+2} \to \chi_J N'_{n_1+n_2+2}$:

$$n = n_1 + n_2 + n_3 + 2$$

number of involved nucleons



$$\begin{aligned}
M^{J_1 J}(1,2,\ldots,n) \; = \; & \frac{i^{n-1}}{(2E_1)^{1/2}(2\pi)^{2(n-1)}(4mp_{\text{lab}})^{n-1}} \int d^3x_1 \cdots d^3x_A \\
& \times \psi^*_{A-1}(\mathbf{x}_2,\ldots,\mathbf{x}_A)\psi_A(\mathbf{x}_1,\ldots,\mathbf{x}_A)\Theta(z_1-z_2)\cdots\Theta(z_1-z_{n_1+1}) \\
& \times \Theta(z_{n_1+2}-z_1)\Theta(z_{n_1+n_2+2}-z_{n_1+2})\cdots\Theta(z_{n_1+n_2+1}-z_1)\Theta(z_{n_1+n_2+2}-z_{n_1+n_2+1}) \\
& \times \Theta(z_{n_1+n_2+3}-z_{n_1+n_2+2})\cdots\Theta(z_n-z_{n_1+n_2+2}) \\
& \times \exp\{-i\Delta^0_{J_1}z_1 - i\mathbf{k}_t\mathbf{b}_1 + i(\Delta^0_{J_1}-\Delta^0_J)z_{n_1+n_2+2}\} \int d^2t_2\cdots d^2t_n \\
& \times \exp\{-i\mathbf{t}_2(\mathbf{b}_2-\mathbf{b}_1) - \cdots - i\mathbf{t}_n(\mathbf{b}_n-\mathbf{b}_1)\} M_{\bar{p}N}(\mathbf{t}_2)\cdots M_{\bar{p}N}(\mathbf{t}_{n_1+1}) \\
& \times M_{J_1;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2-\cdots-\mathbf{t}_n) M_{J_1N}(\mathbf{t}_{n_1+2})\cdots M_{J_1N}(\mathbf{t}_{n_1+n_2+1}) \\
& \times M_{JN';J_1N}(\mathbf{t}_{n_1+n_2+2}) M_{JN}(\mathbf{t}_{n_1+n_2+3})\cdots M_{JN}(\mathbf{t}_n)
\end{aligned}$$

Additive terms contributing to $|M|^2$

Optical theorem:

$$\mathrm{Im} M_{JN}(0) = 2p_{\mathrm{lab}} m \sigma_{JN}^{\mathrm{tot}}$$

Direct term („simple“ Glauber model):

$$\sum_{\mathrm{set1} \neq \mathrm{set2}} \sum_{\psi_{A-1}} M^{J}(1, \mathrm{set1}) M^{J*}(1, \mathrm{set2})$$

$$= \frac{|M_{J;\bar{p}p}(\mathbf{k}_t)|^2}{2E_1} \int d^3X f_1(\mathbf{X}, \mathbf{k}_t, \Delta_J^0) \underbrace{\prod_{i=2}^{A} \left(1 - \sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_i |\phi_i(\mathbf{B}, z_i)|^2 - \sigma_{JN}^{\mathrm{tot}} \int\limits_{Z}^{+\infty} dz_i |\phi_i(\mathbf{B}, z_i)|^2 \right)}_{\simeq \exp\left(-\sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_2 \rho(\mathbf{B}, z_2) - \sigma_{JN}^{\mathrm{tot}} \int\limits_{Z}^{+\infty} dz_2 \rho(\mathbf{B}, z_2) \right)}$$

Interference term:

$$\sum_{\mathrm{set1} \neq \mathrm{set2}} \sum_{\psi_{A-1}} M^{J_1 J}(1, 2, \mathrm{set1}) M^{J*}(1, \mathrm{set2}) + \mathrm{c.c.}$$

$$= \frac{i M_{J;\bar{p}p}^{*}(\mathbf{k}_t)}{2E_1 4 m p_{\mathrm{lab}}} M_{JN';J_1 N}(0) M_{J_1;\bar{p}p}(\mathbf{k}_t) \int d^3X f_1 \left(\mathbf{X}, \mathbf{k}_t, \frac{\Delta_J^0 + \Delta_{J_1}^0}{2} \right) \int\limits_{Z}^{+\infty} dz_2 \, |\phi_2(\mathbf{B}, z_2)|^2 \mathrm{e}^{i(\Delta_{J_1}^0 - \Delta_J^0)(z_2 - Z)}$$

