Hadronic Inputs to the $(g\text{-}2)_\mu$ Puzzle

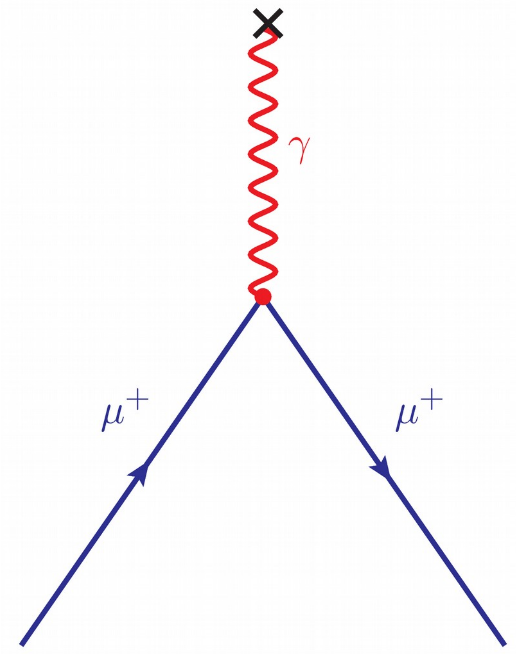
June 4, 2016 | Christoph Florian Redmer

14th International Workshop on Meson Production, Properties, and Interaction
MESON2016, Cracow, Poland

(g-2)$_\mu$

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \mu_B \vec{s}$

Dirac theory: $g_\mu = 2$



$(g\text{-}2)_\mu$

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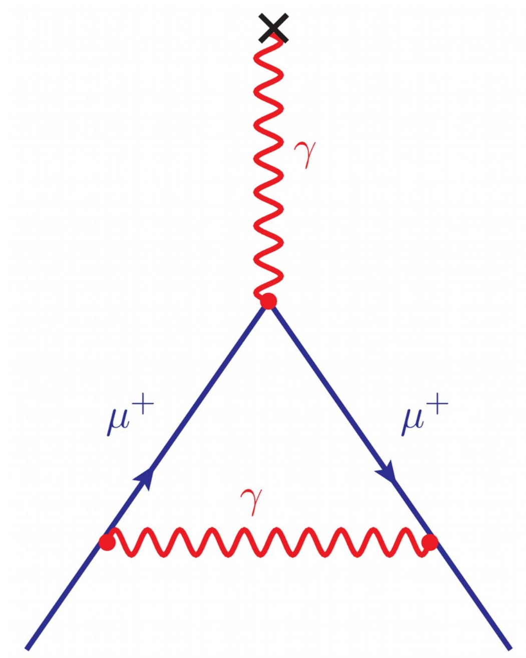
Quantum Field Theory: $g_\mu \neq 2$

Muon anomaly: $a_\mu = \frac{g_\mu - 2}{2}$

$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$

Contribution	in units of 10^{-10}
Schwinger	11620000



J.S. Schwinger (1948):

$$a_\mu^{QED,LO} = \frac{\alpha}{2\pi}$$

$(g\text{-}2)_\mu$

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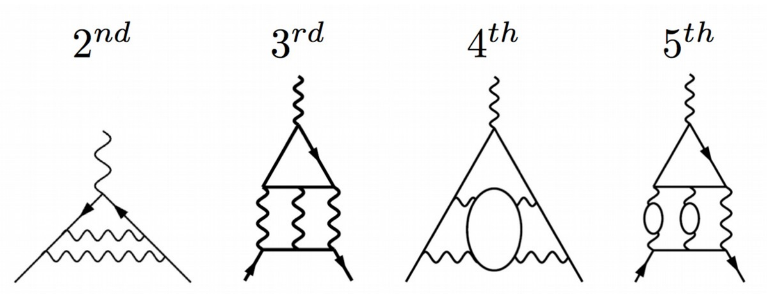
Contribution	in units of 10^{-10}
Schwinger	11620000
QED	11658471.895 ± 0.008

Kinoshita et al., PRL 109 (2012) 111808

Some examples for higher order QED corrections:



Total number of diagrams:

7	72	891	12672

$(g-2)_\mu$

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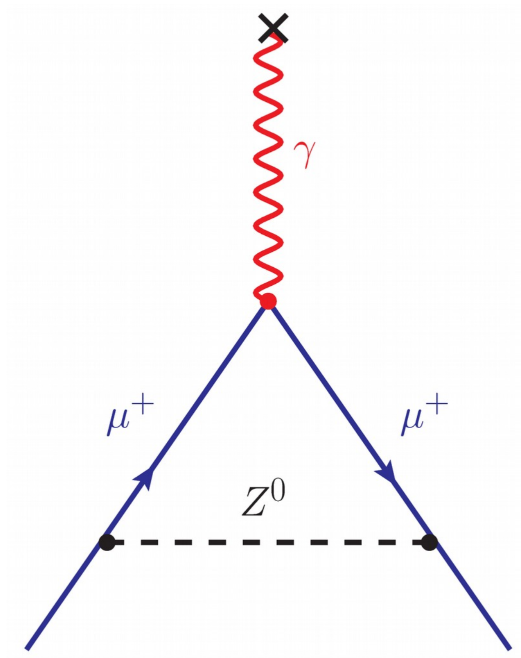

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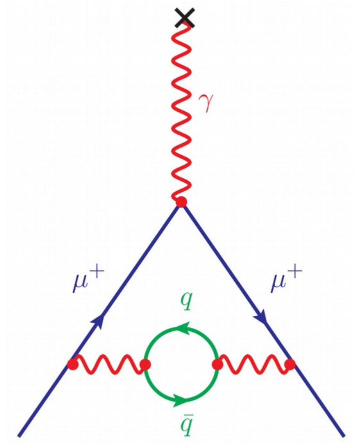
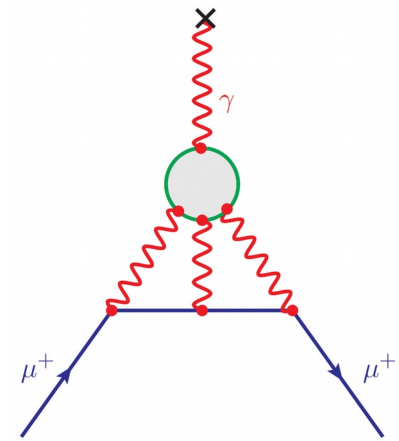
Czarnecki et al., PRD 67 (2003) 073006 + Erratum

$(g\text{-}2)_\mu$

Hadronic Vacuum Polarization





Hadronic Light-by-Light Scattering

Magnetic moment of μ : $\vec{\mu}_\mu = g_\mu \mu_B \vec{s}$

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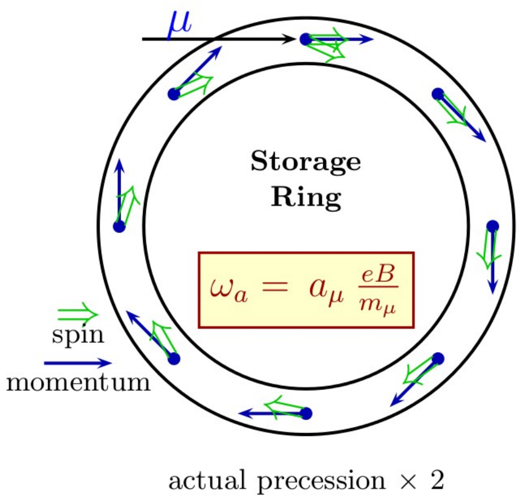
$$a_\mu^{theo} = a_\mu^{QED} + a_\mu^{weak} + a_\mu^{hadr}$$

Contribution	in units of 10^{-10}	
QED	11658471.895	± 0.008
Weak	15.4	± 0.2
HVP(leading order)	692.3	± 4.2
HVP(higher order)	-9.79	± 0.07
HLBL	11.6	± 4.0
Total	11659181.4	± 5.8

