Pion–nucleon scattering: from chiral perturbation theory to Roy–Steiner equations

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Outline

Why is pion–nucleon scattering important?

Chiral perturbation theory

- phase shift analyses with chiral amplitudes

A new dispersive analysis: Roy–Steiner equations

- phase shifts, σ-term, and low-energy constants

in collaboration with M. Hoferichter, J. Ruiz de Elvira, and U.-G. Meißner
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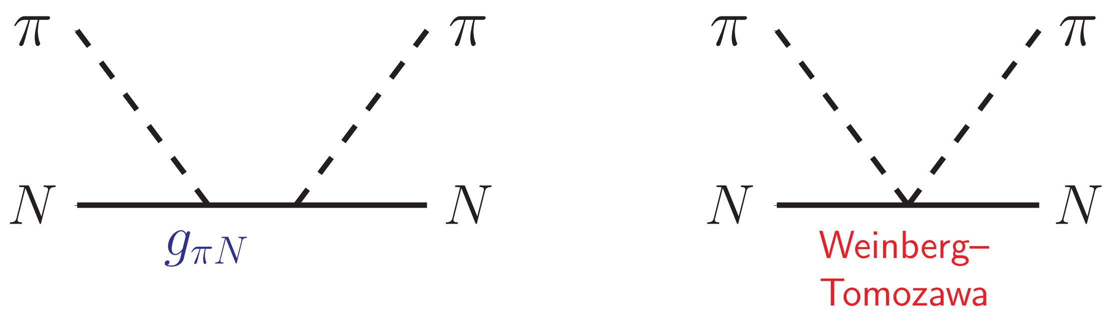
Phys. Rept. 625 (2016) 1, arXiv:1602.07688

Chiral pion–nucleon interaction

- simplest process for chiral pion interaction with nucleons



- leading-order $\mathcal{O}(p) = \mathcal{O}(M_\pi)$ predictions for πN:

  scattering lengths:

$$a^- = \frac{M_\pi m_N}{8\pi(m_N + M_\pi)F_\pi^2} + \mathcal{O}\big(M_\pi^3\big) \qquad a^+ = \mathcal{O}\big(M_\pi^2\big)$$

Weinberg 1966

Goldberger–Treiman relation: $g_{\pi N} = \dfrac{g_A m_N}{F_\pi}$

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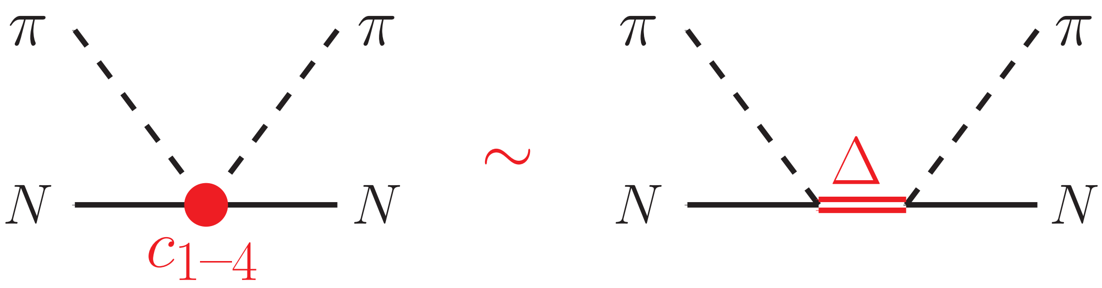
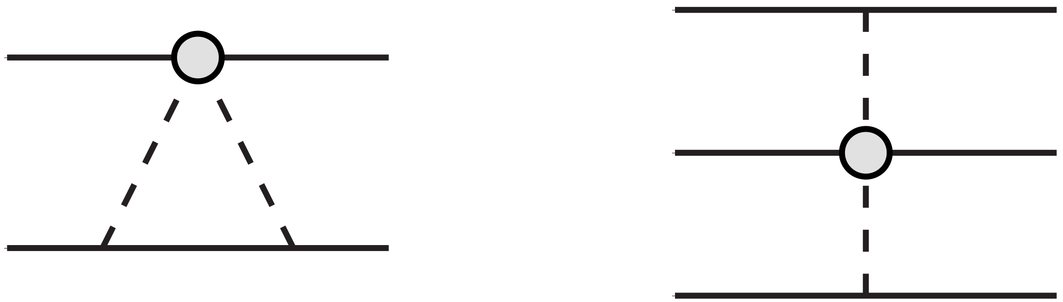


- next-to-leading order $\mathcal{O}(p^2)$: low-energy constants (LECs) c_{1-4} effectively incorporate effects of the $\Delta(1232)$ resonance: low mass $m_\Delta - m_N \approx 2M_\pi$ and strong couplings
- determination of c_i very important for nuclear physics: πN important for NN / determines longest-range $3N$ forces



The pion–nucleon σ-term

- scalar form factor of the nucleon:

$$\langle N(p')|\hat{m}(\bar{u}u + \bar{d}d)|N(p)\rangle = \sigma(t)\bar{u}(p')u(p) \qquad t = (p-p')^2$$

$$\sigma_{\pi N} \equiv \sigma(0) = \frac{\hat{m}}{2m_N}\langle N|\bar{u}u + \bar{d}d|N\rangle \qquad \hat{m} = \frac{m_u + m_d}{2}$$

- $\sigma_{\pi N}$ determines light quark contribution to nucleon mass:
  Feynman–Hellmann theorem

$$\sigma_{\pi N} = \hat{m}\frac{\partial m_N}{\partial \hat{m}} = -4c_1 M_\pi^2 + \mathcal{O}(M_\pi^3)$$

  $\longrightarrow$ at leading order, related to the chiral coupling c_1

- $\sigma_{\pi N}$ determines scalar couplings wanted for
  direct-detection dark matter searches — e.g. Ellis et al. 2008

Extracting LECs from pion–nucleon scattering

Mojžiš 1997, Fettes et al. 1998–2000, Ellis et al. 1998–2003
Alarcón et al. 2011–2013, Chen et al. 2013, Krebs et al. 2012, Gasparyan, Lutz 2010

Strategy:

- fit results of phase shift analyses:
  - ▷ Karlsruhe–Helsinki (KH) $\longrightarrow$ dispersion theory based
    Koch, Pietarinen 1980, Höhler 1983
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- # of parameters: $\mathcal{O}(p^2)$: 4 c_i
  $\mathcal{O}(p^3)$: 4 d_i (+ d_{18} from GT discrepancy)
  $\mathcal{O}(p^4)$: 5 e_i
- use chiral low-energy theorems to extract $\sigma_{\pi N}$

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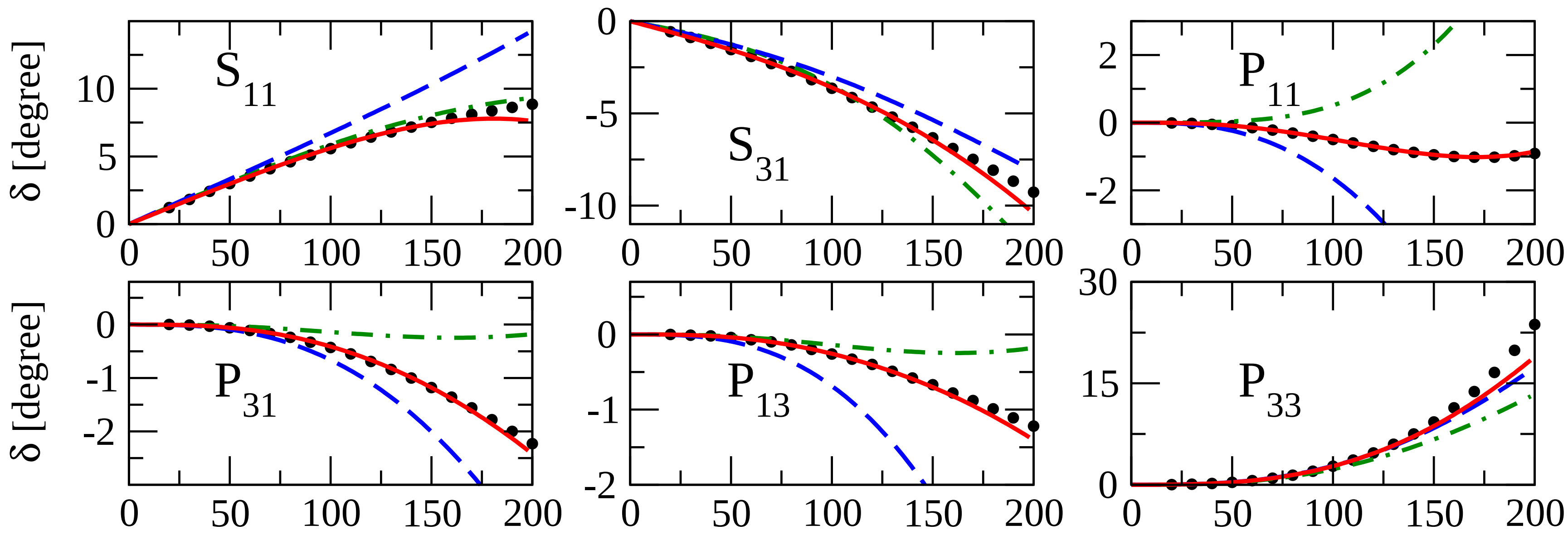
- ChPT obeys unitarity only perturbatively
  de-facto unitarisation to calculate phase shifts from real parts:

$$\delta = \arctan\left(\frac{|\mathbf{p}|}{8\pi\sqrt{s}}\mathrm{Re}\,T\right) \approx \frac{|\mathbf{p}|}{8\pi\sqrt{s}}\mathrm{Re}\,T$$

