Measurement of the charged pion mass using X-ray spectroscopy of exotic atoms

M. Trassinelli, D. F. Anagnostopoulos, G. Borchert, A. Dax, J.-P. Egger, D. Gotta, M. Hennebach, P. Indelicato, Y.-W. Liu, B. Manil, N. Nelms, L. M. Simons, A. Wells

Institut of NanoSciences of Paris, France

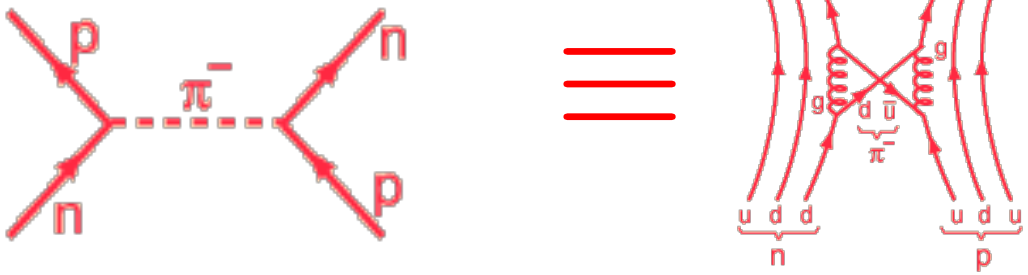
14th International Workshop on Meson Production, Properties and Interaction

2nd - 7th June 2016. Krakow, Poland

MESON 2016

Prediction and discovery of the pion

Predicted in 1935 by Yukawa to describe the strong interaction as exchange of particle [1]



Expected mass ~ 200 m_e

http://hyperphysics.phy-astr.gsu.edu/hbase/forces/funfor.html

Discovered by emulsion photography (Pic du Midi) in 1947 [2]

Estimated mass ~ 1.5 m_μ

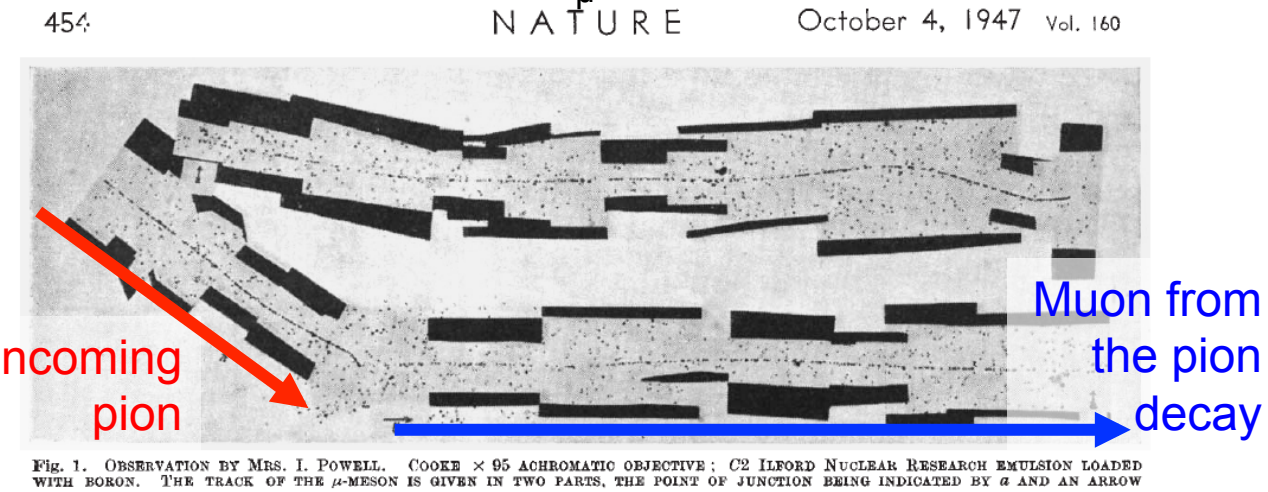


Fig. 1. Observation by Mrs. I. Powell. Cooke × 95 achromatic objective; C2 Ilford Nuclear Research emulsion loaded with boron. The track of the μ-meson is given in two parts, the point of junction being indicated by *a* and an arrow

Charged pion



- m_π = 139.5 MeV/c = 254 m_e
- lifetime=26 ns
- composed by a quark and an antiquark