Kinoshita et al., PRL 109 (2012) 111808
Czarnecki et al., PRD 67 (2003) 073006 + Erratum
Davier et al., EPJC 17 (2011) 1515 + Erratum
Hagiwara et al., CPC 34 (2010) 728
Jegerlehner, Nyffler, Phys.Rept. 477 (2009) 1

Direct Measurement



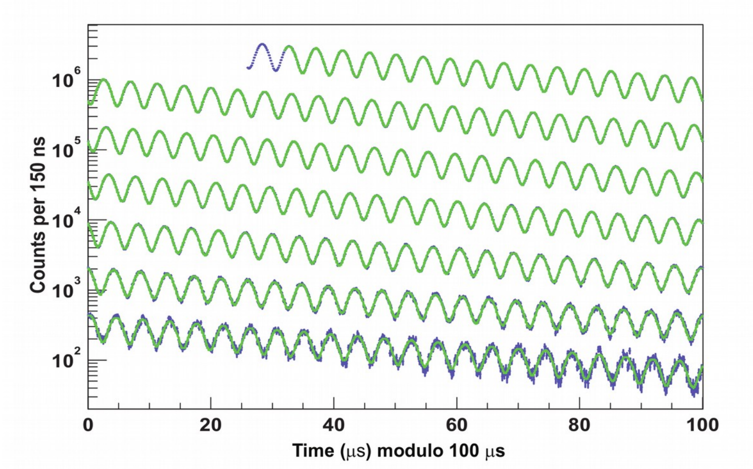
- $\pi^+ \rightarrow \mu^+\nu_\mu$ (longitudinally polarized μ^+ due to P violation)
- Precession in magnetic field and focusing electric field

$$\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu}(a_\mu\vec{B} - [a_\mu - \frac{1}{\gamma^2-1}]\vec{v}\times\vec{E})$$

- Select " Magic γ " to be independent of $\vec{E}$

$$\gamma = \sqrt{1 + 1/a_\mu} = 29.3 \quad \Rightarrow \quad p_\mu = 3.094\mathrm{GeV}/c$$

- Detect e⁺ from $\mu^+ \rightarrow e^+\bar{\nu}_\mu\nu_e$
  - Direction of e⁺ influenced by polarization of μ^+
  - Rate of measured e⁺ modulated with $\vec{\omega}_a$



Bennet et al., PRD 73 (2006) 072003

Direct Measurement

Long History of direct Measurements:

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL	1997	μ^+	11 659 251(150)	13
BNL	1998	μ^+	11 659 191(59)	5
BNL	1999	μ^+	11 659 202(15)	1.3
BNL	2000	μ^+	11 659 204(9)	0.73
BNL	2001	μ^-	11 659 214(9)	0.72
Average			11 659 208.0(6.3)	0.54

Latest High Precision Measurement of a_μ: BNL-E821

Bennet et al., PRD 73 (2006) 072003

$$a_\mu^{exp} = 11\,659\,208.9 \pm 6.3 \cdot 10^{-10}$$

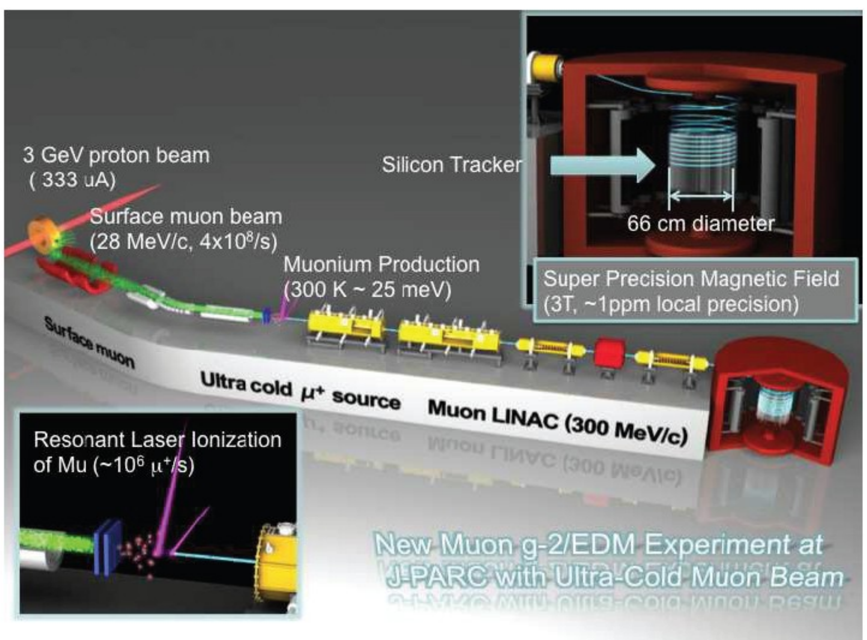
Discrepancy of 3 – 4σ compared to SM predictions!

Hint for New Physics?

New Measurements

Fermilab E989

- Reusing the BNL ring
- Higher statistics
- Improved systematics
- $\delta a_\mu \approx 1.6 \times 10^{-10}$



J-PARC

- Ultra cold muons
- No electric field
- $\delta a_\mu \sim 10 \times 10^{-11}$

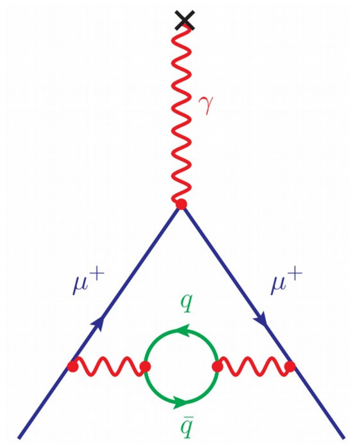
Nucl.Phys.Proc.Suppl.218 (2011) 242

Experiment vs Theory

Improvement of δa_μ by a factor 4 by new experiments

→ Theory has to keep up with the precision!

Hadronic Vacuum Polarization



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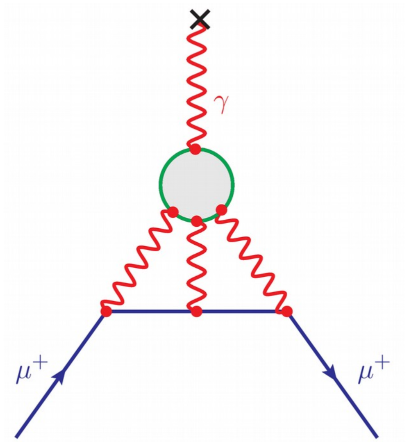


Hadronic Light-by-Light Scattering

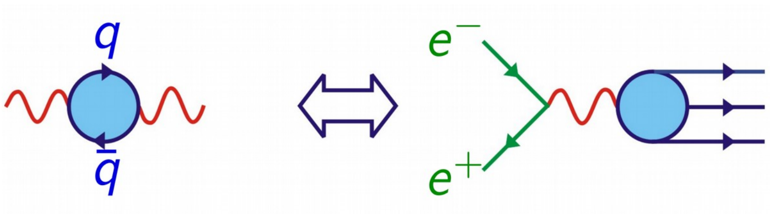
Hadronic contributions completely dominate the uncertainty of the Standard Model prediction!

Challenge: Perturbative methods cannot be applied in the relevant energy regime

Experimental Input needed!

