Exotic Mesons and Baryons in Lattice QCD

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Plan

- Introduction and background
- A consumers guide to Lattice QCD
  - compromises and consequences
- Discussion and selected results
  - new ideas in lattice for progress
  - exotic and hybrid mesons and baryons
    - **excited, exotic and hybrid "single-hadron" states**
    - **scattering states eg X,Y, Z - progress and challenges**
- Summary

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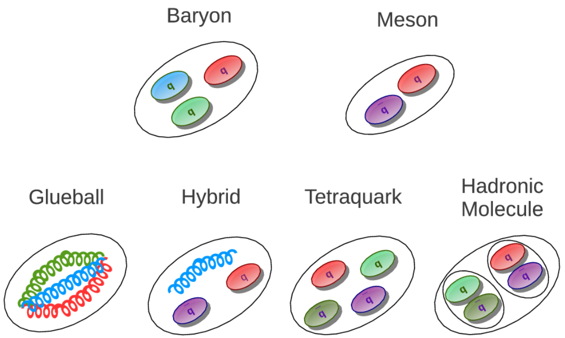
Many details and topics omitted for time constraints - APOLOGIES!

Objects of interest



from M. Cleven thesis

Exotic?

- States **not** in the natural spin-parity series i.e. with $J^{PC} = 0^{+-}, 1^{-+}, 2^{+-}, 3^{-+}, \ldots$
- States **not** simple quark model $q\bar{q}$ or qqq.

Lattice QCD and Exotics

Lattice QCD

- A systematically-improvable non-perturbative formulation of **QCD**
  - a well-defined theory with the lattice as a UV regulator
- Arbitrary precision is in principle possible
  - of course algorithm and theory “wrinkles” make this challenging!
- Starts from first principles - i.e. from the **QCD** Lagrangian with inputs m_q, β.

Challenges

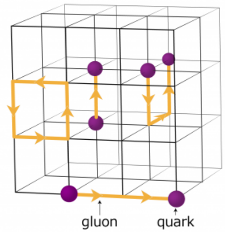
- Exotic states are generally above or close to decay thresholds
- States with quark-model J^{PC} can mix and must be disentangled
- Typically requires significant precision - especially glueballs!

A lattice QCD primer

Start from the QCD Lagrangian:

$$\mathcal{L} = \bar{\Psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\Psi - \frac{1}{4}G^{a}_{\mu\nu}G^{\mu\nu}_{a}$$

Gluon fields on links of a hypercube;
Quark fields on sites: approaches to fermion discretisation - Wilson, Staggered, Overlap.;
Derivatives $\longrightarrow$ finite differences.



Solve the QCD path integral on a finite lattice with spacing $a \neq 0$ estimated stochastically by Monte Carlo. Can only be done effectively in a Euclidean space-time metric (no useful importance sampling weight for the theory in Minkowski space).

Observables determined from (Euclidean) path integrals of the QCD action

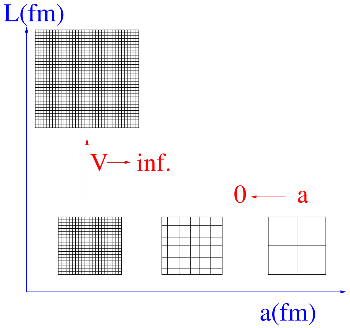
$$\langle \mathcal{O} \rangle = 1/Z \int \mathcal{D}U\mathcal{D}\bar{\Psi}\mathcal{D}\Psi\ \mathcal{O}[U, \bar{\Psi}, \Psi]e^{-S[U,\bar{\Psi},\Psi]}$$

Compromises and the Consequences

1. Working in a finite box at finite grid spacing

- Identify a "scaling window" where physics doesn't change with a or V. Recover continuum **QCD** by extrapolation.

A costly procedure but a regular feature in lattice calculations now



2. Simulating at physical quark masses

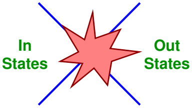
- Computational cost grows rapidly with decreasing quark mass $\longrightarrow m_q = m_{u,d}$ costly. Care needed vis location of decay thresholds and identification of resonances.
- c-quark can be handled relativistically. b-quark with: NRQCD, FNAL etc.

