Recent results on tetra- and penta-quark candidates at LHCb

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On behalf of the LHCb collaboration

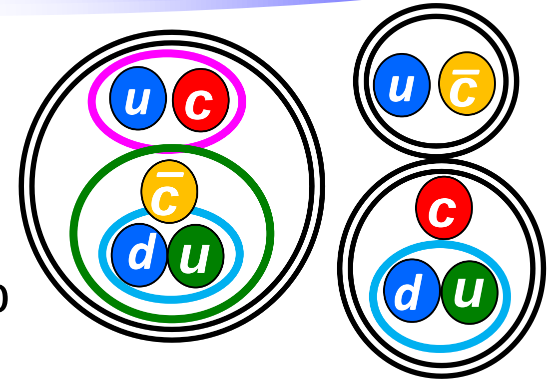
at

Outline

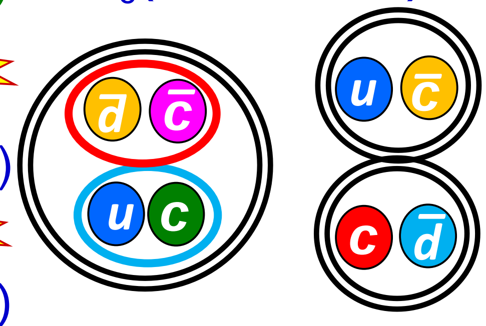
- $\Lambda_b \rightarrow J/\psi\ p\ K^-$ ($P_c(4450)^+ \rightarrow J/\psi\ p$, $P_c(4380)^+ \rightarrow J/\psi\ p$) vs $B^0 \rightarrow \psi'\ \pi^+ K^-$ ($Z_c(4430)^+ \rightarrow \psi'\ \pi^+$) analyzed with two approaches:
  - Amplitude analysis (LHCb-PAPER-2015-029, LHCb-PAPER-2014-014)
  - Model independent approach based on angular moments (LHCb-PAPER-2015-038, **LHCb-PAPER-2016-009**) new
- $\Lambda_b \rightarrow J/\psi\ p\ \pi^-$ ($P_c(4380,4450)^+ \rightarrow J/\psi\ p$, $Z_c(4200)^+ \rightarrow J/\psi\ \pi^+$)
  - Amplitude analysis preliminary! (**LHCb-PAPER-2016-015**) new
- $B^+ \rightarrow J/\psi\ \phi\ K^+$ ($X(4140,4274,\ldots)^+ \rightarrow J/\psi\ \phi$ and other states)
  - Amplitude analysis preliminary! (**LHCb-PAPER-2016-018, LHCb-PAPER-2016-019**) new

$\psi', J/\psi \rightarrow \mu^+\mu^-$

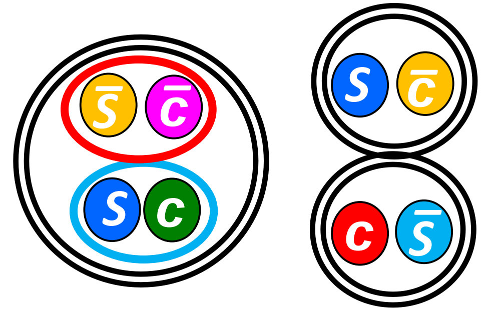
$\phi \rightarrow K^+K^-$



$P_c(4380,4450)^+$?



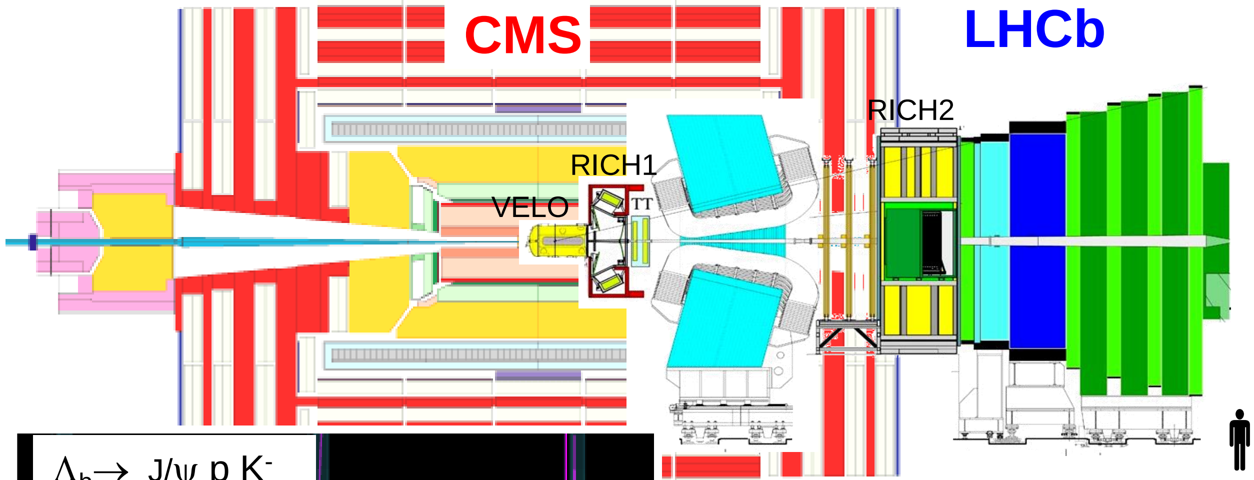
$Z_c(4200,4430)^+$?

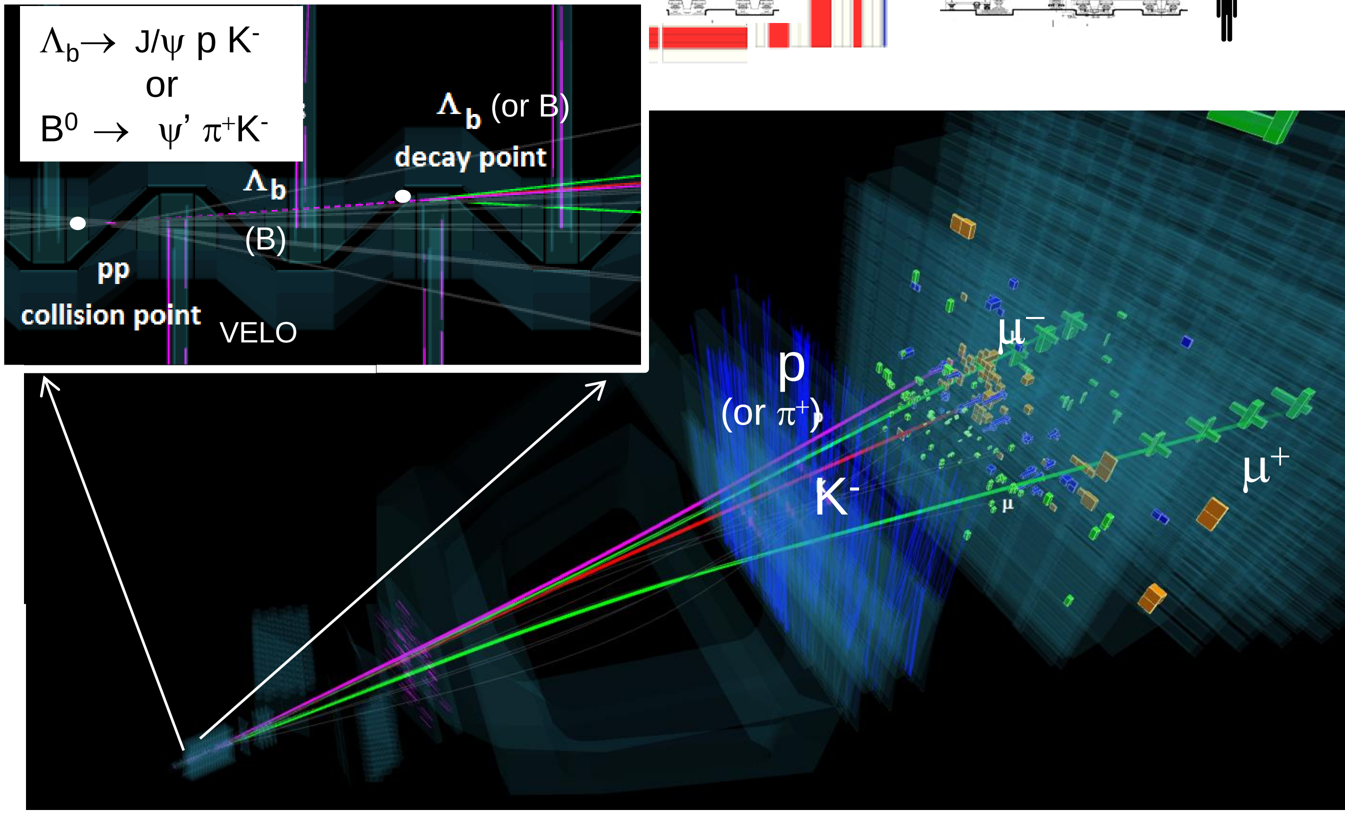


$X(4140,4274)$?

LHCb: first dedicated b,c detector at hadronic collider

- Advantages over e^+e^- B-factories (Belle, BaBar):
  - ~1000x larger b production rate, b decays mostly to c
  - **produce b-baryons at the same time as B-mesons**
  - long visible lifetime of b-hadrons (no backgrounds from the other b-hadron)
- Advantages over ATLAS, CMS, CDF, D0:
  - RICH detectors for π/K/p discrimination (smaller backgrounds)
  - Small event size allows large trigger bandwidth (up to 5 kHz in Run I); all devoted to flavor physics





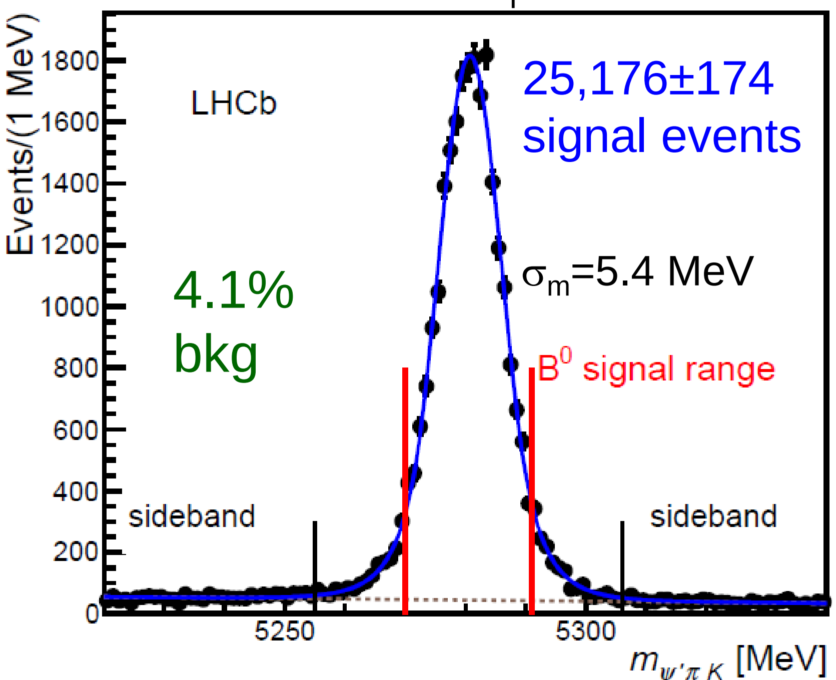
The LHCb detector described in JINST 3 (2008) S08005

LHCb data samples (3 fb^{-1})

LHCb-PAPER-2014-014

PRL 112, 222002 (2014)

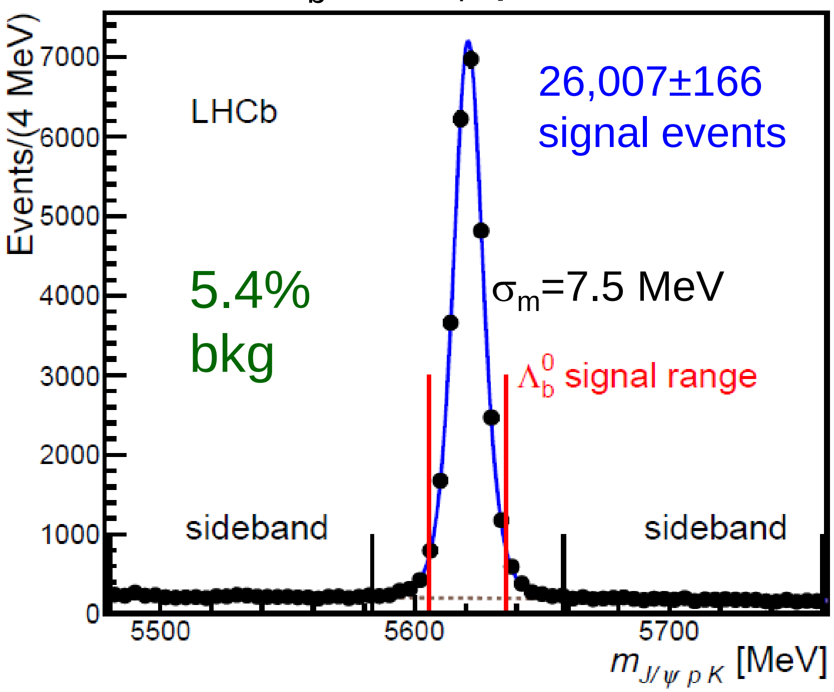
$B^0 \to \psi' \pi^+ K^-$



LHCb-PAPER-2015-029

PRL 115, 07201 (2015)

$\Lambda_b \to J/\psi\ p\ K^-$

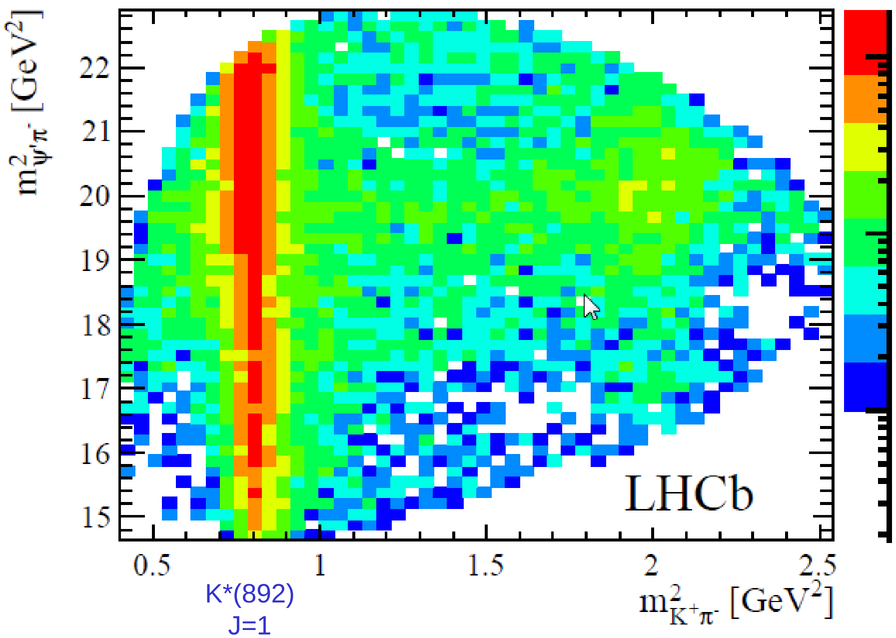


vs. bkg in
Belle: 7.8%

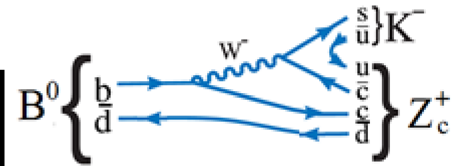
vs
Belle: 2,010±50
BaBar: 2,021±53

- More than a factor of 10 better statistics than at the B factories, at smaller background
- Very comparable signal statistics and bkg levels between the B and Λ_b data samples

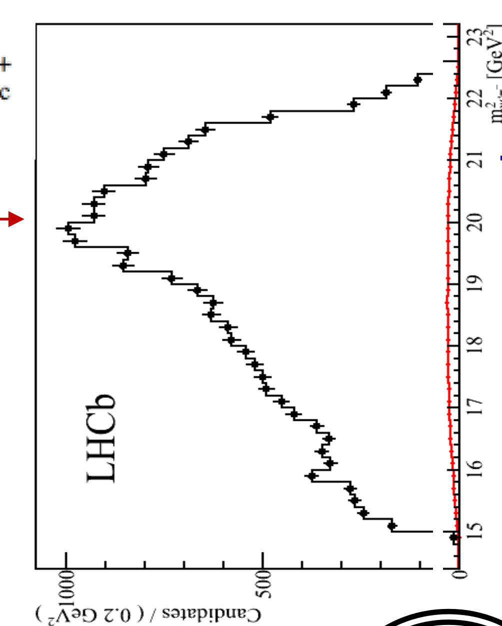
$B^0 \to \psi' \pi^+ K^-$



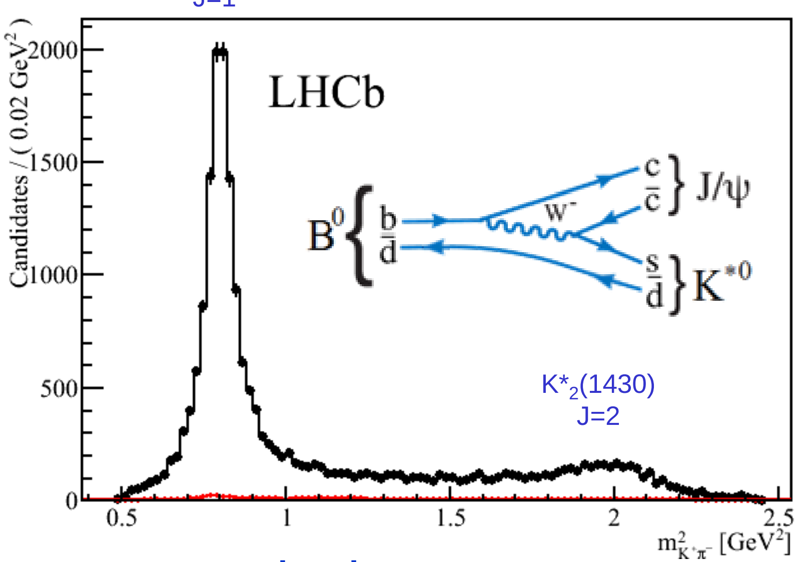
K*(892)
J=1



Z_c (4430)$^+$
$\to J/\psi\pi^+$
?

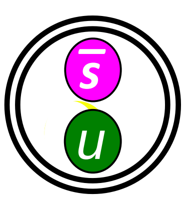


Tetraquark



K*$_2$(1430)
J=2

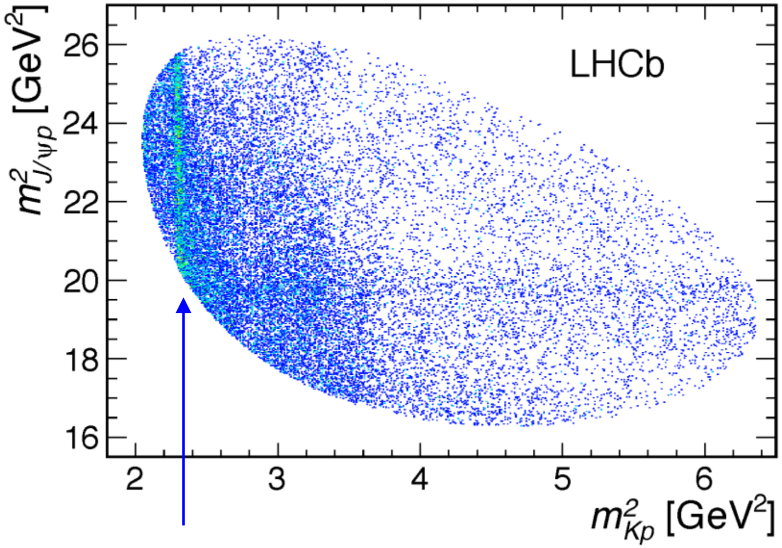
Kaon excitations

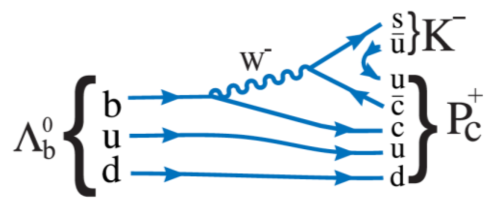


Is it a reflection of interfering K*'s $\to \pi^+ K^-$?

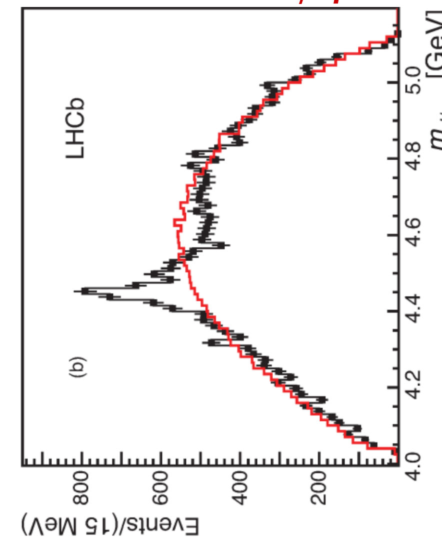
Proper amplitude analysis necessary to check

$\Lambda_b^0 \to J/\psi p K^-$: unexpected structure in $m_{J/\psi p}$

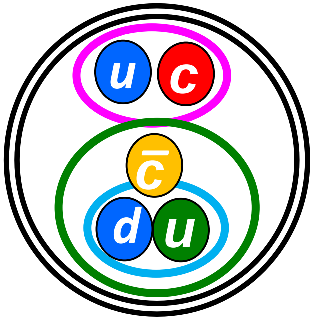




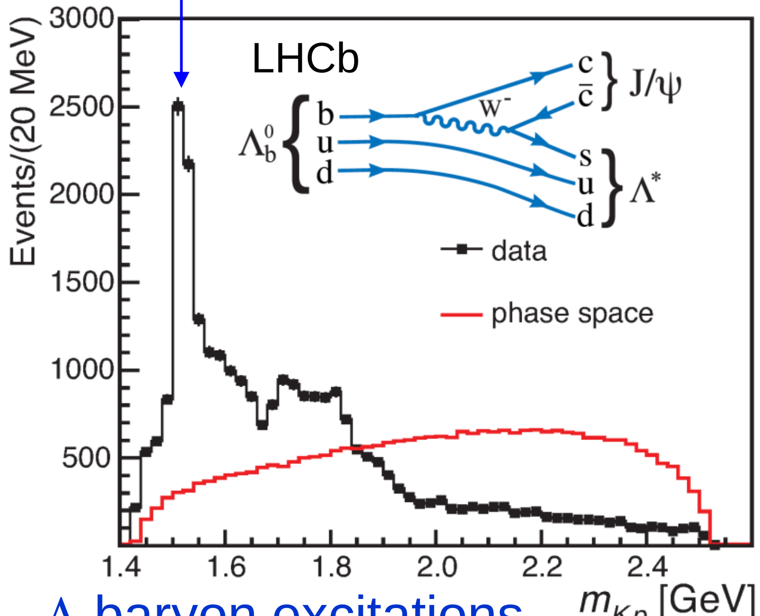
$P_c^+ \to J/\psi\, p$?



Exotic pentaquark

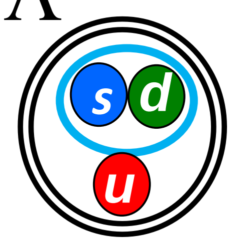


$\Lambda(1520)$ and other Λ^*'s $\to p K^-$



Λ baryon excitations

Λ^*



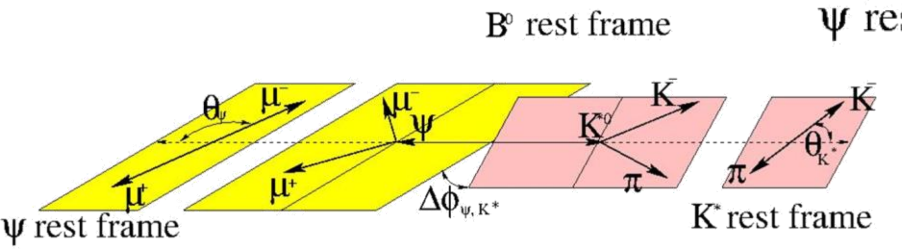
- Unexpected, narrow peak in $m_{J/\psi p}$

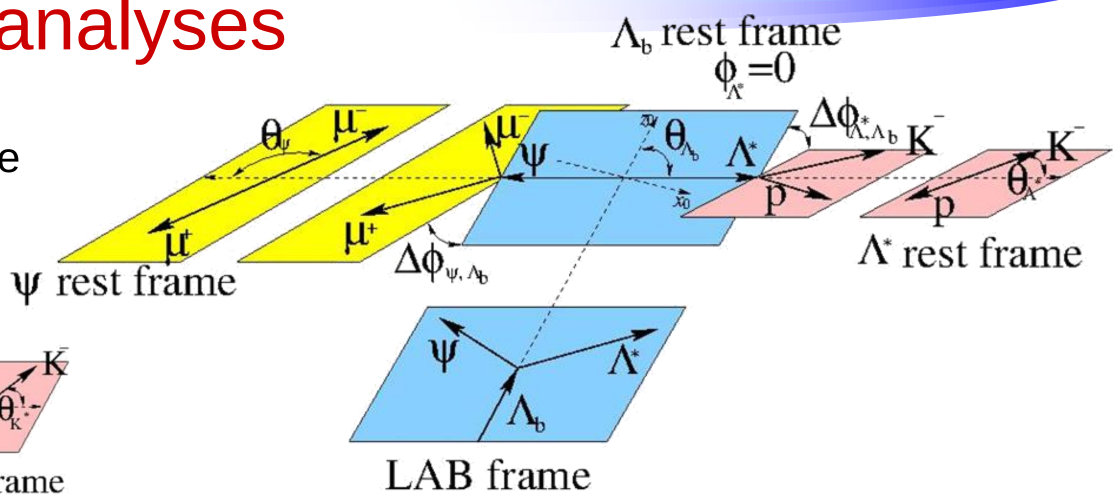
Is it a reflection of interfering Λ^*'s $\to p K^-$?

Proper amplitude analysis necessary to check

Full amplitude analyses

For the best sensitivity and to avoid dependence of efficiency on fit model, use all degrees of freedom in the decay





4D maximum likelihood fit

$$\Omega \equiv (\theta_{K^*}, \theta_{\psi}, \Delta\phi_{\psi,K^*})$$

6D maximum likelihood fit

$$\Omega \equiv (\theta_{\Lambda_b}, \theta_{\Lambda^*}, \Delta\phi_{\Lambda^*,\Lambda_b}, \theta_{\psi}, \Delta\phi_{\psi,\Lambda_b})$$

$$\mathrm{PDF}(\mathrm{m}_{K\pi/Kp}, \Omega) = \left| MatrixEle(\mathrm{m}_{K\pi/Kp}, \Omega \,|\, J_R^P, M_R, \Gamma_R, H_R) \right|^2 \times \mathrm{eff}(\mathrm{m}_{K\pi/Kp}, \Omega) + \mathrm{PDF}_{\mathrm{bkg}}(\mathrm{m}_{K\pi/Kp}, \Omega)$$

Fixed to known values.
M_R, Γ_R varied within errors for systematics.