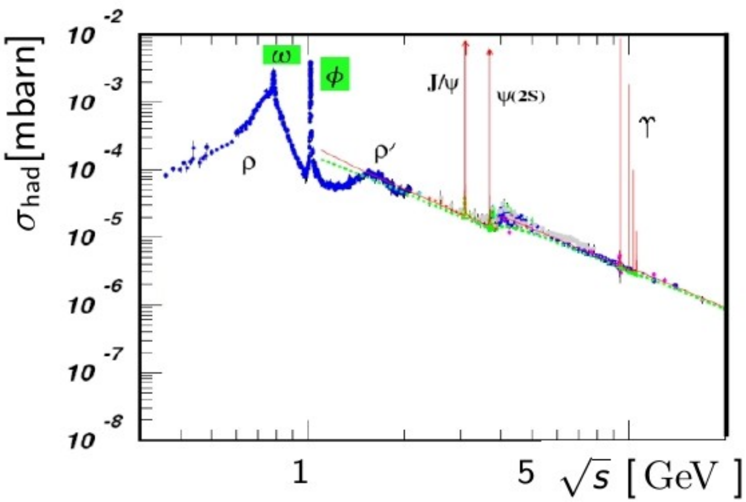
Hadronic Vacuum Polarization

related to hadronic cross sections by optical theorem



Dispersion Integral :

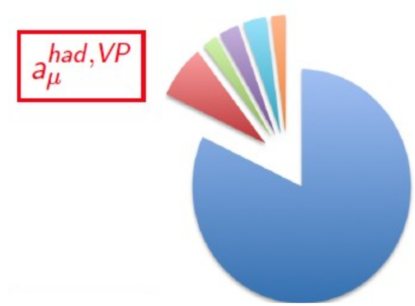
$$a_\mu^{hVP}{}_{LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} K(s)\sigma(e^+e^- \to hadr)ds$$

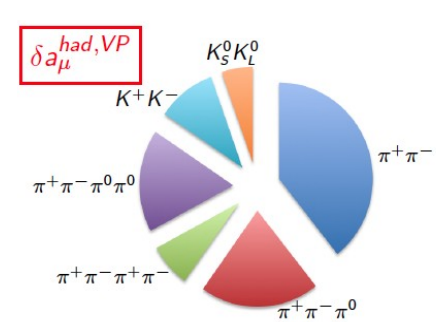


$$K(s) \sim \frac{1}{s}$$

$$\sigma(e^+e^- \to hadr) \sim \frac{1}{s}$$

} Low energy contributions dominate !



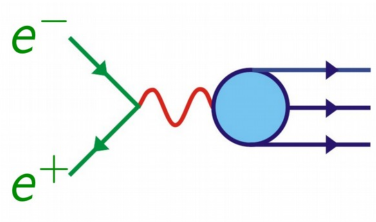


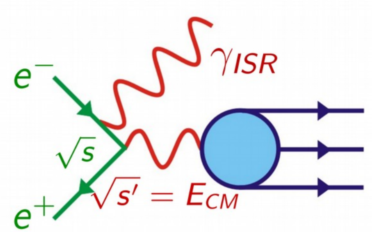
Hadronic Vacuum Polarization

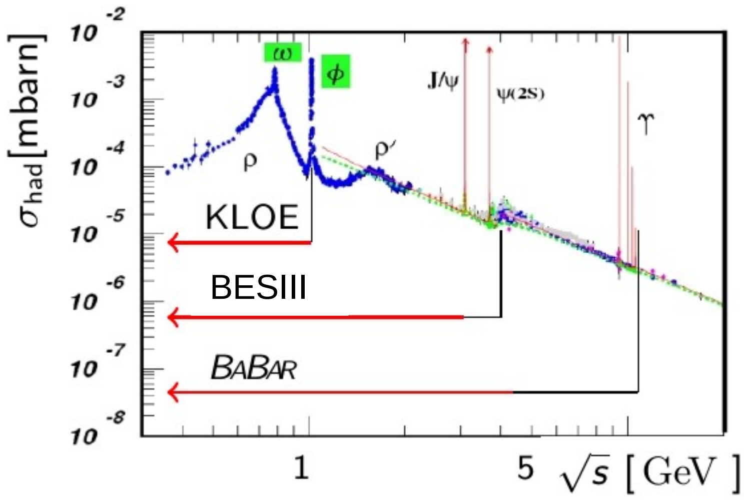
<u>Energy Scan Measurements:</u>

CMD, SND (Novosibirsk)
BES (Beijing)

Next presentation by G. Fedotovich







<u>Initial State Radiation Measurements:</u>

- Photon emitted in initial state
- Measurement a different energy possible

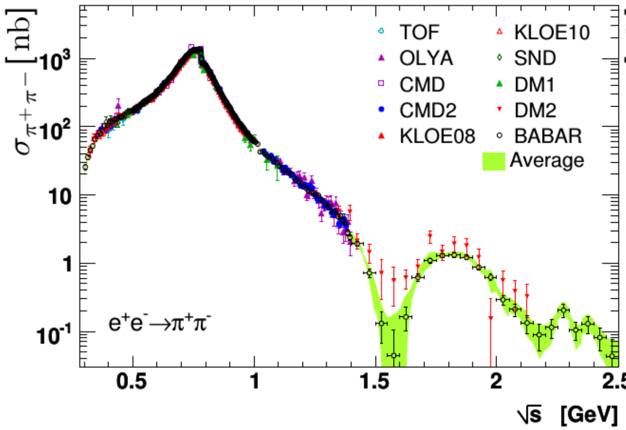
KLOE (Frascati)
BaBar (Stanford)
BESIII (Beijing)

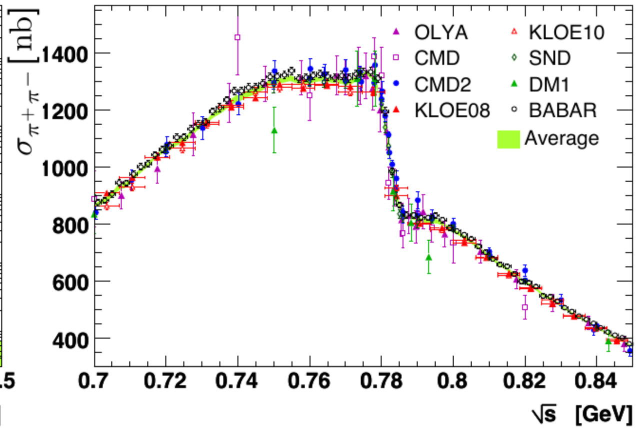
Yesterdays presentation by P.Lukin

Hadronic Vacuum Polarization

$e^+e^- \rightarrow \pi^+\pi^-$

- Accounts for 75% of a_μ^{hVP}
  - Good knowledge important !



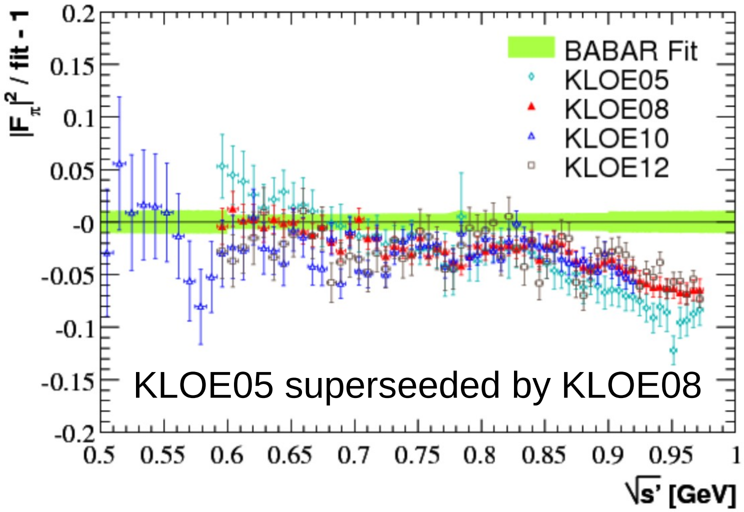


<u>Systematic Uncertainties:</u>

BaBar	0.5%
KLOE	0.8%
CMD	0.8%
SND	1.5%

} Limited by statistics (CMD, SND)

- KLOE and BaBar dominate world average
- systematic differences
  - large uncertainty for a_μ^{hVP}