$$\times \prod_{i=3}^{A} \left(1 - \sigma_{\bar{p}N}^{\mathrm{tot}} \int\limits_{-\infty}^{Z} dz_i |\phi_i(\mathbf{B}, z_i)|^2 + \frac{i[M_{J_1 N}(0) - M_{JN}^{*}(0)]}{4 m p_{\mathrm{lab}}} \int\limits_{Z}^{z_2} dz_i |\phi_i(\mathbf{B}, z_i)|^2 - \sigma_{JN}^{\mathrm{tot}} \int\limits_{z_2}^{+\infty} dz_i |\phi_i(\mathbf{B}, z_i)|^2 \right) + \mathrm{c.c.}$$

$$\Delta_J^0 = \frac{m^2 + E_1^2 + 2E_{\bar{p}} E_1 - m_J^2}{2p_{\mathrm{lab}}}$$

— longitudinal momentum transfer needed for on-shell χ_J

$\bar{p}$ rescattering term:

$$\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^{J}(2,1,\text{set1})M^{J*}(2,1,\text{set2})$$
$$=\frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2|M_{J;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)|^2\,|M_{\bar{p}N}(\mathbf{t}_2)|^2\int d^3X\, f_1(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\Delta_J^0)$$
$$\times\int\limits_{-\infty}^{Z} dz_2\,|\phi_2(\mathbf{B},z_2)|^2\prod_{i=3}^{A}\left(1-\sigma^{\text{tot}}_{\bar{p}N}\int\limits_{-\infty}^{Z} dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma^{\text{tot}}_{JN}\int\limits_{Z}^{+\infty} dz_i|\phi_i(\mathbf{B},z_i)|^2\right)\ .$$

χ_J diagonal rescattering term:

$$\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^{J}(1,2,\text{set1})M^{J*}(1,2,\text{set2})$$
$$=\frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2|M_{JN}(\mathbf{t}_2)|^2\,|M_{J;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)|^2\int d^3X\, f_1(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\Delta_J^0)$$
$$\times\int\limits_{Z}^{+\infty} dz_2\,|\phi_2(\mathbf{B},z_2)|^2\prod_{i=3}^{A}\left(1-\sigma^{\text{tot}}_{\bar{p}N}\int\limits_{-\infty}^{Z} dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma^{\text{tot}}_{JN}\int\limits_{Z}^{+\infty} dz_i|\phi_i(\mathbf{B},z_i)|^2\right)\ .$$

$\chi_{J_1}N_2 \to \chi_J N_2'$ nondiagonal rescattering term:

$$\begin{aligned}
&\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^{J_1J}(1,2,\text{set1})M^{J_1J*}(1,2,\text{set2})\\
&= \frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2|M_{JN';J_1N}(\mathbf{t}_2)|^2\,|M_{J_1;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)|^2\\
&\quad\times\int d^3X f_1(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\Delta^0_{J_1})\int\limits_{Z}^{+\infty}dz_2\,|\phi_2(\mathbf{B},z_2)|^2\\
&\quad\times\prod_{i=3}^{A}\left(1-\sigma^{\text{tot}}_{\bar{p}N}\int\limits_{-\infty}^{Z}dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma^{\text{tot}}_{J_1N}\int\limits_{Z}^{z_2}dz_i|\phi_i(\mathbf{B},z_i)|^2-\sigma^{\text{tot}}_{JN}\int\limits_{z_2}^{+\infty}dz_i|\phi_i(\mathbf{B},z_i)|^2\right)\ .
\end{aligned}$$

Diagonal-nondiagonal rescattering interference term:

$$\begin{aligned}
&\sum_{\text{set1}\neq\text{set2}}\sum_{\psi_{A-1}} M^{J_1J}(1,2,\text{set1})M^{J*}(1,2,\text{set2})+\text{c.c.}\\
&= \frac{1}{(2\pi)^2 2E_1(4mp_{\text{lab}})^2}\int d^2t_2 M_{JN';J_1N}(\mathbf{t}_2)M_{J_1;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)M^*_{JN}(\mathbf{t}_2)M^*_{J;\bar{p}p}(\mathbf{k}_t-\mathbf{t}_2)\\
&\quad\times\int d^3X f_1\left(\mathbf{X},\mathbf{k}_t-\mathbf{t}_2,\frac{\Delta^0_{J_1}+\Delta^0_J}{2}\right)\int\limits_{Z}^{+\infty}dz_2\,\text{e}^{i(\Delta^0_{J_1}-\Delta^0_J)(z_2-Z)}|\phi_2(\mathbf{B},z_2)|^2\\
&\quad\times\prod_{i=3}^{A}\left(1-\sigma^{\text{tot}}_{\bar{p}N}\int\limits_{-\infty}^{Z}dz_i|\phi_i(\mathbf{B},z_i)|^2+\frac{i[M_{J_1N}(0)-M^*_{JN}(0)]}{4mp_{\text{lab}}}\int\limits_{Z}^{z_2}dz_i|\phi_i(\mathbf{B},z_i)|^2\right.\\
&\qquad\qquad\left.-\sigma^{\text{tot}}_{JN}\int\limits_{z_2}^{+\infty}dz_i|\phi_i(\mathbf{B},z_i)|^2\right)+\text{c.c.}\ .
\end{aligned}$$

Elastic $\bar{p}N$ scattering amplitude:

$$M_{\bar{p}N}(\mathbf{q}_t) = 2ip_{\mathrm{lab}} m \sigma^{\mathrm{tot}}_{\bar{p}p}(1 - i\rho_{\bar{p}p})\mathrm{e}^{-B_{\bar{p}p}\mathbf{q}_t^2/2}$$

$$B_{\bar{p}p} = 12.5 \pm 1 \ \mathrm{GeV}^{-2} \ \mathrm{at} \ \sqrt{s} \simeq 3.4 - 7.0 \ \mathrm{GeV}$$

exp. data: **Yu.M. Antipov et al., NPB 57, 333 (1973)**

$$\rho_{\bar{p}p} \simeq -0.05 \ \mathrm{at} \ \sqrt{s} \simeq 3 - 5 \ \mathrm{GeV}$$

Reggeized Pomeron exchange model:
R. Fiore et al., PRD 81, 056001 (2010)

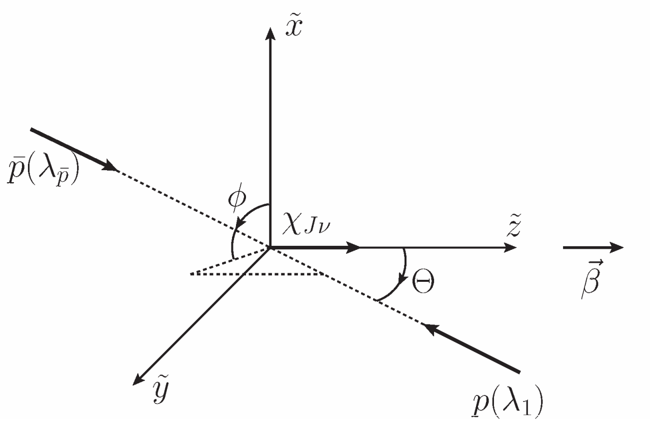
$$\sigma^{\mathrm{tot}}_{\bar{p}p}(p_{\mathrm{lab}}) = 38.4 + 77.6 p_{\mathrm{lab}}^{-0.64} + 0.26 \ln^2(p_{\mathrm{lab}}) - 1.2 \ln(p_{\mathrm{lab}})$$

in mb

in GeV/c

PDG: **L. Montanet et al., PRD 50, 1173 (1994)**

Formation amplitude $\bar{p}(\lambda_{\bar{p}})p(\lambda_1) \to \chi_J(\nu)$



$$\beta = \frac{\mathbf{p}_{\bar{p}} + \mathbf{p}_1}{E_{\bar{p}} + E_1}$$ **— c.m. velocity**

$$\langle J\nu|B|\Theta\phi, \lambda_{\bar{p}}\lambda_1\rangle = \left(\frac{2J+1}{4\pi}\right)^{1/2} B^J_{\lambda_{\bar{p}}\lambda_1} D^J_{\nu\lambda}(\phi, \Theta, -\phi) \ , \quad \lambda = \lambda_{\bar{p}} - \lambda_1 \ , \quad \sum_{\lambda_{\bar{p}}\lambda_1} |B^J_{\lambda_{\bar{p}}\lambda_1}|^2 = 1$$

M. Jacob and G.C. Wick, Ann. Phys. 7, 404 (1959); A.D. Martin et al., PLB 147, 203 (1984); F.L. Ridener et al., PRD 45, 3173 (1992)