Better algorithms for physical light quarks and/or chiral extrapolation. Relativistic m_b in reach

2. Breaking symmetry

- Lorentz symmetry broken at $a \neq 0$ so $SO(4)$ rotation group broken to discrete rotation group of a hypercube.
  Classify states by irreps of O_h and relate by subduction to J values of O_3. Lots of degeneracies in subduction for $J \geq 2$ and physical near-degeneracies. Complicates spin identification.

Spin identification at finite lattice spacing: 0707.4162, 1204.5425



3. Working in Euclidean time.

- Scattering matrix elements not directly accessible from Euclidean QFT [*Maiani-Testa theorem*]. Scattering matrix elements: asymptotic $|\text{in}\rangle$, $|\text{out}\rangle$ states:
  $\langle \text{out}|e^{i\hat{H}t}|\text{in}\rangle \rightarrow \langle \text{out}|e^{-\hat{H}t}|\text{in}\rangle$. Euclidean metric: project onto ground state. Analytic continuation of numerical correlators an ill-posed problem.

Lüscher and generalisations of: method for indirect access.

4. Quenching

No longer an issue: Simulations done with $N_f = 2, 2+1, 2+1+1$.

New (and not so new) ideas for old problems

- **Anisotropic lattices**
  - improving resolution for better measurement
- **Distillation**
  - breakthrough idea for quark propagation enabling precision spectroscopy including for isoscalars and exotics
- **Operator construction & spin id**
  - allows for robust spin assignment at finite lattice spacing and for high spins
- **Extension of scattering methodology to coupled channels**
  - Lüscher's idea from '90s extended to many scenarios enabling resonance/scattering parameters

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Together these ideas have led to rapid recent progress

Lattice Hadron Spectroscopy
precision & pioneering results

(i) Precision spectroscopy of “single-hadron” states
(ii) Exploratory studies of “exotic” and scattering states

A recipe for (meson) spectroscopy

- Construct a basis of local and non-local operators $\bar{\Psi}(x)\Gamma D_i D_j \ldots \Psi(x)$ from *distilled* fields [PRD80 (2009) 054506].
- Build a correlation matrix of two-point functions

$$C_{ij} = \langle 0|\mathcal{O}_i \mathcal{O}_j^\dagger|0\rangle = \sum_n \frac{Z_i^n Z_j^{n\dagger}}{2E_n} e^{-E_n t}$$

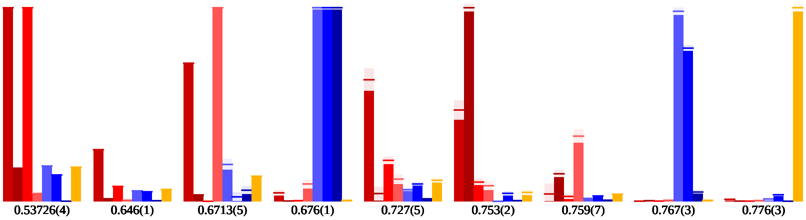
- Ground state mass from fits to $e^{-E_n t}$
- Beyond ground state: Solve generalised eigenvalue problem $C_{ij}(t)v_j^{(n)} = \lambda^{(n)}(t)C_{ij}(t_0)v_j^{(n)}$

- eigenvalues: $\lambda^{(n)}(t) \sim e^{-E_n t}\left[1 + O(e^{-\Delta E t})\right]$ - principal correlator
- eigenvectors: related to overlaps $Z_i^{(n)} = \sqrt{2E_n} e^{E_n t_0/2} v_j^{(n)\dagger} C_{ji}(t_0)$

- operators of definite J^{PC} constructed in step 1 are subduced into the relevant irrep
- a subduced irrep carries a "memory" of continuum spin J from which it was subddduced - it **overlaps** predominantly with states of this J.

J	0	1	2	3	4
A_1	1	0	0	0	1
A_2	0	0	0	1	0
E	0	0	1	0	1
T_1	0	1	0	1	1
T_2	0	0	1	1	1

- Using $Z = \langle 0|\Phi|k\rangle$, helps to identify continuum spins
- For high spins, can look for agreement between irreps
- Data below for T_1^{--} irrep, colour-coding is **Spin 1**, **Spin 3** and **Spin 4**.