Hadronic Vacuum Polarization

$e^+e^- \to \pi^+\pi^-$ measurement at BESIII

Phys.Lett.B753 (2016) 629

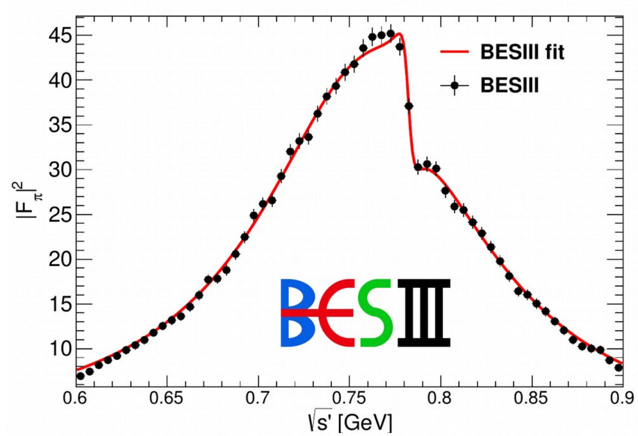
- 2.9 fb^{-1} on ψ(3770) peak
- ISR technique
- $\mu - \pi$ separation with ANN
- careful evaluation of systematics

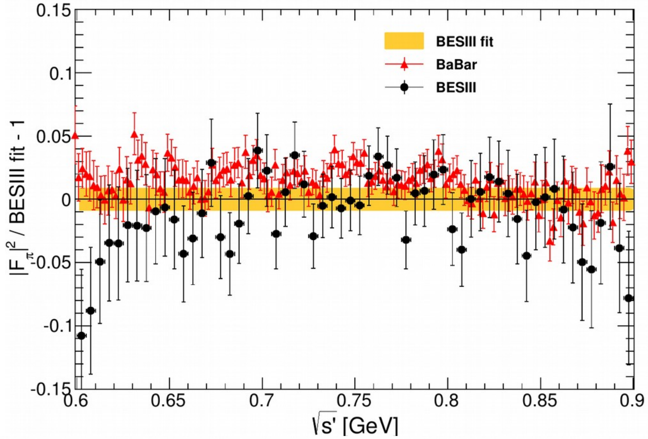
Source	Uncertainty (%)
Photon efficiency	0.2
Tracking efficiency	0.3
Pion ANN efficiency	0.2
Pion e-PID efficiency	0.2
Angular acceptance	0.1
Background subtraction	0.1
Unfolding	0.2
FSR correction δ_{FSR}	0.2
Vacuum polarization correction δ_{vac}	0.2
Radiator function	0.5
Luminosity $\mathcal{L}$	0.5
Sum	**0.9**

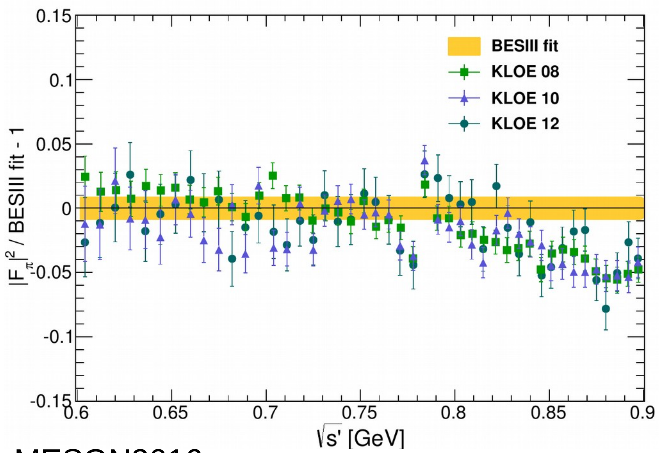
- evaluation for $0.6 \leq m_{\pi\pi} \leq 0.9$
  - 70% of total 2π contribution
  - 50% of a_μ^{hVP} contribution

Comparison to previous measurements:

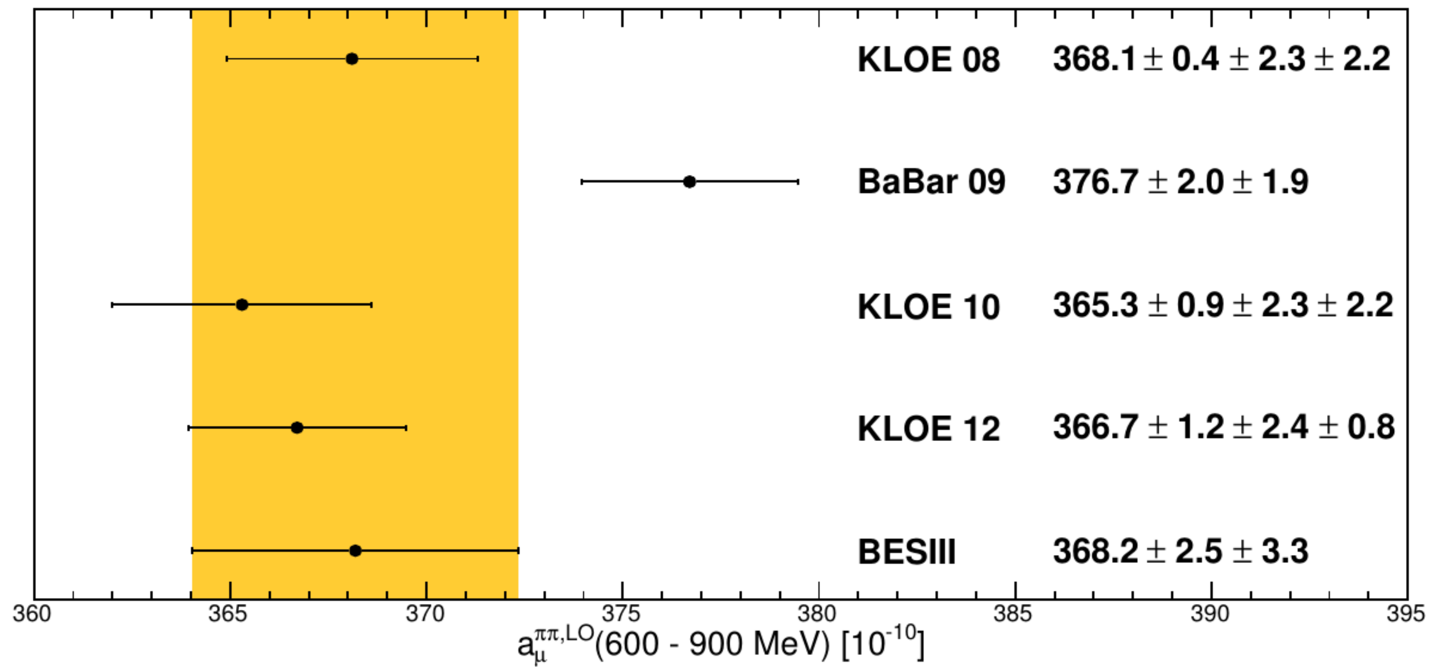
- Systematic shift in pion form factor
  - below ρ/ω interference wrt BaBar
  - above ρ/ω interference wrt KLOE







Hadronic Vacuum Polarization

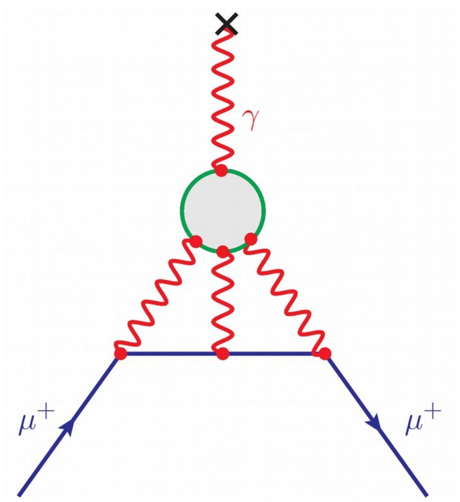


Ablikim et al., Phys.Lett.B753 (2016) 629

- Precision competitive to measurements by BaBar and KLOE
- Good agreement with all KLOE results
- BESIII result confirms $a_\mu^{\text{theo,SM}} - a_\mu^{\text{exp}} > 3\sigma$
- Effect on a_μ^{hVP} under evaluation by theory groups