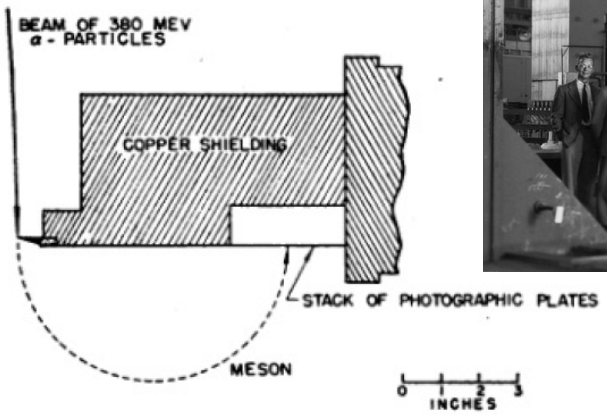
[1] H. Yukawa, Proc. Phys. and Math. Soc. Japan **17**, 48 (1935)
[2] C.M.G. Lattes *et al.*, Nature **160**, 453-456 and 486-492 (1947)

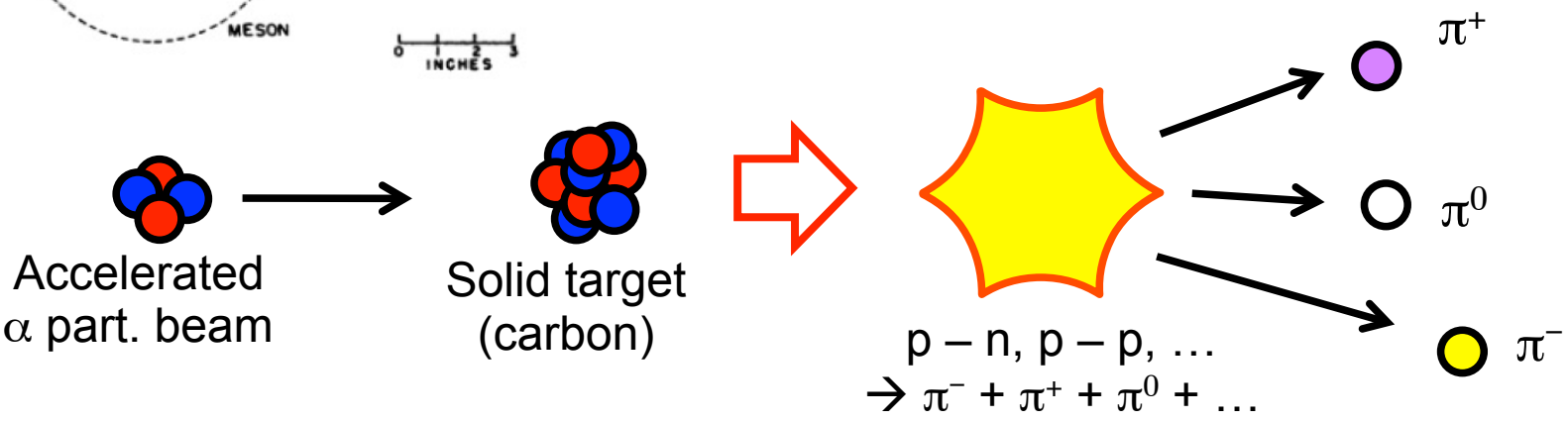
Pion production and first mass measurements



First controlled pion production at the 184” cyclotron in Berkeley in 1948 [1]