Convergence of the chiral expansion

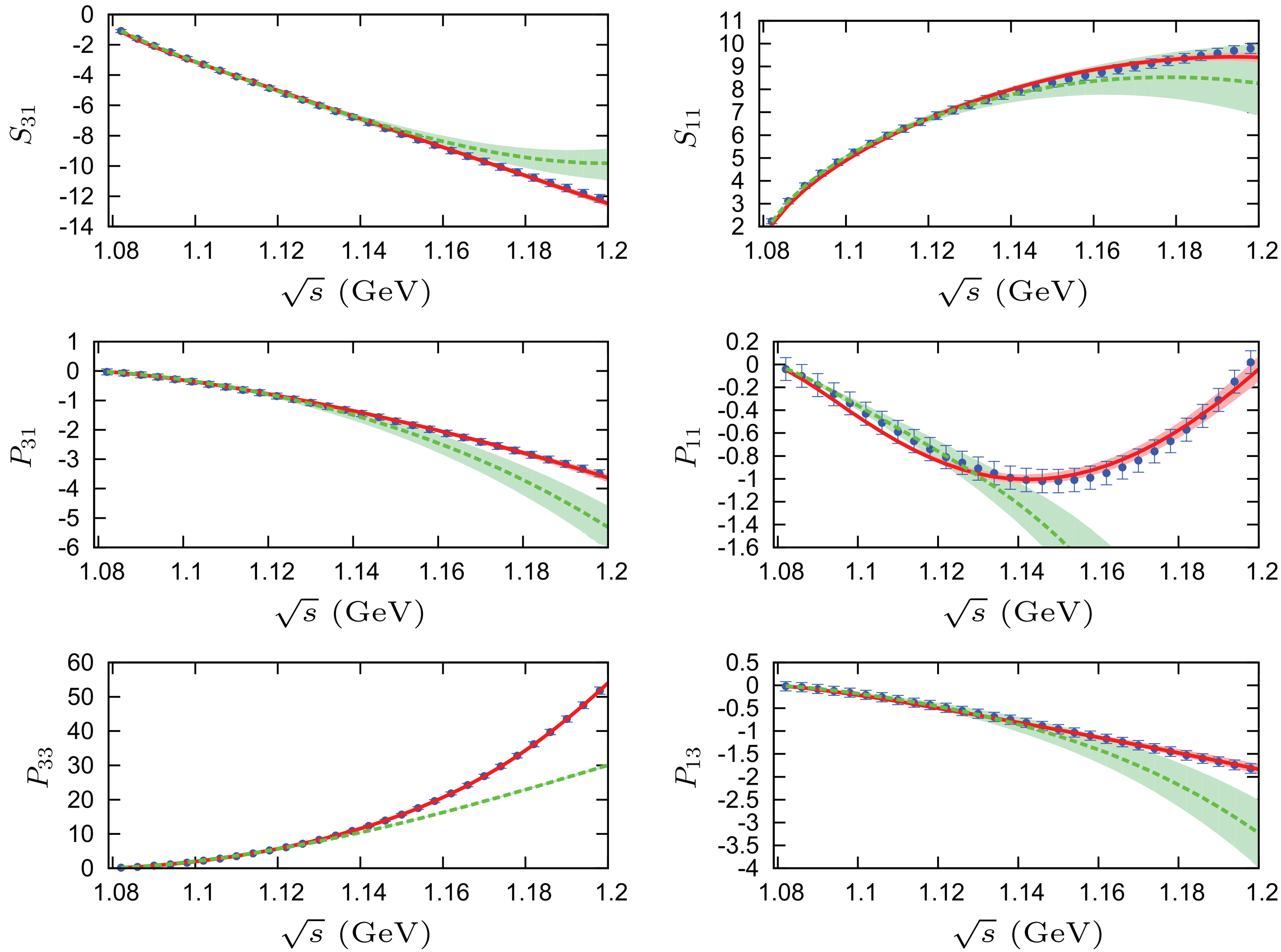


$\mathcal{O}(p^2)$ vs. $\mathcal{O}(p^3)$ vs. $\mathcal{O}(p^4)$

Krebs, Gasparyan, Epelbaum 2012

- fitted up to $p_{\text{Lab}} = 150\,$MeV $\hat{=} \sqrt{s} \approx 1.13\,$GeV,
  maximum energy shown $p_{\text{Lab}} = 200\,$MeV $\hat{=} \sqrt{s} \approx 1.17\,$GeV
- convergence assessed using LECs from highest-order fit
- D-waves also fitted

ChPT with and without explicit $\Delta(1232)$



$\mathcal{O}(p^3)$ / $\mathcal{O}(\delta^3)$ Alarcón, Martin Camalich, Oller 2013

fit range: $\sqrt{s_{\text{max}}} = 1.13$ GeV / $\sqrt{s_{\text{max}}} = 1.20$ GeV

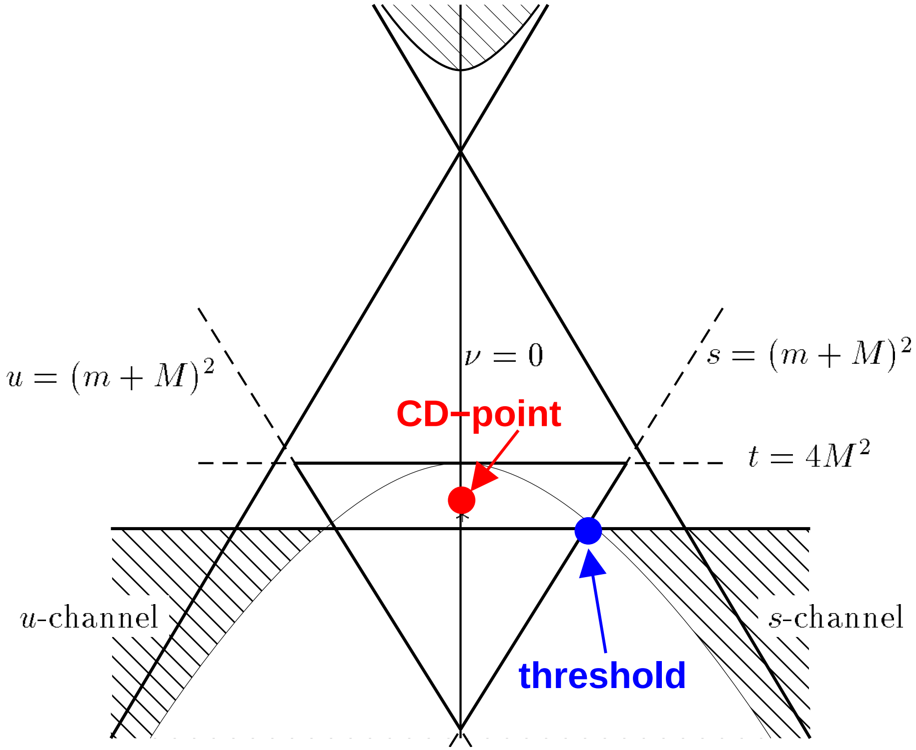
On the chiral extractions of $\sigma_{\pi N}$

The Cheng–Dashen theorem

- isoscalar amplitude at CD point related to scalar form factor

$$\underbrace{F_\pi^2 \bar{D}^+(s=u, t=2M_\pi^2)}_{F_\pi^2(d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_\sigma} + \Delta_R$$

$|\Delta_R| \lesssim 2\,$MeV Bernard et al. 1996



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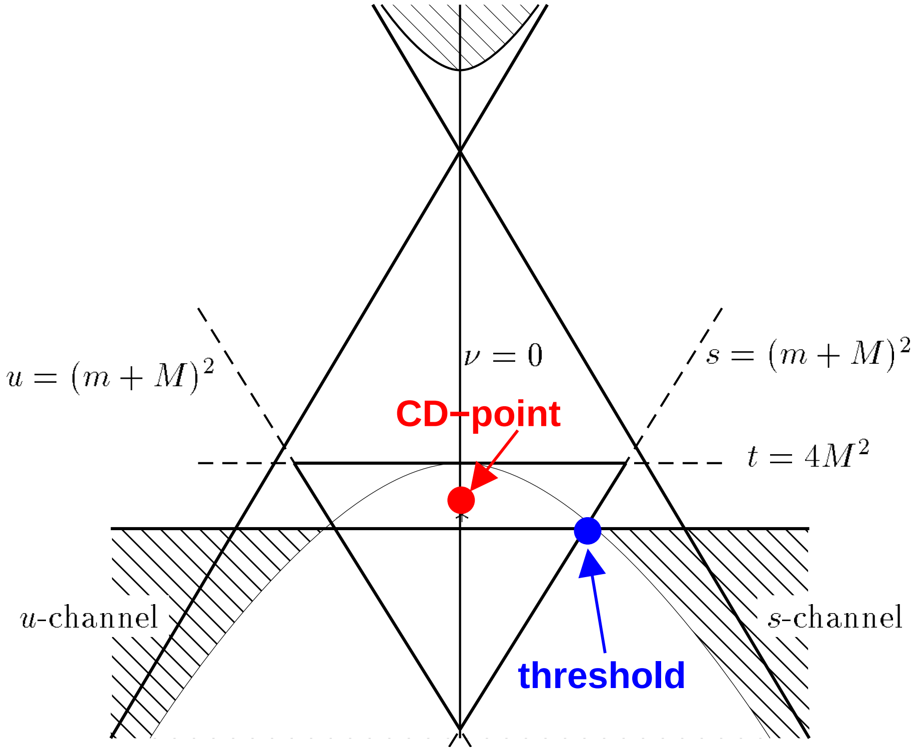