1-3 independent **complex** helicity couplings H_R per K^* resonance

4-6 independent **complex** helicity couplings H_R per Λ^* resonance

- Dealing with baryons results in larger number of helicity couplings per resonance to determine from data (nuisance parameters)

Model of conventional resonances

Well established states from PDG

Only natural parities in decays to Kπ

No high-M_0 high-J^P

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# of complex couplings Red.	Ext.
NR	0^+	—	—	1	1
$K^*(800)^0$	0^+	682	547	1	1
$K^*(892)^0$	0^+	896	49	3	3
$K^*(1410)^0$	1^-	1414	232	3	3
$K^*(1430)^0$	0^+	1425	270	1	1
$K_2^*(1430)^0$	2^+	1432	109	3	3
$K^*(1680)^0$	1^-	1717	322	3	3
$K_3^*(1780)^0$	3^-	1776	159	0	3
Total # of free parameters				28	34

No constraint on parity in decays to Kp

No high-M_0 high-J^P & limit L

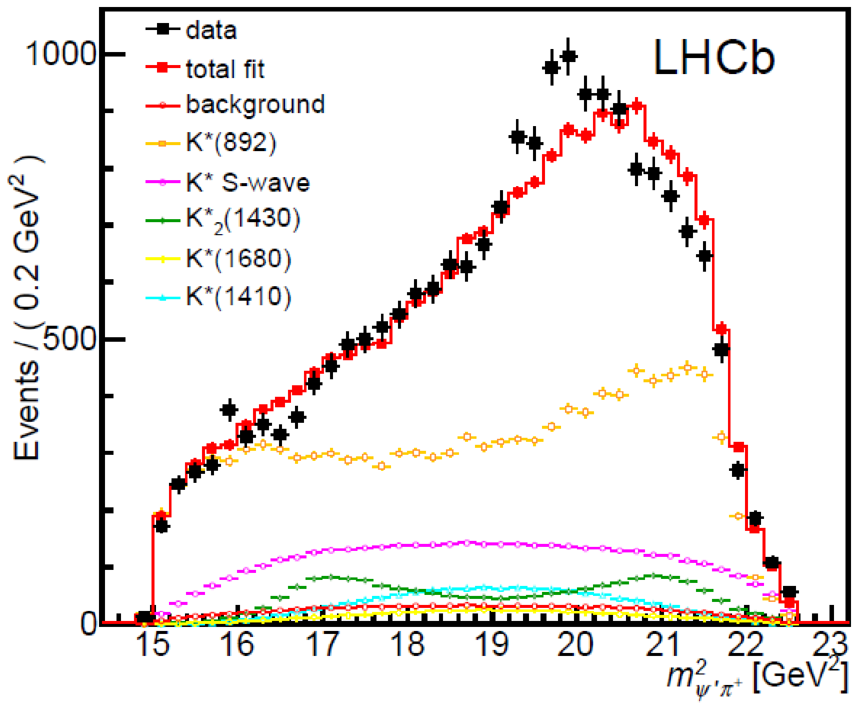
All states all L

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# of complex couplings Red.	Ext.
$\Lambda(1405)$	$1/2^-$	1405	50	3	4
$\Lambda(1520)$	$3/2^-$	1520	16	5	6
$\Lambda(1600)$	$1/2^+$	1600	150	3	4
$\Lambda(1670)$	$1/2^-$	1670	35	3	4
$\Lambda(1690)$	$3/2^-$	1690	60	5	6
$\Lambda(1800)$	$1/2^-$	1800	300	4	4
$\Lambda(1810)$	$1/2^+$	1810	150	3	4
$\Lambda(1820)$	$5/2^+$	1820	80	1	6
$\Lambda(1830)$	$5/2^-$	1830	95	1	6
$\Lambda(1890)$	$3/2^+$	1890	100	3	6
$\Lambda(2100)$	$7/2^-$	2100	200	1	6
$\Lambda(2110)$	$5/2^+$	2110	200	1	6
$\Lambda(2350)$	$9/2^+$	2350	150	0	6
$\Lambda(2585)$	$5/2^-?$	2585	200	0	6
Total # of free parameters				64	146

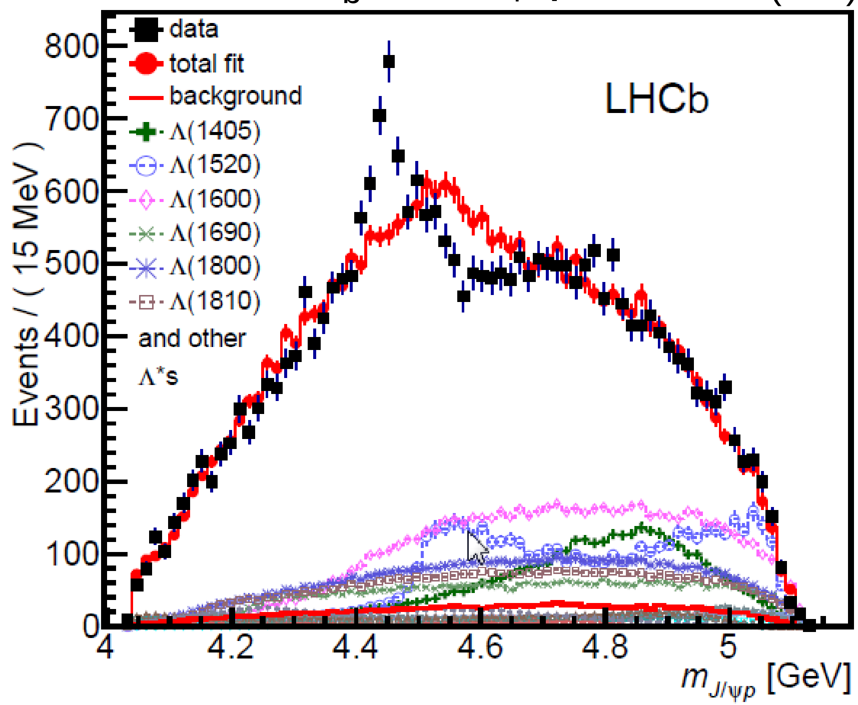
- Dealing with baryons results in more than doubling of known states to include

Fits with conventional resonances only

$B^0 \to \psi' \pi^+ K^-$

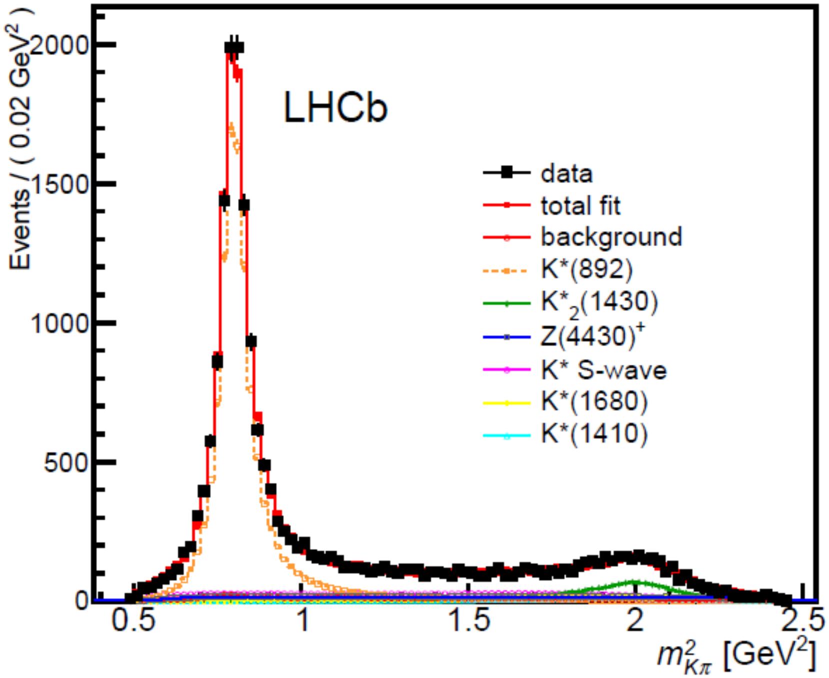


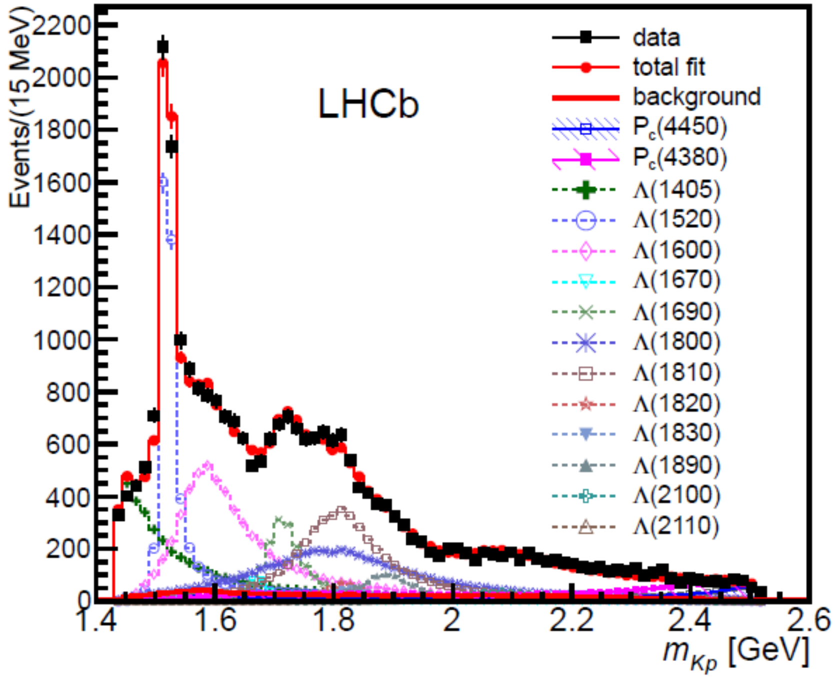
$\Lambda_b^0 \to J/\psi\ pK^-$ (Ext.)



- Cannot describe the data with the conventional resonances alone

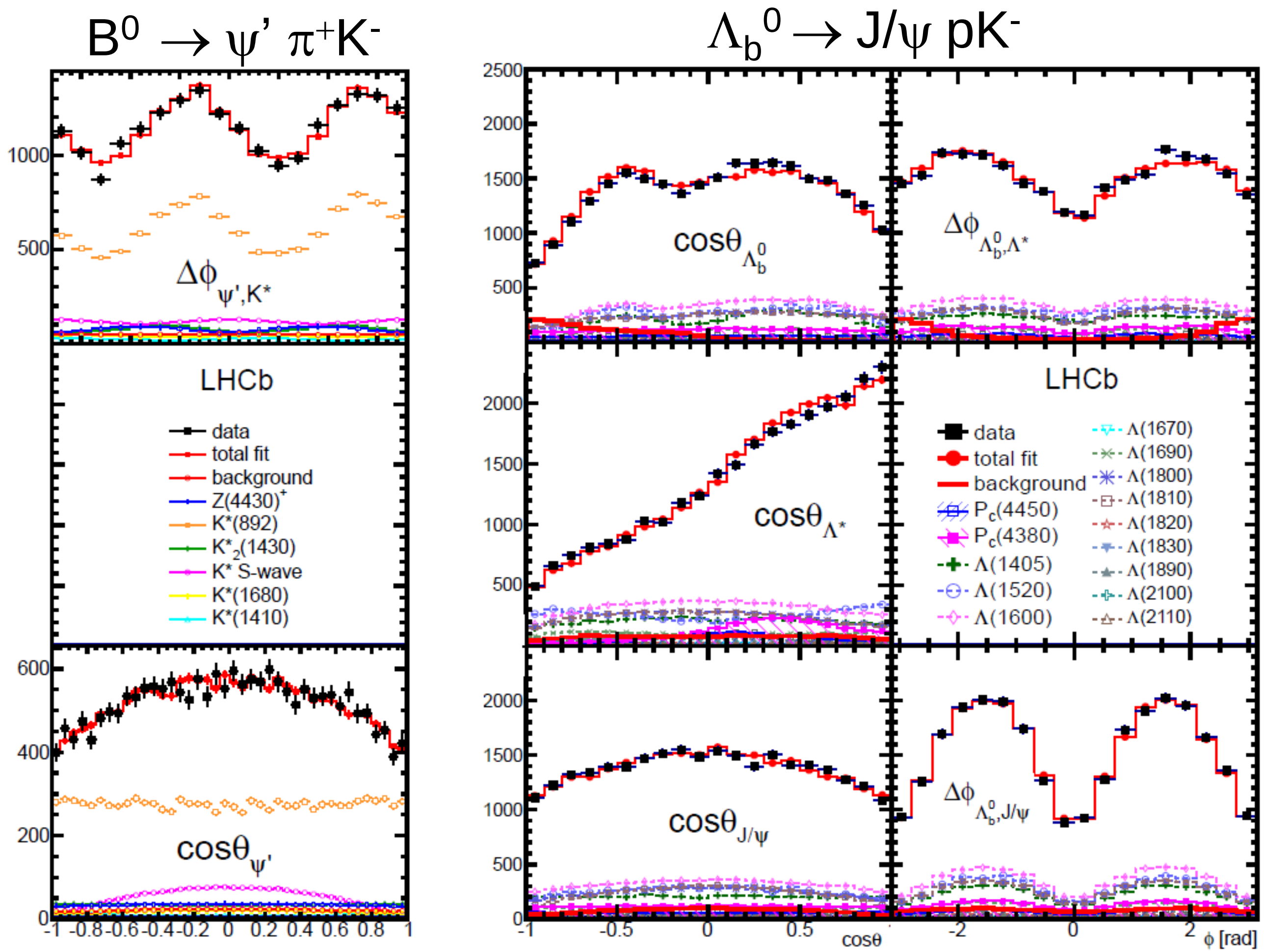
Fits including exotic hadrons





- The models based on well established conventional resonances describe these projections well (without or with exotics):
  - They dominate the rate
  - If exotics present (as shown above) they spread across wide range of these masses

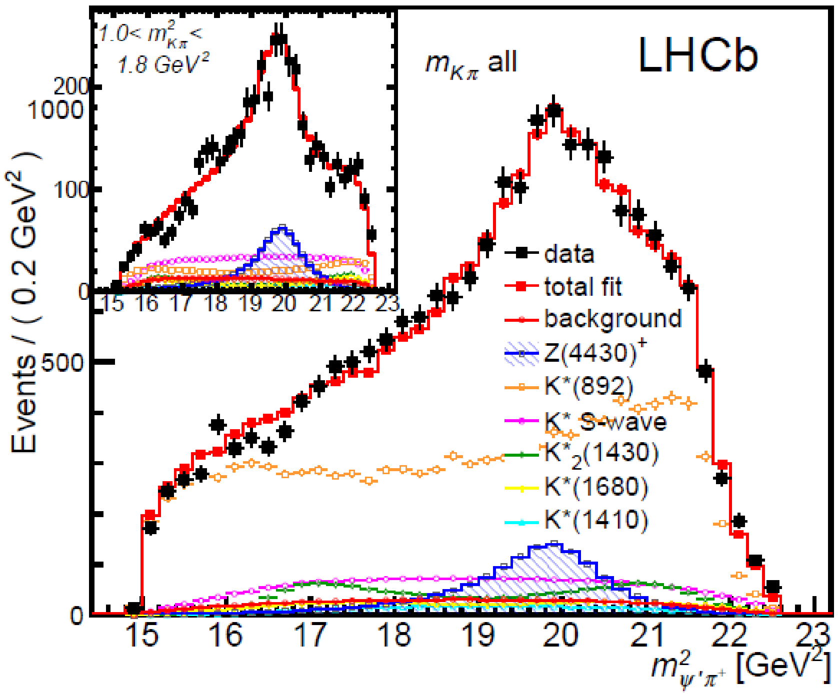
Fitting decay angles important for resolving overlapping resonances



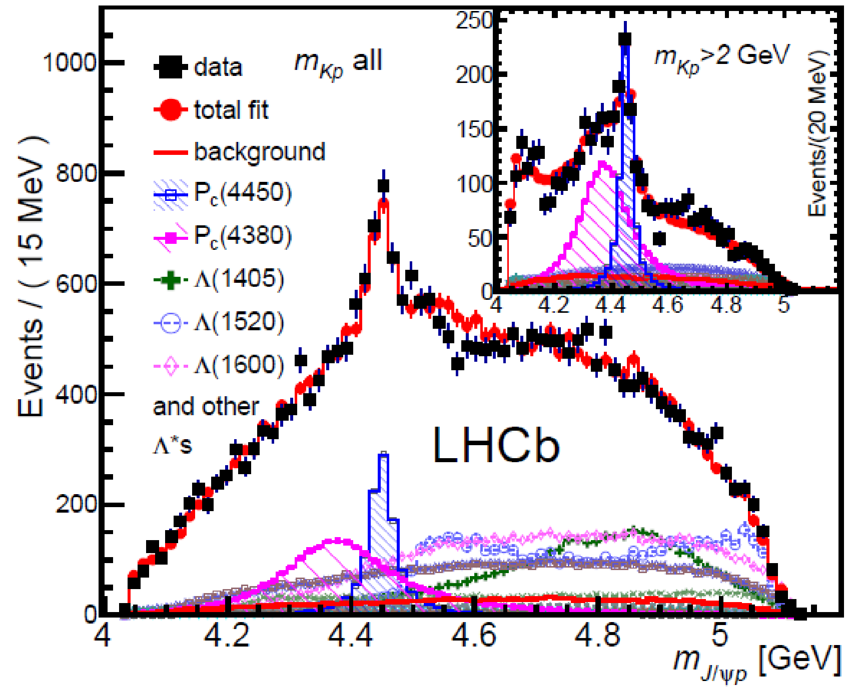
- They greatly increase discrimination power between resonances of different J^P
- Without using full decay phase-space difficult to do efficiency correction correctly

Exotic hadrons

$B^0 \to \psi' \pi^+ K^-$



$\Lambda_b^0 \to J/\psi\ pK^-$ (Red.)



State	Mass (MeV)	Width (MeV)	Fit frac. (%)	Sig.
$Z_c(4430)^+$	$4475\pm 7^{+15}_{-25}$	$172\pm13^{+37}_{-34}$	$5.9\pm0.9^{+1.5}_{-3.3}$	14σ
Belle	$4485\pm22^{+28}_{-11}$	$200\pm46^{+26}_{-35}$	$10.3\pm3.5^{+4.3}_{-2.3}$	5σ

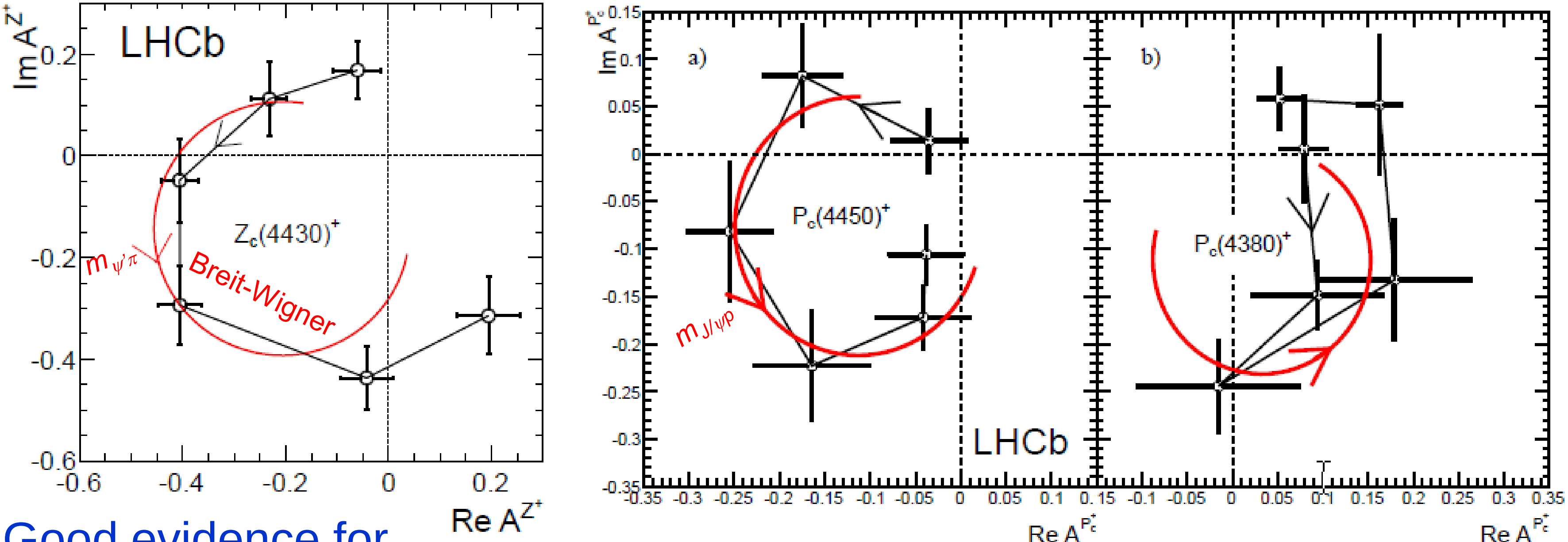
State	Mass (MeV)	Width (MeV)	Fit fract. (%)	Sig.
$P_c(4450)^+$	4449.8±1.7±2.5	39± 5±19	4.1±0.5±1.1	12σ
$P_c(4380)^+$	4380 ±8±29	205±18±86	8.4±0.7±4.2	9σ

- $J^P=1^+$ at 9.7σ incl. syst. (in Belle at 3.4σ)
- Best fit has $J^P=(3/2^-, 5/2^+)$, also $(3/2^+, 5/2^-)$ & $(5/2^+, 3/2^-)$ are preferred. $(5/2^-, 3/2^+)$ cannot be ruled out within systematics

Argand diagrams: exotic hadron amplitudes without Breit-Wigner assumption

Exotic hadron amplitudes for 6 $m_{\psi'\pi}/m_{J/\psi p}$ bins near the peak mass

(all other model parameters fitted simultaneously)



Good evidence for resonant character

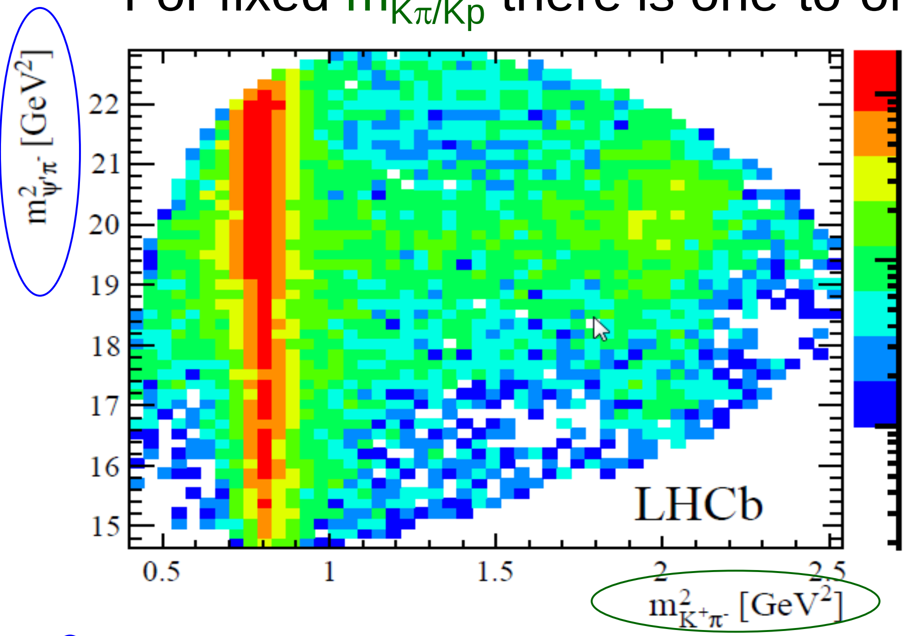
Large errors

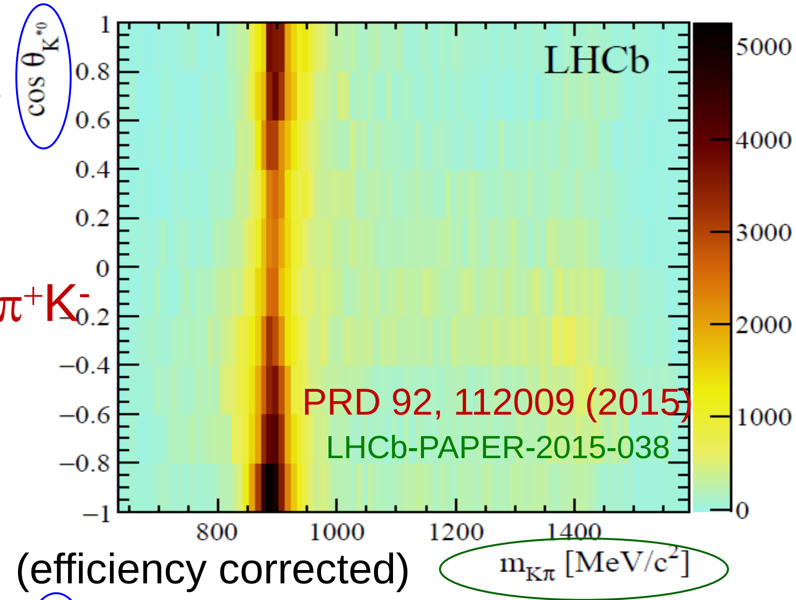
Such studies make exotic hadron amplitude model-independent, but the results are still dependent on the model of conventional hadrons. Simultaneous PWA of the latter is not possible since exotics reflect into variables characterizing conventional hadrons.

However, we can assume exotics are not present and test for their presence in model-independent way - next 6 slides.