Estimated mass:
$m_\pi = 313 \pm 16\ m_e$

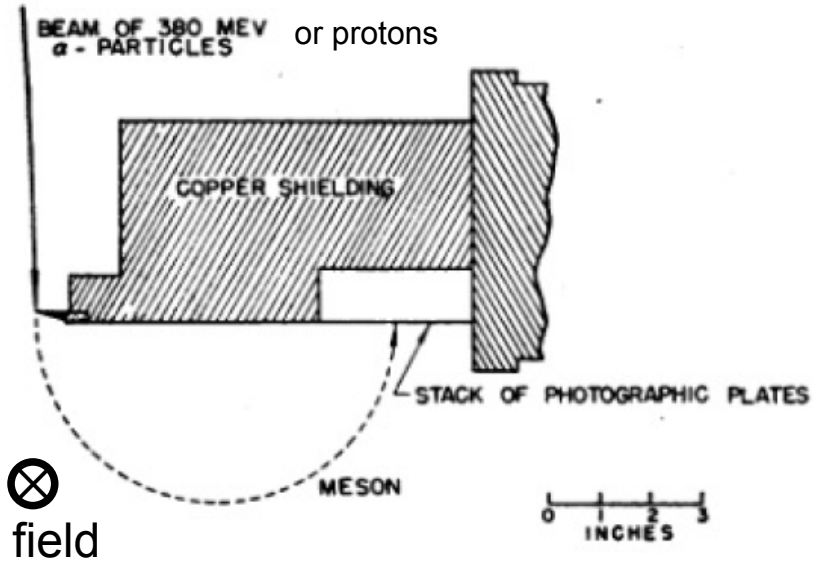


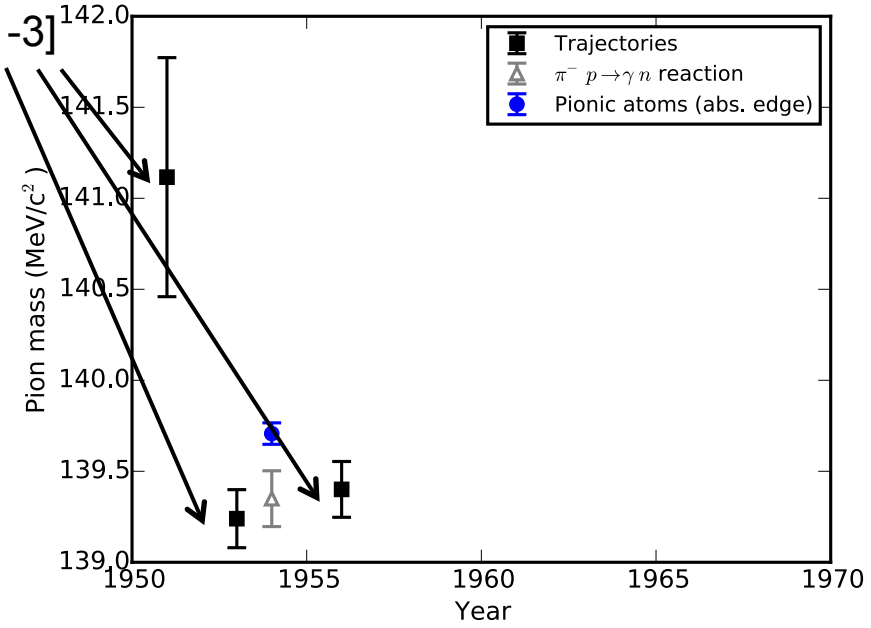


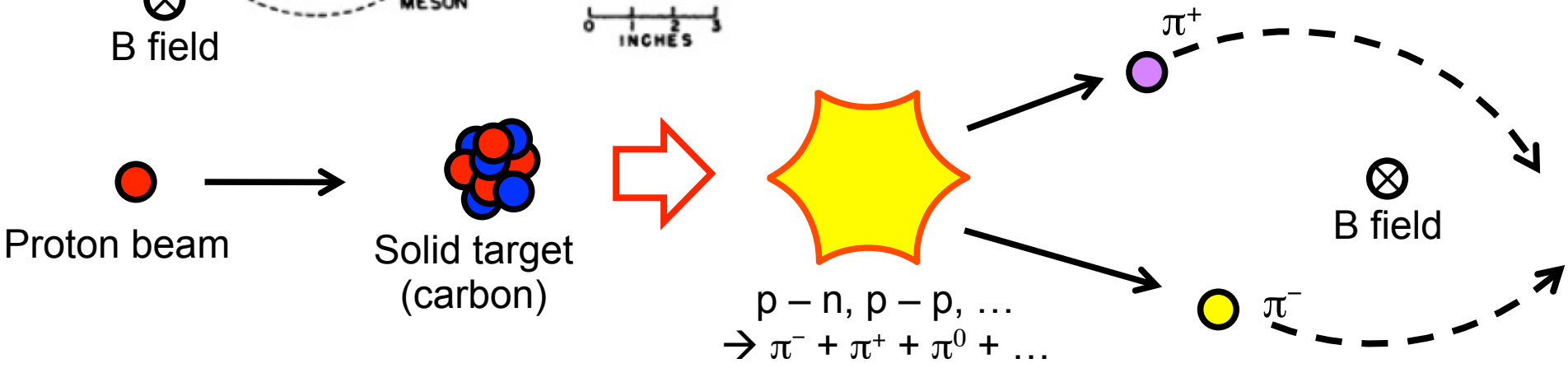
[1] E. Gardner and C. Lattes, Science **107**, 270-271 (1948)