Invariant amplitude: $M_{J\nu;\lambda_{\bar{p}}\lambda_1} = \kappa_J \langle J\nu|B|\Theta\phi, \lambda_{\bar{p}}\lambda_1\rangle$,

$$\kappa_J = \left(\frac{64\pi^2 m_J^2 \Gamma_{\chi_J \to \bar{p}p}}{\sqrt{m_J^2 - 4m^2}}\right)^{1/2} .$$

Symmetries of helicity amplitudes F.L. Ridener et al., PRD 45, 3173 (1992):

Charge conjugation invariance: $B^J_{\lambda_{\bar{p}}\lambda_1} = \eta_c(-1)^J B^J_{\lambda_1\lambda_{\bar{p}}}$, $\eta_c = (-1)^{L+S}$

Parity invariance: $B^J_{\lambda_{\bar{p}}\lambda_1} = \eta_p(-1)^J B^J_{-\lambda_{\bar{p}},-\lambda_1}$, $\eta_p = (-1)^{L+1}$

For χ-states $L = S = 1$, $\eta_c = \eta_p = 1$.

$$|B^J_{-+}|^2 = |B^J_{+-}|^2 = |B_1|^2 \ , \quad B_1 \equiv B^J_{+-}$$

$$|B^J_{++}|^2 = |B^J_{--}|^2 = |B_0|^2/2 \ , \quad B_0/\sqrt{2} \equiv B^J_{++}$$

Norma: $2|B_1|^2 + |B_0|^2 = 1$

$B_1 = 0$ for $J = 0$

$B_0 = 0$ for $J = 1$ (from C-parity)

$|B_0|^2 = 0.13 \pm 0.08$ for $J = 2$ from angular distributions for $\bar{p}p \to \chi_{c2} \to J/\psi\gamma \to e^+e^-\gamma$

M. Ambrogiani et al., PRD 65, 052002 (2002)

Proton occupation numbers:

$$2n(\mathbf{X},\mathbf{p}) = \sum_{N_1} f_1(\mathbf{X},\mathbf{p})$$

spin factor

sum over all occupied proton states

$$n(\mathbf{X},\mathbf{p}) = (1-P_2)\Theta(p_F-p)+\boxed{\frac{(2\pi)^3}{2}\rho_p a_2|\psi_D(p)|^2\Theta(p-p_F)}$$

high momentum tail due to short range NN correlations (SRC)

L. Frankfurt, M. Strikman, Phys. Rep. 76, 215 (1981);
L. Frankfurt, M. Sargsian, M. Strikman,
Int. J. Mod. Phys. A23, 2991 (2008)

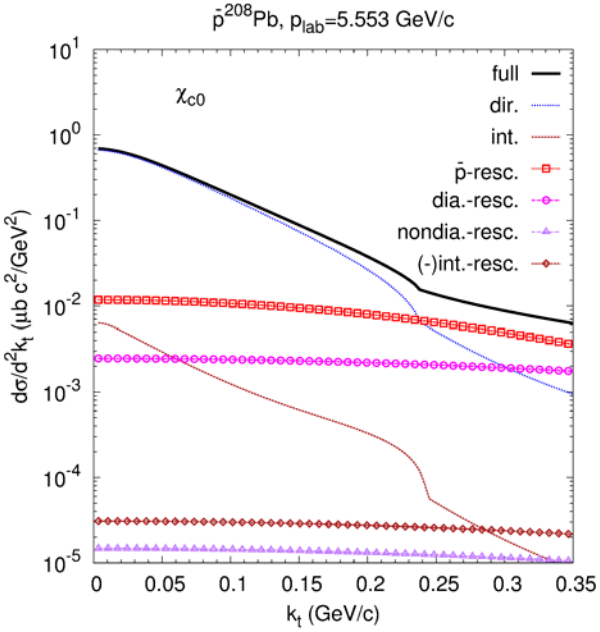
$p_F(\mathbf{X}) = (3\pi^2\rho_p)^{1/3}$ **— proton Fermi momenum**