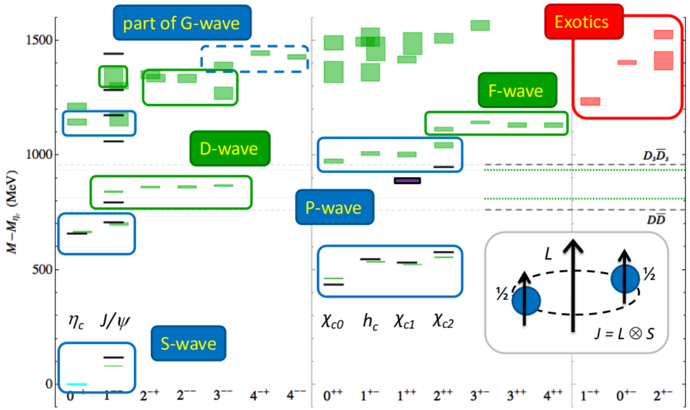
Single hadron states: Charmonium exotics

Precision calculation of high spin ($J \geq 2$) and exotic states is relatively new

Caveat Emptor

- Only single-hadron operators
- Physics of multi-hadron states appears to need relevant operators
- No continuum extrapolation
- $m_\pi \sim 400$MeV $\leftarrow$ already changing

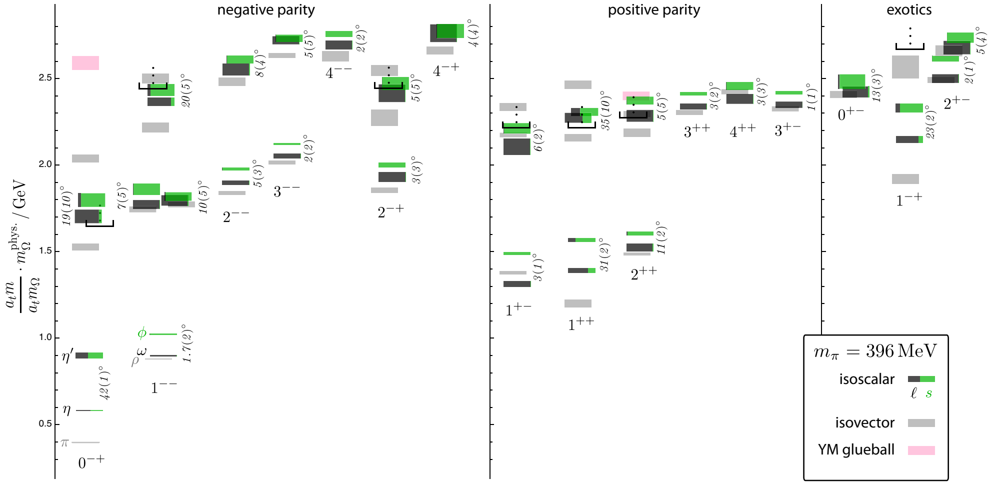
Charmonium



from HSC 2012

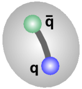
$\longrightarrow$ Expect improvements now methods established