Rectangular Dalitz plane: variables of conventional hadrons

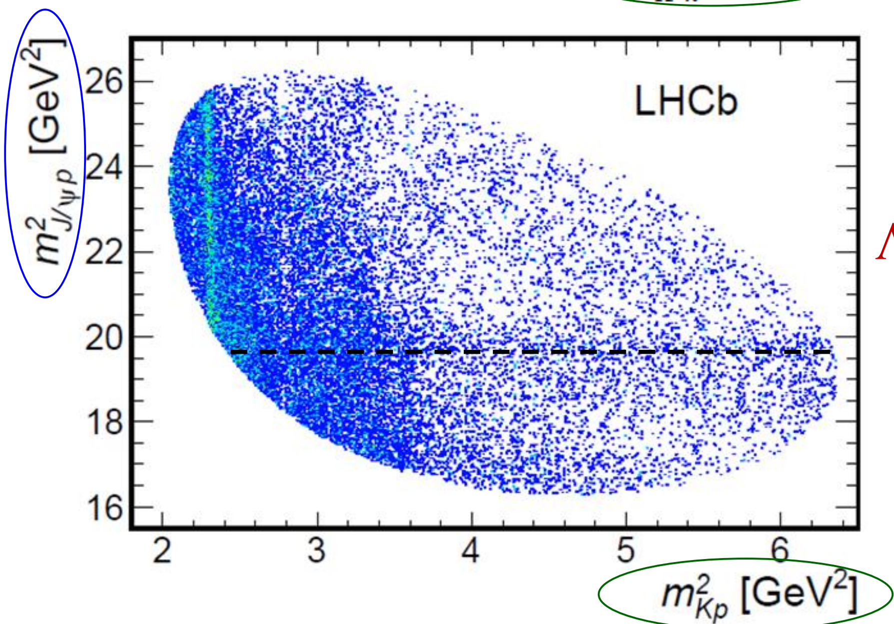
- For fixed $m_{K\pi/Kp}$ there is one-to-one relation between $m_{\psi\pi/\psi p}$ and $\cos\theta_{K^*/\Lambda^*}$



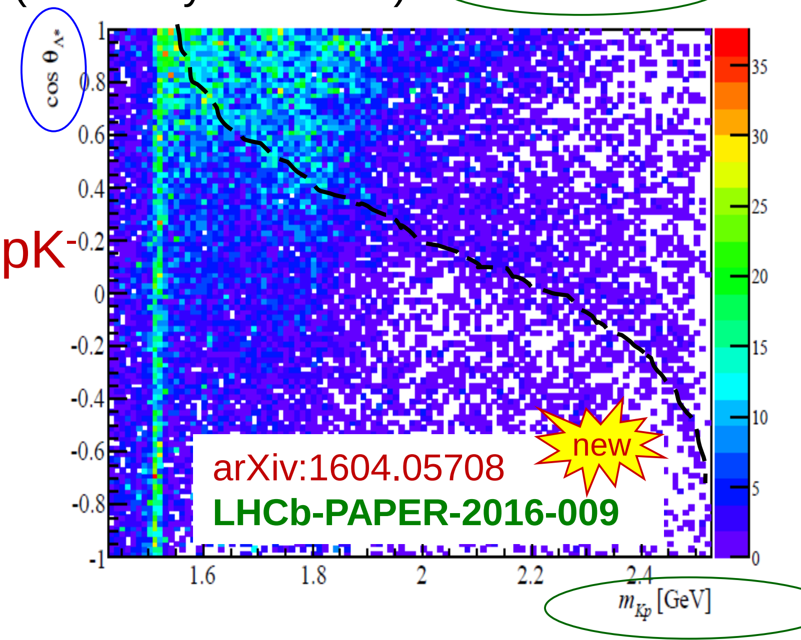


$B^0 \to \psi'\pi^+K^-$

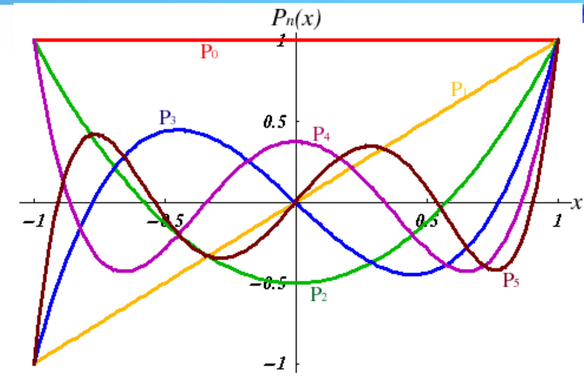
(efficiency corrected)



$\Lambda_b^0 \to J/\psi pK^-$



Legendre moments



$$\frac{dN}{d\cos\theta} = \sum_{l=0}^{l_{\max}} \left\langle P_l^U \right\rangle P_l(\cos\theta) \qquad \theta = \theta_{K^*} \quad \text{or} \quad \theta_{\Lambda^*}$$

$$\left\langle P_l^U \right\rangle = \int_{-1}^{+1} \frac{dN}{d\cos\theta} P_l(\cos\theta) d\cos\theta \propto \sum_{i=1}^{n_{events}} \frac{1}{\varepsilon_i} P_l(\cos\theta_i)$$

Decomposition into $\langle P_l \rangle$ corresponds to decomposition into "frequencies"

With $l_{\max} \to \infty$ can reproduce any $\frac{dN}{d\cos\theta}$

Smooth cosθ structures produce low rank moments

Sharp cosθ structures produce low and high rank moments

The sharper the structure the higher l_{max} required

K*/Λ* can contribute only to low-rank moments

$l_{\max} = J_1 + J_2$ for interfering resonances

In K*/Λ*-only hypothesis ($\mathbf{H_0}$)

$$l_{\max} = 2J_{\max}$$

J_{max} is the highest spin of K*/Λ* resonance possible

Reflections of exotic hadrons can contribute to low **and high** rank moments:

- **Detecting non-zero moments above $2J_{max}$ signals presence of exotics**
- The narrower the peak the higher the $\mathbf{2J_{max}}$ required. **The sensitivity is better for narrower exotic hadrons.**
- Exotic hadron contributions spread over wide range of $m_{K\pi}/m_{Kp}$. An effective way of testing H_0 is to **aggregate the information about $\cos\theta_{K\pi/Kp}$ moments in a function of $m_{\psi'\pi}/m_{J/\psi p}$.**

Setting highest rank of Legendre moments

The sensitivity of the method improves by considering $l_{max}(m_{K\pi}/m_{Kp}) = 2\,J_{max}(m_{K\pi}/m_{Kp})$ dependence:

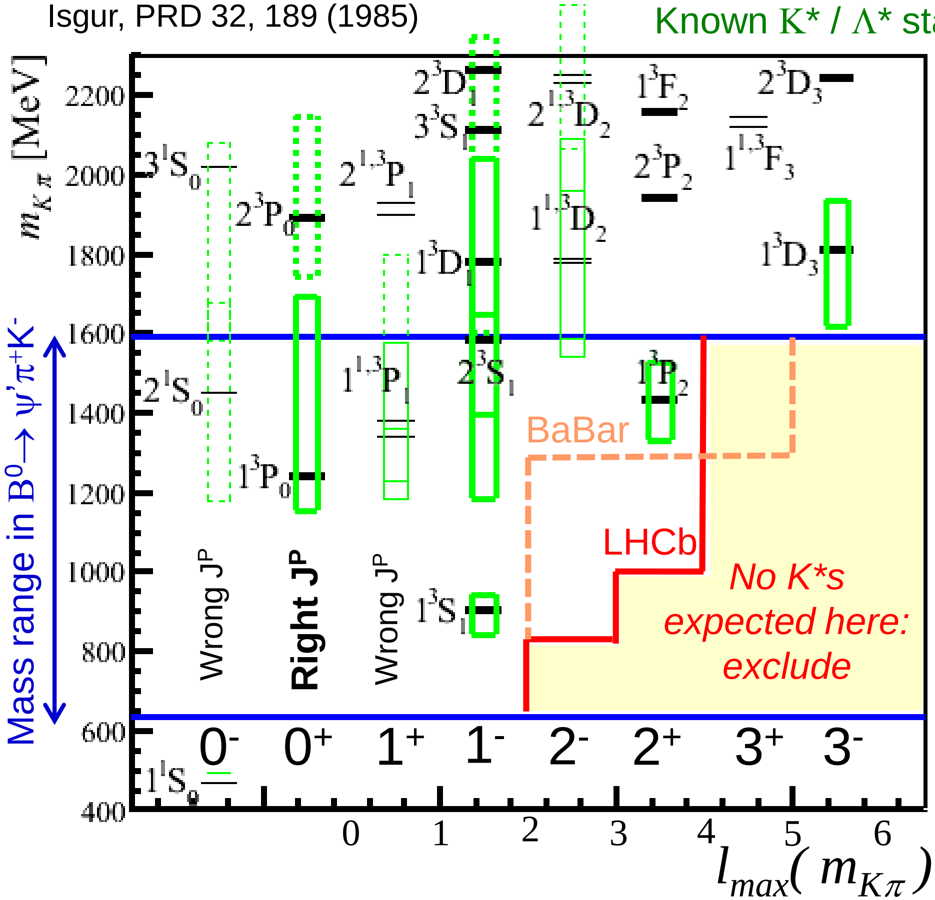
it can be set from **know K*/Λ* resonances**, **quark model predictions** as a guide

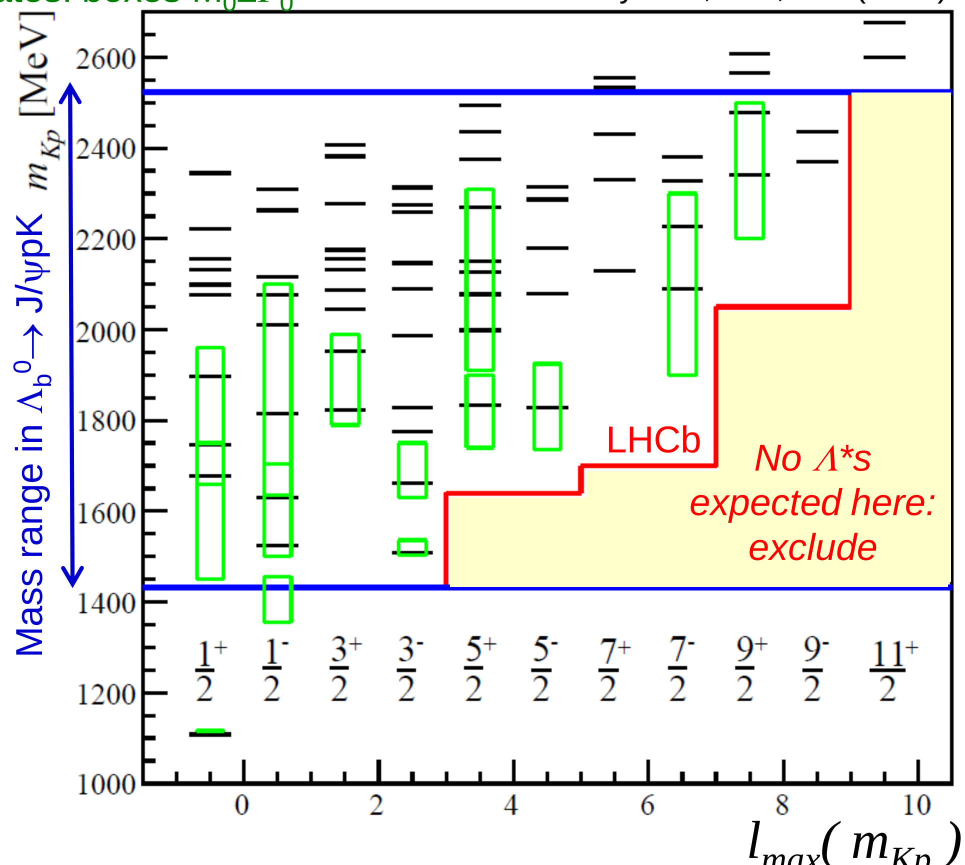
Much fewer known states than predicted!

K* mass predictions by Godfrey-Isgur, PRD 32, 189 (1985)

Known K* / Λ* states: boxes $M_0 \pm \Gamma_0$

Λ* mass predictions by Loring-Metsch-Petry EPJ, A10, 447 (2001)





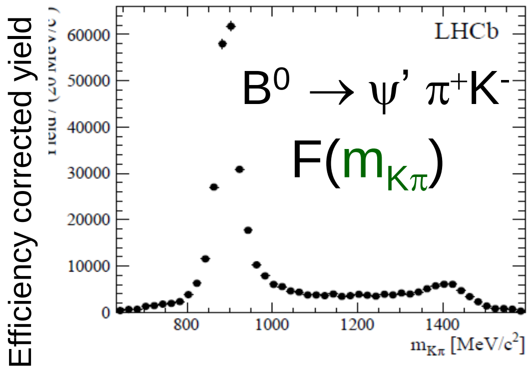
- Because the J/ψ mass is smaller than ψ' mass, must allow for higher excitations in the $\Lambda_b^0 \to J/\psi pK$ analysis, higher l_{max}

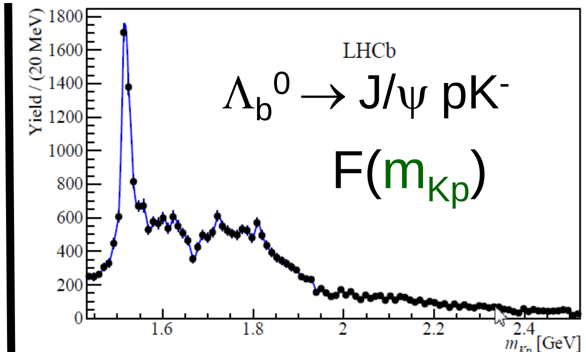
Test the hypothesis (H_0) that the data contain only conventional hadrons

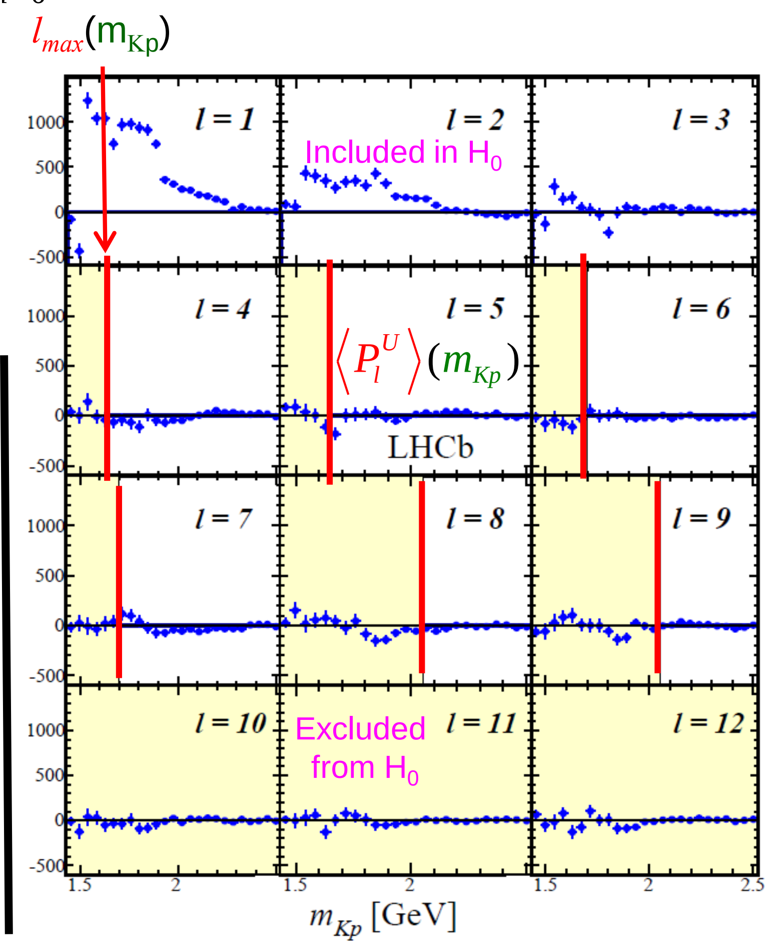
Form a model of the data implementing this hypothesis:

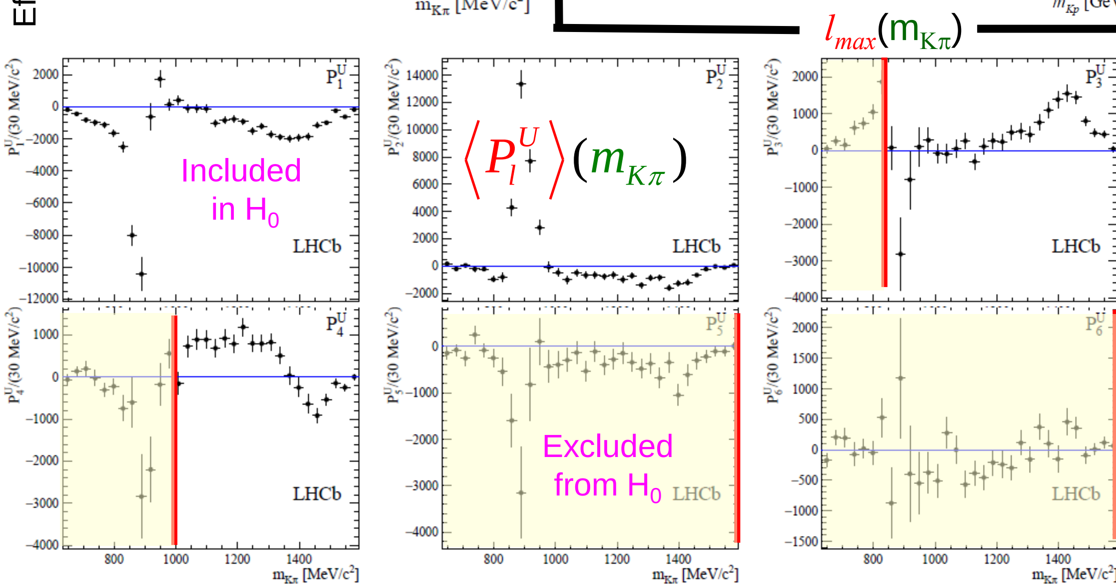
$$\mathrm{PDF}(m_{K\pi/Kp},\cos\theta_{K^*/\Lambda^*}\,|H_0) = F(m_{K\pi/Kp})\; F(\cos\theta_{K^*/\Lambda^*}\,|\; m_{K\pi/Kp})$$

$$F(\cos\theta_{K^*/\Lambda^*}\,|\,m_{K\pi/Kp}) = \sum_{l=0}^{l_{\max}(m_{K\pi/Kp})} \left\langle P_l^U \right\rangle(m_{K\pi/Kp})\; P_l(\cos\theta_{K^*/\Lambda^*})$$







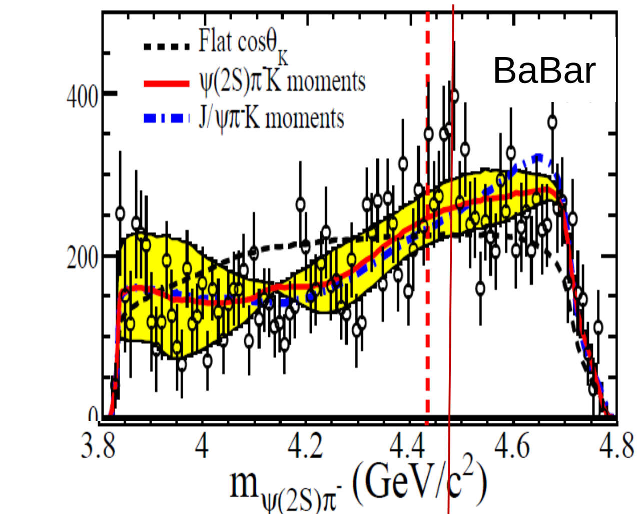


Test H_0 model on $m_{\psi'\pi/\psi p}$ distribution

$$\mathrm{PDF}(m_{\psi'\pi/\psi p}|\mathrm{H}_0)=\int dm_{K\pi/Kp}\,\mathrm{PDF}(m_{K\pi/Kp},\cos\theta_{K^*/\Lambda^*}(m_{\psi'\pi/\psi p})\,|\,\mathrm{H}_0)\left|\frac{\partial\cos\theta_{K^*/\Lambda^*}(m_{\psi'\pi/\psi p})}{\partial m_{\psi'\pi/\psi p}}\right|$$

$B^0 \to \psi' \pi^+ K^-$

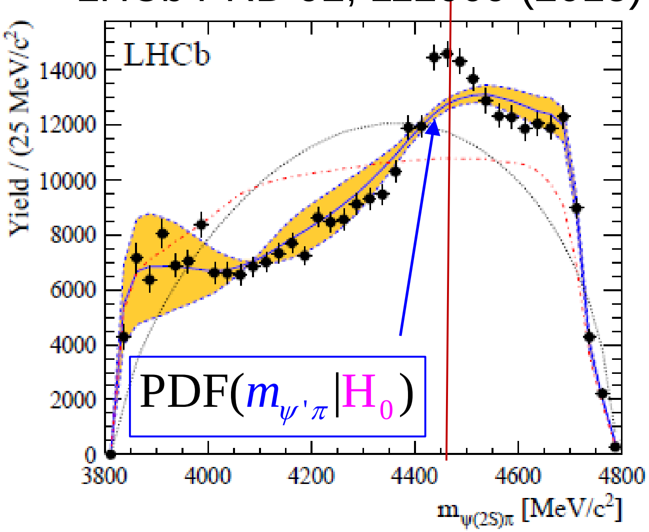
BaBar PRD 79, 112001 (2009)



BaBar did not have enough statistics to see Z(4430) this way.

Negative results like this impossible to interpret without amplitude analysis since Z-K* interfere!

LHCb PRD 92, 112009 (2015)



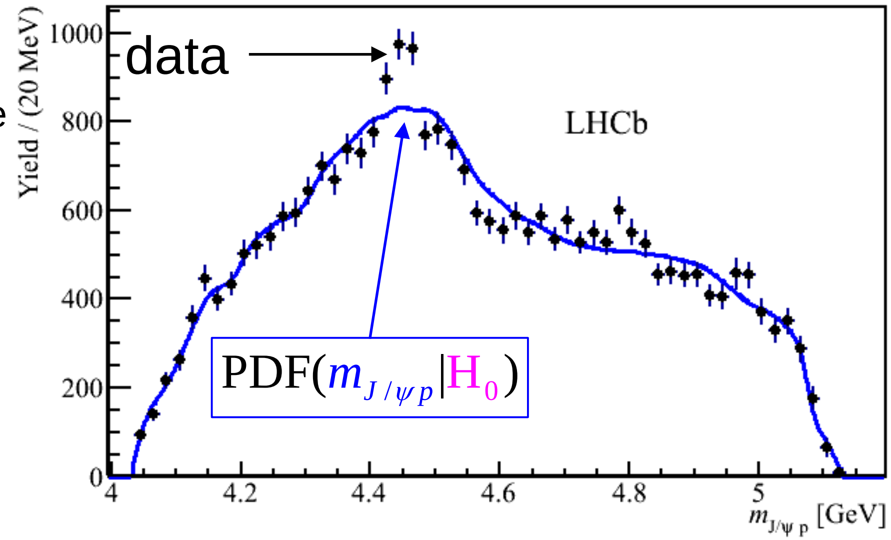
LHCb data inconsistent with K* contributions alone

$\Lambda_b \to J/\psi\, p\, K^-$

arXiv:1604.05708

LHCb-PAPER-2016-009

new



LHCb data inconsistent with Λ^* contributions alone

This model independent proof of the presence of exotic hadron contributions is especially important for the Λ_b data, because of the difficulties in construction of a complete model of Λ excitations

Rejection of H_0 can be quantified

Test variable:
(quasi) log-likelihood-ratio

$$\Delta(-2\ln L) \equiv -2 \sum_{i=1}^{n_{events}} \frac{1}{\varepsilon_i} \ln \frac{PDF(m_{\psi'\pi}|H_0)}{PDF(m_{\psi'\pi}|H_1)}$$

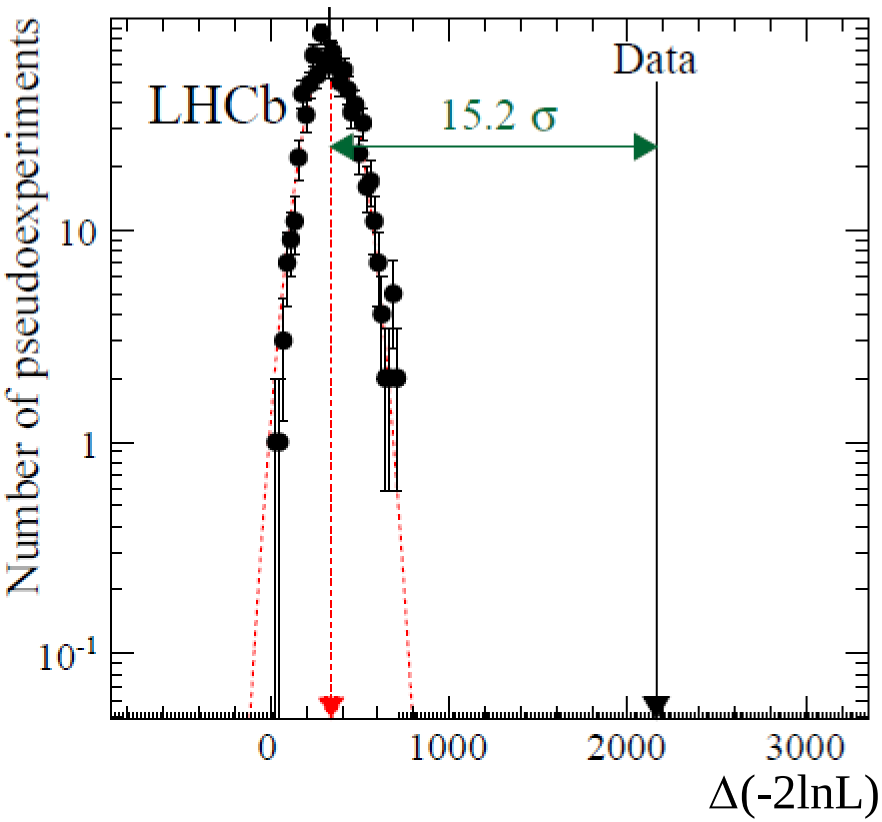
$$\Delta(-2\ln L) \equiv -2 \sum_{i=1}^{n_{events}} \ln \frac{PDF(m_{J/\psi p}|H_0)/I_{H_0}}{PDF(m_{J/\psi p}|H_1)/I_{H_1}}$$

$$I_H = \int PDF(m_{J/\psi p}|H)\varepsilon(m_{J/\psi p})\, d\, m_{J/\psi p}$$

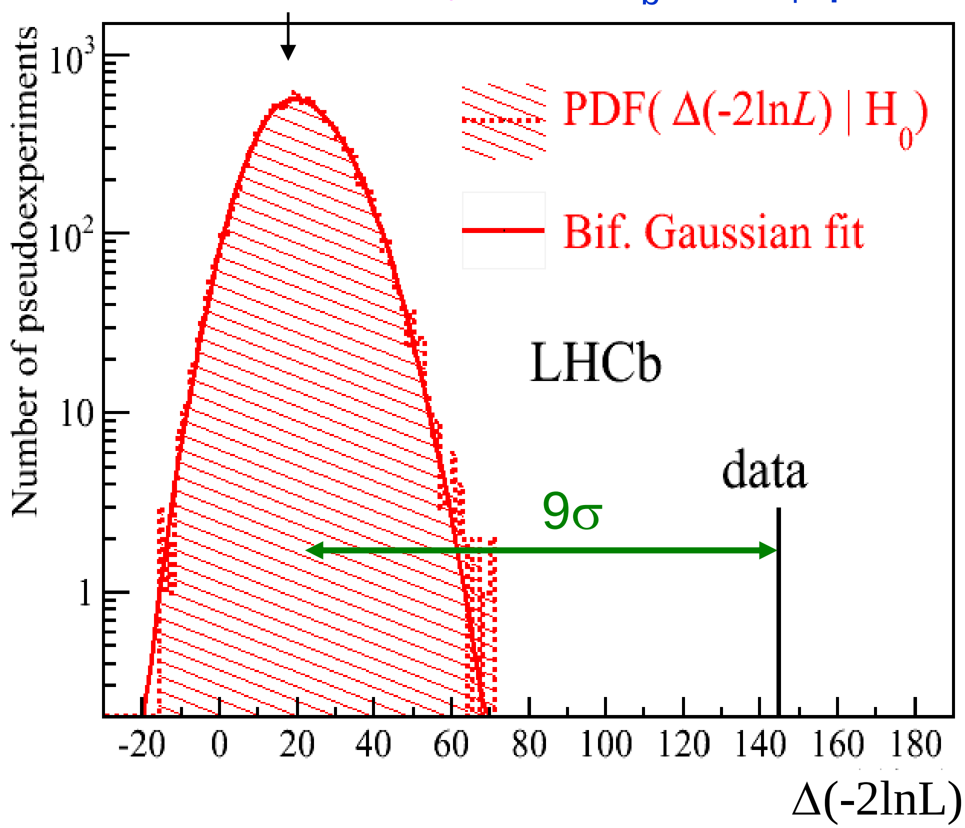
H_0: $l_{max}(m_{K\pi})$ H_1: l_{max}^{H1}=30 H_0: $l_{max}(m_{Kp})$ H_1: l_{max}^{H1}=31