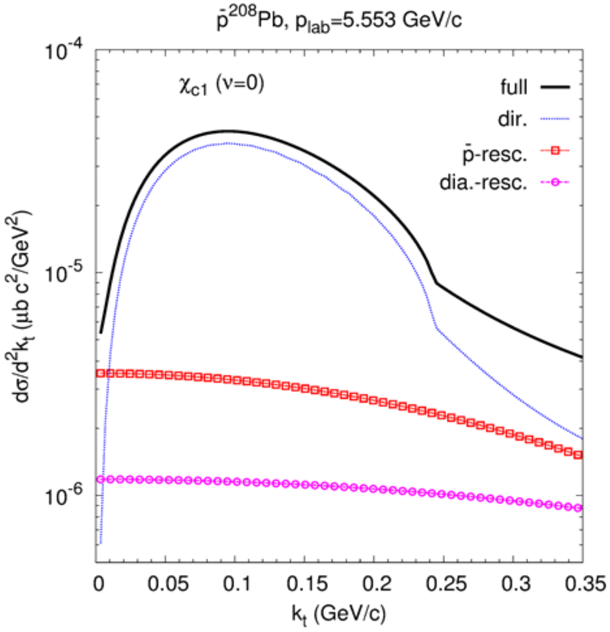
$P_2 \simeq 0.25$ **— proton fraction above Fermi surface,** $P_2 = 4\pi a_2 \int\limits_{p_F}^{+\infty} dp p^2|\psi_D(p)|^2$

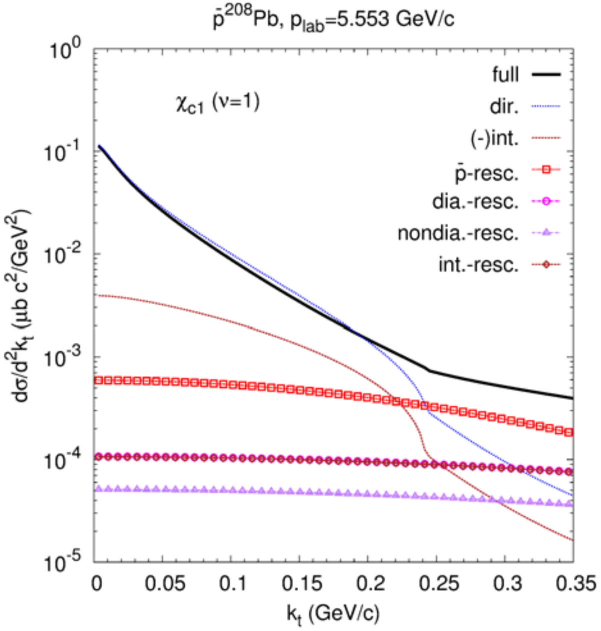
$\psi_D(p)$ **— deuteron wave function,** $4\pi \int\limits_{0}^{+\infty} dp p^2|\psi_D(p)|^2 = 1$

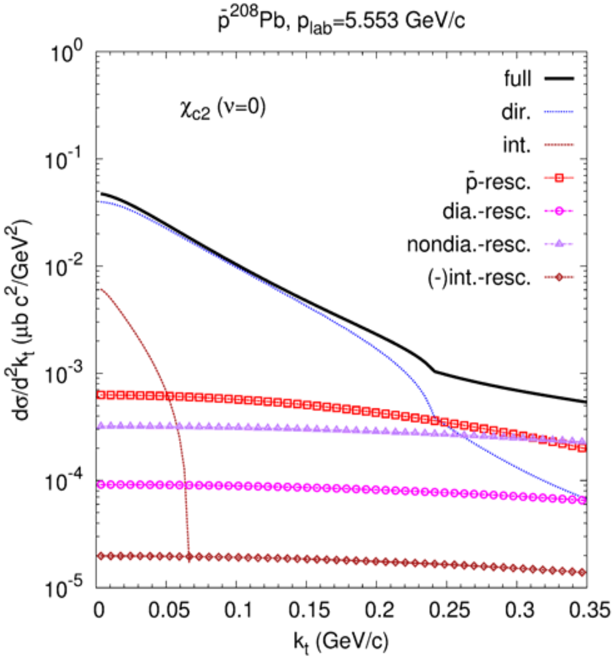
Paris potential: M. Lacombe et al., PRC 21, 861 (1980)