First mass measurements

- Particle trajectories analysis Barkas et al. [1-3]



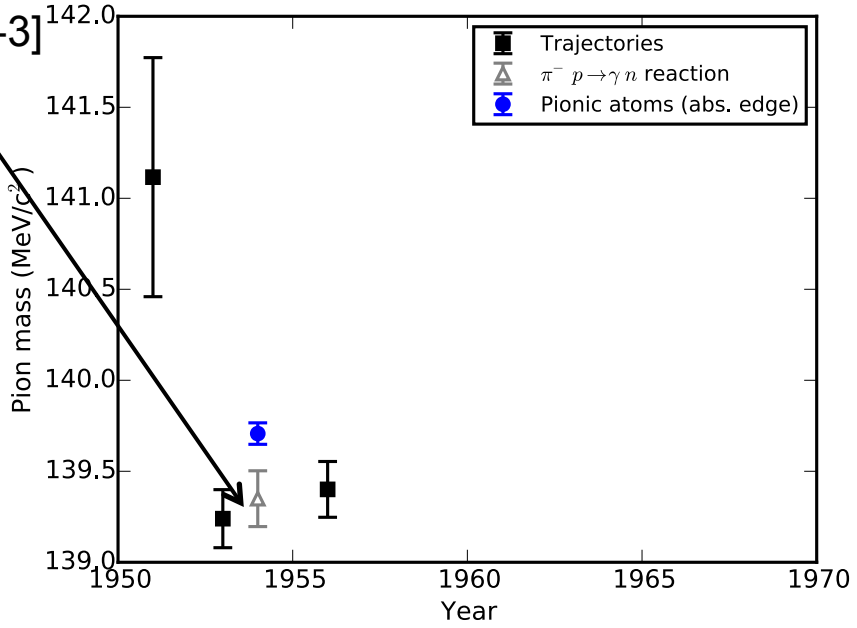


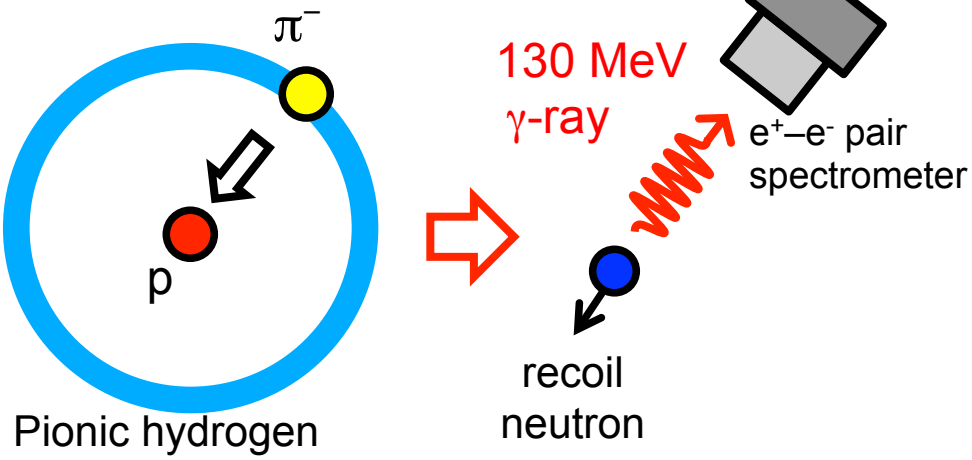


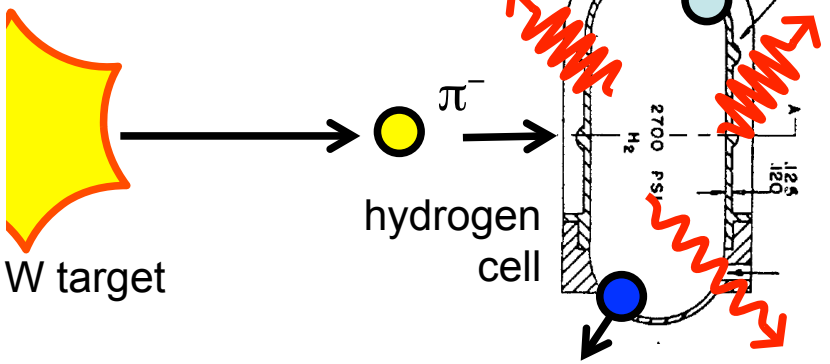
First mass measurements

- Particle trajectories analysis Barkas et al. [1-3]
- $\pi^- + p \rightarrow n + \gamma$ reaction Crowe et al. [4]