- ChPT fulfils all these relations perturbatively only
  is known to fail at one loop for Δ_D, Δ_σ: Gasser, Leutwyler, Sainio 1991
  curvature d_{02}^+ not reproduced at one loop   Alarcón et al. 2013
- we're lucky: $\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)$ MeV cancels to large extent

$\longrightarrow$ one-loop ChPT does not describe pion–nucleon scattering accurately in the whole low-energy region

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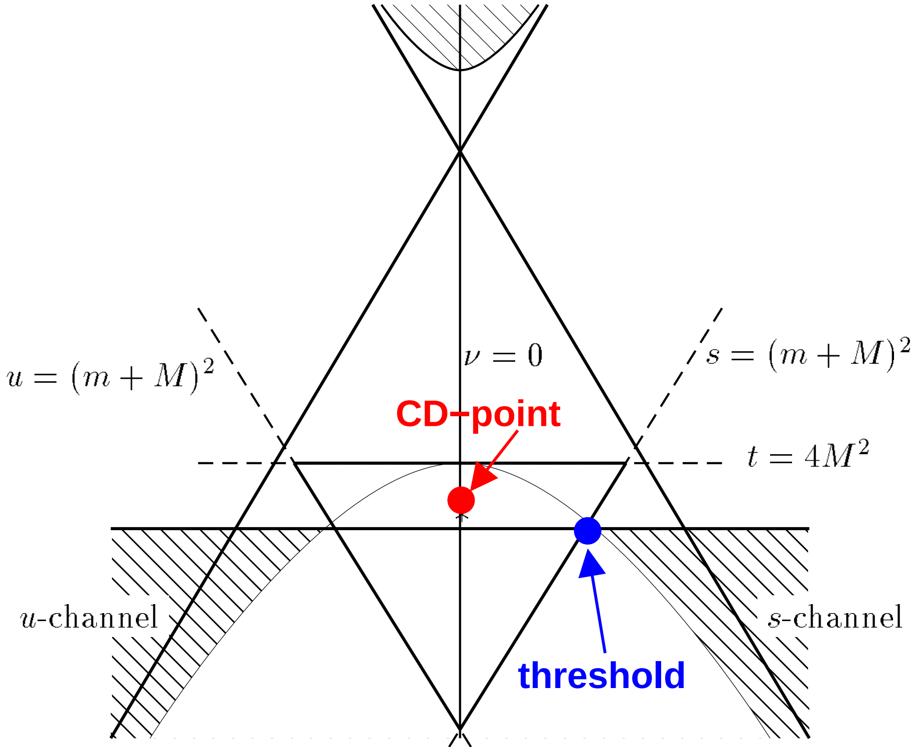

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$\longrightarrow$ update dispersive analysis, Roy–Steiner equations

Hoferichter, Ruiz de Elvira, BK, Meißner

The well-known paradigm: $\pi\pi$ Roy equations

Roy equations = coupled system of partial-wave dispersion relations + crossing symmetry + unitarity

- twice-subtracted fixed-t dispersion relation:

$$T(s,t) = c(t) + \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} ds' \left\{ \underbrace{\frac{s^2}{s'^2(s'-s)}}_{s\text{-channel cut}} + \underbrace{\frac{u^2}{s'^2(s'-u)}}_{u\text{-channel cut}} \right\} \mathrm{Im}T(s',t)$$

- subtraction function $c(t)$ determined from crossing symmetry

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- subtraction function $c(t)$ determined from crossing symmetry
- project onto partial waves $t_J^I(s)$ (angular momentum J, isospin I)
  expand $\mathrm{Im}T(s',t)$ in partial waves

$$t_J^I(s) = \text{polynomial}(a_0^0, a_0^2) + \sum_{I'=0}^{2}\sum_{J'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s,s')\,\mathrm{Im}t_{J'}^{I'}(s')$$

kernel functions $K_{JJ'}^{II'}(s,s')$ known analytically

Roy 1971

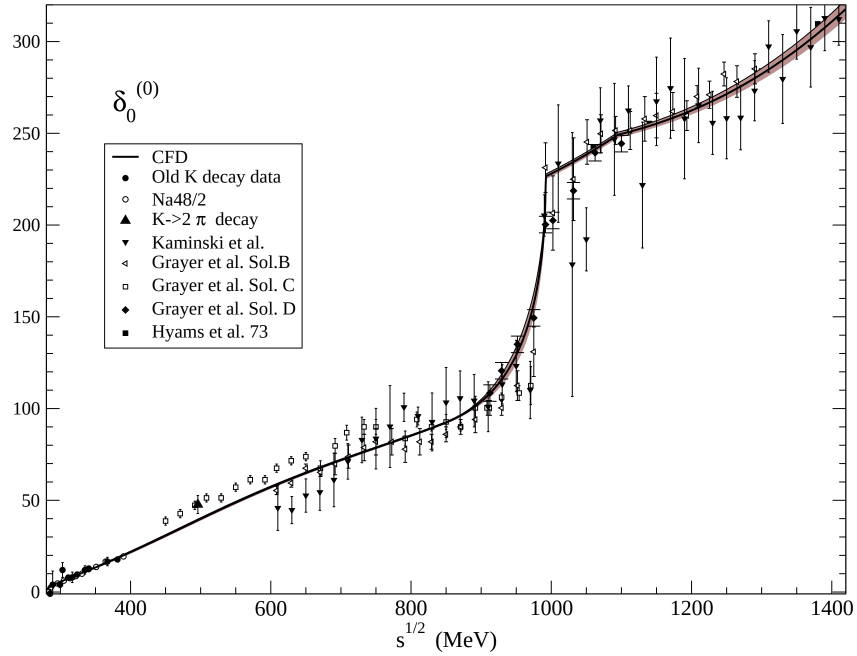
$\pi\pi$ Roy equations

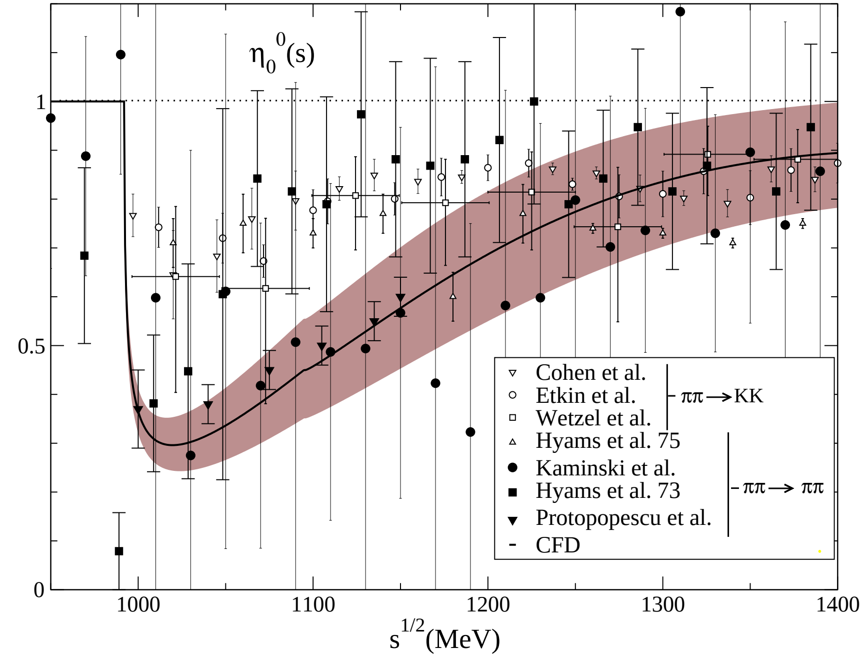
- elastic unitarity:

$$t_J^I(s) = \frac{e^{2i\delta_J^I(s)} - 1}{2i\sigma} \qquad \sigma = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