This variable tests a significance of moments between $l_{max}(m_{K\pi/Kp})$ and l_{max}^{H1}

PDF(Δ(-2lnL) |H_0) $B^0 \rightarrow \psi' \pi^+ K^-$



PDF(Δ(-2lnL) |H_0) $\Lambda_b \rightarrow J/\psi\, p\, K^-$



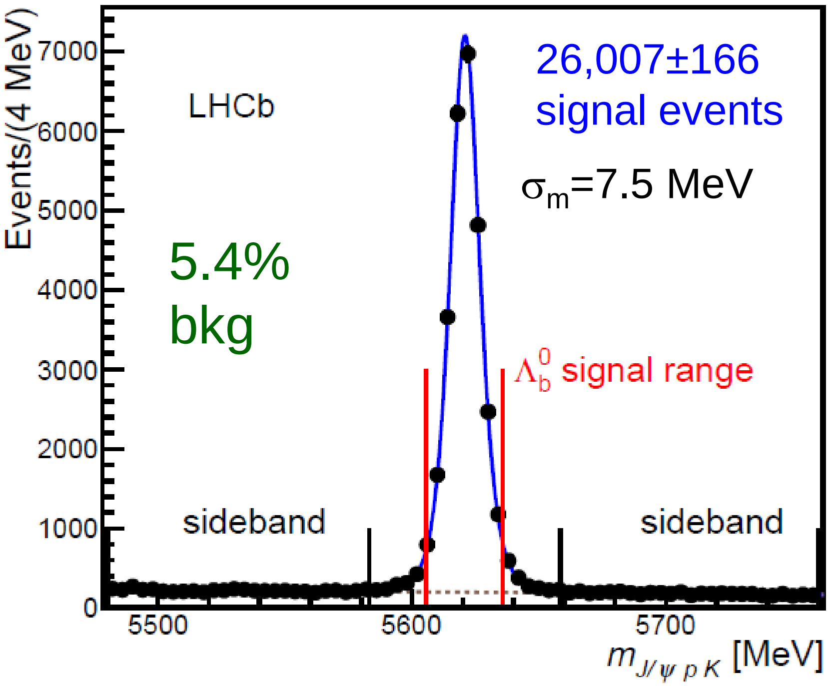
However, this approach cannot characterize exotics – amplitude analysis is still necessary.

Cabibbo suppressed vs favored Λ_b decays

LHCb-PAPER-2015-029

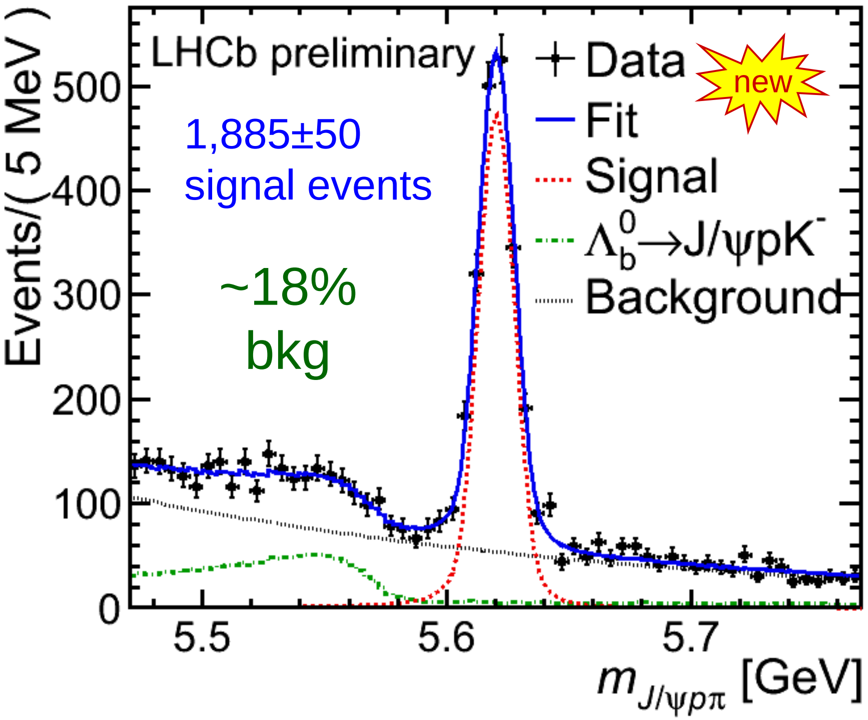
PRL 115, 07201 (2015)

$\Lambda_b \to J/\psi\, p\, K^-$



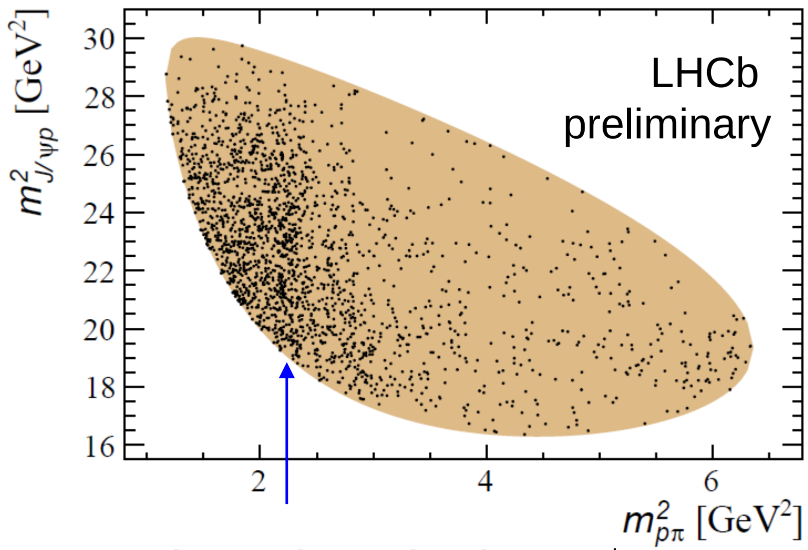
LHCb-PAPER-2016-015 (in preparation)

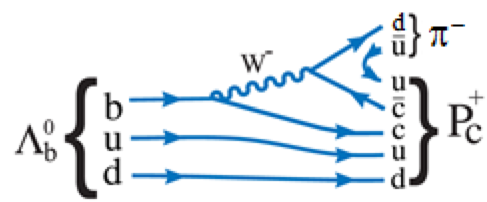
$\Lambda_b \to J/\psi\, p\, \pi^-$



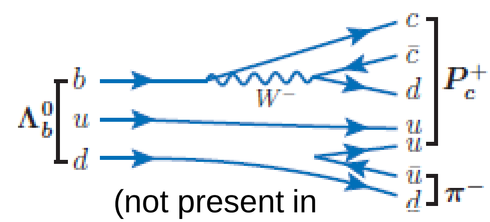
- More than a factor of 10 lower signal statistics in $\Lambda_b \to J/\psi\, p\, \pi^-$ analysis than in $\Lambda_b \to J/\psi\, p\, K^-$
- Relative background fraction higher by more than a factor of 3

$\Lambda_b^0 \rightarrow J/\psi p \pi^-$: Cabibbo suppressed

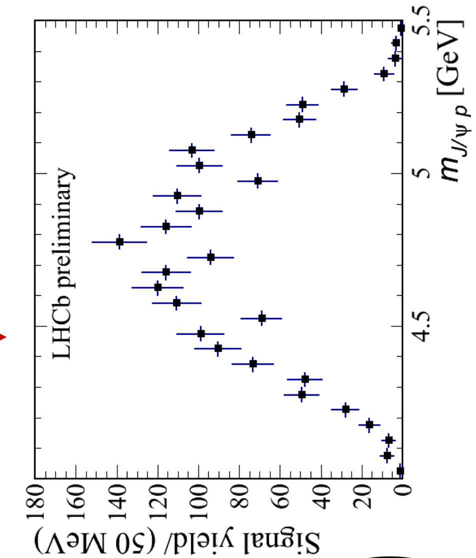




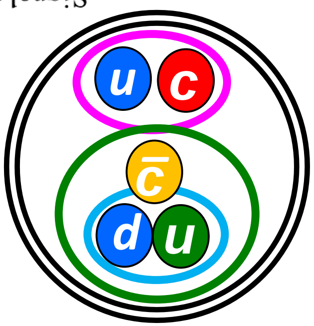
$P_c^+ \rightarrow J/\psi\ p$?



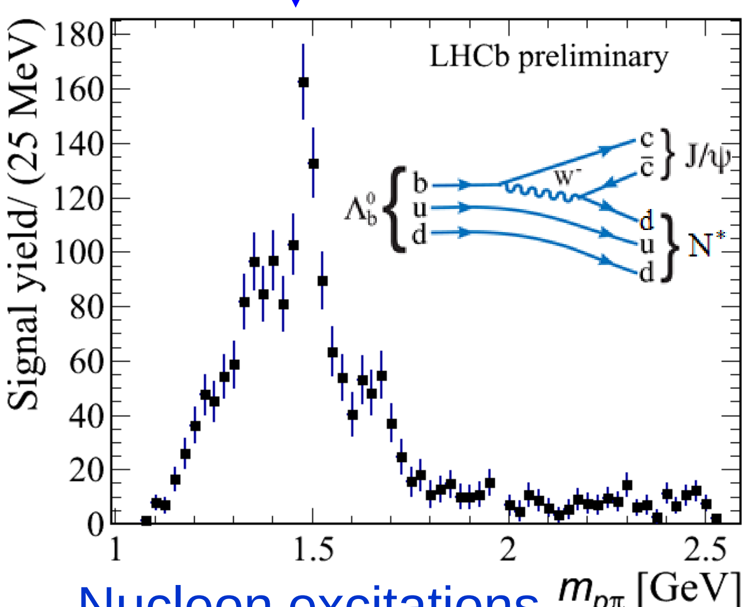
(not present in Cabibbo favored case)



Exotic pentaquark

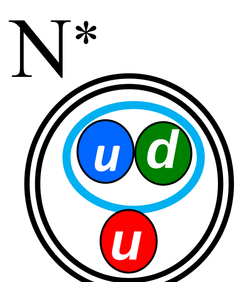


N(1535) and other N*'s → p π^-



Nucleon excitations

- No obvious structure in $m_{J/\psi p}$



Statistics is low.

Proper amplitude analysis necessary to check for consistency with Cabibbo favored $\Lambda_b^0 \rightarrow J/\psi p K^-$

Model of conventional N* resonances

Better established states from PDG

$\Lambda_b \to J/\psi\, p\, \pi^-$

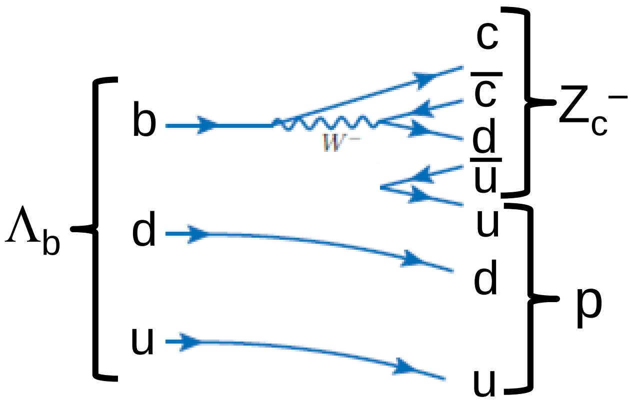
Only significant states limit L | All states limit L

State	J^P	M_0 (MeV)	Γ_0 (MeV)	# of complex couplings Red.	# of complex couplings Ext.
NR $p\pi$	$1/2^-$	-	-	4	4
$N(1440)$	$1/2^+$	1430	350	3	4
$N(1520)$	$3/2^-$	1515	115	3	3
$N(1535)$	$1/2^-$	1535	150	4	4
$N(1650)$	$1/2^-$	1655	140	1	4
$N(1675)$	$5/2^-$	1675	150	3	5
$N(1680)$	$5/2^+$	1685	130	0	3
$N(1700)$	$3/2^-$	1700	150	0	3
$N(1710)$	$1/2^+$	1710	100	0	4
$N(1720)$	$3/2^+$	1720	250	3	5
$N(1875)$	$3/2^-$	1875	250	0	3
$N(1900)$	$3/2^+$	1900	200	0	3
$N(2190)$	$7/2^-$	2190	500	0	3
$N(2300)$	$1/2^+$	2300	340	0	3
$N(2570)$	$5/2^-$	2570	250	0	3
Total # of free parameters				40	106

- Use reduced model for central values, extended for systematics and significance of exotic contributions
- Because of insufficient statistics forced to neglect higher orbital angular momenta for most of the N* states
- Almost as many free parameters in the fit as in $\Lambda_b \to J/\psi\, p\, K^-$ with 14 times smaller statistics and 3 times higher relative bkg

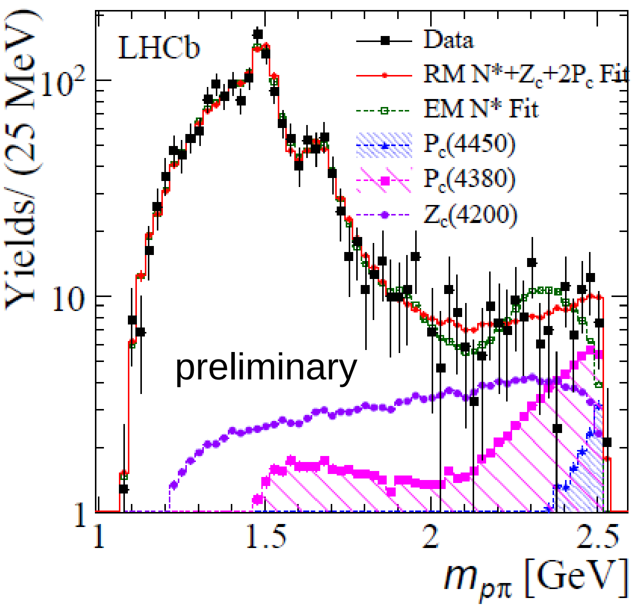
Exotic hadron contributions to $\Lambda_b \to J/\psi\, p\, \pi^-$

- Open ended search for exotic hadrons in $\Lambda_b \to J/\psi\, p\, \pi^-$ with the present statistics is not possible
- Test data for presence of previously observed states:
  - $P_c(4380)^+$,$P_c(4450)^+ \to J/\psi\, p$ observed by LHCb in $\Lambda_b \to J/\psi\, p\, K^-$
    - Masses, widths, P_c^+- decay helicity couplings fixed (fit only their production couplings – 4 free parameters). Varied within the errors for the systematics.
    - Fix J^P assignments to (3/2-,5/2+). Use other possible assignments for the systematics.
    - Significance determined including all systematic effects.
  - $Z_c(4200)^+ \to J/\psi\, \pi^+$ observed by Belle in $B^0 \to J/\psi\, \pi^+ K^-$ [PRD, 90, 112009 (2014)]
    - Mass, width fixed. Varied for the systematics.
    - Helicity couplings are free (10 fit parameters).
    - $J^P=1^+$ well determined.

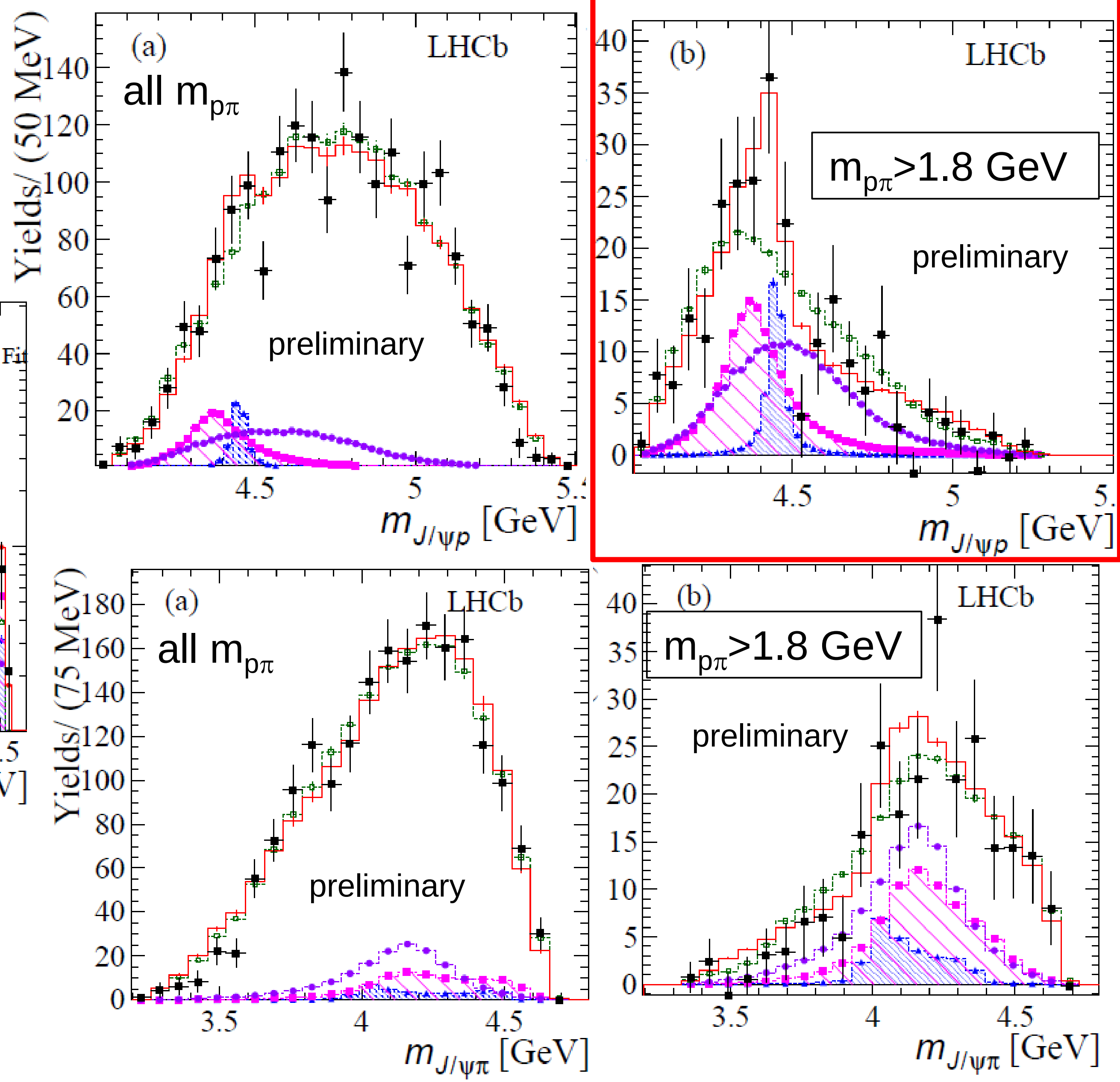


Amplitude fits to $\Lambda_b \to J/\psi\, p\, \pi^-$

- Significance of $P_c(4380)^+$, $P_c(4450)^+$, $Z_c(4200)^-$ taken together is 3.1σ



- Evidence for exotic hadron contributions to $\Lambda_b \to J/\psi\, p\, \pi^-$!



Results for $\Lambda_b \to J/\psi\, p\, \pi^-$

- Significance of $P_c(4380)^+$,$P_c(4450)^+$, $Z_c(4200)^-$ taken together is 3.1σ (including systematic uncertainty) – evidence for exotic hadrons.
- Individual exotic hadron contributions are not significant. For example, significance of $P_c(4380)^+$ plus $P_c(4450)^+$ is <1.7σ - no independent confirmation of the P_c^+ states (it increases to an evidence level, 3.3σ, if assume production of $Z_c(4200)^-$ is negligible).

State	Fit fraction (%)	$BR(\Lambda_b \to P_c^+ \pi)/BR(\Lambda_b \to P_c^+ K^-)$
$Z_c(4200)^-$	$7.7 \pm 2.8^{+3.4}_{-4.0}$	---
$P_c(4380)^+$	$5.1 \pm 1.5^{+2.1}_{-1.6}$	$0.050 \pm 0.016^{+0.020}_{-0.016} \pm 0.025$
$P_c(4450)^+$	$1.6\ ^{+0.8}_{-0.6}\ ^{+0.6}_{-0.5}$	$0.033\ ^{+0.016}_{-0.014}\ ^{+0.011}_{-0.009} \pm 0.009$

Expected if the additional internal W emission diagram negligible: $0.07 \sim 0.08$

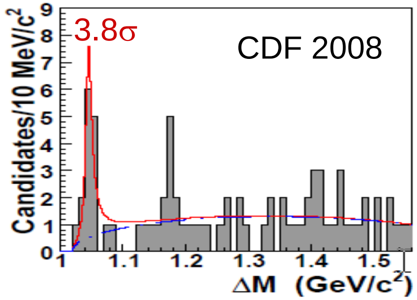
H.-Y. Cheng and C.-K. Chua, PRD, 92 (2015) 096009, arXiv:1509.03708.

The $\Lambda_b \to J/\psi\, p\, \pi^-$ data are consistent with the presence of $P_c(4380)^+$,$P_c(4450)^+$ at the level expected from $\Lambda_b \to J/\psi\, p\, K^-$ measurement and Cabibbo suppression.

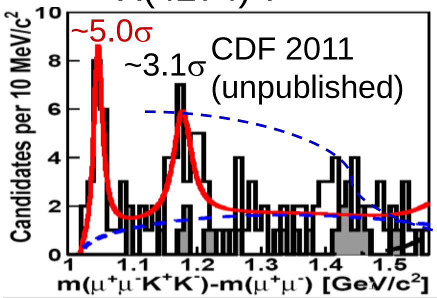
Confusing experimental situation concerning X→J/ψφ states

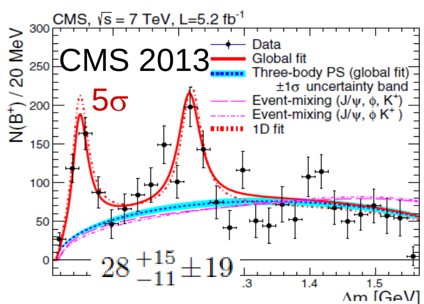
X(4140) ?

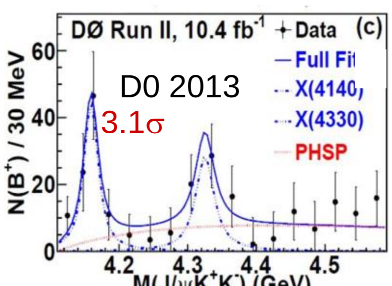
B→J/ψφK

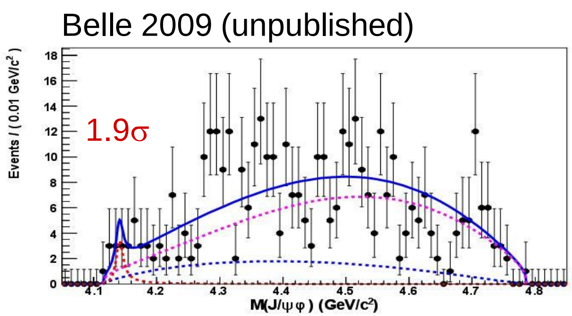


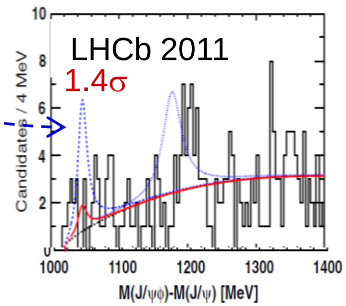
X(4274) ?