Hadronic Light-by-Light

a_μ^{hLBL} not directly related to measurable quantities



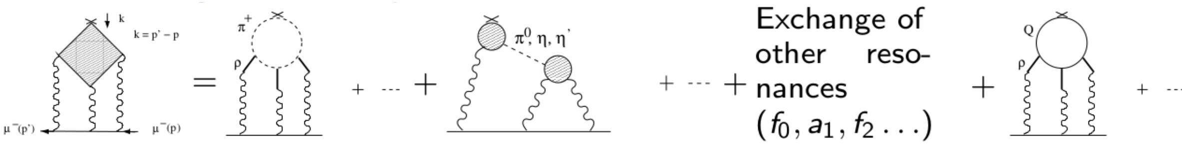
- Interaction of virtual mesons with real/virtual photons
  - ChPT at lowest energies
  - pQCD at high energies
  - Intermediate region ?

- “classic” approach: Hadronic models
  - “Glasgow Consensus” arXiv:0901.0306
  - Jegerlehner,Nyffeler Phys.Rep.477 (2009) 1

- Models can be validated with experimental data
- Error estimates for a_μ^{hLBL} are model dependent

Relevant Processes and Energies

Counting scheme for contributions to a_μ^{hLBL} (de Rafael, Phys.Lett.B322 (1994) 239)



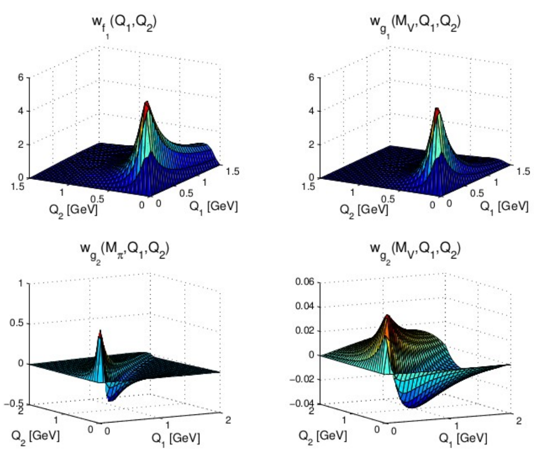
Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C

Dominating contributions:
- Pion loop
- PS meson exchange

2D integral representation for pion-pole contribution (Knecht, Nyffeler PRD65 (2002) 073035)

$$a_\mu^{\mathrm{HLbL};\pi^0} = \int_0^\infty dQ_1 \int_0^\infty dQ_2 \sum_i w_i(Q_1, Q_2)\, f_i(Q_1, Q_2)$$

- Universal weight functions w_i
- Form factor dependence f_i
- $Q_i^2 = -q_i^2$

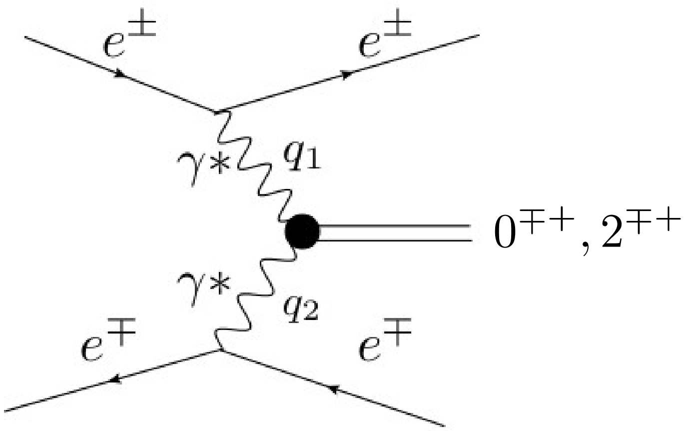


relevant momentum region to measure Transition Form Factor
0.25 – 1.25 GeV

space-like Transition Form Factors

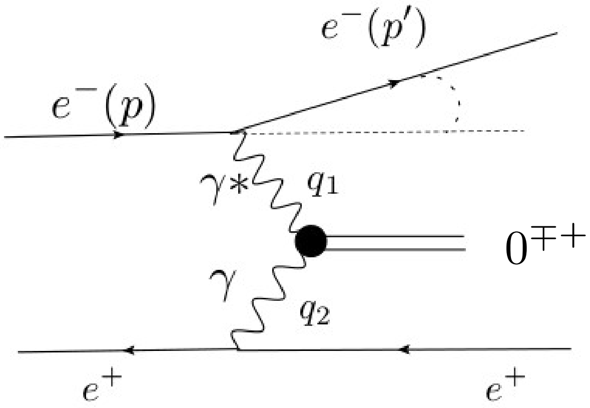
Can be investigated at e⁺e⁻ colliders:

- Exchange of two photons in e⁺e⁻ collisions
- Pseudoscalar, axial, and tensor states accessible
- $\sigma \propto \alpha^2 \ln^2 E$
- $\sigma \propto F^2(Q_1^2, Q_2^2)$, with $Q_i^2 = -q_i^2$



- Forward peaked kinematic
  - Experimentally challenging

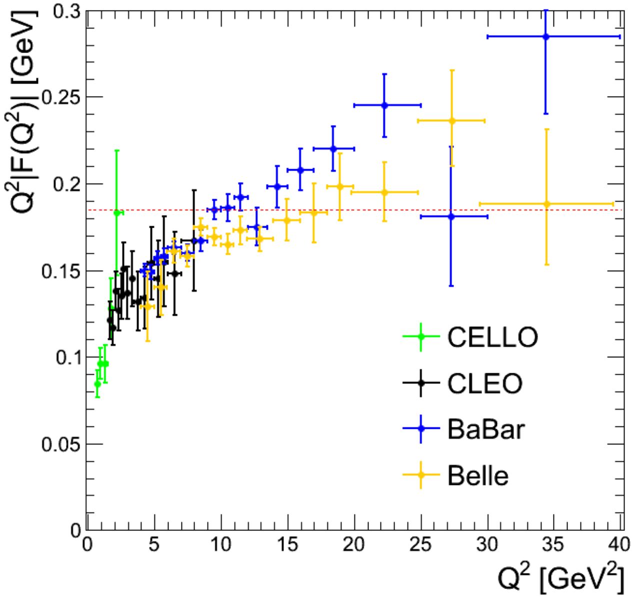
- Single-tag to study momentum dependence
  - Detect only one scattered lepton
  - Require small virtuality for second photon
    - $F^2(Q_1^2, Q_2^2) \Rightarrow F^2(Q_1^2, 0) \Rightarrow F^2(Q^2)$



- TFF should factorize at lowest energies

$$F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2) \sim F_{\pi\gamma^*\gamma}(Q_1^2, 0) \times F_{\pi\gamma^*\gamma}(0, Q_2^2)$$

π^0 Transition Form Factor



- Results from B-factories cover large Q^2 ($5 < Q^2$ [GeV^2] < 40)
  - Discrepancy for π^0 between BaBar and Belle
- Data scarce at lowest Q^2
  - Region of relevance for (g-2)µ

CELLO: Z.Phys.C49 (1991) 401
CLEO: Phys.Rev.D57 (1998) 33
BaBar: Phys.Rev.D80 (2009) 052002
Belle: Phys.Rev.D86 (2012) 092007