$\longrightarrow$ coupled integral equations for phase shifts

- example: $\pi\pi$ $I = 0$ S-wave phase shift & inelasticity

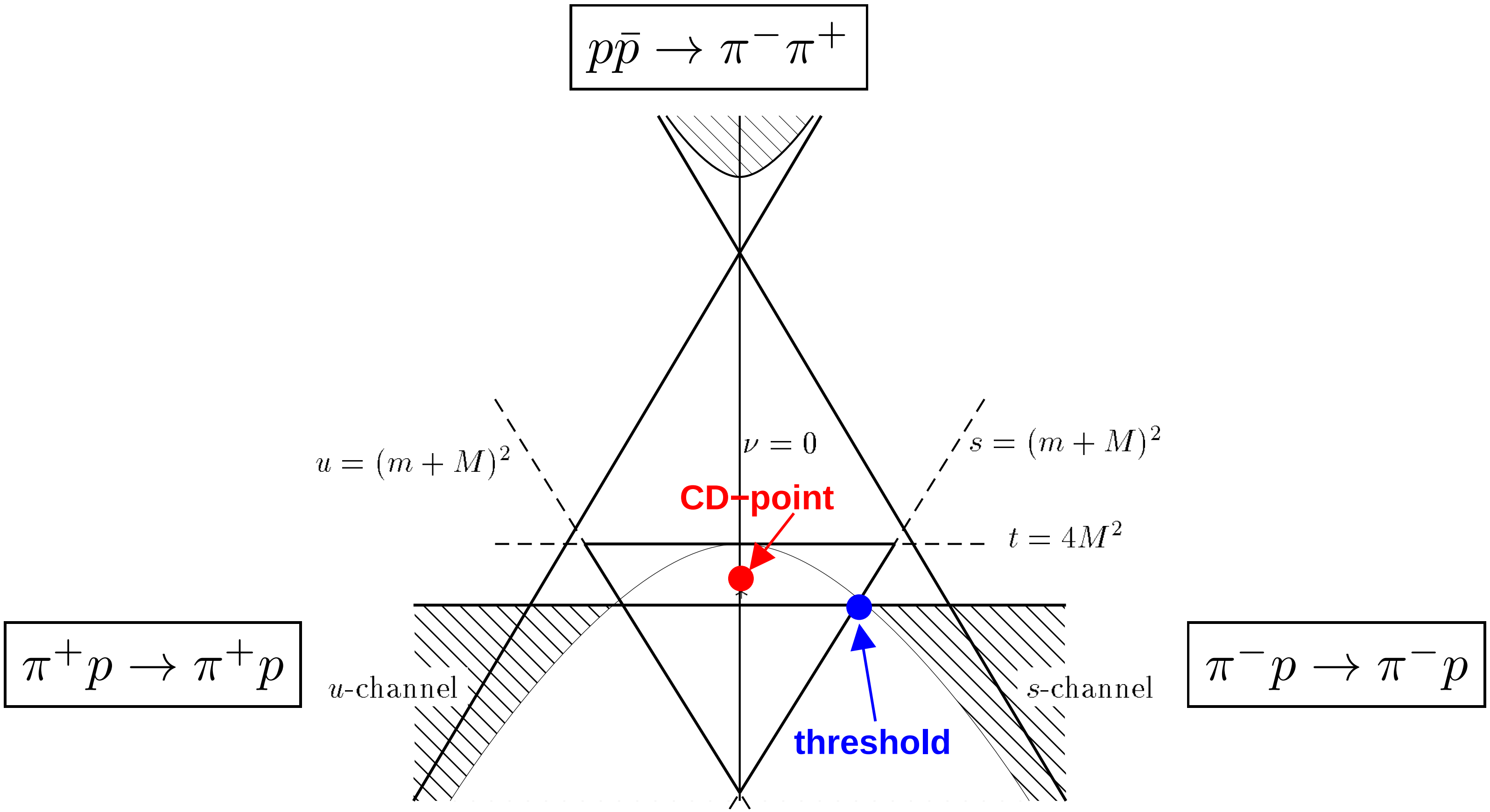




García-Martín et al. 2011

$\longrightarrow$ strong constraints on data from analyticity and unitarity!

Pion–nucleon scattering, crossing symmetry



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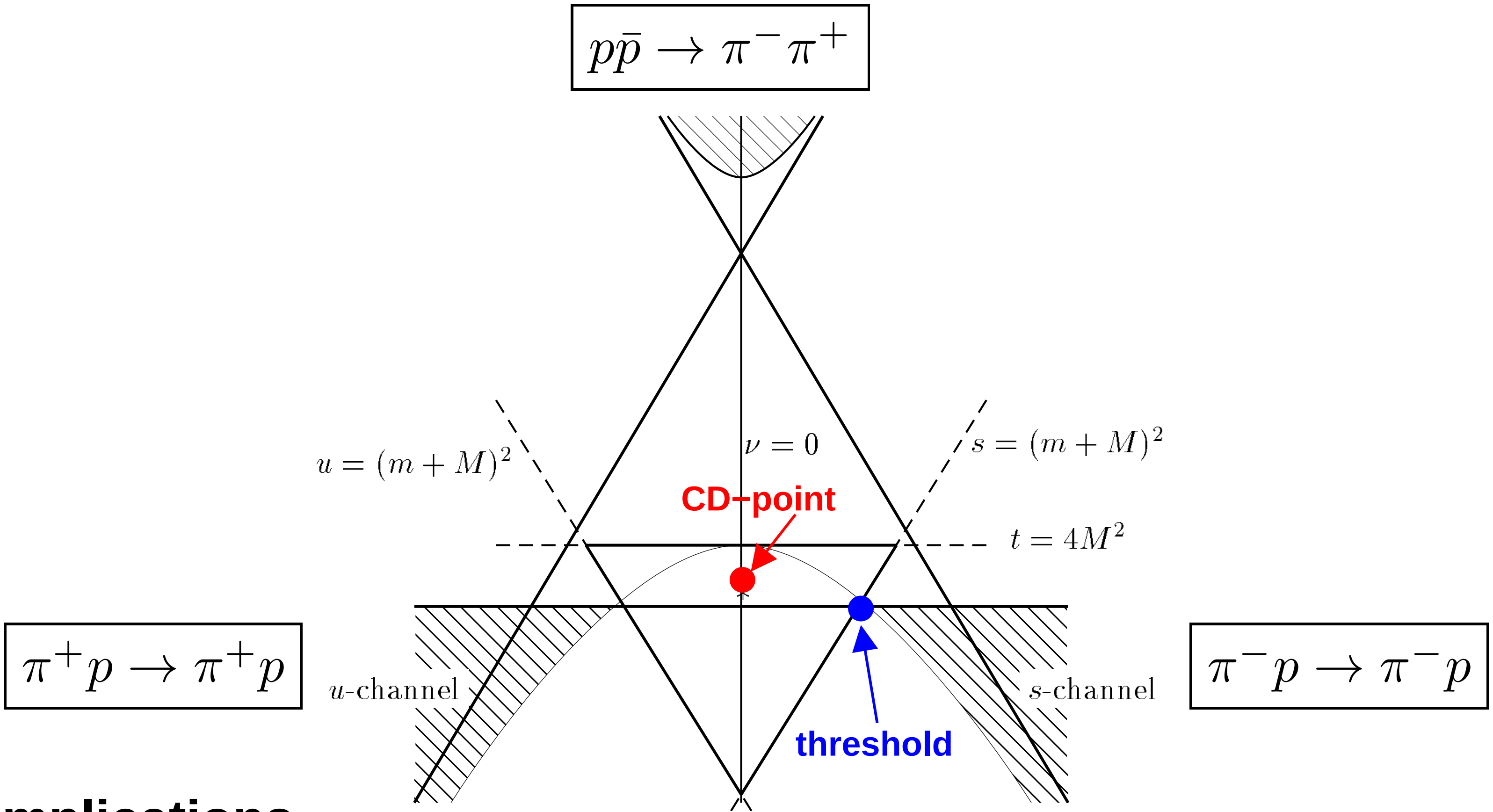

Complications

- crossing links two different processes, $\pi N \to \pi N$ and $\pi\pi \to \bar{N}N$
  $\longrightarrow$ use hyperbolic (instead of fixed-t) DR (Roy–Steiner)
- large pseudophysical region in the t-channel: $t = 4M_\pi^2 \longrightarrow 4m_N^2$, $\bar{K}K$ intermediate states ($f_0(980)$)

Roy–Steiner equations for pion–nucleon scattering

Limited range of validity:

$$\sqrt{s} \leq \sqrt{s_m} = 1.38\,\text{GeV} \qquad \sqrt{t} \leq \sqrt{t_m} = 2.00\,\text{GeV}$$

Input / constraints:

- S-, P-waves above matching point $s > s_m$ ($t > t_m$)
- inelasticities
- higher waves (D-, F-. . .)
- scattering lengths from hadronic atoms Baru et al. 2011

Output:

- S- and P-waves at low energies $s < s_m$, $t < t_m$
- subthreshold parameters
  - ▷ pion–nucleon σ-term
  - ▷ nucleon form factor spectral functions

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Important analysis steps:

- full analytic system Ditsche, Hoferichter, BK, Meißner 2012
- improved t-channel S-wave ($\pi\pi \leftrightarrow \bar{K}K \leftrightarrow \bar{N}N$) Hoferichter, Ditsche, BK, Meißner 2012
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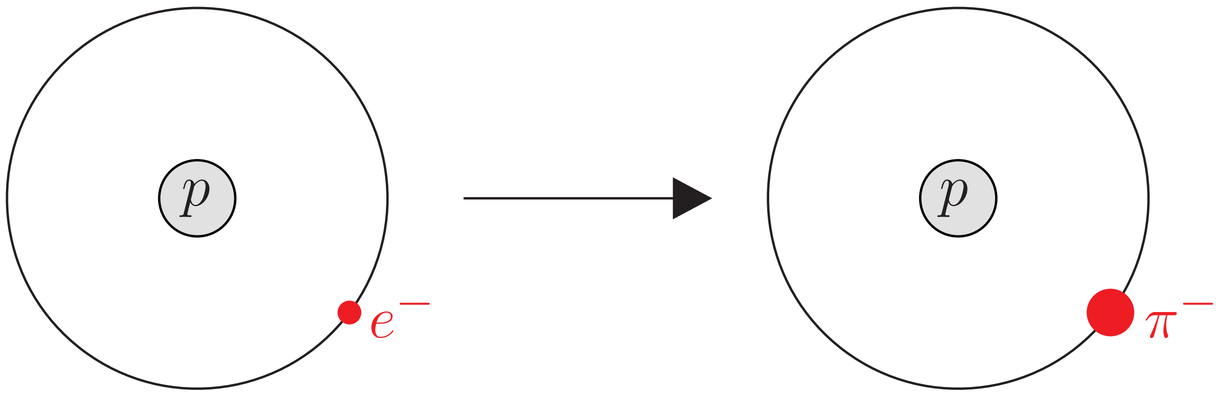
Pionic atoms and pion–nucleon scattering lengths

cf. Gasser, Lyubovitskij, Rusetsky 2008

- pionic hydrogen πH, pionic deuterium πD: atoms with $e^- \to \pi^-$
  calculate energy levels as for hydrogen in quantum mechanics!