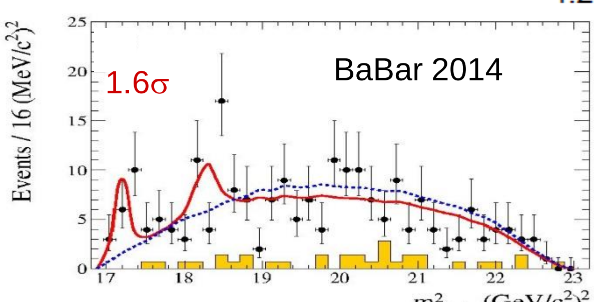






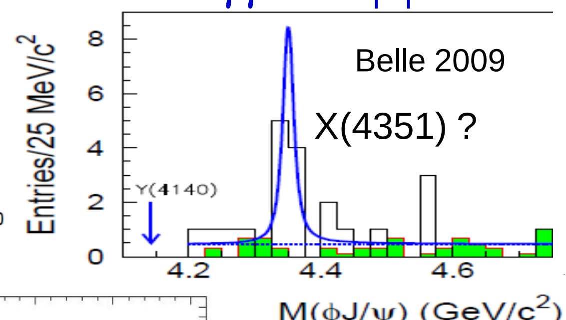




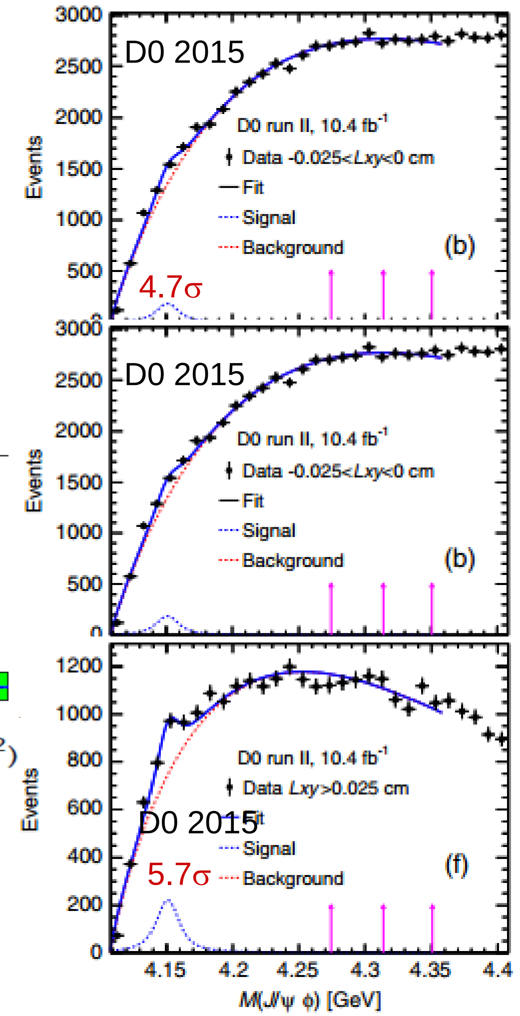


(refs in the tables in backup slides)

γγ→J/ψφ



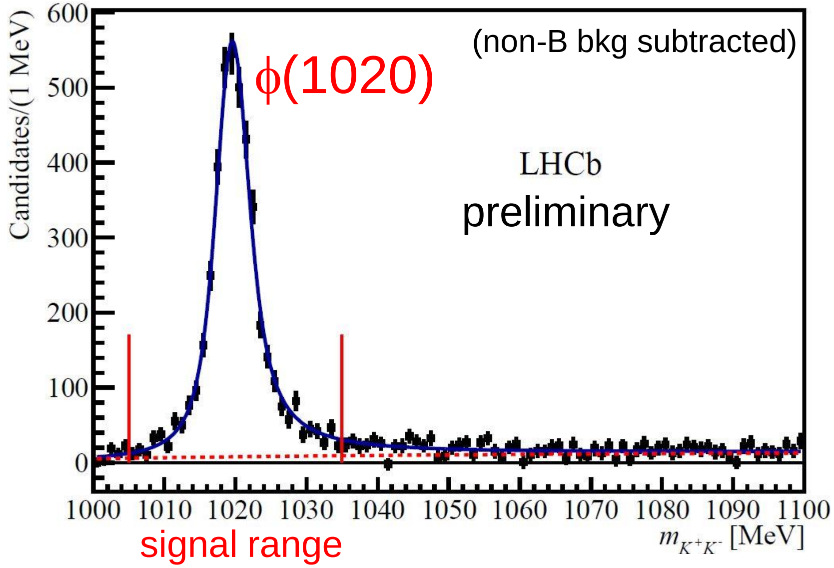
pp̄→J/ψφ…

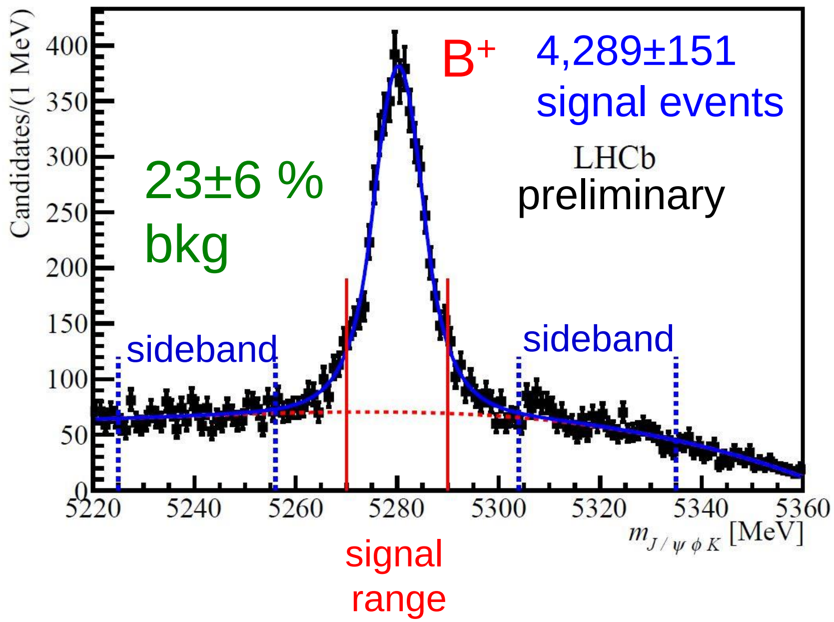


- Some experiments saw narrow X(4140) [i.e. Y(4140)], some didn't.
- Possibly 2nd J/ψφ structure in B decays, X(4274), but seen at inconsistent mass. No published claim of its significance.
- Possibly X(4351) state seen in γγ collisions

LHCb $B^+ \to J/\psi\ \phi\ K^+$ data samples (3 fb^{-1})

$B^+ \to J/\psi\ \phi\ K^+$





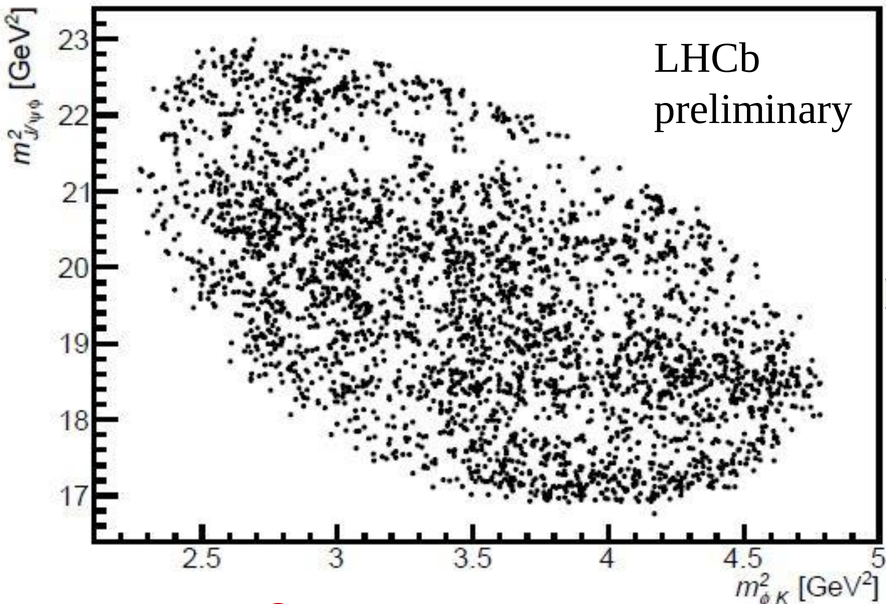
LHCb-PAPER-2016-018
LHCb-PAPER-2016-019
In preparation

new

- Statistically, the most powerful $B \to J/\psi\ \phi\ K$ sample analyzed so far

Use sidebands to subtract background

$B^+ \to J/\psi\, \phi\, K^+$

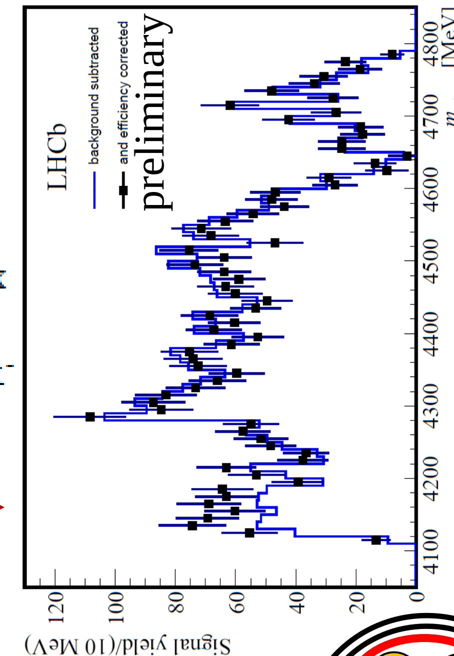


X(?)

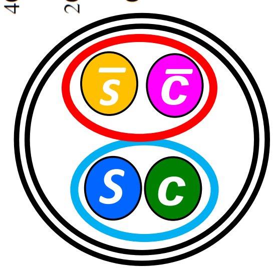
X(?)

X(4274) ?

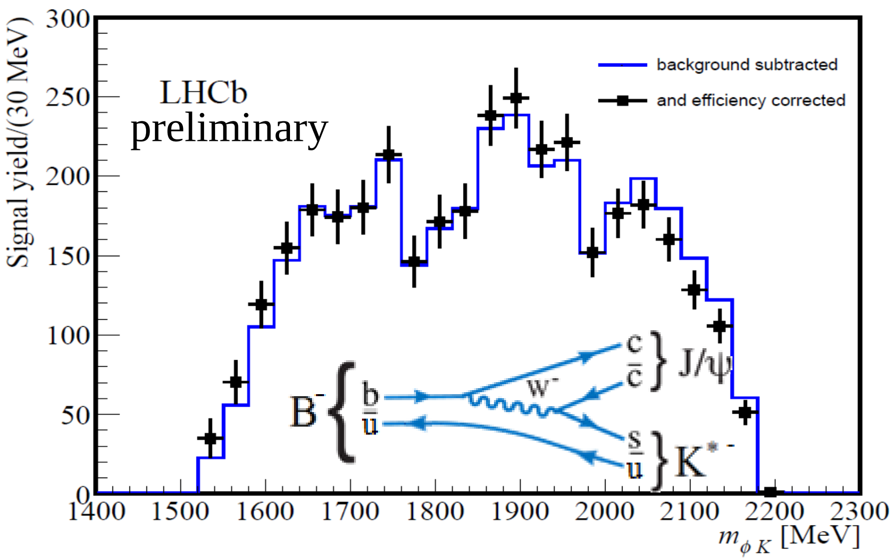
X(4140) ?
$\to J/\psi\phi$



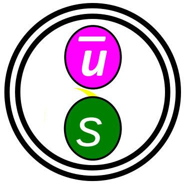
Tetraquark



?



Kaon excitations

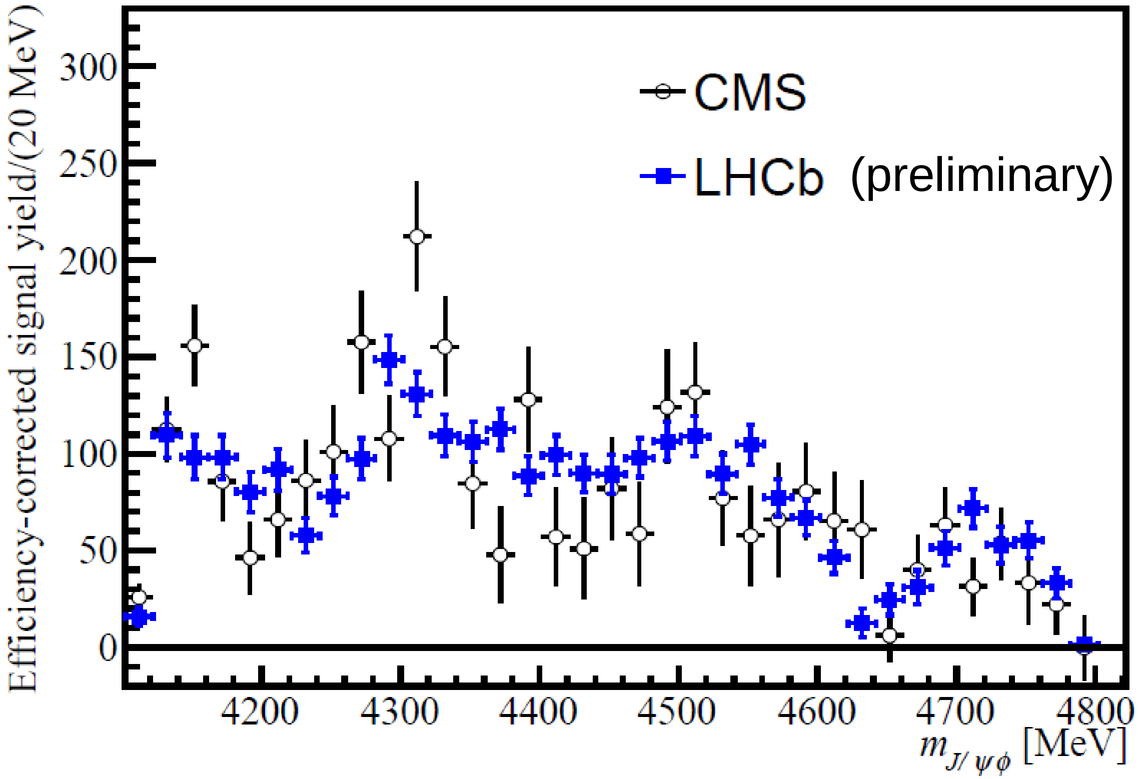


Are these reflections of interfering $K^{*-} \to \phi K^-$?

Proper amplitude analysis necessary to check

LHCb vs CMS data

- Compare $m_{J/\psi\phi}$ to the CMS data (the previous best sample).
- Non-B background subtracted, corrected for signal efficiency.



Used publically available CMS background-free distribution and CMS efficiency dependence on $m_{J/\psi\phi}$

LHCb efficiency corrections via 6D parameterization of efficiency in all dimensions of the decay phase-space.

Normalized to the same area.

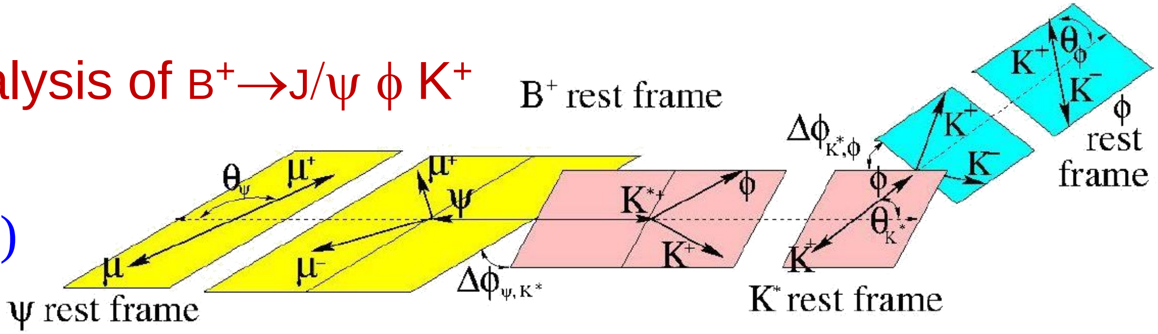
The vertical scale is arbitrary.

- LHCb data more precise.
- Qualitative agreement over the full mass range.

Amplitude analysis needed

- Amplitude analysis is needed to demonstrate that the $X \to J/\psi\ \phi$ peaks are not due to reflections of interfering kaon excitations ("K*") decaying to ϕK^+
  - Smoothness of $m_{\phi K}$ spectrum does not mean that there are no kaon excitations in the data. The narrowest known K* state in the relevant mass range is 150 MeV. Many overlapping resonances expected. Only analysis of the masses in correlation with the decay angles can disentangle them.
- All previous analyses performed naïve 1D mass fits to $m_{J/\psi\phi}$
  - Ad hoc assumptions about kaon contributions (e.g. 3-body phase-space distribution, incoherent)
  - No sensitivity to J^{PC} of X structures

Perform first amplitude analysis of $B^+ \to J/\psi\ \phi\ K^+$



$$\Omega \equiv (\theta_{K^*}, \theta_\psi, \Delta\phi_{\psi,K^*}, \theta_\phi, \Delta\phi_{K^*,\phi})$$

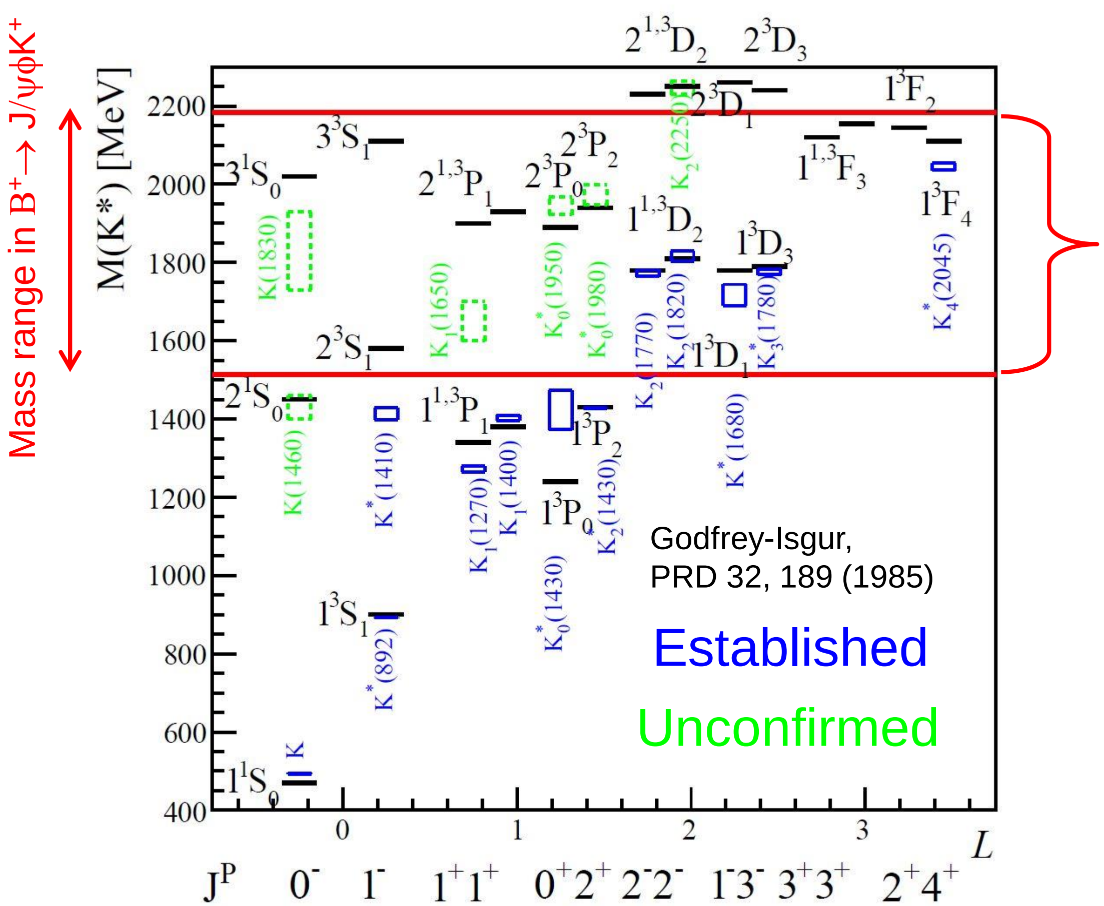
$$\left| MatrixEle(\mathrm{m}_{K\phi}, \Omega \,|\, J_R^P, M_R, \Gamma_R, H_R) \right|^2$$

6D maximum likelihood fit

1-4 independent **complex** helicity couplings H_R per K^* resonance

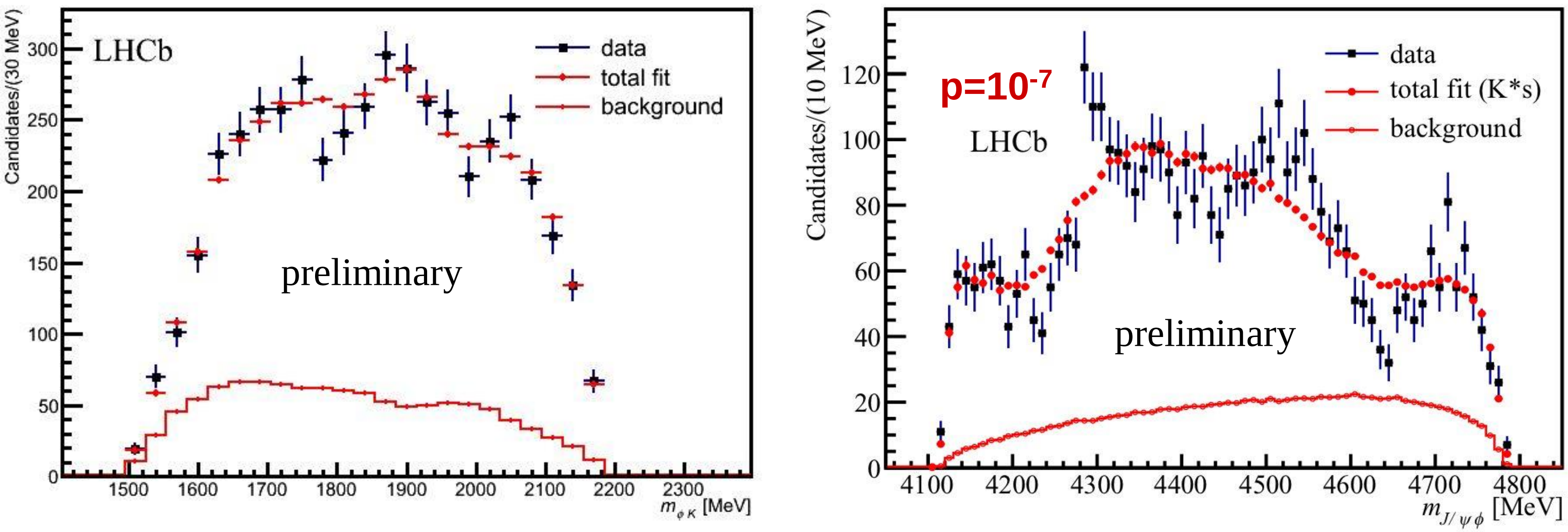
Model of conventional K* resonances

- All K* states (except 0^{++}) between kinematic boundaries are allowed to decay to ϕK but may not have been seen in experiment because previous searches are typically old scattering experiments with low statistics at high masses
- All known excited states in this mass range are broad: Γ~150-400 MeV



- Guidance from quark model was used to inform choices for K* sector
- Try both known and unknown K* states
- No constraints placed on mass or width parameters (**fits don't depend on predictions or previous measurements**)
- Take K* contributions greater than ~2σ significance.

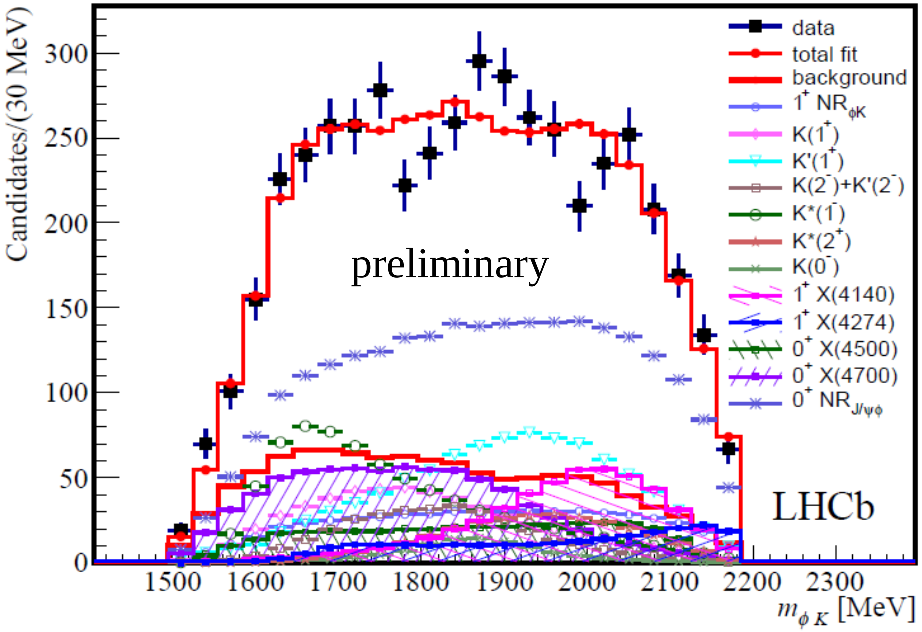
Amplitude fits with kaon excitations only

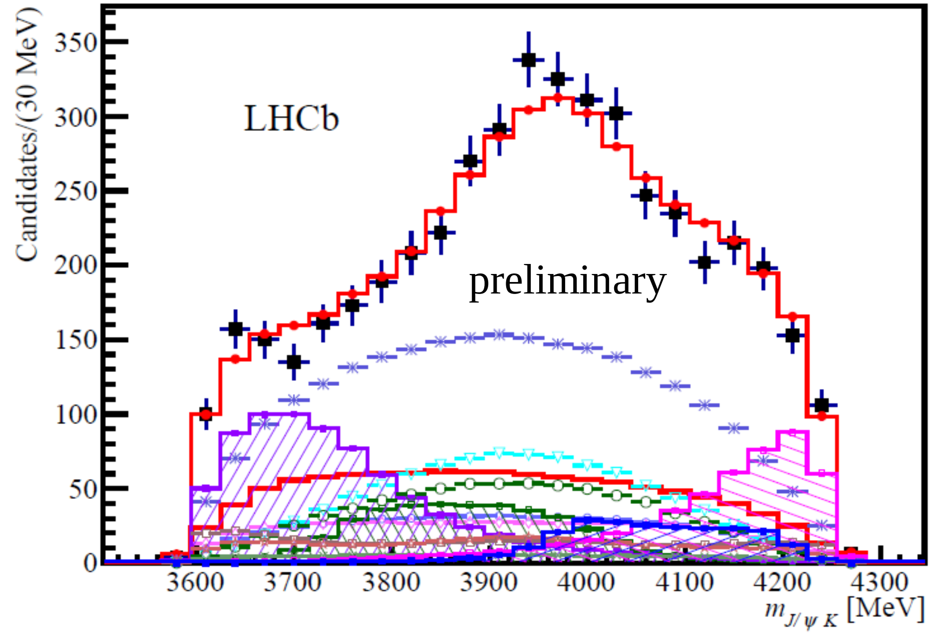


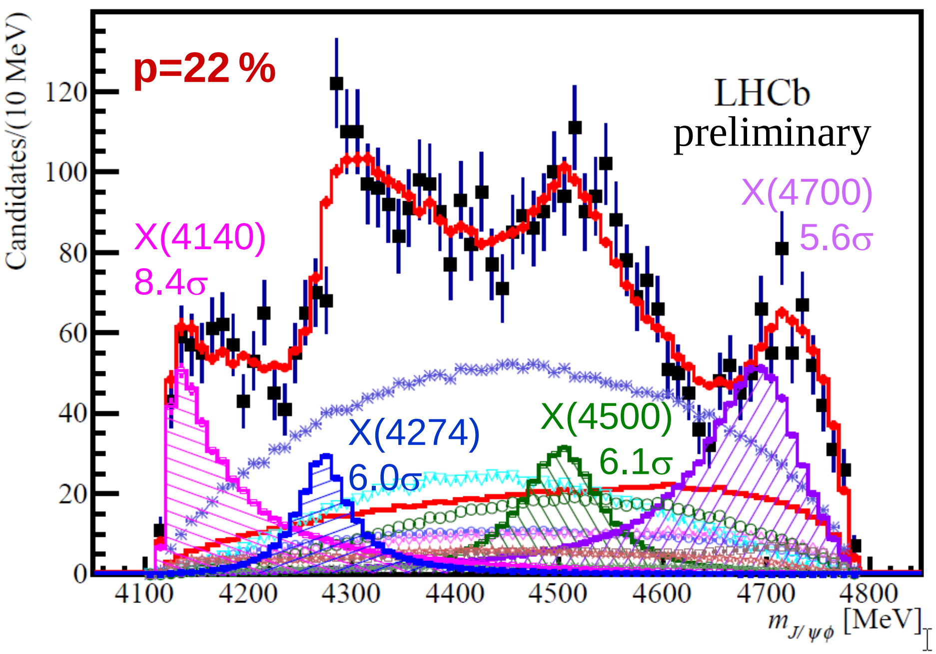
- Fits without exotic contributions were tried:
  - Example: two $2P_{1+}$, two $2D_{1-}$, and one of 1^3F_{3+}, 1^3D_{1-}, 3^3S_{1-}, 3^1S_{0-}, 2^3P_{2+}, 1^3F_{2+}, 1^3D_{3-}, 1^3F_{4+}. Contained 104 free parameters.