π^- stopped in a hydrogen cell
→ **pionic hydrogen** production
→ nuclear absorption of the pion

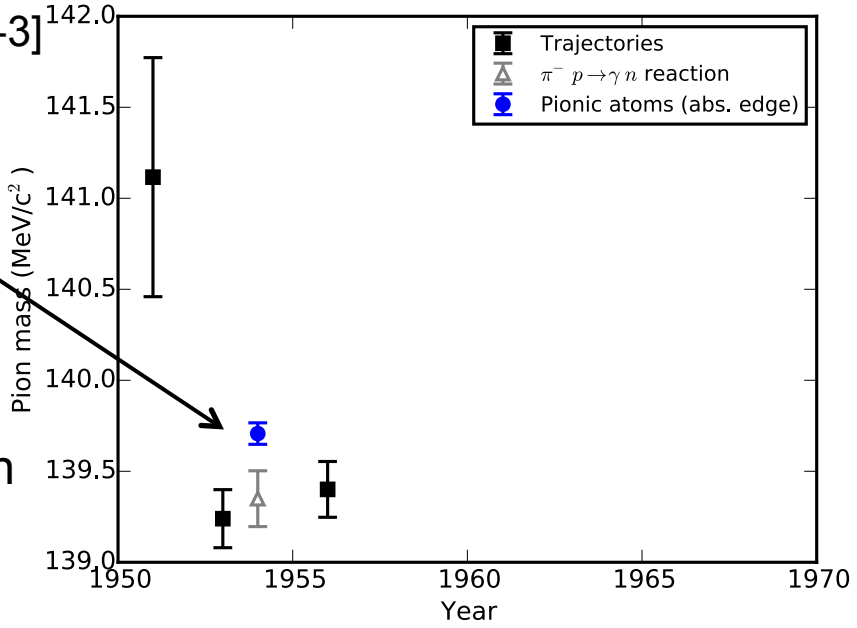






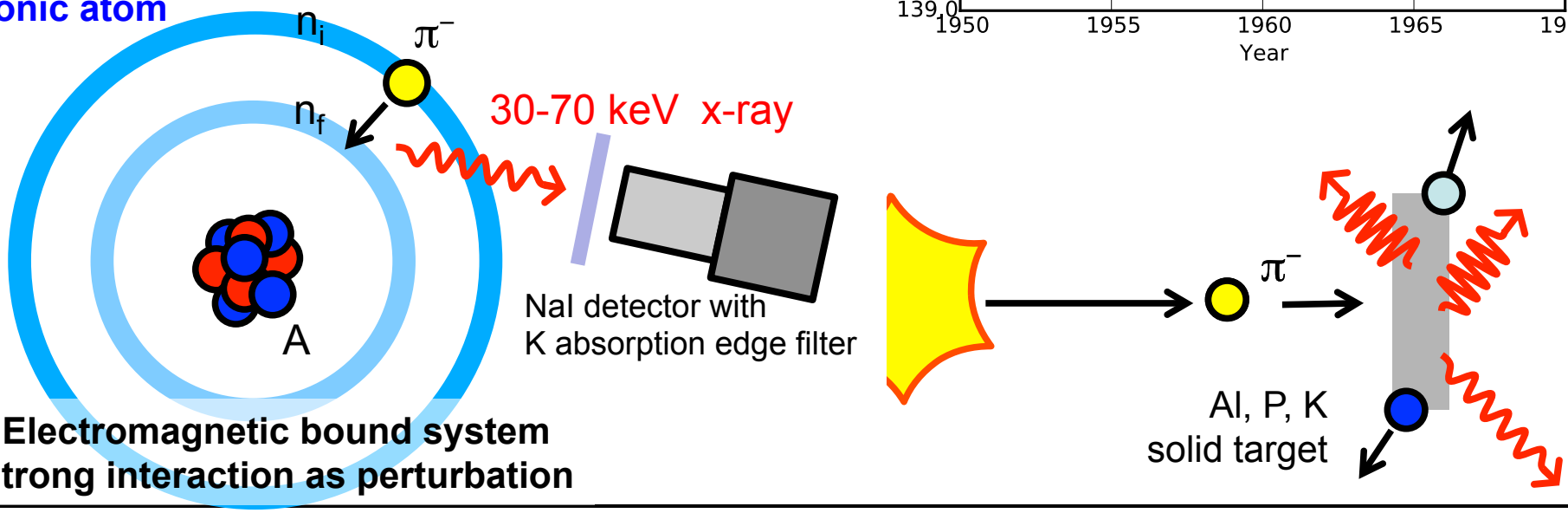
First mass measurements

- Particle trajectories analysis Barkas et al. [1-3]
- $\pi^- + p \rightarrow n + \gamma$ reaction Crowe et al. [4]
- Pionic atoms emission Stearns et al. [5]



π^- stopped in a solid target (Al, P, K)
→ **pionic atoms** production
→ X-ray from atomic level de-excitation