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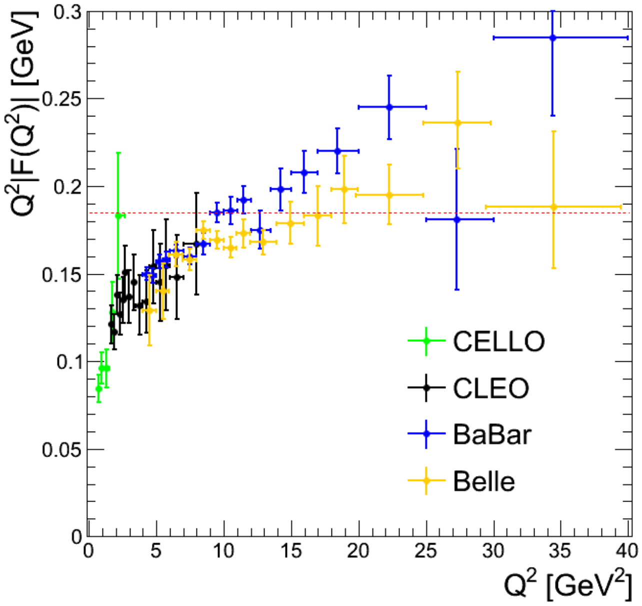

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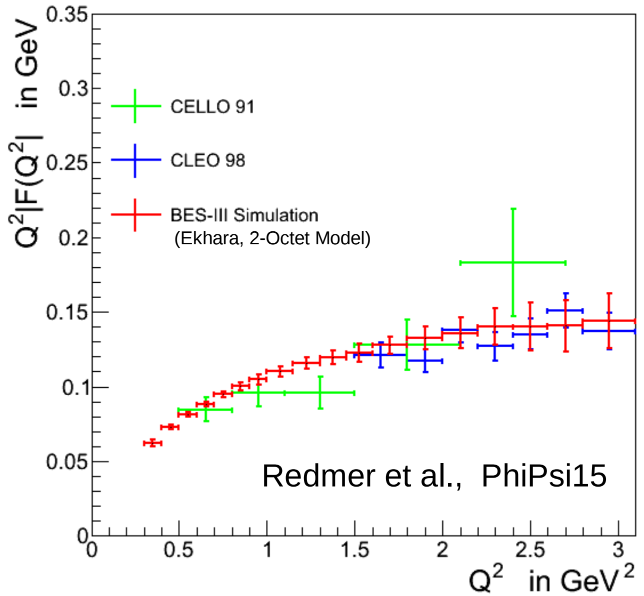
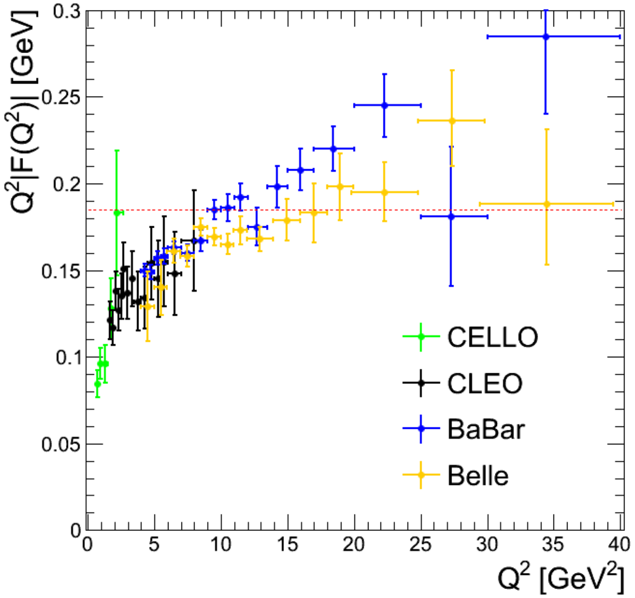


Prospects for BESIII

- 2.9 fb^{-1} analyzed at ψ(3770) peak
- Covering $0.3 < Q^2$ [GeV^2] < 3.1
- Unprecedented statistical accuracy expected for $Q^2 < 1.5$ GeV^2

π^0 Transition Form Factor



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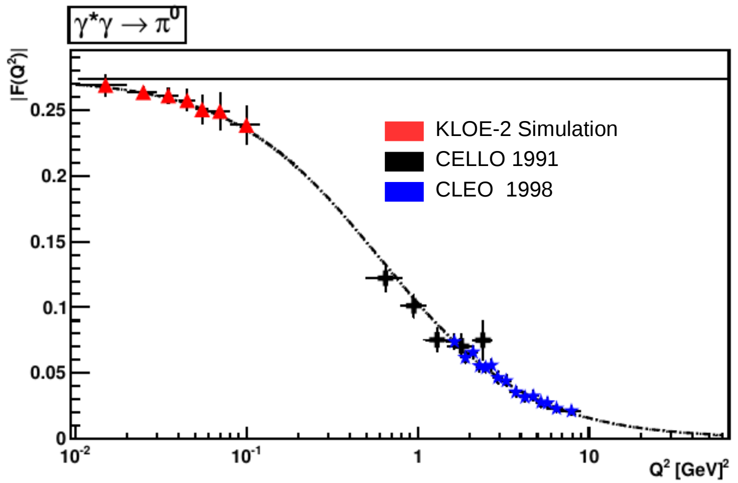
Prospects for KLOE-2

- Special tagging detectors installed
- Covering $0.01 < Q^2$ [GeV^2] < 0.1
- 6% statistical accuracy expected from 5 fb^{-1} at ϕ peak



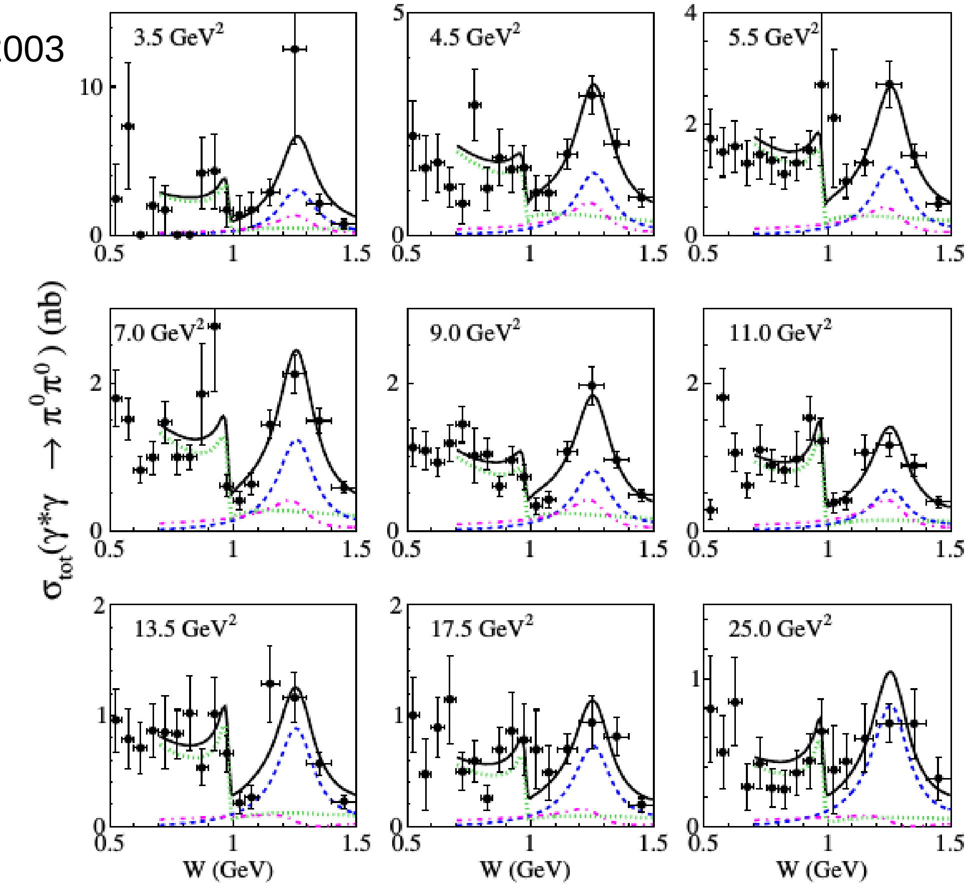
Babusci et al., EPJC 72 (2012) 1917

$\gamma^*\gamma^* \to \pi\pi$

First single-tag measurement by Belle

Phys.Rev.D93 (2016) 032003

- $\gamma^*\gamma \to \pi^0\pi^0$
- 759 fb^{-1}
- 3 < Q^2 [GeV^2] < 30
- 0.5 < W [GeV/c^2] < 2.1
- $|\cos\theta^*| < 1.0$
- Determination of partial-wave amplitudes
  - $|S|^2$
  - $|D_0|^2$
  - $|D_2|^2$