- Further K* additions, including states not predicted by the quark model, does not change the conclusion that non-K* contributions are needed to adequately describe all distributions

Amplitude fit including 4 exotic X resonances

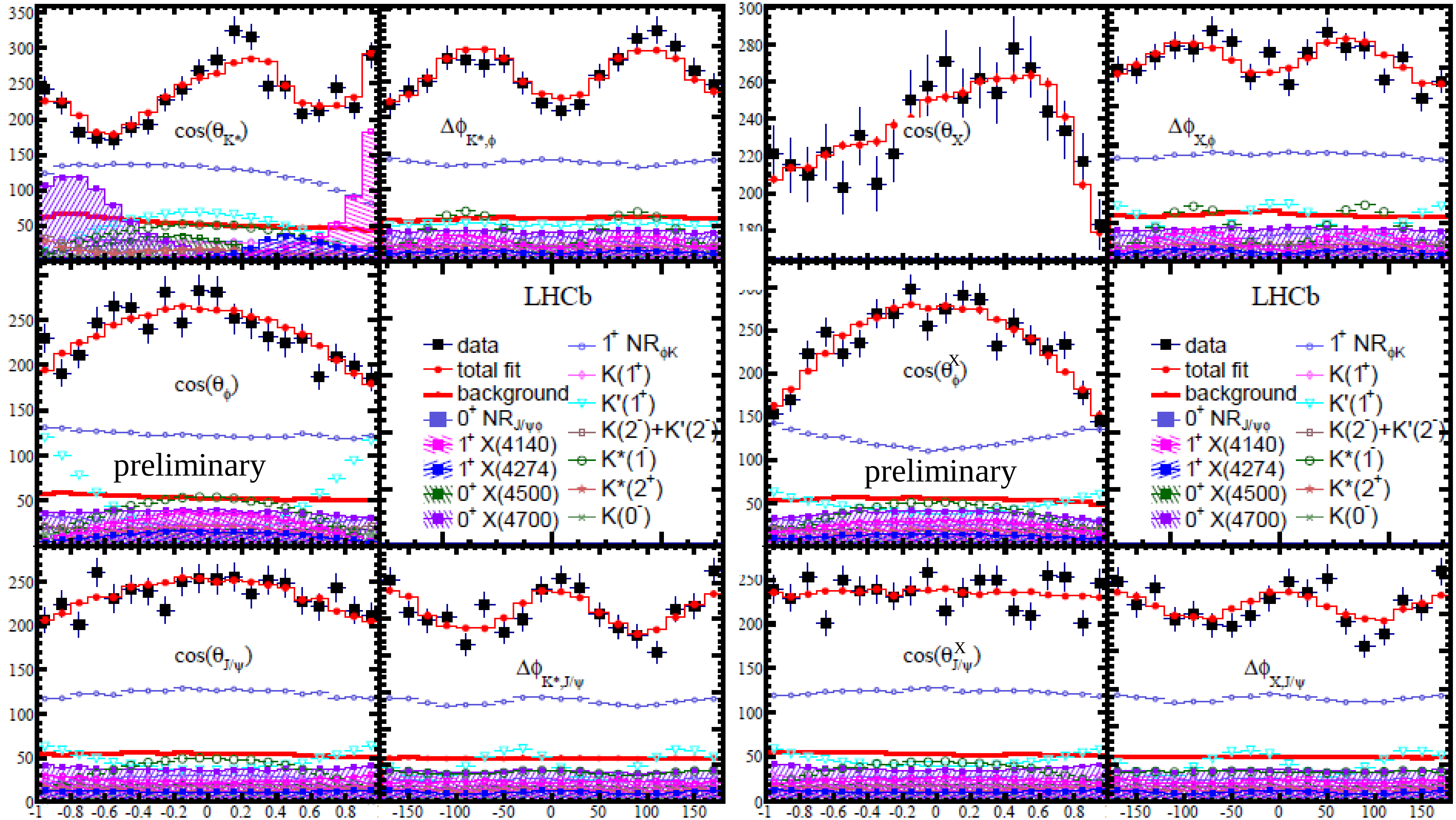






- We considered adding possible exotic X→J/ψ φ and Z^+→J/ψ K^+ states as well as removing insignificant or implausible (Γ>1000 or <100 MeV) conventional K^{*+}→φ K^+ states leading us to a default model
- Only X states give very significant improvements in fit qualities over the models with K*s alone
- The default fit model is shown here.

Fitted angles

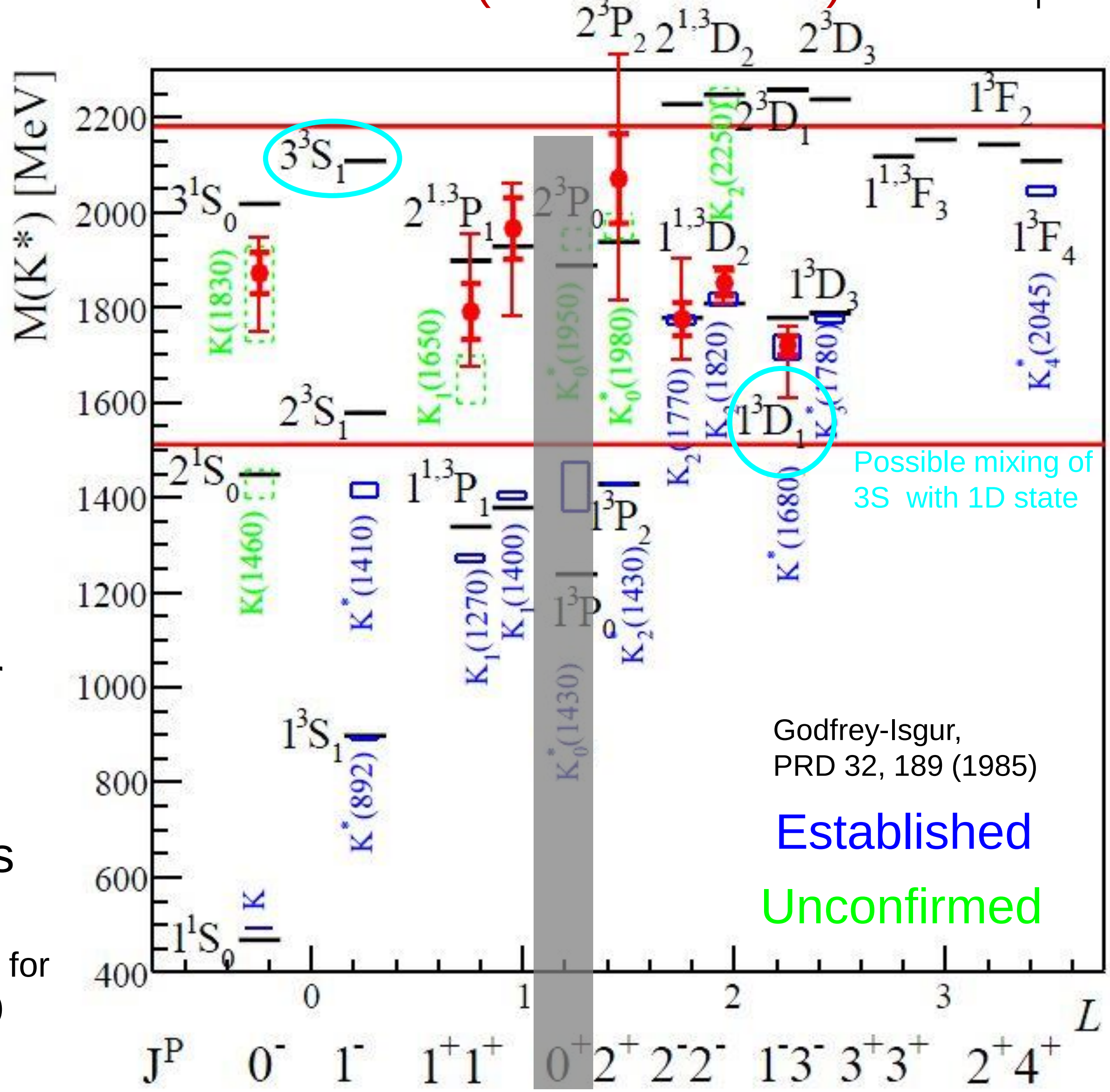


- Fit quality is good in all fitted variables

K* results

Default model: 6 K* resonances (of 4 different J^P) + 1 NR ϕK

Our results are given by the red points with error bars



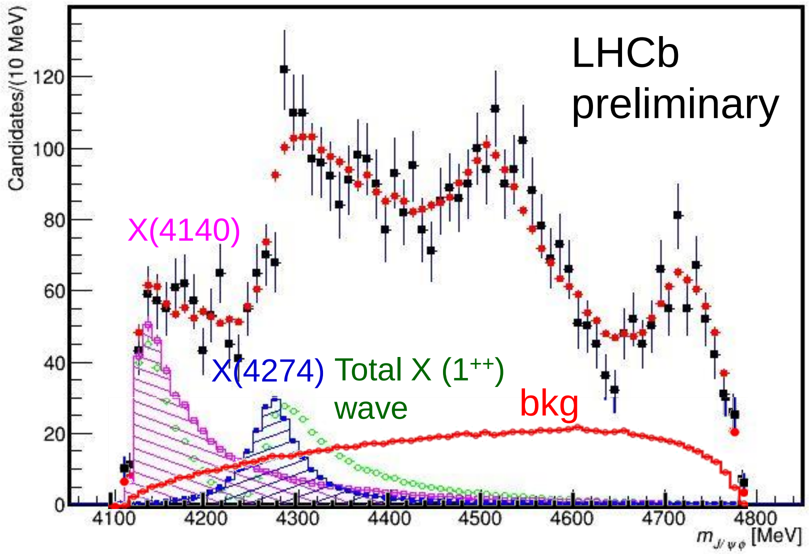
Forbidden

High spin states (3-4) not observed but also expected to be suppressed by orbital angular momentum barrier in B decay

Excellent agreement between our results and both theory and previous experiments

(see backup slide for numerical results)

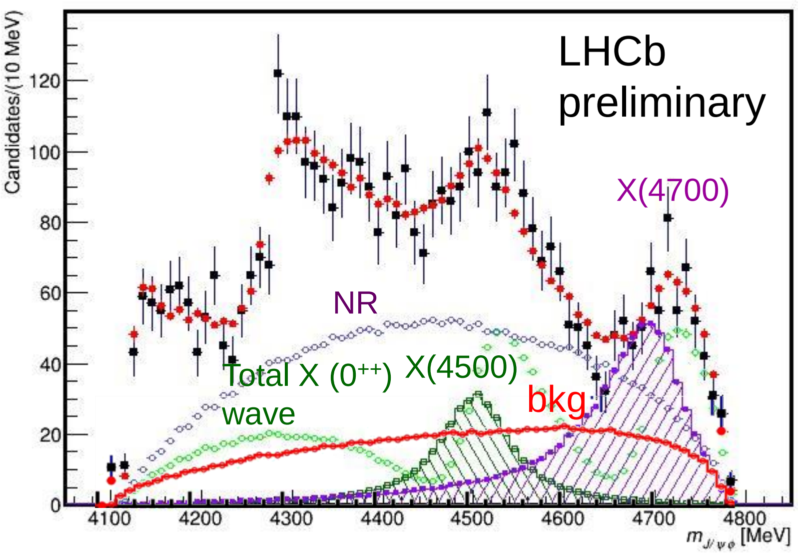
X(1++)



Contribution	sign. or Ref.	M_0 MeV	Γ_0 MeV	Fit results F.F. %
All $X(1^+)$				$16\pm3\ ^{+\ 6}_{-\ 2}$
$X(4140)$	8.4σ	$4146.5\pm4.5\,^{+4.6}_{-2.8}$	$83\pm21\,^{+21}_{-14}$	$13\pm3.2\,^{+4.8}_{-2.0}$
ave.		4146.9 ± 2.3	17.8 ± 6.8	
$X(4274)$	6.0σ	$4273.3\pm8.3\,^{+17.2}_{-3.6}$	$56\pm11\,^{+8}_{-11}$	$7.1\pm2.5\,^{+3.5}_{-2.4}$
CDF		$4274.4\,^{+8.4}_{-6.7}\pm1.9$	$32\,^{+22}_{-15}\pm8$	
CMS		$4313.8\pm5.3\pm7.3$	$38\,^{+30}_{-15}\pm16$	

- Significant X(4140) 8.4σ,
  - mass consistent with the previous measurements, but the width substantially larger
  - $J^{PC}=1^{++}$ determined at 5.7σ including systematic errors
- Significant X(4274) 6.0σ,
  - Consistent with the unpublished CDF results. First significant claim for this structure.
  - $J^{PC}=1^{++}$ determined at 5.8σ including systematic errors

X(0^{++})

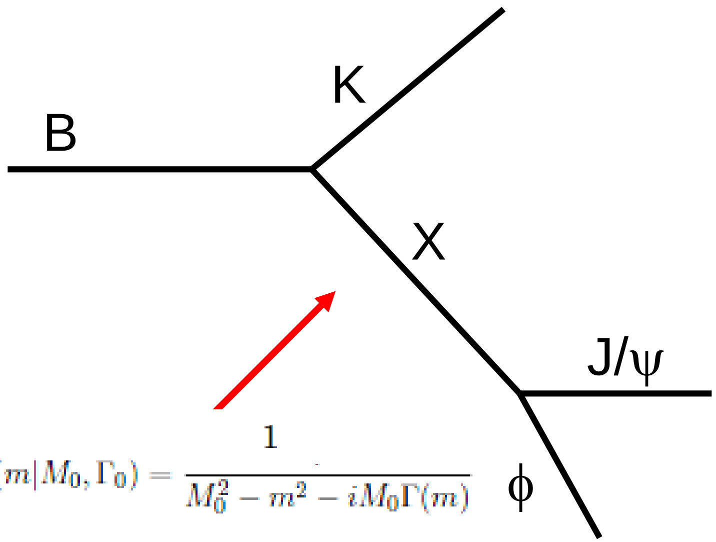


Contri-bution	sign.	M_0 MeV	Γ_0 MeV	F.F. %
All $X(0^+)$				$28\pm\ 5\,^{+7}_{-7}$
$NR_{J/\psi\phi}$	6.4σ			$46\pm11\,^{+11}_{-21}$
$X(4500)$	6.1σ	$4506\pm11\,^{+12}_{-15}$	$92\pm21\,^{+21}_{-20}$	$6.6\pm2.4\,^{+3.5}_{-2.3}$
$X(4700)$	5.6σ	$4704\pm10\,^{+14}_{-24}$	$120\pm31\,^{+42}_{-33}$	$12\pm\ 5\,^{+\ 9}_{-\ 5}$

- Significant structures at higher masses, best described by two new 0^{++} resonances X(4500),X(4700):
  - Significances of 6.1σ, 5.6σ
  - $J^{PC}=0^{++}$ determined at 4.0σ, 4.5σ, respectively

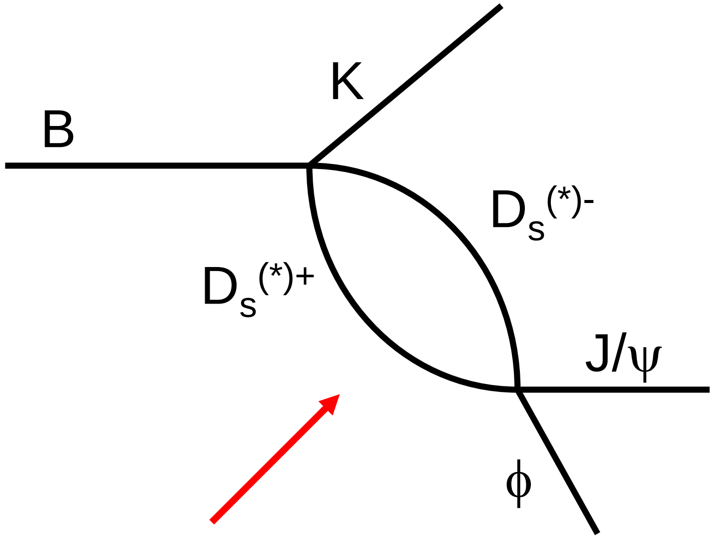
X exotics as $D_s^{(*)}\overline{D_s}^{(*)}$ cusps?

Breit-Wigner Model



$$BW(m|M_0,\Gamma_0) = \frac{1}{M_0^2 - m^2 - iM_0\Gamma(m)}$$

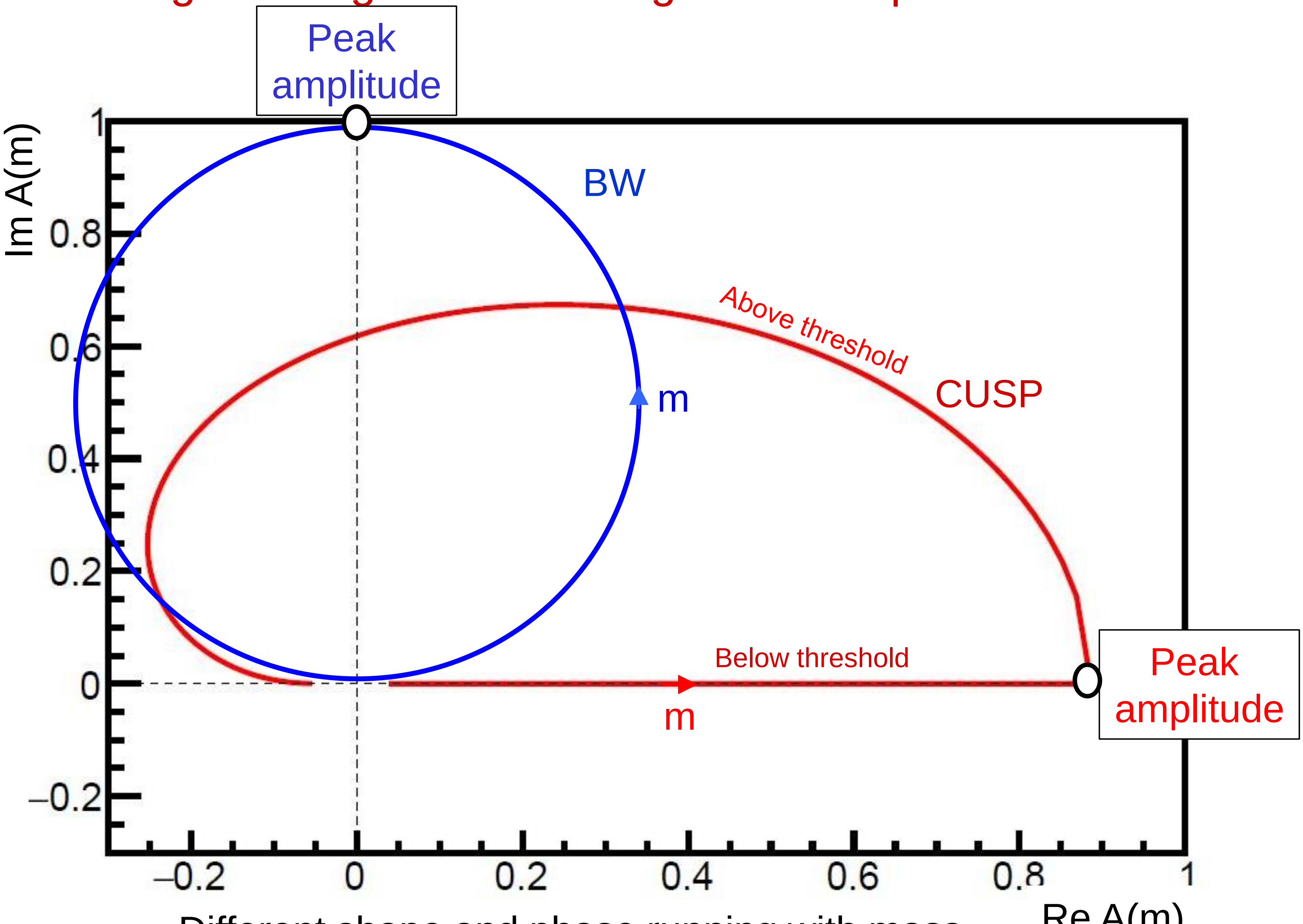
Cusps Model



$$\Pi(m|\beta_0)= \int \frac{d^3q}{(2\pi)^3} \frac{e^{-2q^2/\beta_0^2}}{m - M_A - M_B - \frac{q^2}{2\mu_{AB}} + i\,\epsilon}$$

Cusp model by
E. S. Swanson, arXiv:1504.07952
(see also PRD91, 034009 (2015))

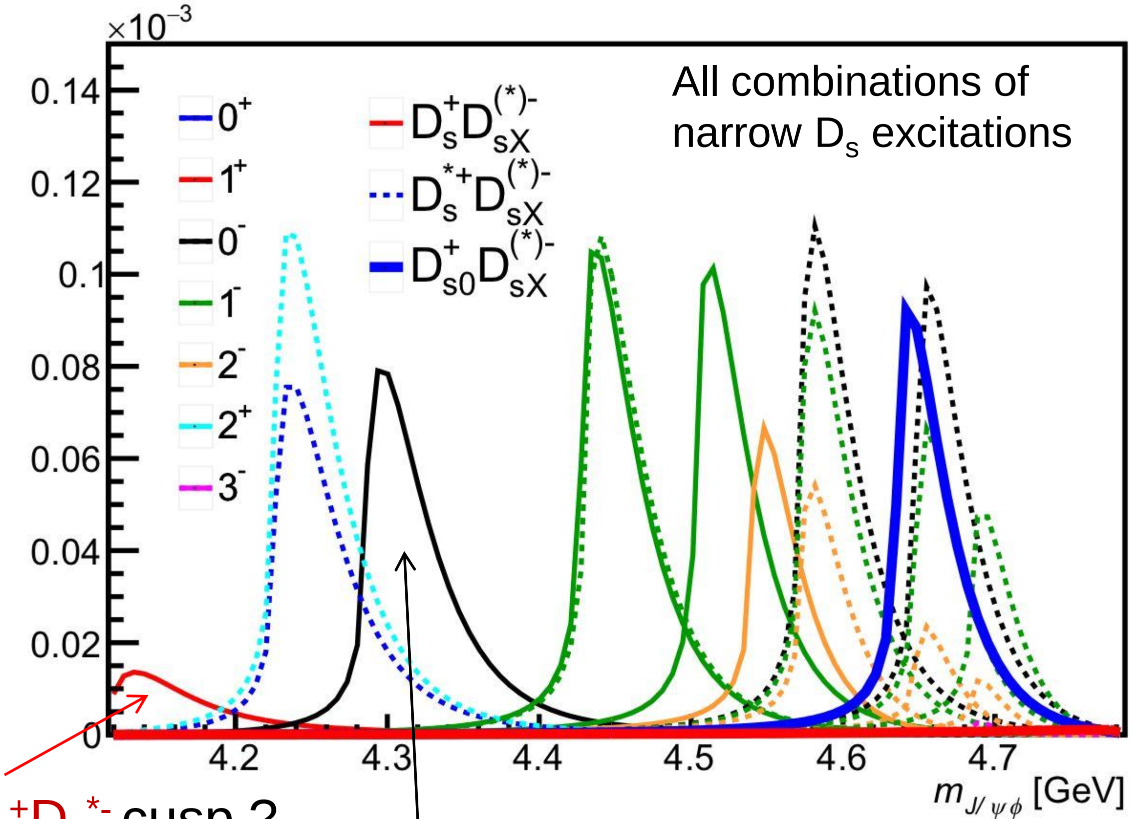
Argand diagram: Breit-Wigner vs cusp



Different shape and phase running with mass

Cusps

- Cusp peaks at the sum of masses of the virtual narrow-$D_{sX}^{(*)}$ pairs.
- Width of cusp in Swanson model is controlled with a free parameter (β_0)
- J^P of cusp determined by J^Ps of virtual D_s pairs (cusps occur in S-wave)

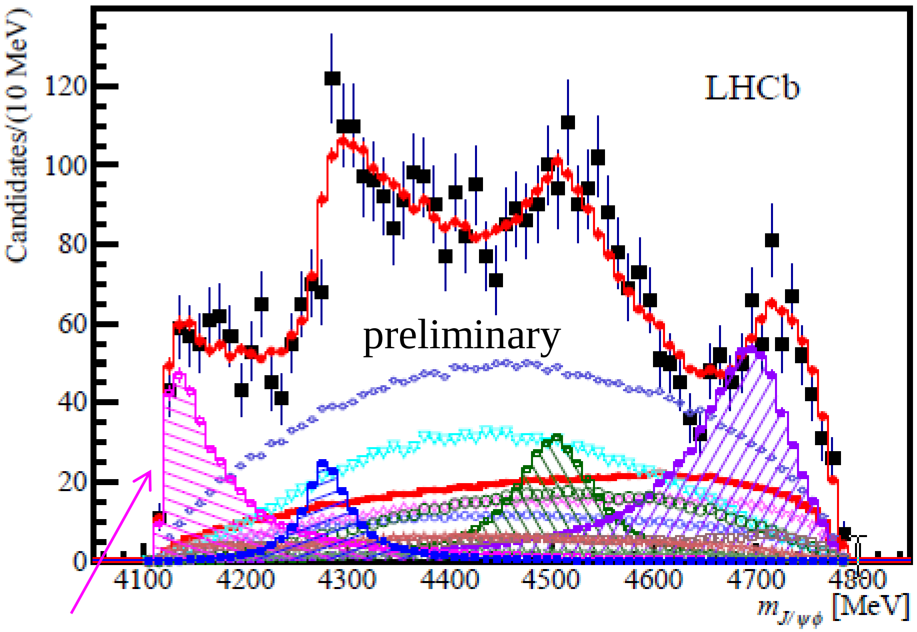


(β_0=0.3 GeV)

Is X(4140) a $D_s^+D_s^{*-}$ cusp ?
Right $J^P=1^+$

Is X(4274) a $D_s^+D_{s0}^{*-}$ cusp ?
Wrong $J^P=0^-$

Is X(4140) a $D_s^+ D_s^{*-}$ cusp ?