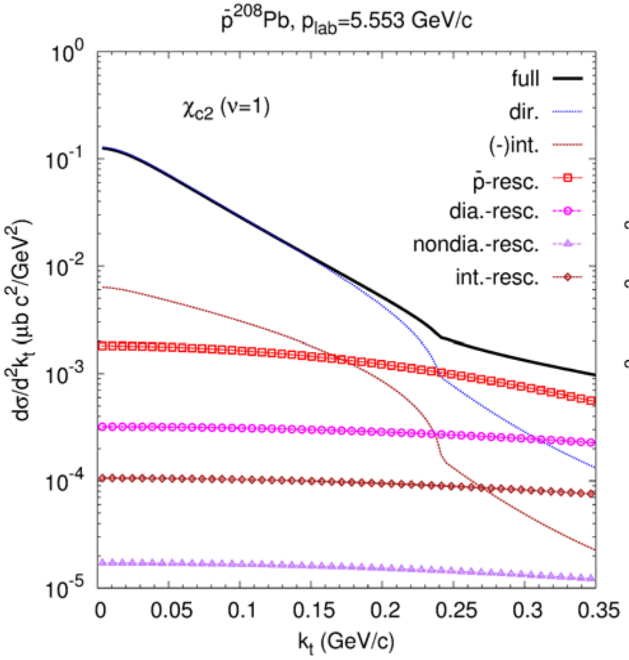
$\bar{p}p \to \chi_{c1}$ **on-shell**

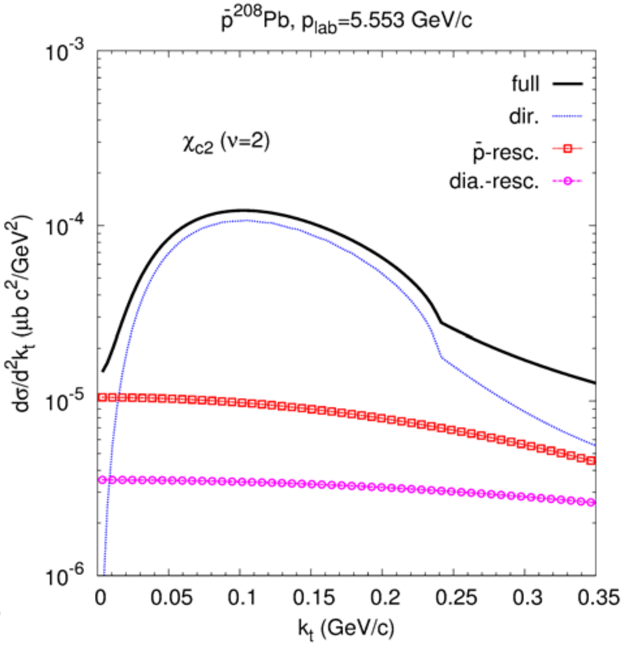






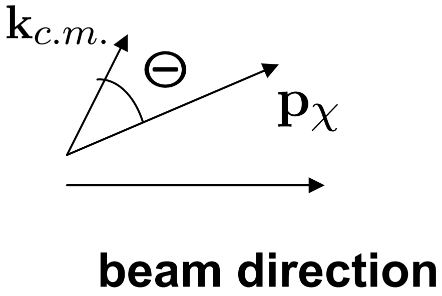






How to measure $\mathcal{R}$ at PANDA ?

- **Use decay** $\chi_c \to J/\Psi\gamma \to e^+e^-\gamma$
- **Double trigger on the photon energy in** $e^+e^-\gamma$ **c.m. frame (E$_\gamma$=303, 389 and 430 MeV for** χ_{c0} **,**χ_{c1} **and** χ_{c2} **, respectively) and on** $M_{\text{inv}}(e^+e^-\gamma)$
- **Determine the angle** Θ **between photon momentum in** χ_c **c.m. frame and the** χ_c **momentum in lab. frame**



Distribution in Θ **:**

$$W_J(\Theta) = \sum_{\nu=\pm 1,0} P_{J\nu} W_{J\nu}(\Theta) \ ,$$

$$P_{J\nu} = \frac{\chi_{J\nu}}{\chi_{J0} + 2\chi_{J1}} \ ,$$

$$P_{20} = \mathcal{R}|B_0|^2, \ P_{2,\pm 1} = (1 - \mathcal{R}|B_0|^2)/2$$

$$W_{J\nu}(\Theta) \propto \sum_{\nu'=0}^{J} |A^J_{\nu'}|^2([d^J_{\nu\nu'}(\Theta)]^2 + [d^J_{\nu,-\nu'}(\Theta)]^2) \ .$$

F.L. Ridener et al., PRD 45, 3173 (1992)

$a_1, a_2, \ldots, a_{J+1}$ **- multipole amplitudes of E1,M2,… transitions**