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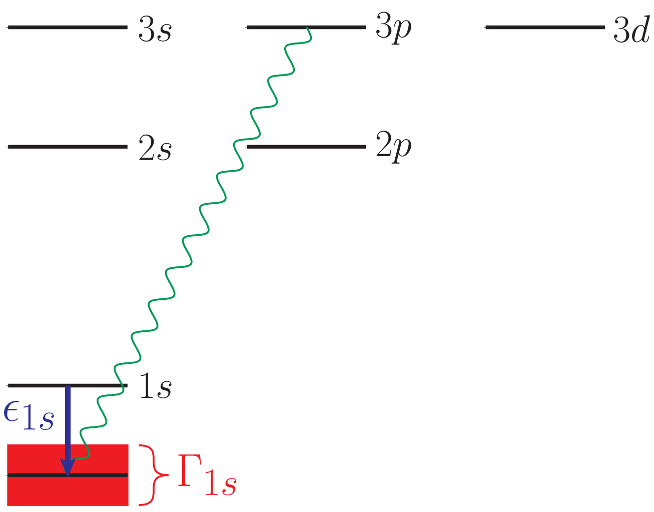

- energy levels perturbed by strong interactions:
  - ▷ ground state instable, decays:
    $\pi^- p \to \pi^0 n \quad \longrightarrow \quad$ width Γ_{1s}
  - ▷ ground state energy shift ϵ_{1s}
- linked to πN scattering at threshold:

$$\epsilon_{1s} \;\propto\; T(\pi^- p \to \pi^- p) \quad \propto\; a_0^+ + a_0^-$$
$$\Gamma_{1s} \;\propto\; |T(\pi^- p \to \pi^0 n)|^2 \propto\; |a_0^-|^2$$

Deser, Goldberger, Baumann, Thirring 1954

- πD: add. information from energy shift (diff. isospin combination)

Pionic atoms and pion–nucleon scattering lengths

Measurements of πH and πD PSI 1995-2010

$\epsilon_{1s} = (7.120 \pm 0.012)$ eV $\quad \Gamma_{1s} = (0.823 \pm 0.019)$ eV $\quad \epsilon_{1s}^{D} = (2.356 \pm 0.031)$ eV

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Theory to match this accuracy requires

- isospin breaking in πN Hoferichter, BK, Meißner 2009
- three-body corrections in πD Weinberg 1992, ...
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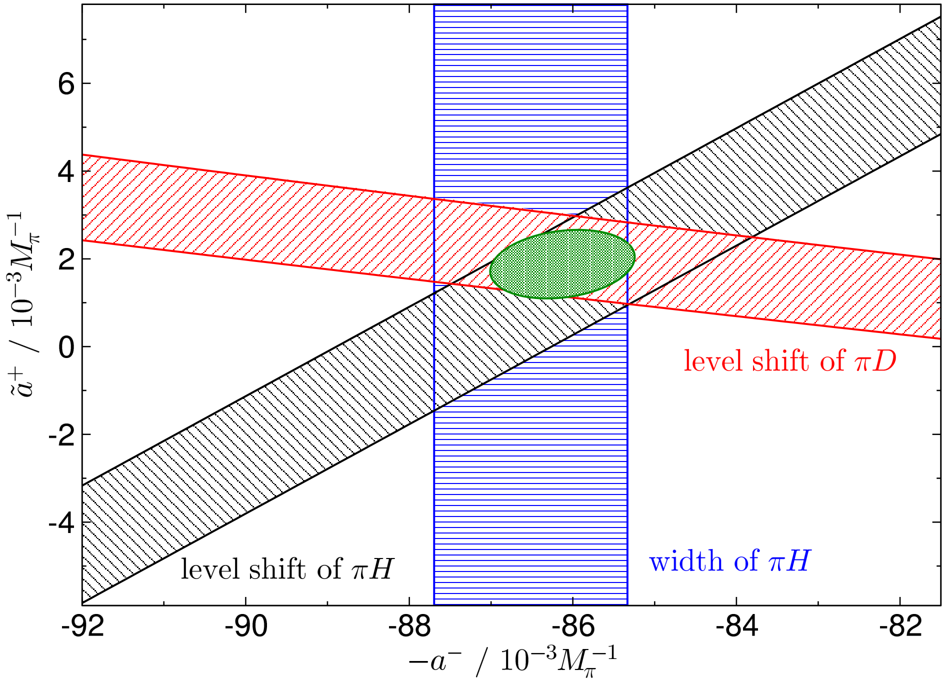

$$a_0^- = (86.1 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}$$
$$a_0^+ = (7.6 \pm 3.1) \cdot 10^{-3} M_\pi^{-1}$$

but: $\frac{1}{2}(a_{\pi^- p} + a_{\pi^+ p})$
$= (-1.1 \pm 0.9) \cdot 10^{-3} M_\pi^{-1}$

$\longrightarrow$ large isospin-breaking effects in isoscalar sector

Baru et al. 2011

Solving the coupled system: paradigms, uncertainties

An update on Karlsruhe–Helsinki (KH) with modern input

- πN scattering lengths extracted from hadronic atoms
- Goldberger–Miyazawa–Oehme sum rule from those:

  $$g_{\pi N}^2/4\pi = 13.7 \pm 0.2$$ Baru et al. 2011

  in perfect agreement with NN extractions Navarro Pérez et al. 2016

  compare: $g_{\pi N}^2/4\pi = 14.28$ Höhler 1983

  $\longrightarrow$ check: always reproduce KH results with KH input
- modern s-channel partial waves from SAID above s_m

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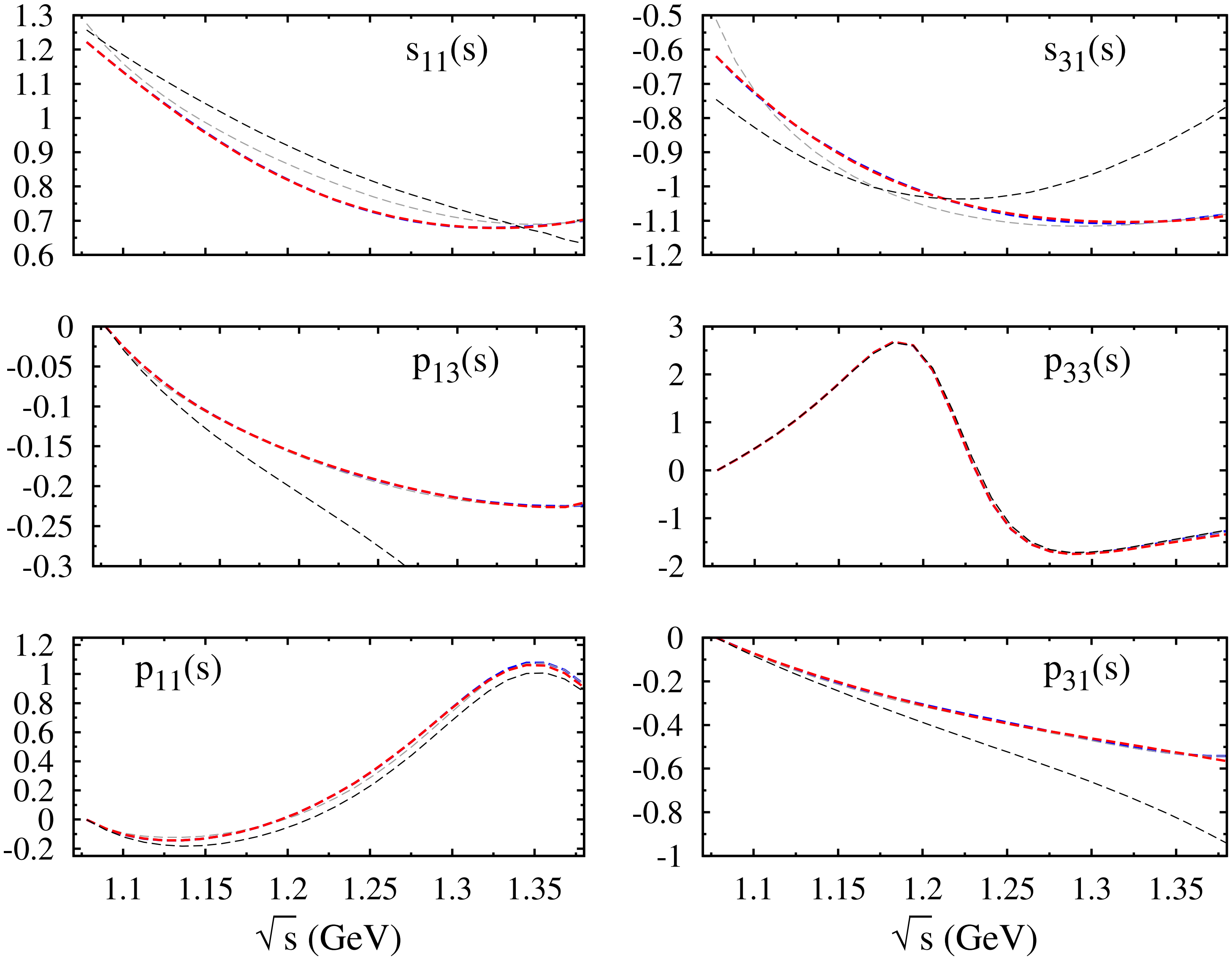
Dominant uncertainties