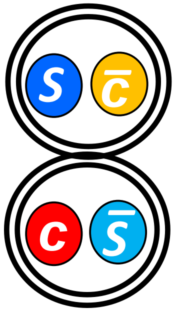
$D_s^+ D_s^{*-}$ cusp

β_0=297±20 MeV
vs 300 MeV used by Swanson

- The cusp is preferred by 1.6-3σ over the Breit-Wigner amplitude for X(4140) from the fit likelihood ratio
- No success in describing any other $J/\psi\phi$ mass structures as a cusp

Theoretical interpretations of X(4140),X(4274)

Molecular models

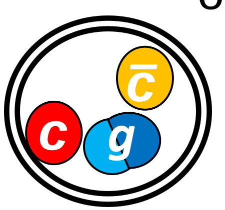


- The determination of the quantum numbers of X(4140) as $J^{PC}=1^{++}$ rules out many interpretations. Namely, 0^{++} or 2^{++} $D_s^*\overline{D}_s^*$ molecules. The large width is also not expected for true molecular bound states.

- However, X(4140) may be a 1^{++} $D_s\overline{D}_s^*$ cusp (form of rescattering)

Hybrid models

- Hybrid charmonium states proposed for X(4140) would have $J^{PC}=1^{-+}$. Thus they are also ruled out.



Tightly-bound tetraquark models

- There are tetraquark models which predict states with $J^{PC}=0^{-+}$, 1^{-+} or 0^{++}, 2^{++} near X(4140); these can be ruled out.

- A tetraquark model implemented by Stancu [JP G37, 075017 (2010), arXiv:0906.2485] correctly assigns 1^{++} to X(4140) and predicts a second 1^{++} state at a mass not much higher than X(4274)

- A Lattice calculation by Padmanth et al [PRD92, 034501 (2015)], based on a diquark tetraquark model, found no evidence for a 1^{++} tetraquark below 4.2 GeV

Summary

- We have demonstrated that exotic hadron contributions are present in $B^0 \to \psi'\pi^+K^-$ and $\Lambda_b \to J/\psi\ p\ K^-$ decays with the model independent approach.
- Using amplitude analysis we have confirmed $Z_c(4430)^+ \to \psi'\pi^+$ in $B^0 \to \psi'\pi^+K^-$ and demonstrated its resonant character with Argand diagram.
- Using amplitude analysis we have observed two pentaquark $P_c(4450)^+$,$P_c(4380)^+ \to J/\psi\ p$ candidates in $\Lambda_b \to J/\psi\ p\ K^-$

preliminary

- Using amplitude analysis we have found **3.1σ evidence for exotic hadron contributions in $\Lambda_b \to J/\psi\ p\ \pi^-$**, but confusion between $P_c(4450)^+$,$P_c(4380)^+$ and $Z_c(4200)^- \to J/\psi\ \pi^-$ contributions prevents establishing either pentaquark or $Z_c(4200)^-$ in these decays. We have demonstrated that the **$\Lambda_b \to J/\psi\ p\ \pi^-$ data are consistent with the $P_c(4450)^+$,$P_c(4380)^+$ rate measured in $\Lambda_b \to J/\psi\ p\ K^-$ and Cabibbo suppression.**

preliminary

- The first full amplitude analysis of **$B^+ \to J/\psi\ \phi\ K^+$** decays has been performed. The data cannot be described by a model that contains only excited kaon states decaying into $\phi\ K^+$ and **four $J/\psi\ \phi$ structures are observed, each with significance over 5σ. The quantum numbers of these structures are determined with significance of at least 4σ.** The lightest is best described as a $D_s^+D_s^{*-}$ cusp, but a resonant interpretation is also possible with mass consistent with, but width much larger than, previous measurements of the claimed X(4140) state. We have also contributed to kaon spectroscopy for higher-mass excitations.

BACKUP SLIDES

Confusing experimental situation concerning X→J/ψϕ states

X(4140) summary

Year	Experiment luminosity	Ref	$B \to J/\psi\,\phi K$ statistics	$X(4140)$ peak mass [MeV]	width [MeV]	sign.	fraction %
2008	CDF 2.7 fb^{-1}	PRL 102,242002	58 ± 10	$4143.0 \pm 2.9 \pm 1.2$	$11.7\,^{+8.3}_{-5.0} \pm 3.7$	3.8σ	
2009	Belle	LP2009 (unpub.)	325 ± 21	4143.0 fixed	11.7 fixed	1.9σ	
2011	CDF 6.0 fb^{-1}	arXiv:1101.6058 (unpub.)	115 ± 12	$4143.4\,^{+2.9}_{-3.0} \pm 0.6$	$15.3\,^{+10.4}_{-6.1} \pm 2.5$	5.0σ	$14.9 \pm 3.9 \pm 2.4$
2011	LHCb 0.37 fb^{-1}	PRD85, 091103	346 ± 20	4143.4 fixed	15.3 fixed	1.4σ	< 7 @ 90%CL
2013	CMS 5.2 fb^{-1}	PL, B734, 261	2480 ± 160	$4148.0 \pm 2.4 \pm 6.3$	$28\,^{+15}_{-11} \pm 19$	5.0σ	10 ± 3 (stat.)
2013	D0 10.4 fb^{-1}	PRD89, 012004	215 ± 37	$4159.0 \pm 4.3 \pm 6.6$	$19.9 \pm 12.6\,^{+1.0}_{-8.0}$	3.1σ	$21 \pm 8 \pm 4$
2014	BaBar 422 fb^{-1}	PRD91, 012003	189 ± 14	4143.4 fixed	15.3 fixed	1.6σ	< 13.3 @ 90%CL
2015	D0 10.4 fb^{-1}	PRL, 115, 232001	$p\bar{p} \to J/\psi\,\phi ...$	$4152.5 \pm 1.7\,^{+6.2}_{-5.4}$	$16.3 \pm 5.6 \pm 11.4$	4.7σ (5.7σ)	
Average				4146.9 ± 2.3	17.8 ± 6.8		

X(4274-4351) summary

Year	Experiment luminosity	Ref	$B \to J/\psi\,\phi K$ statistics	$X(4274-4351)$ peaks(s) mass [MeV]	width [MeV]	sign.	fraction [%]
2011	CDF 6.0 fb^{-1}	arXiv:1101.6058 (unpub.)	115 ± 12	$4274.4\,^{+8.4}_{-6.7} \pm 1.9$	$32.3\,^{+21.9}_{-15.3} \pm 7.6$	3.1σ	
2011	LHCb 0.37 fb^{-1}	PRD85, 091103	346 ± 20	4274.4 fixed	32.3 fixed		< 8 @ 90%CL
2013	CMS 5.2 fb^{-1}	PL, B734, 261	2480 ± 160	$4313.8 \pm 5.3 \pm 7.3$	$38\,^{+30}_{-15} \pm 16$		
2013	D0 10.4 fb^{-1}	PRD89, 012004	215 ± 37	4328.5 ± 12.0	30 fixed		
2014	BaBar 422 fb^{-1}	PRD91, 012003	189 ± 14	4274.4 fixed	32.3 fixed	1.2σ	< 18.1 @ 90%CL
2010	Belle 825 fb^{-1}	PRL 104, 112004	$\gamma\gamma \to J/\psi\,\phi$	$4350.6\,^{+4.6}_{-5.1} \pm 0.7$	$13\,^{+18}_{-9} \pm 4$	3.2σ	

Amplitude fit results to $B^+ \to J/\psi\, \phi\, K^+$

LHCb Preliminary!

Contri-bution	sign. or Ref.	M_0 MeV	Γ_0 MeV	F.F. %	f_L	$f_\perp$
		Fit results				
all $K(1^+)$	8.0σ			$42\pm 8^{+5}_{-9}$		
$NR_{\phi K}$				$16\pm13^{+35}_{-6}$	0.52 ± 0.29	0.21 ± 0.16
$K(1^+)$	7.6σ	$1793\pm59^{+153}_{-101}$	$365\pm157^{+138}_{-215}$	$12\pm10^{+17}_{-6}$	0.24 ± 0.21	0.37 ± 0.17
2^1P_1	[45]	1900				
$K_1(1650)$	[36]	1650 ± 50	150 ± 50			
$K'(1^+)$	1.9σ	$1968\pm65^{+70}_{-172}$	$396\pm170^{+174}_{-178}$	$23\pm20^{+31}_{-29}$	0.04 ± 0.08	0.49 ± 0.10
2^3P_1	[45]	1930				
all $K(2^-)$	5.6σ			$11\pm 3^{+2}_{-5}$		
$K(2^-)$	5.0σ	$1777\pm35^{+122}_{-77}$	$217\pm116^{+221}_{-154}$		0.64 ± 0.11	0.13 ± 0.13
1^1D_2	[45]	1780				
$K_2(1770)$	[36]	1773 ± 8	188 ± 14			
$K'(2^-)$	3.0σ	$1853\pm27^{+18}_{-35}$	$167\pm 58^{+83}_{-72}$		0.53 ± 0.14	0.04 ± 0.08
1^3D_2	[45]	1810				
$K_2(1820)$	[36]	1816 ± 13	276 ± 35			
$K^*(1^-)$	8.5σ	$1722\pm20^{+33}_{-109}$	$354\pm 75^{+140}_{-181}$	$6.7\pm1.9^{+3.2}_{-3.9}$	0.82 ± 0.04	0.03 ± 0.03
1^3D_1	[45]	1780				
$K^*(1680)$	[36]	1717 ± 27	322 ± 110			
$K^*(2^+)$	5.4σ	$2073\pm94^{+245}_{-240}$	$678\pm311^{+1153}_{-559}$	$2.9\pm0.8^{+1.7}_{-0.7}$	0.15 ± 0.06	0.79 ± 0.08
2^3P_2	[45]	1940				
$K_2^*(1980)$	[36]	1973 ± 26	373 ± 69			
$K(0^-)$	3.5σ	$1874\pm43^{+59}_{-115}$	$168\pm 90^{+280}_{-104}$	$2.6\pm1.1^{+2.3}_{-1.8}$	1.0	
3^1S_0	[45]	2020				
$K(1830)$	[36]	~ 1830	~ 250			
All $X(1^+)$				$16\pm3^{+6}_{-2}$		
$X(4140)$	8.4σ	$4146.5\pm4.5^{+4.6}_{-2.8}$	$83\pm21^{+21}_{-14}$	$13\pm3.2^{+4.8}_{-2.0}$		
ave.	Table 1	4146.9 ± 2.3	17.8 ± 6.8			
$X(4274)$	6.0σ	$4273.3\pm8.3^{+17.2}_{-3.6}$	$56\pm11^{+8}_{-11}$	$7.1\pm2.5^{+3.5}_{-2.4}$		
CDF	[27]	$4274.4^{+8.4}_{-6.7}\pm1.9$	$32^{+22}_{-15}\pm8$			
CMS	[24]	$4313.8\pm5.3\pm7.3$	$38^{+30}_{-15}\pm16$			
All $X(0^+)$				$28\pm 5^{+7}_{-7}$		
$NR_{J/\psi\,\phi}$	6.4σ			$46\pm11^{+11}_{-21}$		
$X(4500)$	6.1σ	$4506\pm11^{+12}_{-15}$	$92\pm21^{+21}_{-20}$	$6.6\pm2.4^{+3.5}_{-2.3}$		
$X(4700)$	5.6σ	$4704\pm10^{+14}_{-24}$	$120\pm31^{+42}_{-33}$	$12\pm 5^{+9}_{-5}$		

$$f_L = \frac{\left|A_{\lambda=0}^{B\to J/\psi K^*}\right|^2}{\left|A_{\lambda=-1}^{B\to J/\psi K^*}\right|^2+\left|A_{\lambda=0}^{B\to J/\psi K^*}\right|^2+\left|A_{\lambda=+1}^{B\to J/\psi K^*}\right|^2},$$

$$A_\perp^{B\to J/\psi K^*} = \frac{A_{\lambda=+1}^{B\to J/\psi K^*}-A_{\lambda=-1}^{B\to J/\psi K^*}}{\sqrt{2}},$$

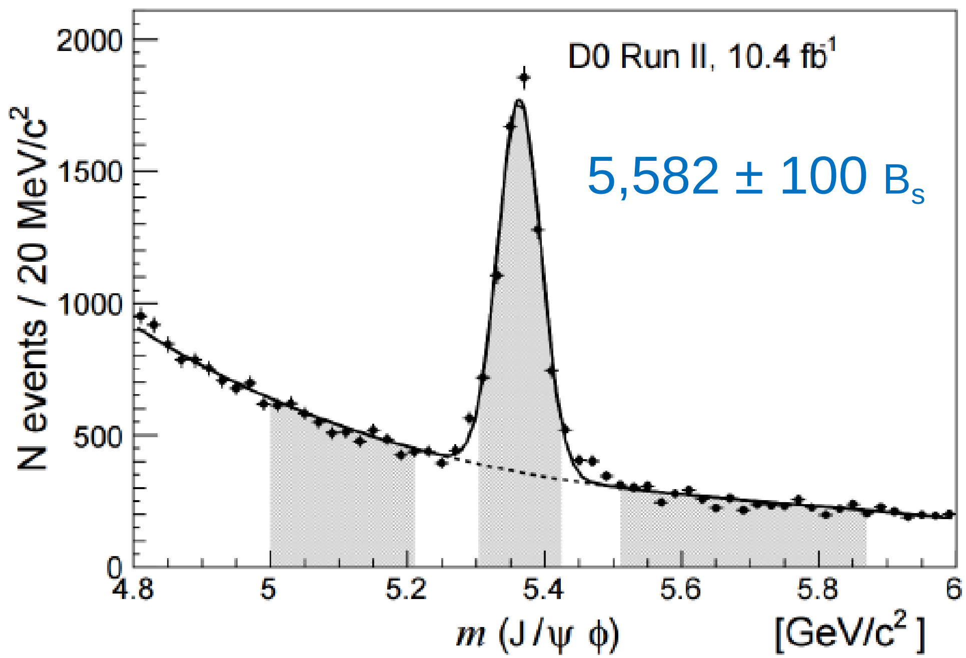
$$f_\perp = \frac{\left|A_\perp^{B\to J/\psi K^*}\right|^2}{\left|A_{\lambda=-1}^{B\to J/\psi K^*}\right|^2+\left|A_{\lambda=0}^{B\to J/\psi K^*}\right|^2+\left|A_{\lambda=+1}^{B\to J/\psi K^*}\right|^2}.$$

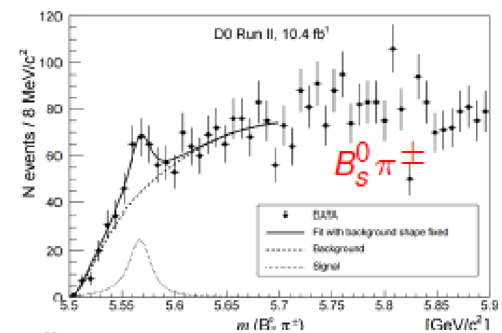
LHCb-PAPER-2016-019 in preparation

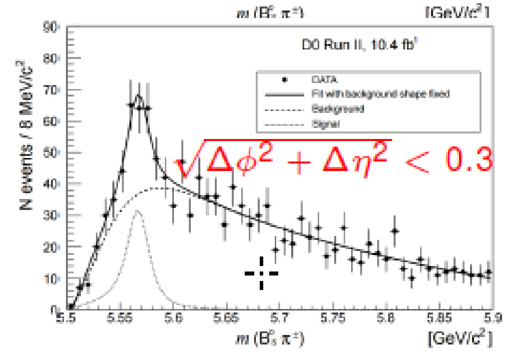
$X(5568)^{\pm} \to B_s \pi^{\pm}$ from D0

- $X(5568)^{\pm} \to B_s^0 \pi^{\pm}$ decay reported by D0 in February with a significance of 5.1σ
- Signal implies large production rate within D0 acceptance

$$\rho_X^{\mathrm{D0}} \equiv \left.\frac{\sigma(p\bar{p} \to X + \text{anything}) \times \mathcal{B}(X \to B_s^0 \pi)}{\sigma(p\bar{p} \to B_s^0 + \text{anything})}\right|_{\mathrm{D0Acc.}}$$
$$= (8.6 \pm 1.9 \pm 1.4)\%$$





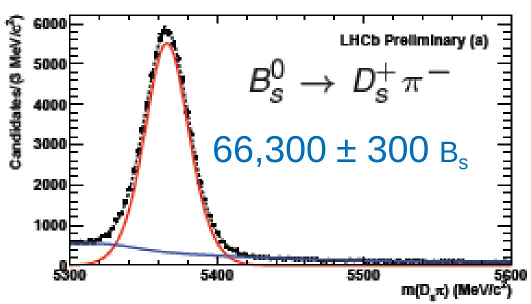


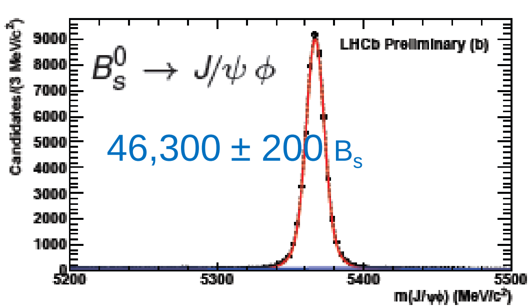
arXiv:1602.07588

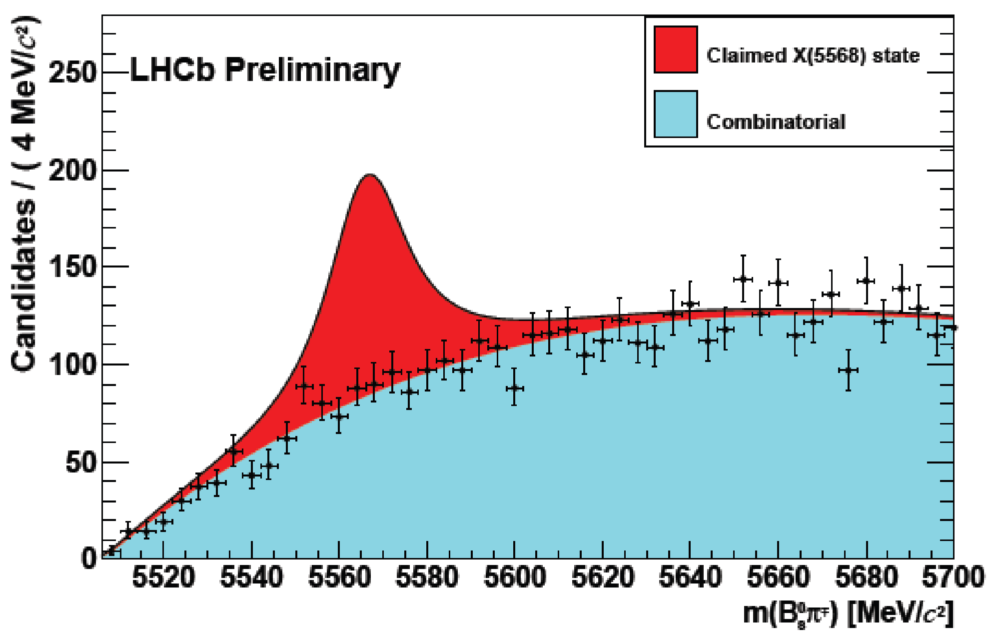
No $X(5568)^{\pm} \to B_s \pi^{\pm}$ in LHCb data

- LHCb search first reported at Moriond
- Study based on large clean samples of B_s^0 decays
- (Right) no peak observed in $m(B_s^0 \pi)$ from $X(5568)$
- Upper limits set on production in the LHCb acceptance

$$\rho_{X,}^{\mathrm{LHCb}} < \begin{cases} 0.009\ (0.010)\ @\ 90\ (95)\ \%\ \mathrm{CL} \\ 0.016\ (0.018)\ @\ 90\ (95)\ \%\ \mathrm{CL} \end{cases} \quad p_{\mathrm{T}\,B_s^0} > \begin{cases} 5\,\mathrm{GeV}/c \\ 10\,\mathrm{GeV}/c \end{cases}$$



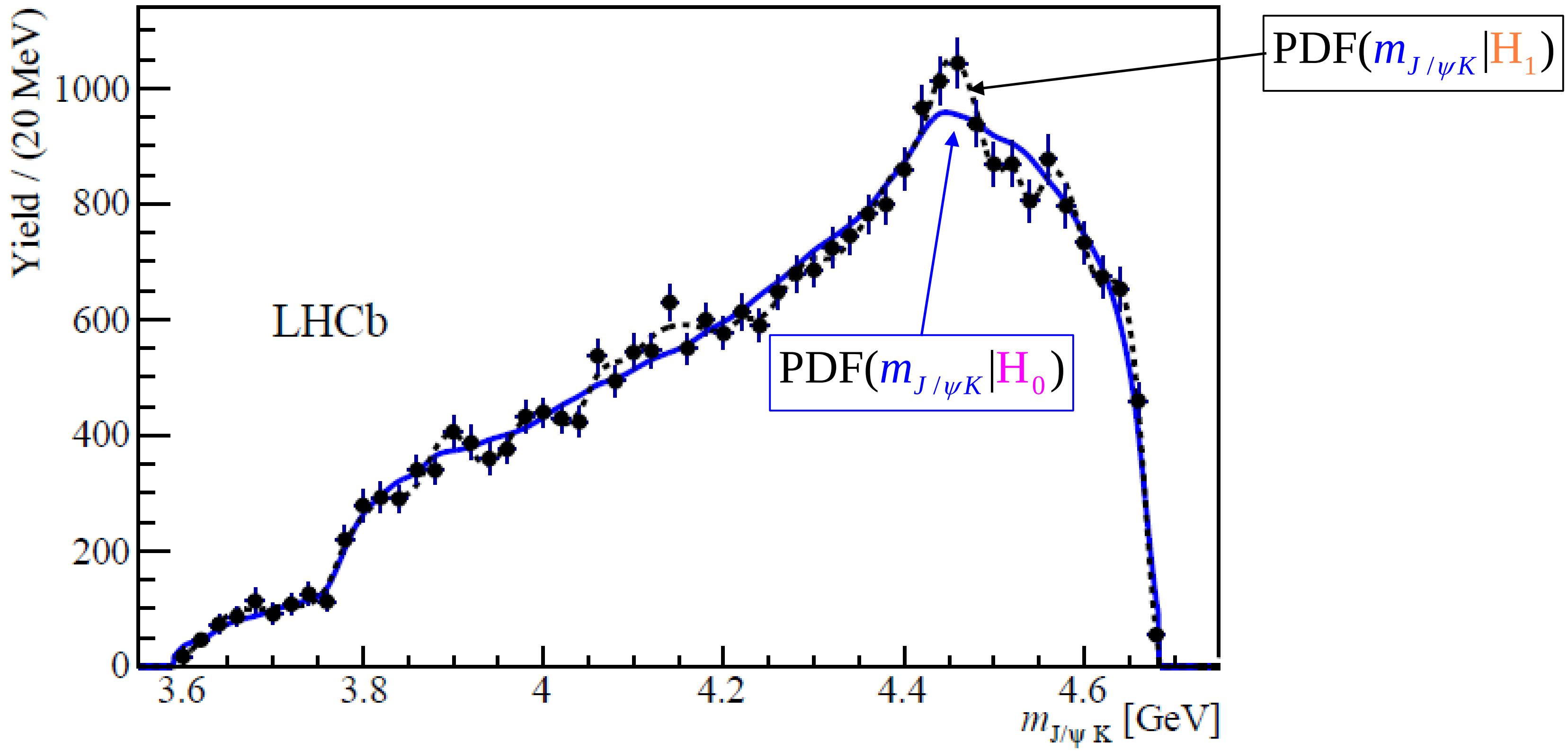




LHCb-CONF-2016-004

Model independent analysis: $J/\psi K^-$

LHCb-PAPER-2016-009 arXiv:1604.05708



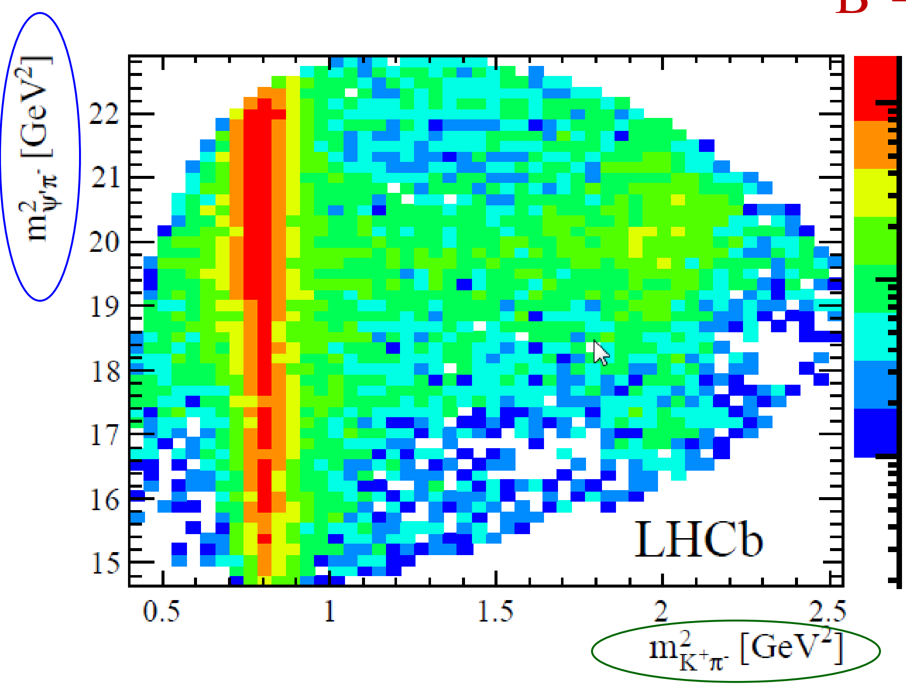
- Rule out the Λ^*-only hypothesis at 5.3σ (vs 9σ using $m_{J/\psi p}$)
- Points to exotic structructures in $J/\psi p$ being more likely than in $J/\psi K$