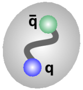
Single-hadron states: light exotics



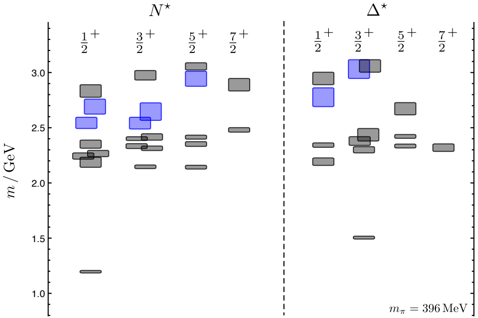
from HSC 2010

Hybrids

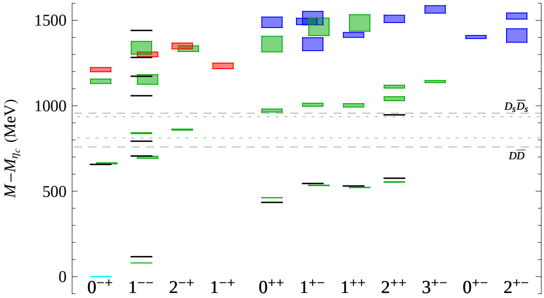




Expect a large overlap with operators $\mathcal{O} \sim F_{\mu\nu}$



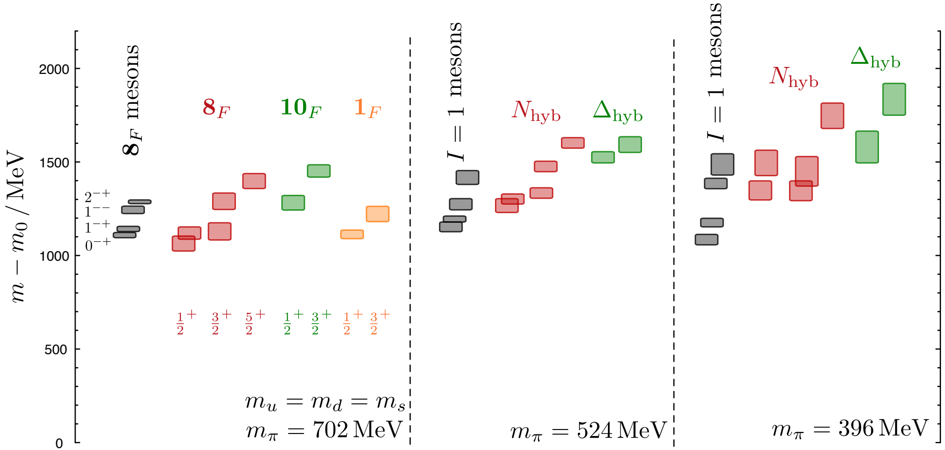
from HSC



$c\bar{c}$ *from HSC*

Lightest hybrid supermultiplet and excited hybrid supermultiplet same pattern and scale in meson and baryon, heavy and light[*HadSpec*:1106.5515] sectors.

Energy scale for hybrids



$m_0 = m_\rho$ for mesons and $m_0 = m_N$ for baryons.

Exploratory studies of scattering states

Needed for

- States, particularly exotics, close to or above thresholds

Characterised by

- New methods (developed/applied in last 5 years)
    - algorithmic: distillation allows access to all elements of propagators *and* construction of sophisticated basis of operators.
    - theoretical: spin-identification; construction of multi-hadron operators and mesons in flight; scattering below inelastic thresholds; coupled-channels (new in '14).
- Generally high statistics, improved actions etc - results can be very precise.
- Systematic errors not all controlled in exploratory studies: e.g. no continuum extrapolation, relatively heavy pions ...

Rapid progress in the last 5 years!

Scattering in a Euclidean theory

Lose direct access to scattering in (Euclidean) lattice calculations

Lüscher found a way to extract scattering information in the elastic region from **LQCD** .
[NPB354, 531-578 (1991)]

- related lattice energy levels in a finite volume to a decomposition of the scattering amplitude in partial waves in infinite volume

$$\det\left[\cot\delta(E_n^*) + \cot\phi(E_n, \vec{P}, L)\right] = 0$$

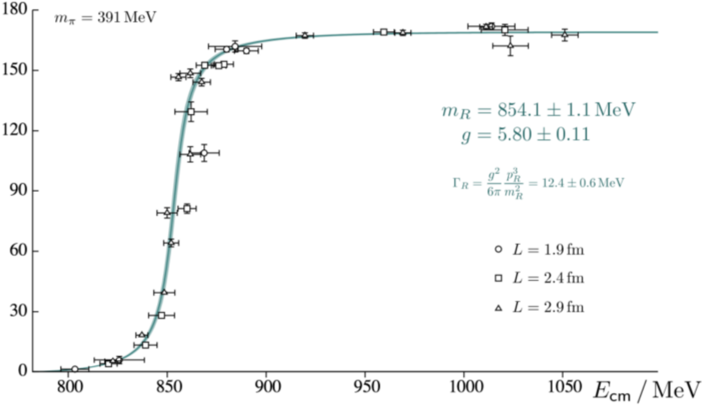
and $\cot\phi$ a known function (containing a generalised zeta function).

- The idea dates from the quenched era. To use it in a dynamical simulation need energy levels at extraordinary precision. This is why it has taken a while ...