- near threshold: S-wave scattering lengths
- intermediate energies: significant correlations between 10 subtraction constants = subthreshold parameters ("flat minima")
- "large" energies: matching point uncertainties
- rather well under control: high-energy input, higher partial waves

Results: *s*-channel solution

LHS+RHS of Roy–Steiner eqs. *before* / LHS+RHS *after* fit/iteration



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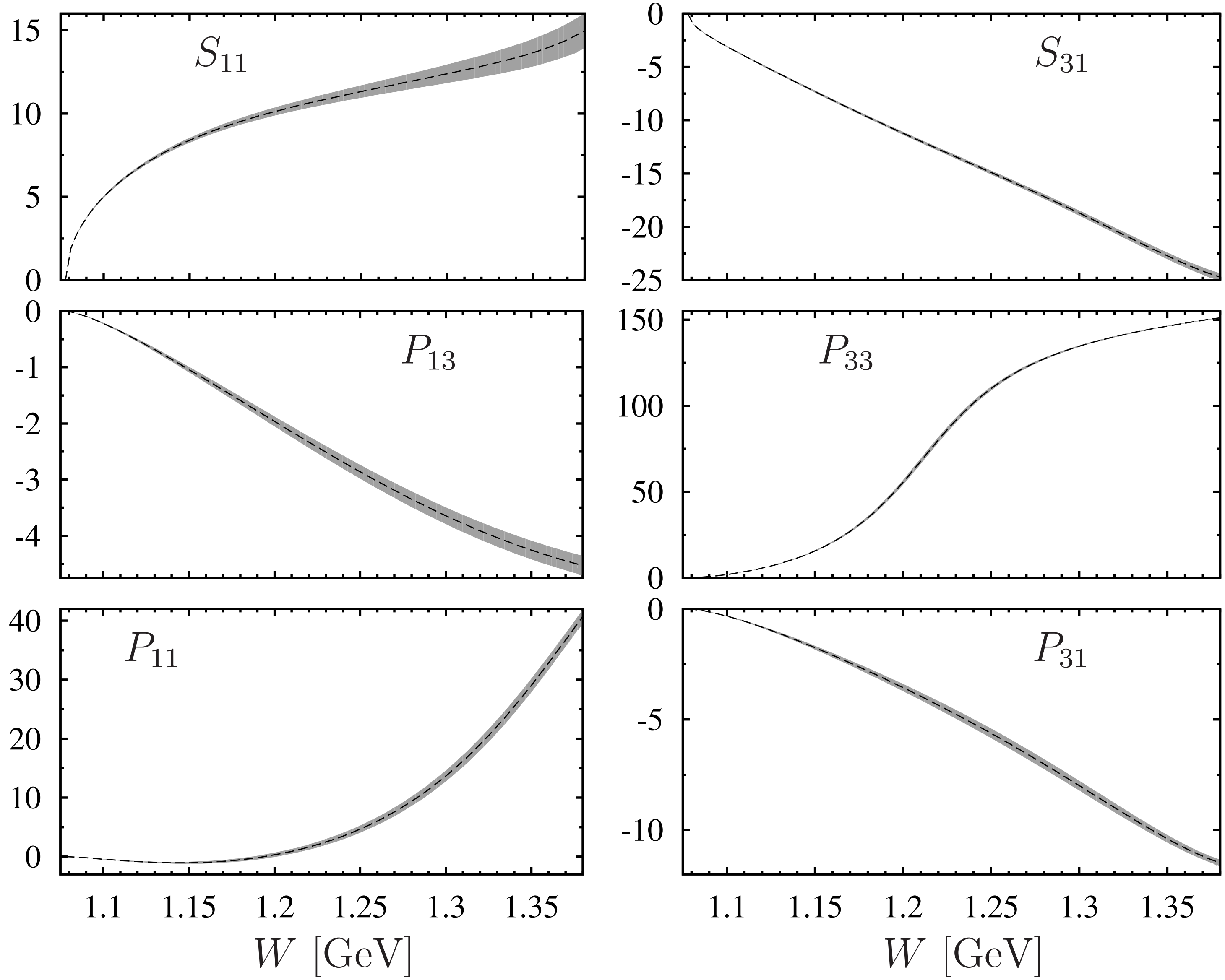
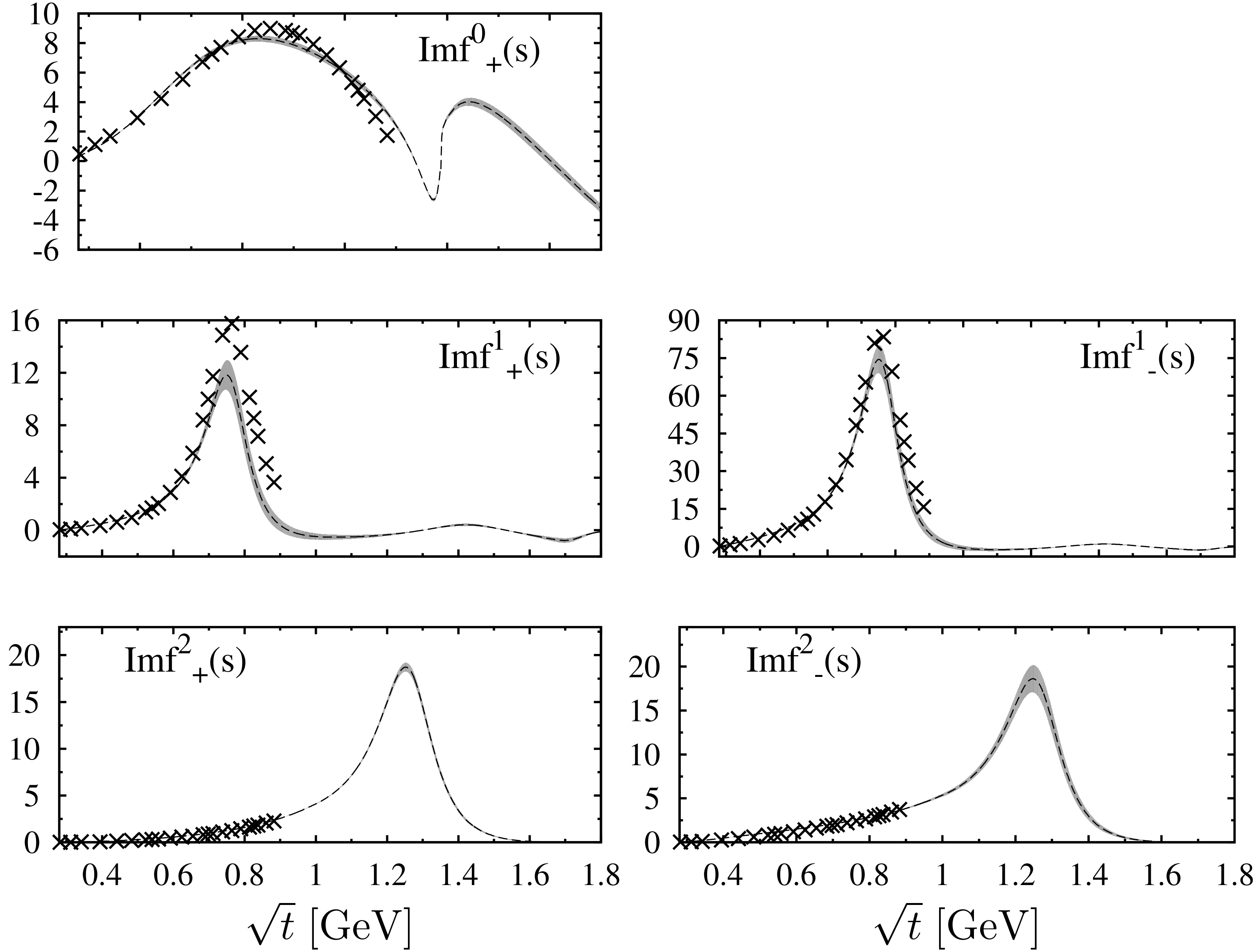


Hoferichter, Ruiz de Elvira, BK, Meißner 2015

Results: *t*-channel S-, P-, D-waves (compared to KH)



Results for the σ-term

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- subthreshold parameters output of the Roy–Steiner equations

$$d_{00}^+ = -1.36(3) M_\pi^{-1} \qquad \text{[KH:} \quad -1.46(10) M_\pi^{-1}\text{]}$$

$$d_{01}^+ = \quad 1.16(2) M_\pi^{-3} \qquad \text{[KH:} \quad 1.14(2) M_\pi^{-3}\text{]}$$

- $\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2)$ MeV Hoferichter et al. 2012
- $|\Delta_R| \lesssim 2$ MeV Bernard, Kaiser, Meißner 1996
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Hoferichter, Ruiz de Elvira, BK, Meißner 2015

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Hoferichter, Ruiz de Elvira, BK, Meißner 2015

- KH input $\longrightarrow$ $\sigma_{\pi N} \approx 46$ MeV — Gasser, Leutwyler, Sainio 1991
- compare also $\sigma_{\pi N} \approx (64 \pm 8)$ MeV — Pavan et al. 2002

Nucleon strangeness

- relate $\sigma_{\pi N}$ to strangeness content of the nucleon:

$$\sigma_{\pi N} = \frac{\hat{m}}{2m_N} \frac{\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle}{1-y} , \qquad y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

$(m_s - \hat{m})(\bar{u}u + \bar{d}d - 2\bar{s}s) \subset \mathcal{L}_{\text{QCD}}$ produces SU(3) mass splittings:

$$\sigma_{\pi N} = \frac{\sigma_0}{1-y} , \qquad \sigma_0 = \frac{\hat{m}}{m_s - \hat{m}}\big(m_\Xi + m_\Sigma - 2m_N\big) \simeq 26 \text{ MeV}$$