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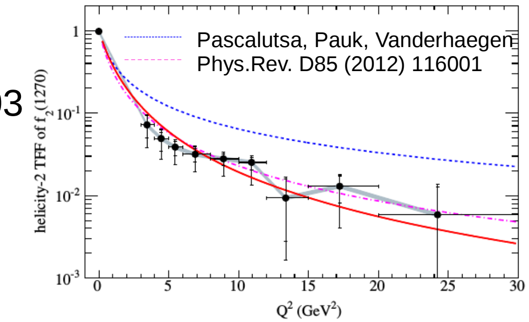
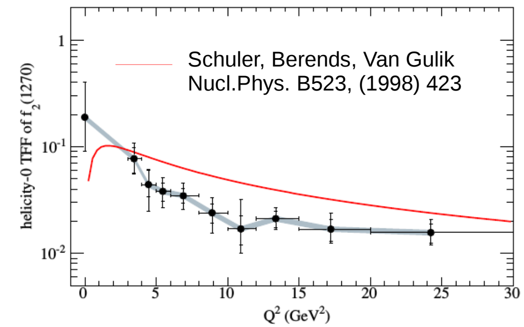
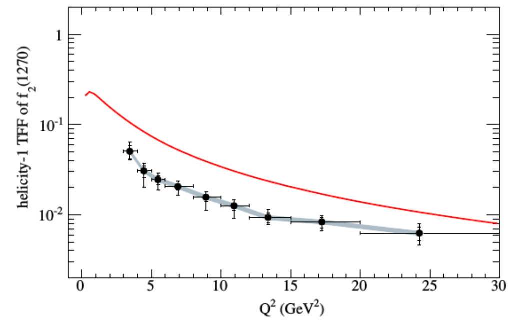
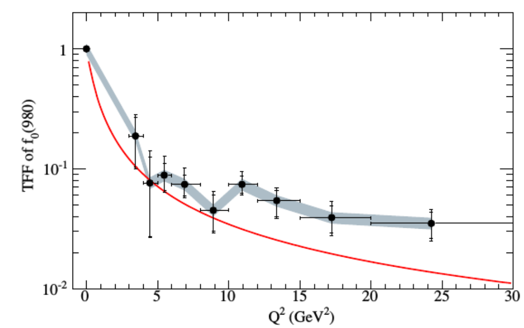
- $|\cos\theta^*| < 1.0$
- Determination of partial-wave amplitudes
- Measurement of TFF for $f_2(1270)$ and $f_0(980)$







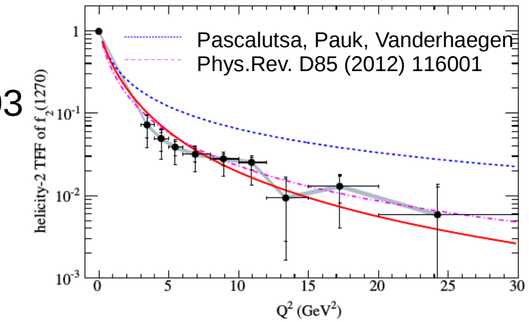


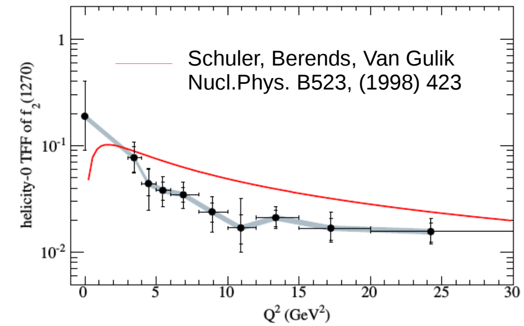
$\gamma^*\gamma^* \to \pi\pi$

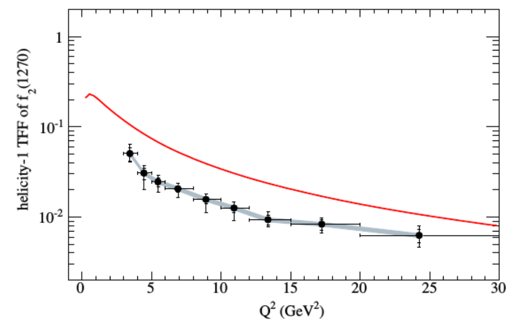
First single-tag measurement by Belle

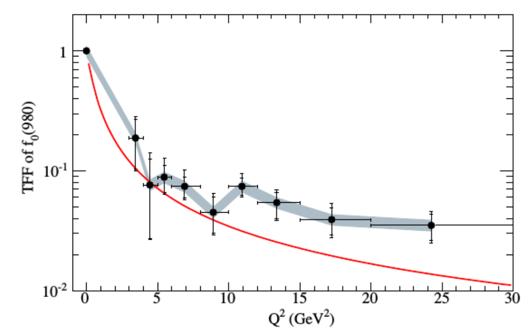
Phys.Rev.D93 (2016) 032003

- $\gamma^*\gamma \to \pi^0\pi^0$
- 759 fb^{-1}
- $3 < Q^2$ [GeV^2] < 30
- $0.5 < W$ [GeV/c^2] < 2.1
- $|\cos\theta^*| < 1.0$
- Determination of partial-wave amplitudes
- Measurement of TFF for $f_2(1270)$ and $f_0(980)$









Prospects for BESIII: $\gamma^*\gamma \to \pi^+\pi^-$

- $0.2 < Q^2$ [GeV^2] < 2.0
- $m_{\pi+\pi-} < M$ [GeV] < 2.0
- $|\cos\theta^*| < 1.0$

Data Driven Approaches

- Padé – Approximants
  - Parametrize TFF by series of rational approximants
  - Fit free parameters to experimental data
  - Estimate for systematic uncertainty provided
  - Space-like and time-like data can be used

Escribano, Masjuan, et al.
PRD 86 (2012) 094021
EPJC 75 (2015) 414

- Dispersive approaches to a_μ^{hLBL}
  - Describe dominating contributions with dispersion relations
  - Relate to measurable quantities
  - Reduce model dependency
  - Give more reliable error estimates
    - Goal 10 – 20 %

Bern (Colangelo, Hoferichter, et al.)

JHEP 1409 (2014) 091
PLB 738 (2014) 6
EPJ C74 (2014) 3180
JHEP 1509 (2015) 074

Mainz (Pauk, Vanderhaegen, et al.)