Using Lüscher's idea

Now in use to determine resonance parameters

$\pi\pi \to \pi\pi$



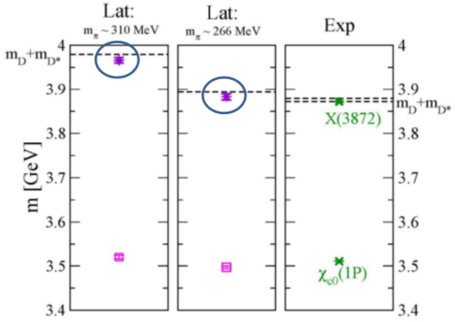
$m_\pi = 391$ MeV

from Dudek, Edwards, Thomas in *Phys.Rev.* D87 (2013) 034505

Many talks at Lattice 2015 & probably(!) 2016

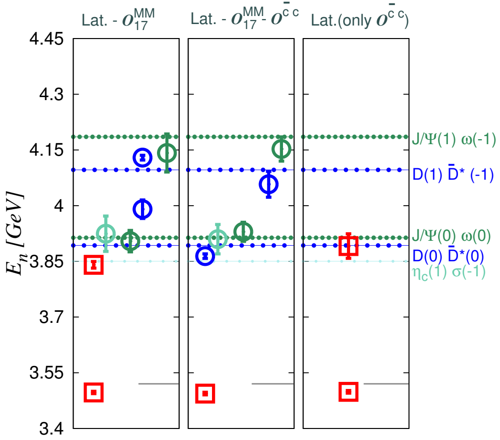
X(3872) - a first look

Prelovsek & Leskovec 1307.5172



ground state: $\chi_{c1}(1P)$
$D\bar{D}^*$ scattering mx: pole just below thr.
Threshold $\sim m_{u,d}$ *and* m_c *discretisation?*

Padmanath, Lang, Prelovsek 1503.03257



X(3872) not found if $c\bar{c}$ not in basis.

Also results from Lee et al 1411.1389
Within 1MeV of $D^0\bar{D}^{0*}$, 8MeV of D^+D^* thresholds: isospin breaking effects important?

Z_c^+

An “exotic” hadron i.e. does not fit in the quark model picture.

There are a number of exploratory calculations on the lattice.

Challenges:

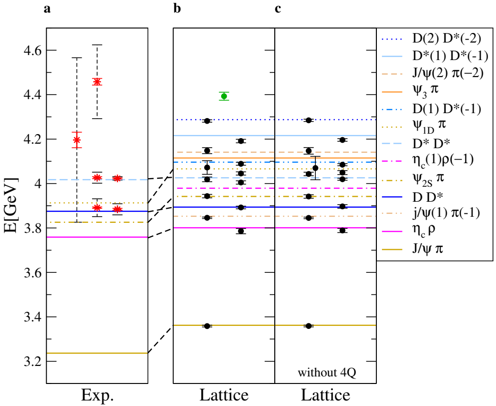
- The Z_c^+ (and most of the XYZ states) lies above several thresholds and so decay to several two-meson final states
- requires a coupled-channel analysis for a rigorous treatment
- on a lattice the number of relevant coupled-channels is large for high energies.

State of the art in coupled-channel analysis:

- Lüscher: $K\pi$, $K\eta$ [HSC 2014,2015]
- HALQCD: Z_c [preliminary results]

Z_c^+ - First look on the lattice

Prelovsek, Lang, Leskovec, Mohler: 1405.7615



- 13 expected 2-meson e'states found (black)
- no additional state below 4.2GeV
- no Z_c^+ candidate below 4.2GeV

Similar conclusion from Lee et al [1411.1389] and Chen et al [1403.1318]

Why no eigenstate for Zc? Is Z_c^+ a coupled channel effect? What can other groups say? Work needed!

Many other states being investigated

Tetraquarks:

- Double charm tetraquarks ($J^P = 1^+, I = 0$) by HALQCD [PLB712 (2012)]
  - attractive potential, no bound tetraquark state
- Charm tetraquarks: variational method with DD^*, D^*D^* and tetraquark operators finds no candidate.

Y(4140)

- Ozaki and Sasaki [1211.5512] - no resonant Y(4140) structure found
- Padmanath, Lang, Prelovsek [1503.03257] considered operators: $c\bar{c}$, $(\bar{c}s)(\bar{s}c)$, $(\bar{c}c)(\bar{s}s)$, $[\bar{c}\bar{s}][cs]$ in $J^P = 1^+$. Expected 2-particle states found and χ_{c1}, $X(3872)$ **not** Y(4140).