Rectangular Dalitz plane

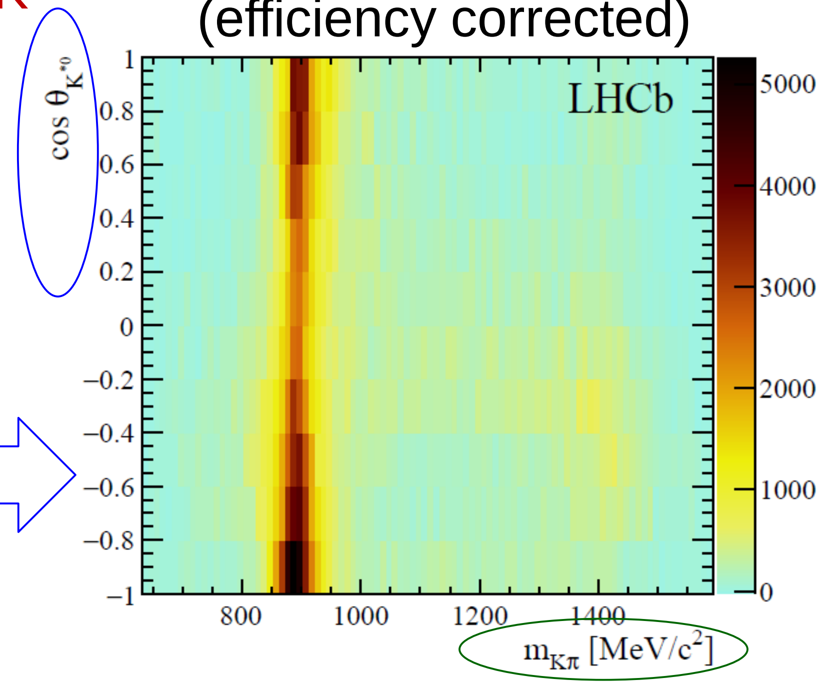
LHCb-PAPER-2015-038
LHCb PRD 92, 112009 (2015)

- For fixed $m_{K\pi}$ there is one-to-one relation between $m_{\psi\pi}$ and $\cos\theta_{K*}$

$B^0 \to \psi' \pi^+ K^-$



(efficiency corrected)



$$m^2_{\psi\pi} = m^2_{\psi} + m^2_{\pi} + 2(E_{\psi}E_{\pi} + p_{\psi}p_{\pi}\cos\theta_{K*})$$

$$p^2_{\psi} = E^2_{\psi} - m^2_{\psi} \qquad p^2_{\pi} = E^2_{\pi} - m^2_{\pi}$$

$$E_{\psi} = \frac{m^2_B - m^2_{\psi} - m^2_{K\pi}}{2m_{K\pi}} \qquad E_{\pi} = \frac{m^2_{K\pi} + m^2_{\pi} - m^2_{K}}{2m_{K\pi}}$$

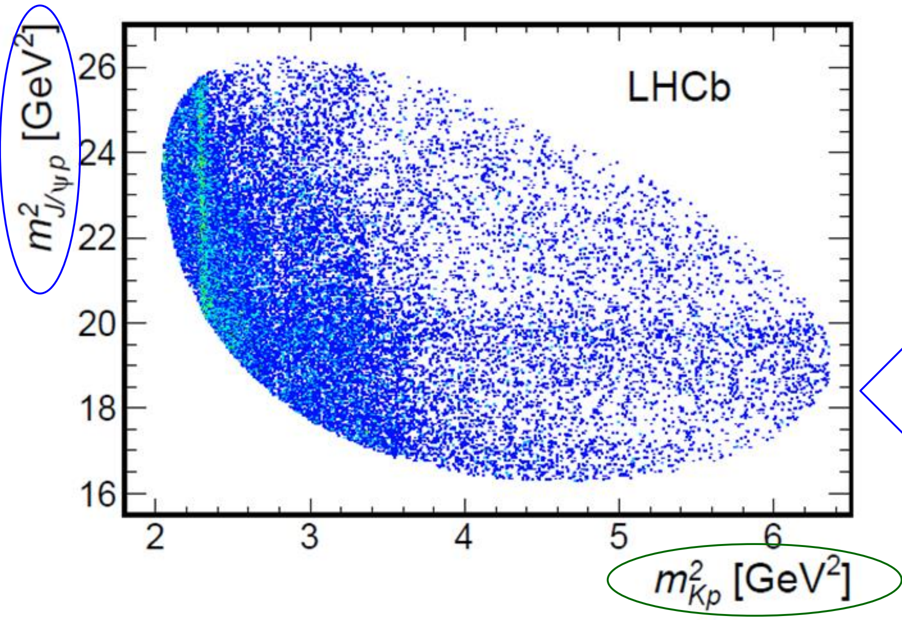
new

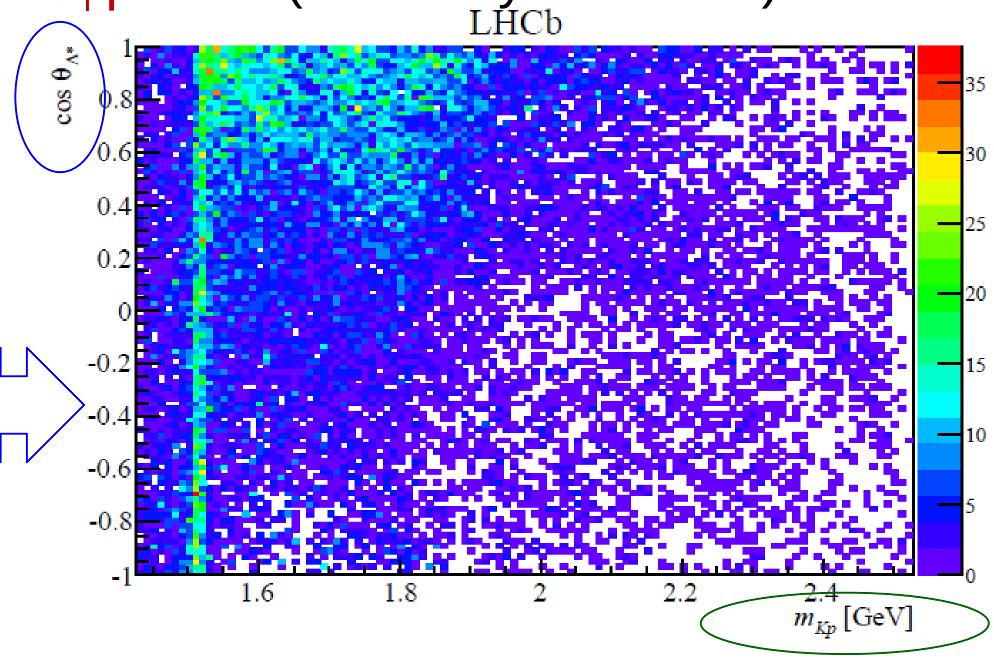
Rectangular Dalitz plane

LHCb-PAPER-2016-009 arXiv:1604.05708

- For fixed m_{Kp} there is one-to-one relation between $m_{J/\psi p}$ and $\cos\theta_{\Lambda*}$

$\Lambda_b^0 \to J/\psi p K^-$ (efficiency corrected)





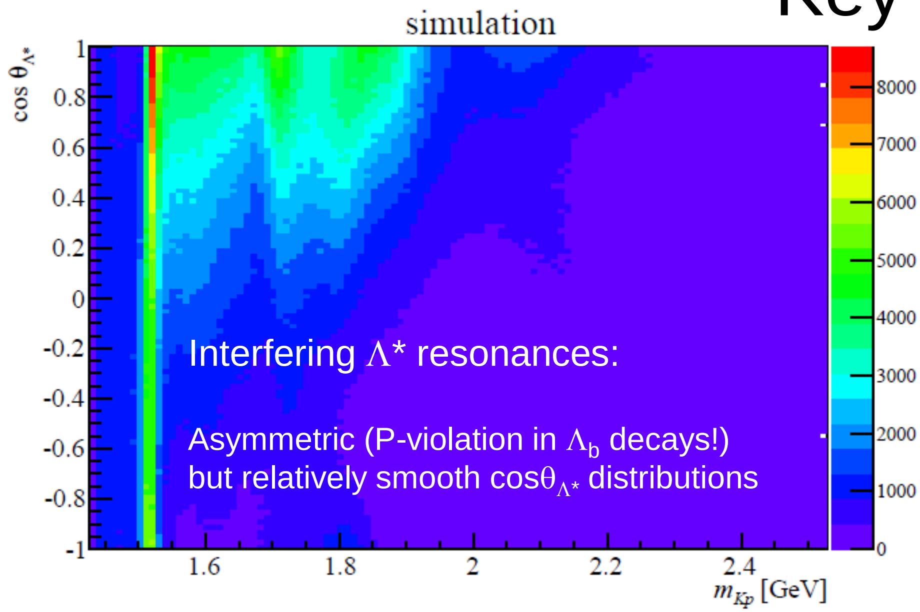
$$m^2_{\psi p} = m^2_{\psi} + m^2_{p} + 2(E_{\psi}E_{p} + p_{\psi}p_{p}\cos\theta_{\Lambda*})$$

$$p^2_{\psi} = E^2_{\psi} - m^2_{\psi} \qquad p^2_{p} = E^2_{p} - m^2_{p}$$

$$E_{\psi} = \frac{m^2_{\Lambda_b} - m^2_{\psi} - m^2_{Kp}}{2m_{Kp}} \qquad E_{p} = \frac{m^2_{Kp} + m^2_{p} - m^2_{K}}{2m_{Kp}}$$

Legendre Moments

Key idea:



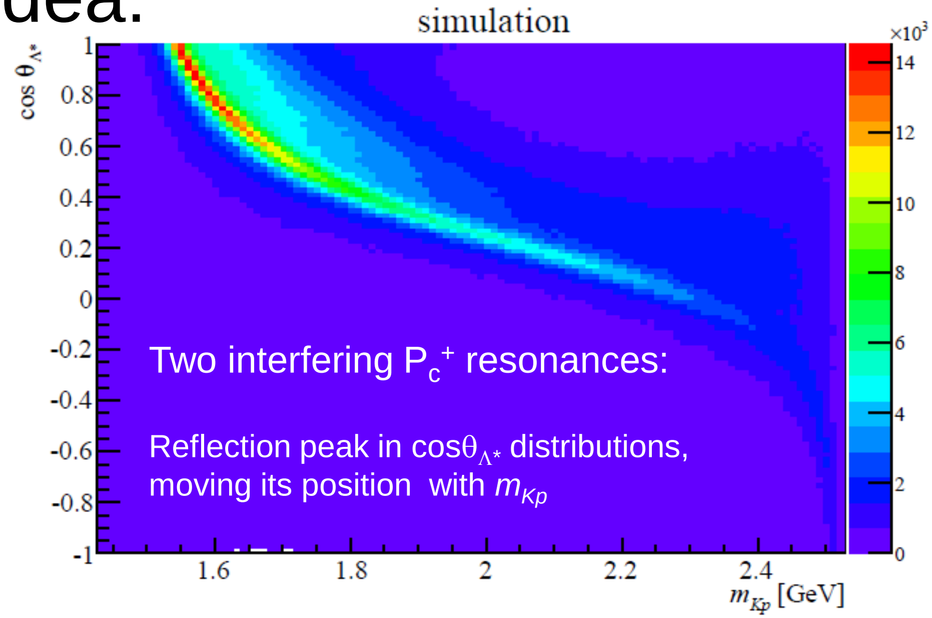
K*/Λ* can contribute only to low-rank moments

$l_{\max} = J_1 + J_2$ for interfering resonances

In K*/Λ*-only hypothesis (H_0)

$$l_{\max} = 2J_{\max}$$

J_{max} is the highest spin of K*/Λ* resonance possible

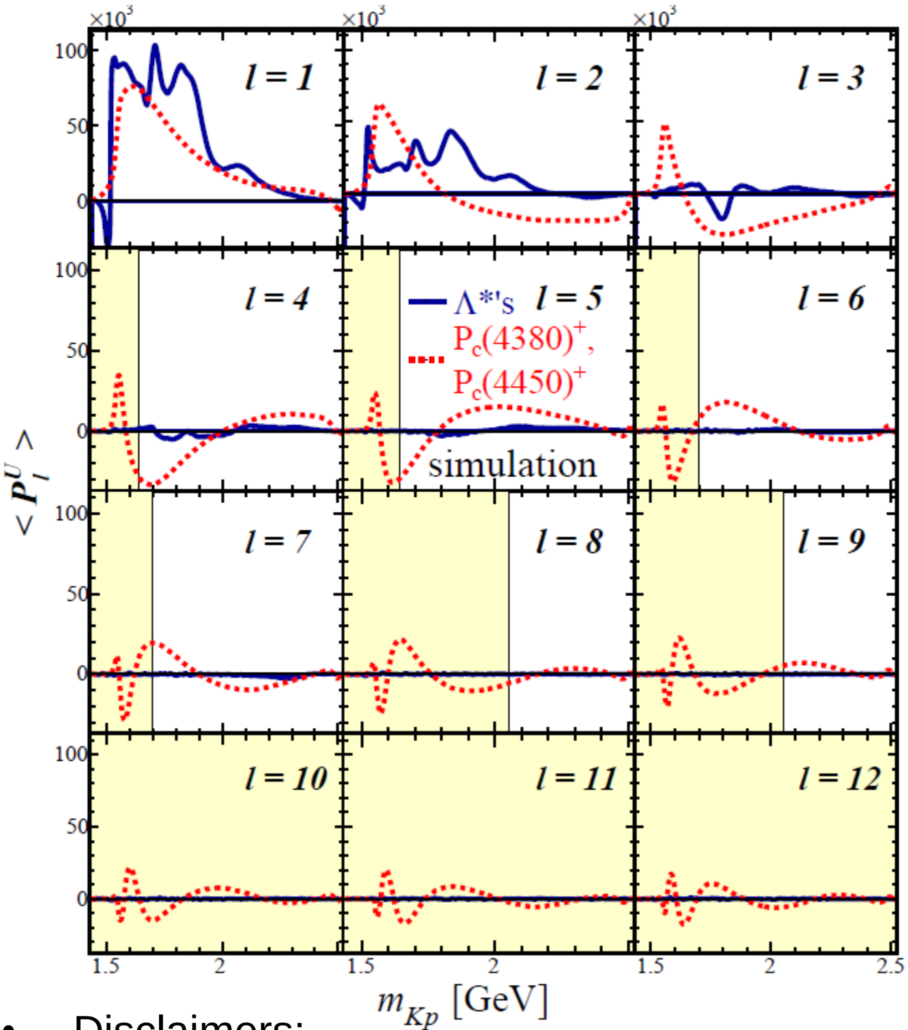


Reflections of exotic hadrons can contribute to low **and high** rank moments:

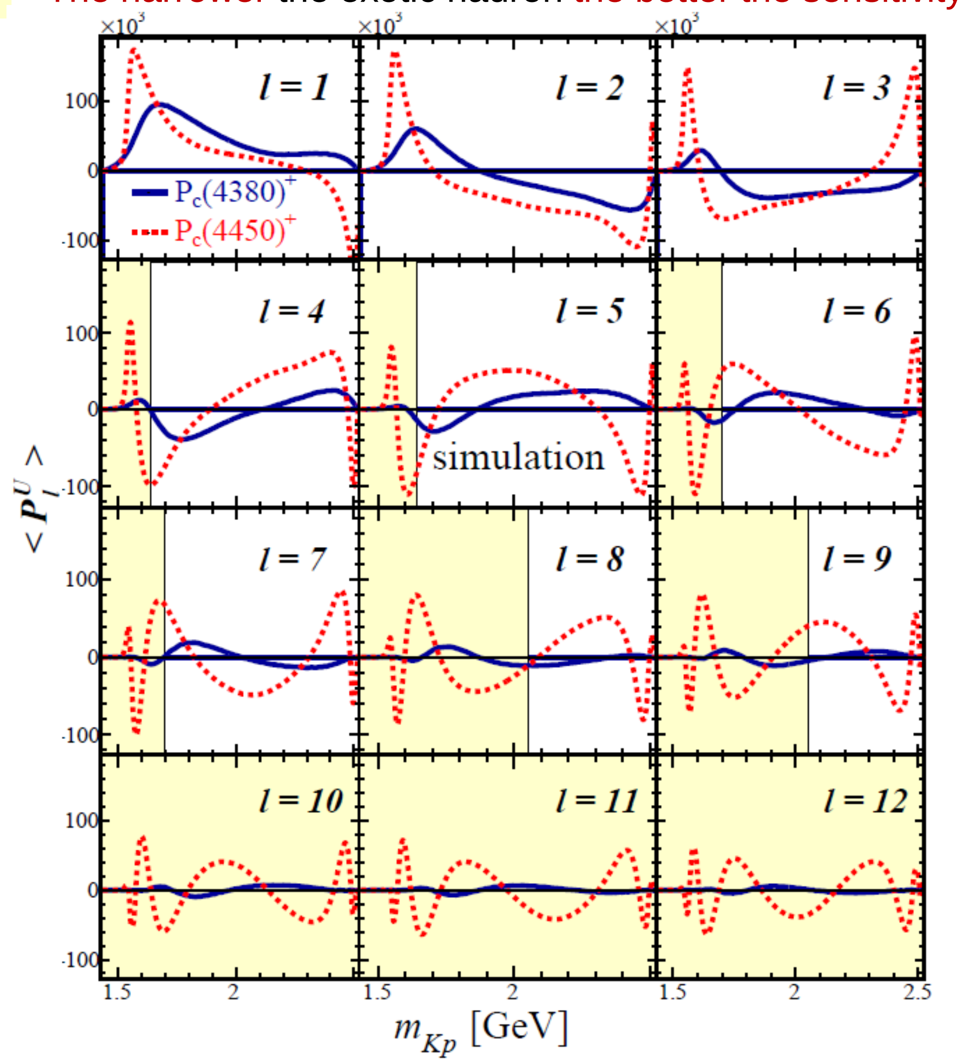
- **Detecting non-zero moments above l_{max} signals presence of exotics**
- The narrower the peak the higher the l_{max} required. **The sensitivity is better for narrower exotic hadrons.**
- Exotic hadron contributions spread over wide range of $m_{K\pi}/m_{Kp}$. An effective way of testing H_0 is to **aggregate the information about cos$\theta_{K\pi/Kp}$ moments in a function of $m_{\psi'\pi}/m_{J/\psi p}$.**

Illustrations using amplitude models of $\Lambda_b^0 \to J/\psi p K^-$

Only exotic hadrons can contribute to excluded moments



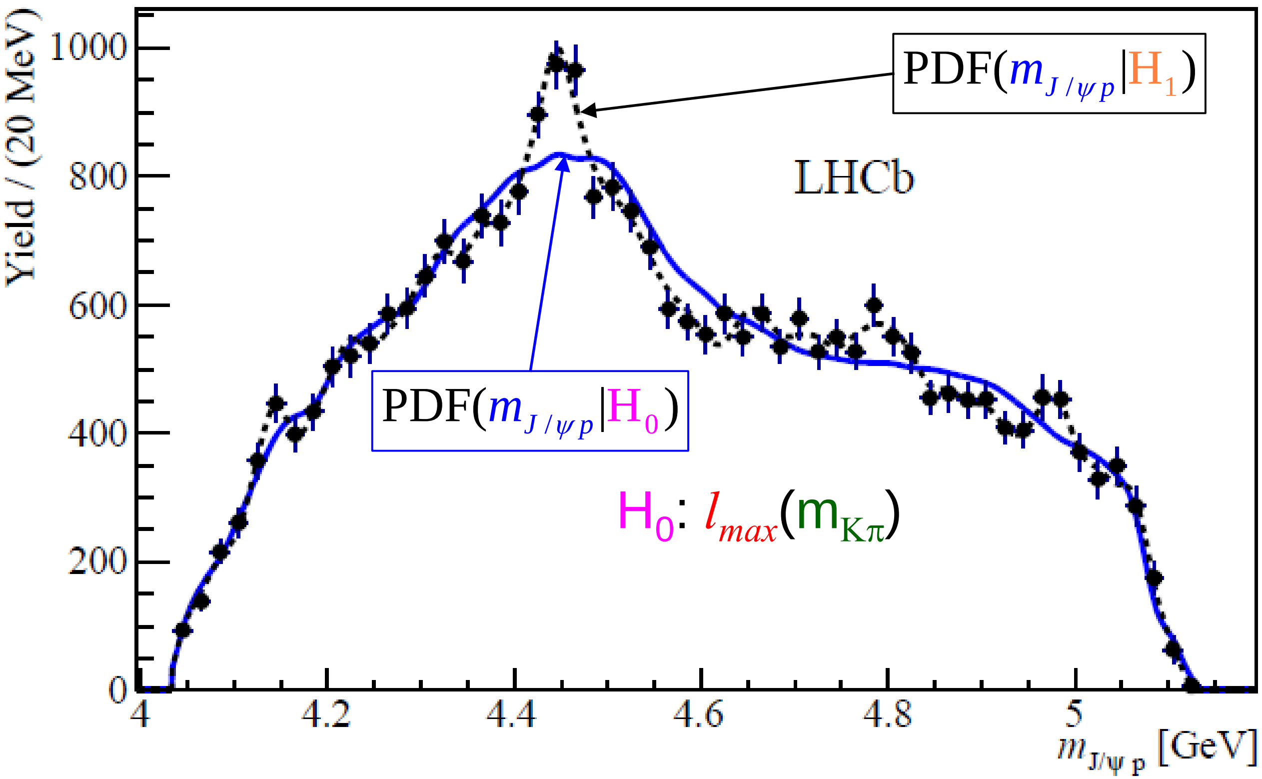
The narrower the exotic hadron the better the sensitivity



- Disclaimers:
  - these are high statistics simulations to eliminate any statistical fluctuations (vertical scale is arbitrary)
  - exotic hadron contributions are usually only a few % fit fractions, thus the amplitudes of the red curves is expected to be small in the real data

In preparation for quantitative test

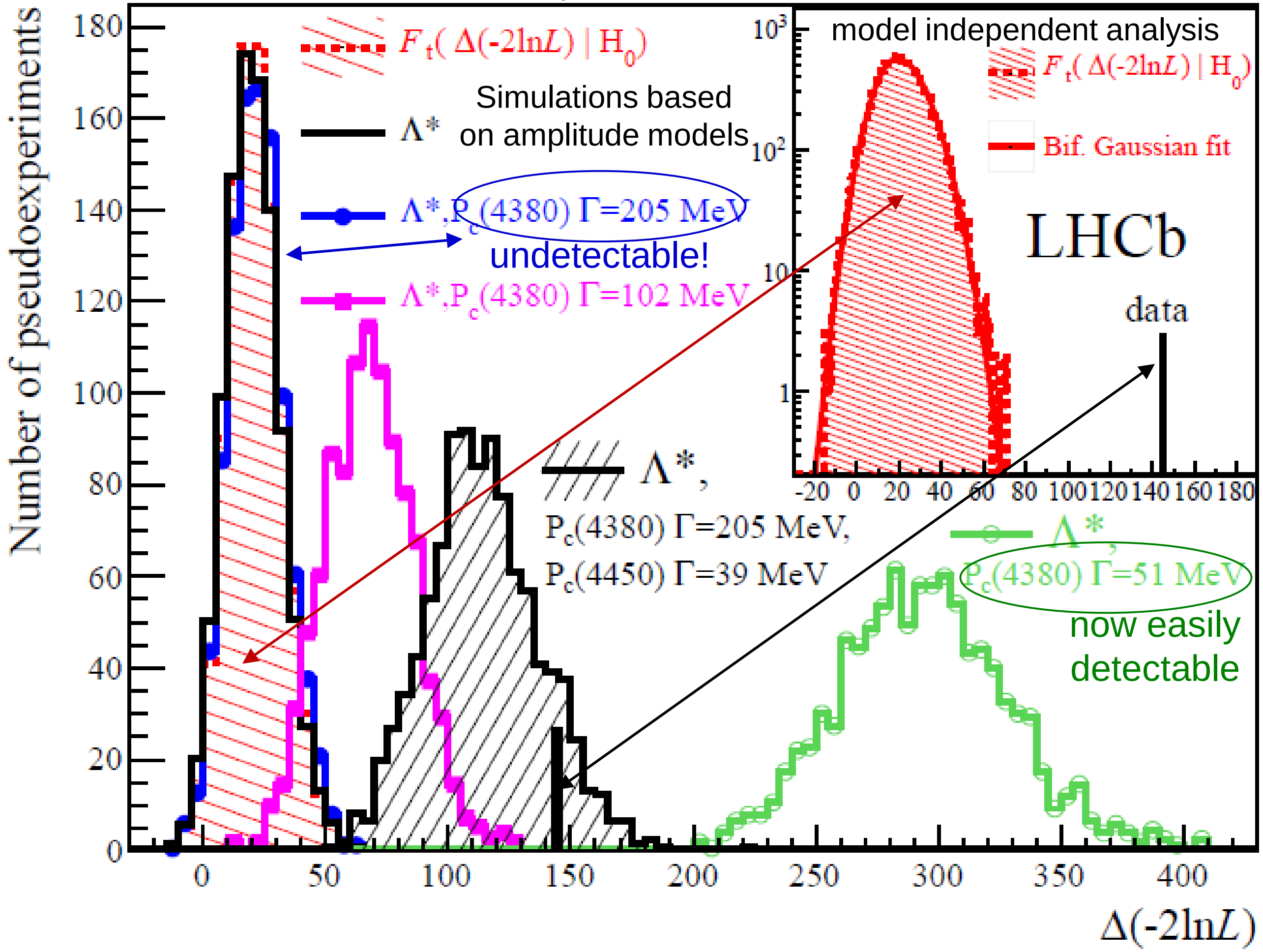
H_1: l_{max}^{H1}=31



- Creating H_1 hypothesis helps since exotic hadrons will generate higher moments than can be accommodated in H_0 (Λ*-only hypothesis), but not very high moments:
  - Very high moments driven by statistical fluctuations
  - Looking for significance of moments with ranks just above l_{max}($m_{K\pi}$) is more sensitive than looking at any rank moments above l_{max}($m_{K\pi}$)

The data vs amplitude simulations

(F_t means PDF)



- The data point falls in the region predicted by the full amplitude model (i.e. Λ*s+2P$_c$s) [speaks to the quality of the amplitude model]
- The sensitivity of the method depends dramatically on a P$_c$ width; P$_c$(4380)$^+$ does not contribute much to the model independent result [know it from amplitude simulations]