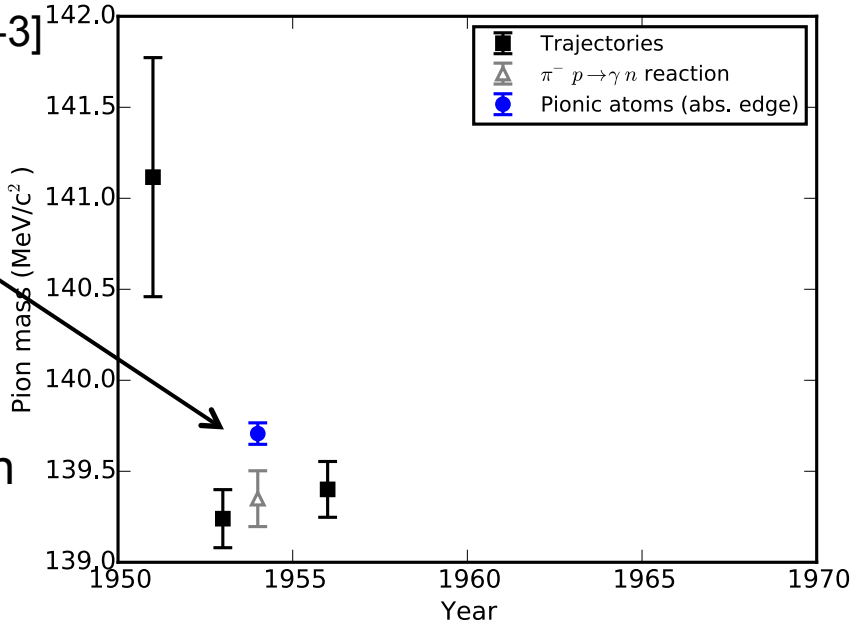
Pionic atom



Electromagnetic bound system
Strong interaction as perturbation

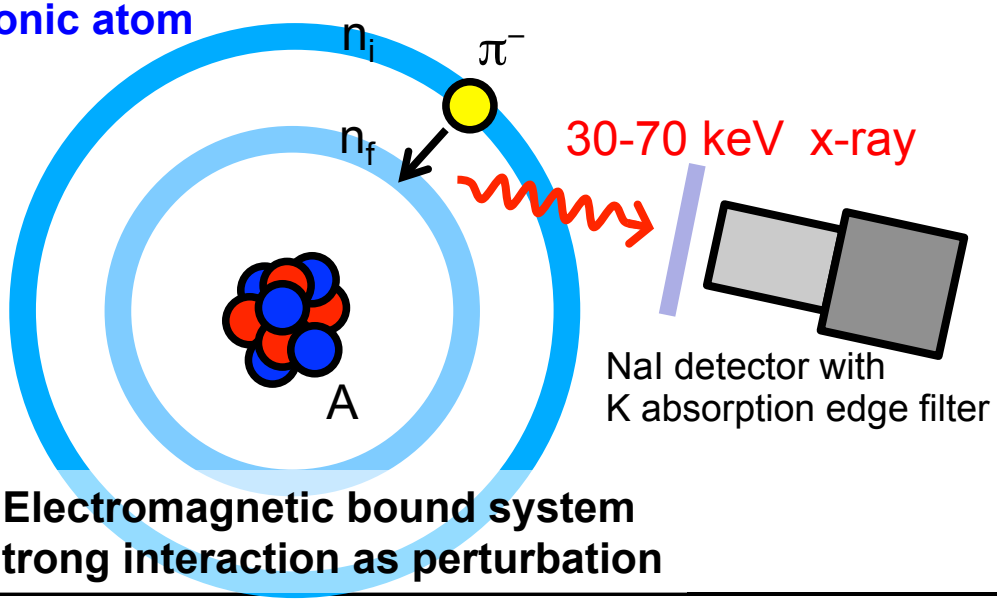
First mass measurements

- Particle trajectories analysis Barkas et al. [1-3]
- $\pi^- + p \rightarrow n + \gamma$ reaction Crowe et al. [4]
- Pionic atoms emission Stearns et al. [5]



π^- stopped in a solid target (Al, P, K)