⋮

See Prelovsek @ Charm2015 for more

Exotic baryonic objects

H-dibaryons:

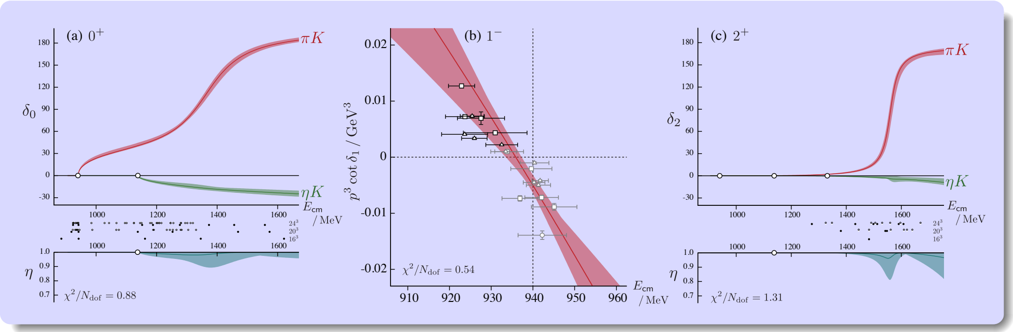
- A bound 6-quark state (udsuds) first proposed by Jaffe (1977) in MIT bag-model - at 81MeV below $\Lambda\Lambda$ threshold.
- Lattice calculations [NPLQCD, HALQCD] find H-dibaryon bound but at quark masses larger than physical pion.
- Extracting resonance parameters from $\Lambda\Lambda$
- A linear chiral extrapolation does not discriminate between bound/unbound at the physical pion mass. Does suggest a state in I=0, J=0, s=-2 ($\Lambda\Lambda$) that is just bound/unbound.

More work to be done for good understanding

Even more recent progress

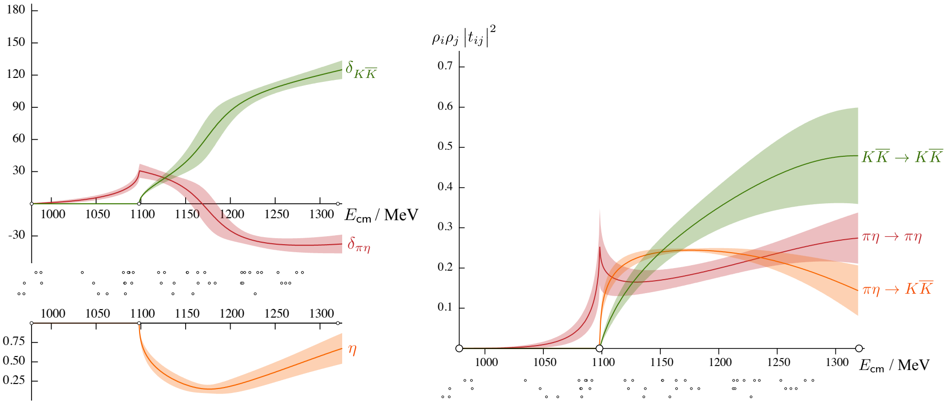
- Generalised for: moving frames; non-identical particles; multiple two-particle channels, particles with spin, by many authors.
- The precision and robustness of some numerical implementations is now very impressive. *[See talks at Lattice 2015 & 2016]*
- First coupled-channel resonance in a lattice calculation

$\pi K \to \eta K$ by D. Wilson et al 1406.4158 and 1507.02599



A0

Is a_0 a $q\bar{q}$ state or dominated by a $K\bar{K}$ molecular configuration?



HSC, Wilson et al 1602.05122

- Phase shifts, inelasticity and amplitudes (for $m_\pi \sim 400$MeV)
- Find an S-wave resonance in a two-coupled channel region - $\pi\eta$, $K\bar{K}$, includes limited 3-channel scattering - $(\pi\eta, K\bar{K}, \pi\eta')$. Resonance pole has large coupling to $K\bar{K}$.

Summary & Outlook

- Precision lattice calculations of excited and exotic hadron states available
  - includes hybrids and other exotics treated as "single-hadron" states.
- Studies of resonances, including multiple two-particle channels underway
  - Expect significant progress in next few years in e.g X,Y,Z simulations
  - Numerically precise but control of other systematics is a challenge
- Many challenges remain e.g. no general framework for extracting scattering amplitudes involving more than two hadrons. Clever ideas needed!

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Thanks for listening!