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- conclusion:
  - ▷ $\sigma_{\pi N} = (59.1 \pm 3.5)$ MeV not incompatible with small y
  - ▷ chiral convergence of σ_0 (hence $\langle N|\bar{s}s|N\rangle$) very doubtful

Comparison to lattice results – a puzzle (1)

- 4 new lattice calculations of $\sigma_{\pi N}$ at physical M_π since Hoferichter, Ruiz de Elvira, BK, Meißner 2015

$\sigma_{\pi N}$ [MeV]	collaboration	tension to RS
$38(3)(3)$	BMW 2015	3.8σ
$44.4(3.2)(4.5)$	χQCD 2015	2.2σ
$37.2(2.6)\binom{+1.0}{-0.6}$	ETMC 2016	4.9σ
$35.0(6.1)$	RQCD 2016	3.4σ

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- robust correlation between $\sigma_{\pi N}$ and scattering lengths:

$$\sigma_{\pi N} = (59.1 \pm 3.1)\,\text{MeV} + \sum_I c_I \big(a_0^I - \bar{a}_0^I\big),$$

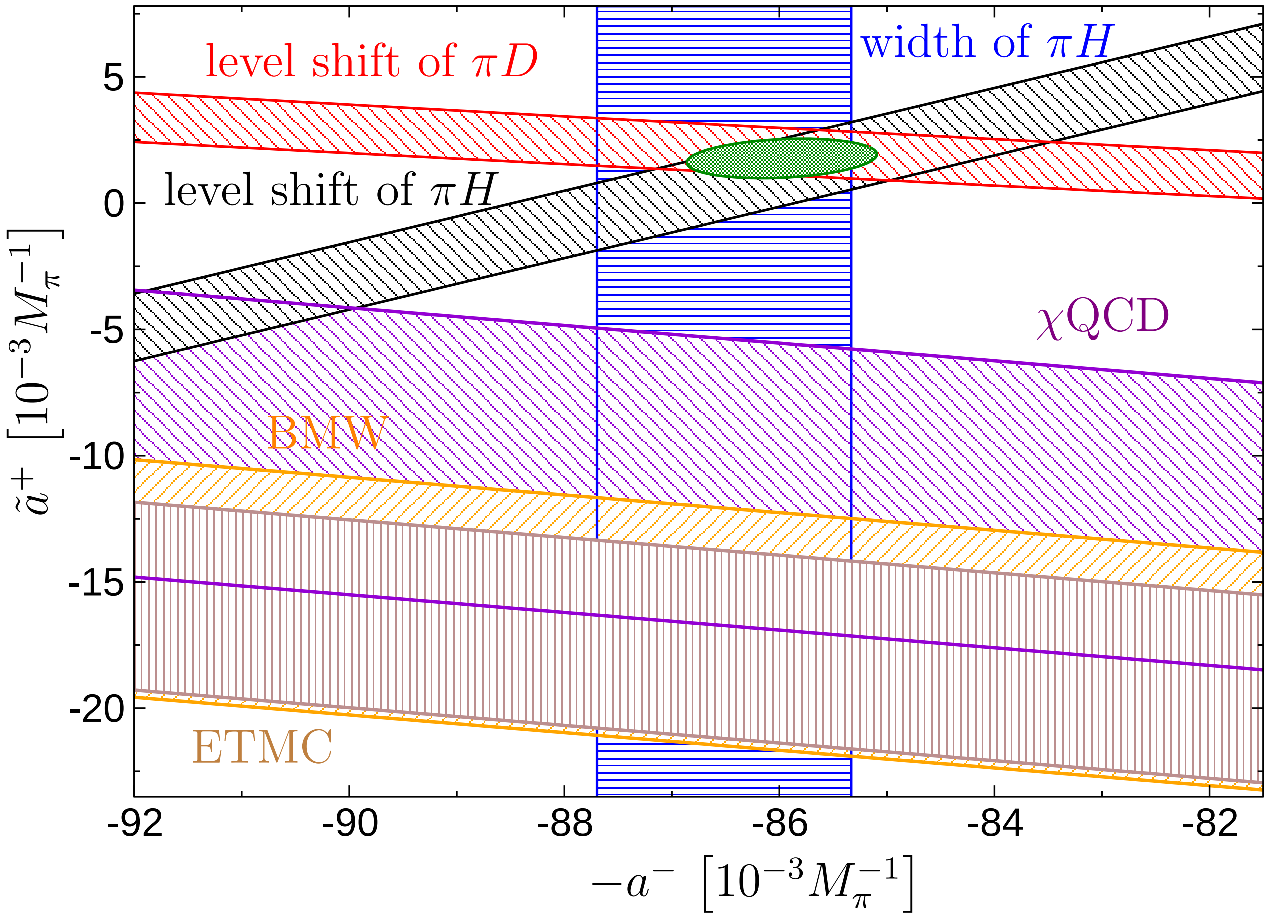
$$c_{1/2} = 0.242\,\text{MeV} \times 10^3 M_\pi \qquad c_{3/2} = 0.874\,\text{MeV} \times 10^3 M_\pi$$

$$\bar{a}_0^{1/2} = (169.8 \pm 2.0) \times 10^{-3}\, M_\pi^{-1} \qquad \bar{a}_0^{3/2} = (-86.3 \pm 1.8) \times 10^{-3}\, M_\pi^{-1}$$

$\longrightarrow$ expansion around reference values from πH and πD

Comparison to lattice results – a puzzle (2)

- lattice $\sigma_{\pi N}$ as additional constraint in scattering lengths plane



$\longrightarrow$ lattice $\sigma_{\pi N}$ clearly at odds with hadronic atoms results

$\longrightarrow$ suggestion: determine πN scattering lengths on the lattice

Hoferichter, Ruiz de Elvira, BK, Meißner 2016

Chiral low-energy constants

- chiral expansion expected to work best at subthreshold point:
  - ▷ maximal distance from threshold singularities
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	LO	NLO	NNLO
c_1 [GeV^{-1}]	-0.74 ± 0.02	-1.07 ± 0.02	-1.11 ± 0.03
c_2 [GeV^{-1}]	1.81 ± 0.03	3.20 ± 0.03	3.13 ± 0.03
c_3 [GeV^{-1}]	-3.61 ± 0.05	-5.32 ± 0.05	-5.61 ± 0.06
c_4 [GeV^{-1}]	2.17 ± 0.03	3.56 ± 0.03	4.26 ± 0.04

$\longrightarrow$ subthreshold errors tiny, chiral expansion dominates uncertainty

Summary

Pion–nucleon Roy–Steiner equations

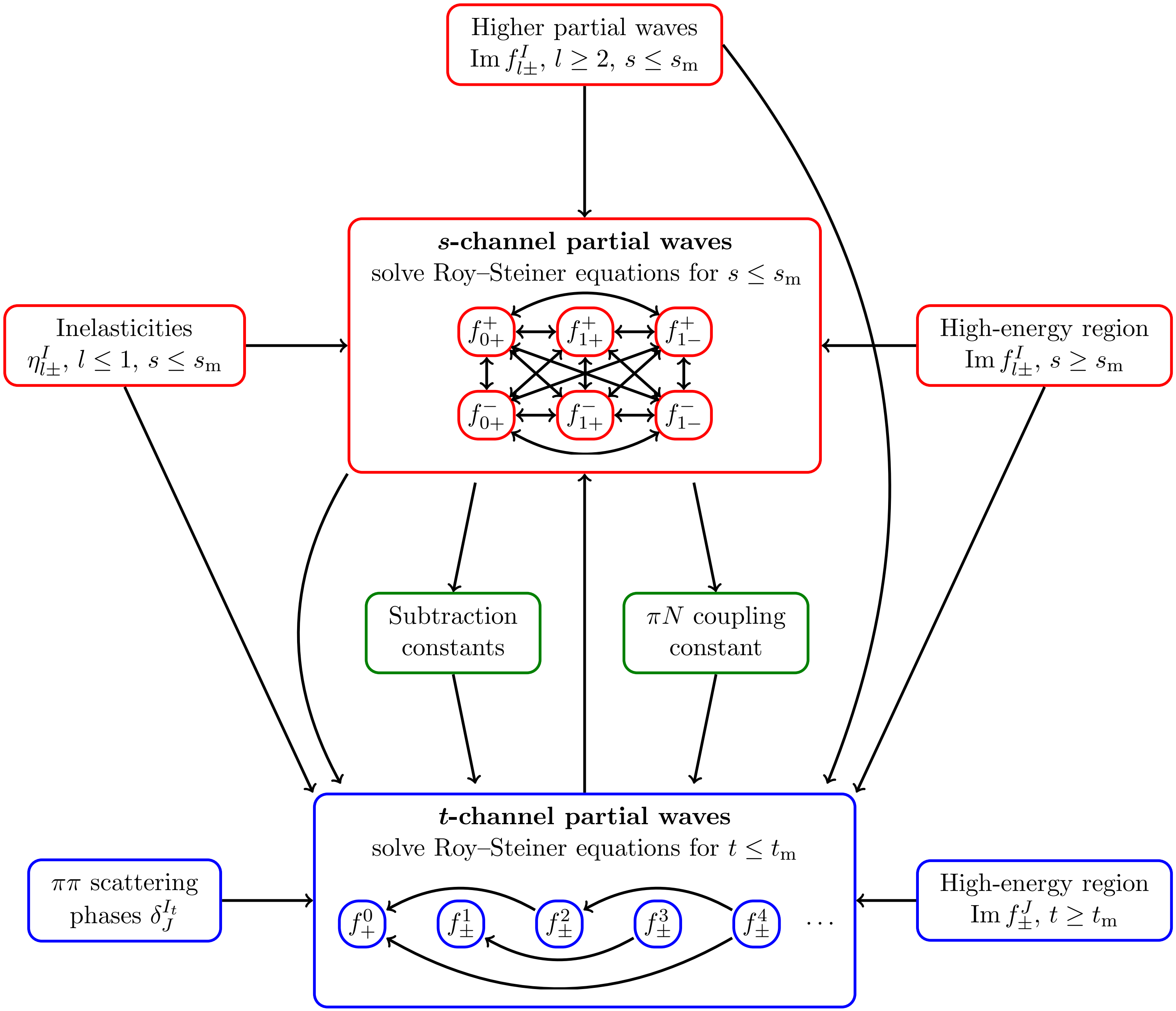
- allow to determine low-energy πN scattering with precision
  - ▷ obeying analyticity, unitarity, crossing symmetry
  - ▷ new input on scattering lengths from hadronic atoms
- provide πN phase shifts with systematic uncertainties
- similarly: t-channel $\pi\pi \to N\bar{N}$ spectral functions
- phenomenological determination of sigma term:

$$\sigma_{\pi N} = 59.1 \pm 3.5\,\text{MeV}$$

  currently at odds with lattice QCD results
- consistency check: Karlsruhe–Helsinki input leads to Karlsruhe–Helsinki results
- chiral low-energy constants obtained algebraically from subtreshold coefficients $\longrightarrow$ to be used in chiral NN potentials

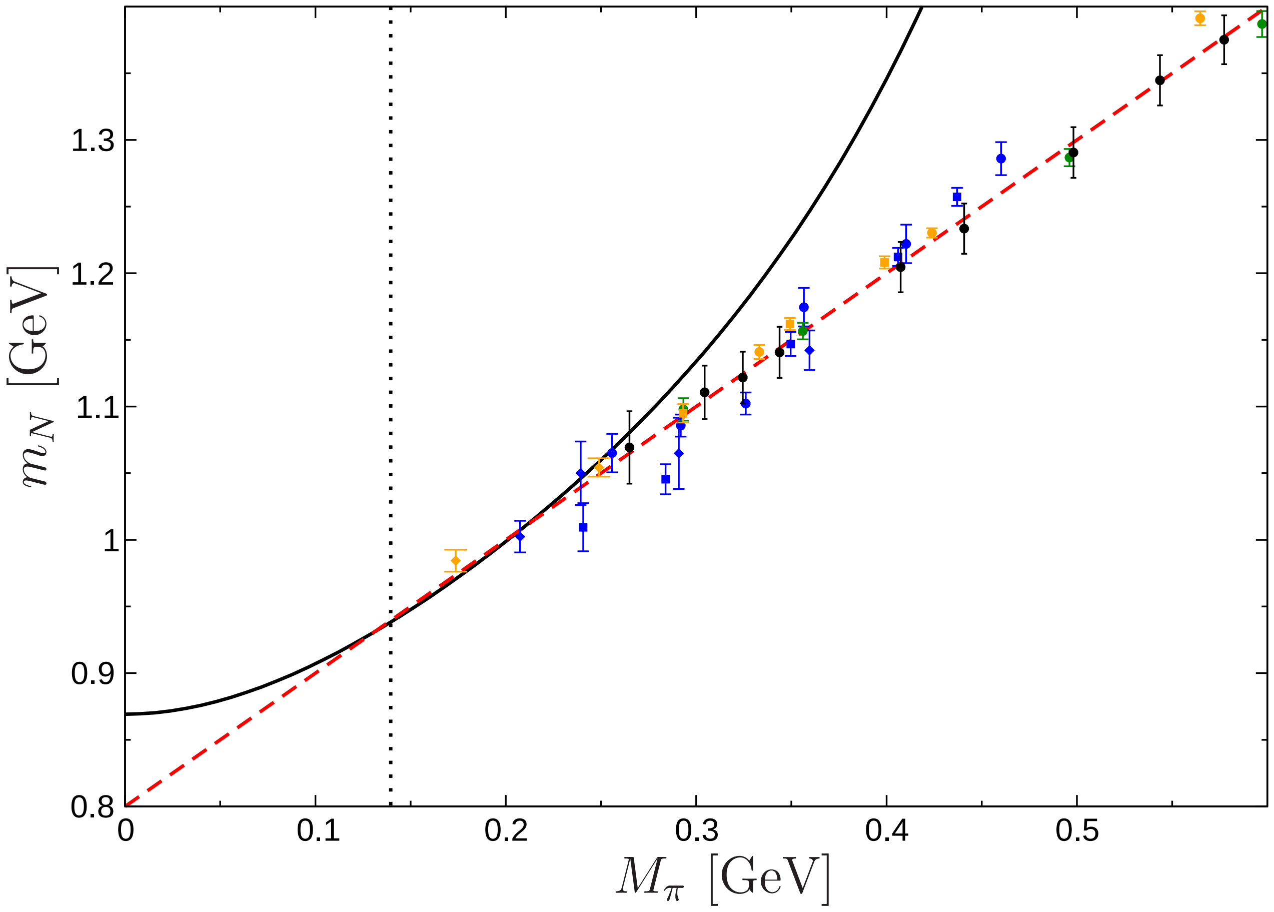
Spares

Roy–Steiner equations: information flowchart



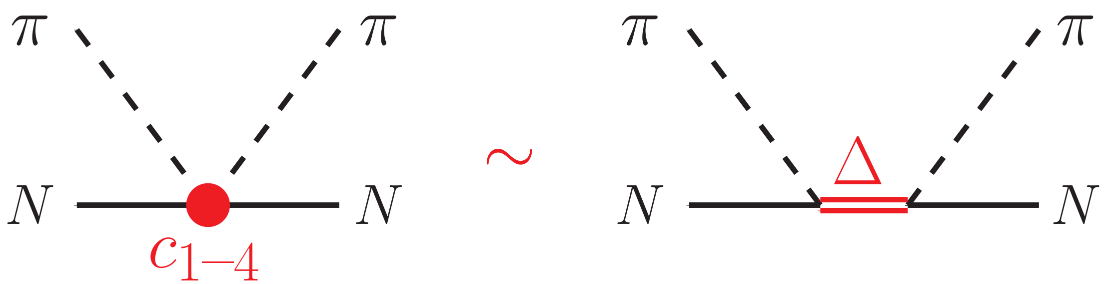
The “ruler plot” vs. ChPT

- pion mass dependence of m_N, using
  - ▷ c_1 from subthreshold matching to Roy–Steiner solution
  - ▷ combination of e_i from $\sigma_{\pi N}$



thanks to A. Walker-Loud for providing the lattice data

Including the $\Delta(1232)$ explicitly in ChPT



- large Δ effects slow down convergence of chiral series:

$$c_2^\Delta \approx 3.8 \qquad c_3^\Delta \approx -3.8 \qquad c_4^\Delta \approx 1.9$$

Bernard, Kaiser, Meißner 1997

- N and Δ become degenerate in the large-N_c limit
  $\longrightarrow$ include Δ as explicit degrees of freedom Jenkins, Manohar 1991
- consistent EFT counting scheme: ϵ-expansion Hemmert et al. 1998

$$p = \mathcal{O}(\epsilon) \qquad M_\pi = \mathcal{O}(\epsilon) \qquad m_\Delta - m_N = \mathcal{O}(\epsilon)$$

- alternative: δ counting Pascalutsa, Phillips 2003

$$p = \mathcal{O}(\delta) \qquad M_\pi = \mathcal{O}(\delta) \qquad m_\Delta - m_N = \mathcal{O}(\delta^{1/2})$$

$\longrightarrow$ loops with Δ shifted to higher orders

Recent chiral phase-shift analyses

- $\mathcal{O}(p^3)$ IR + unitarisation $\not\Delta$ [KH, GW] Alarcón et al. 2011
- $\mathcal{O}(p^3)$ EOMS $\not\Delta$, $\mathcal{O}(\delta^3)$ Δ [KH, GW, EM] Alarcón et al. 2013
  $\longrightarrow \sigma_{\pi N} = 59(7)$ MeV [GW, EM] ($43(5)$ MeV [KH]) Alarcón et al. 2012
- $\mathcal{O}(p^4)$ EOMS ($\not\Delta$), $\mathcal{O}(p^4, \delta^3)$ (Δ) [GW] Y.-H. Chen et al. 2013
  $\longrightarrow \sigma_{\pi N} = 52(7)$ MeV ($\not\Delta$), $45(6)$ MeV (Δ) $^{*)}$
- $\mathcal{O}(p^4)$ EOMS, NN counting [KH, GW] Krebs et al. 2012
  $\longrightarrow \sigma_{\pi N}$ large [GW] or small [KH]
- πN + NN fits to observables using amplitudes by Krebs et al.
  $\longrightarrow \sigma_{\pi N}$ large Wendt et al. 2014
- $\mathcal{O}(p^3)$ N/D unitarisation, CDD-poles for Δ and $N(1440)$ [KH, GW]
  $\longrightarrow \sigma_{\pi N} \approx 77$ MeV ("puzzle") Gasparyan, Lutz 2010

$^{*)}$ including lattice information; Δ amplitude may violate positivity constraints inside the Mandelstam triangle Sanz-Cillero et al. 2014