→ **pionic atoms** production
→ X-ray from atomic level de-excitation

Pionic atom



Electromagnetic bound system
Strong interaction as perturbation

Z = atomic number,
m =reduced mass,
α = fine structure constant.

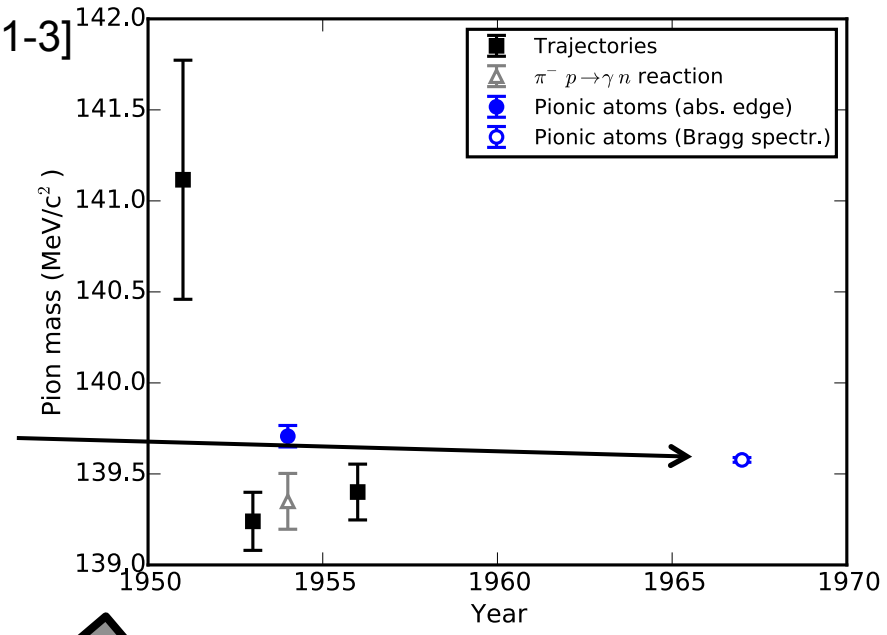
$$E_{\text{X-ray}} = mc^2 \frac{(Z\alpha)^2}{2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) + \mathcal{O}(Z^4\alpha^4)$$