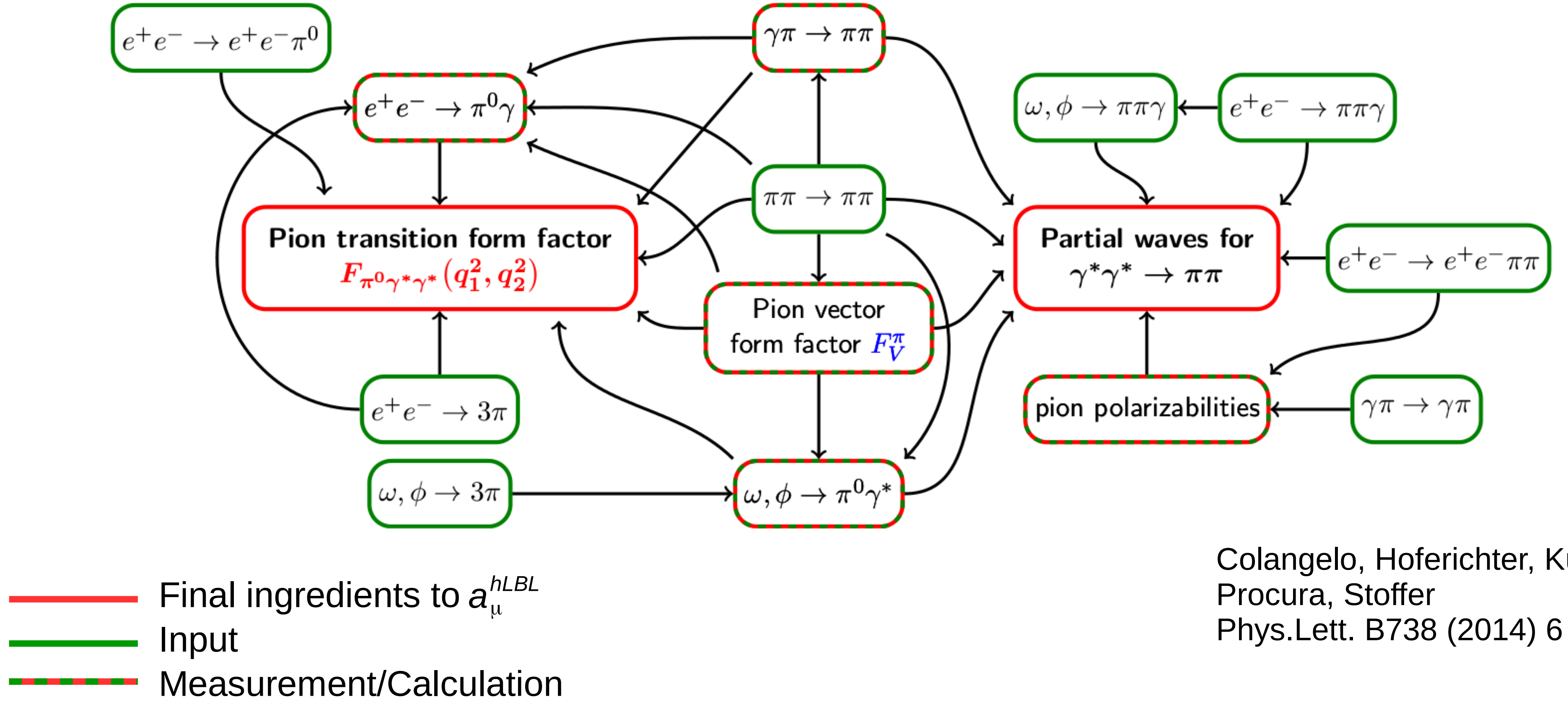
PRD 90 (2014) 113012
hep-ph:1403.7503

Dispersive Approach

Measurable quantities needed:

- Space-like TFF $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ for arbitrary virtualities
- Partial waves for $\gamma^*\gamma^* \to \pi\pi$

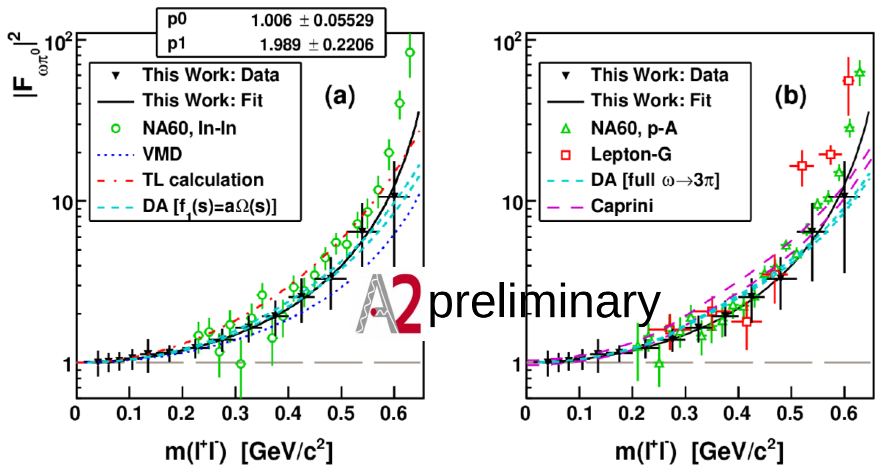
Both can be constructed from other input:



Colangelo, Hoferichter, Kubis, Procura, Stoffer
Phys.Lett. B738 (2014) 6

Time-like double-virtual TFF

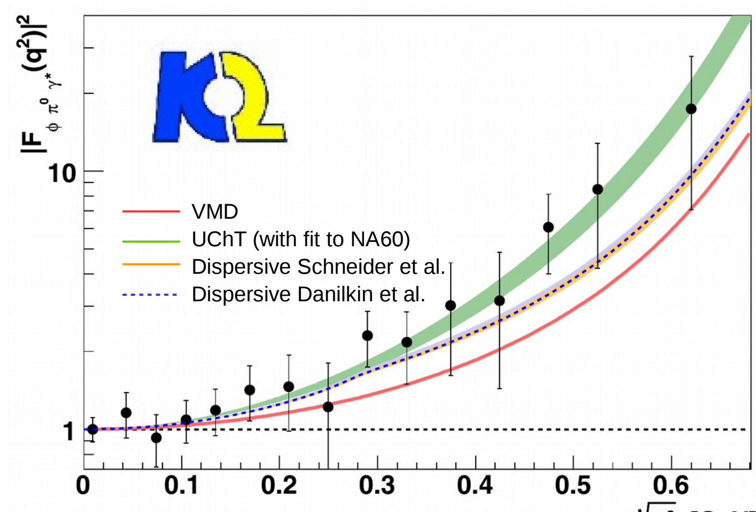
$\omega \to \pi^0 e^+ e^-$



Courtesy of S.Prakhov (JGU Mainz &UCLA)

- $\gamma p \to p\pi^0 e^+ e^-$
- Non-magnetic spectrometer
- ~ 1100 events reconstructed
- Consistent with NA60 (within errors)
- New production run planned

$\phi \to \pi^0 e^+ e^-$



PLB article in press; arXiv:1601:06565

- 1.7 fb^{-1} at ϕ peak
- ~ 9500 events reconstructed
- Signif. improved precision of $\mathcal{B}(\phi \to \pi^0 e^+ e^-)$
- First measurement of TFF
  - Best described by UChT with fit to NA60 data

Tension between data and VMD persists!

Summary

- SM prediction of a_μ limited by hadronic contributions
- Experimental input needed to solve the puzzle
  - hadronic Vacuum Polarization a_μ^{hVP}
    - direct relation to hadronic cross sections $\sigma(e^+e^- \to \text{hadrons})$
    - High precision data needed ⟶ BaBar, KLOE, BESIII, CMD-3
  - hadronic Light-by-Light scattering a_μ^{hLbL}
    - Realistic error estimates from data-driven approaches
      - Transition form factor $F_{\pi\gamma^*\gamma^*}(Q_1^2, Q_2^2)$ and partial waves $\gamma^*\gamma^* \to \pi\pi$
        - Can also be constructed from other exp. input
    - To compete with upcoming experimental accuracy $\delta a_\mu^{\text{exp}} \sim 1.6 \times 10^{-10}$ contributions of η and η' become relevant!

$a_\mu^{\text{hLbL},\eta} \sim 1.5 \times 10^{-10}$

$a_\mu^{\text{hLbL},\eta'} \sim 1.5 \times 10^{-10}$

Knecht,Nyffeler
Phys.Rev.D65 (2002) 073034