First mass measurements

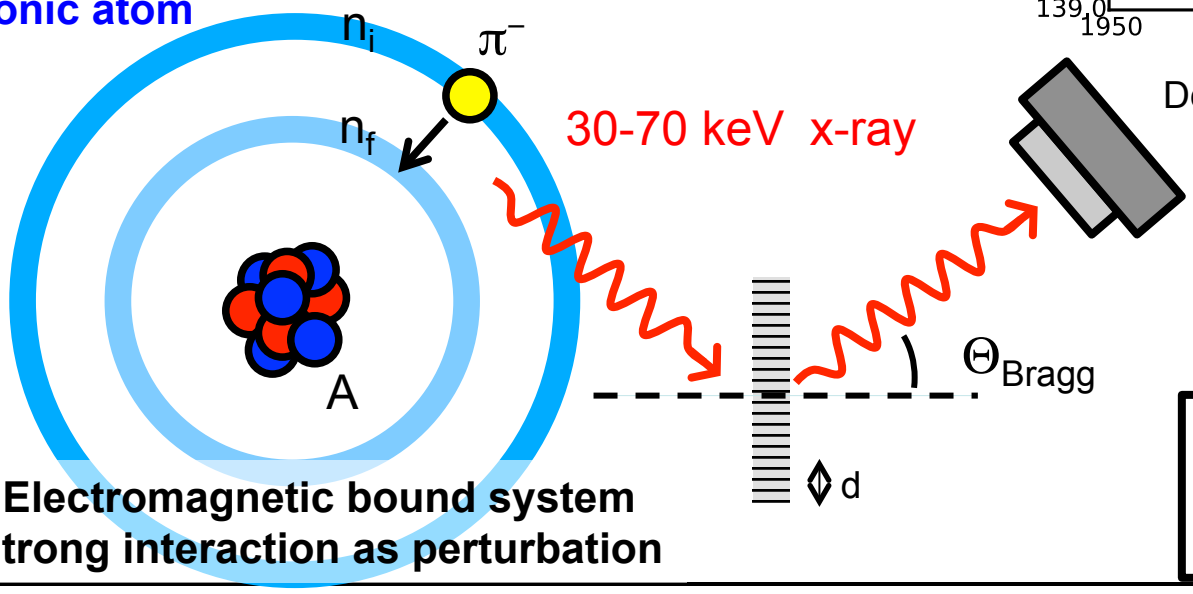
- Particle trajectories analysis Barkas et al. [1-3]
- $\pi^- + p \rightarrow n + \gamma$ reaction Crowe et al. [4]
- Pionic atoms emission Stearns et al. [5]

Higher pion beam intensity

→ **Bragg spectroscopy** Shafer et al. [6]



Pionic atom

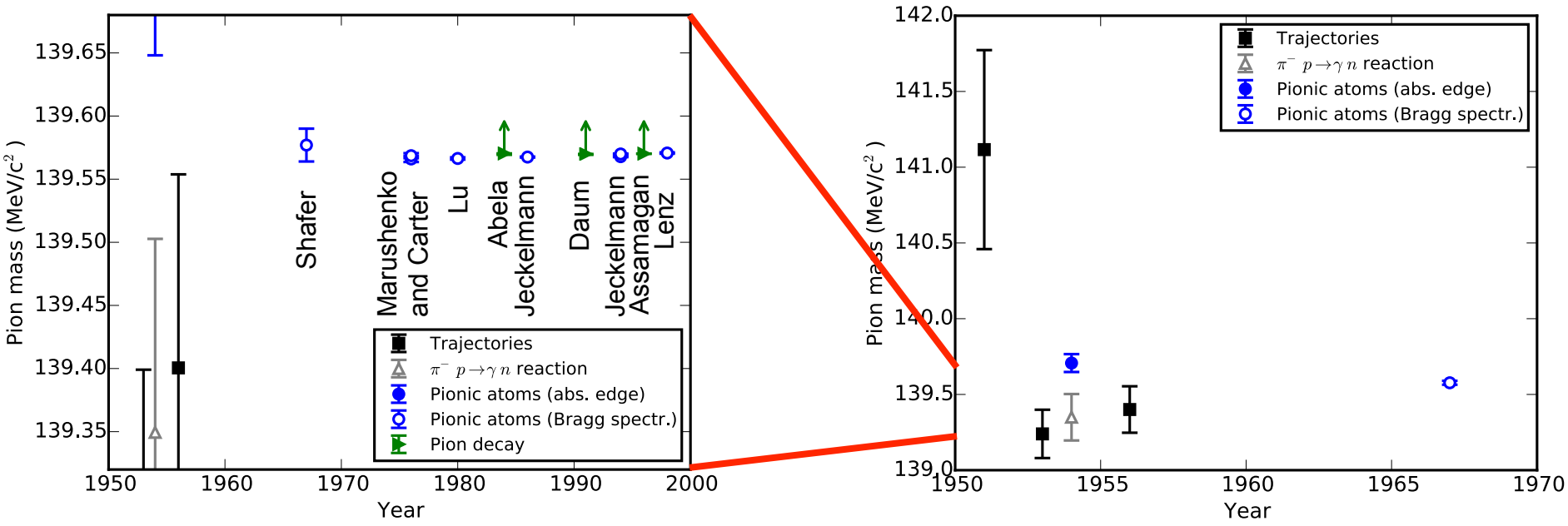


Electromagnetic bound system
Strong interaction as perturbation

The Bragg law

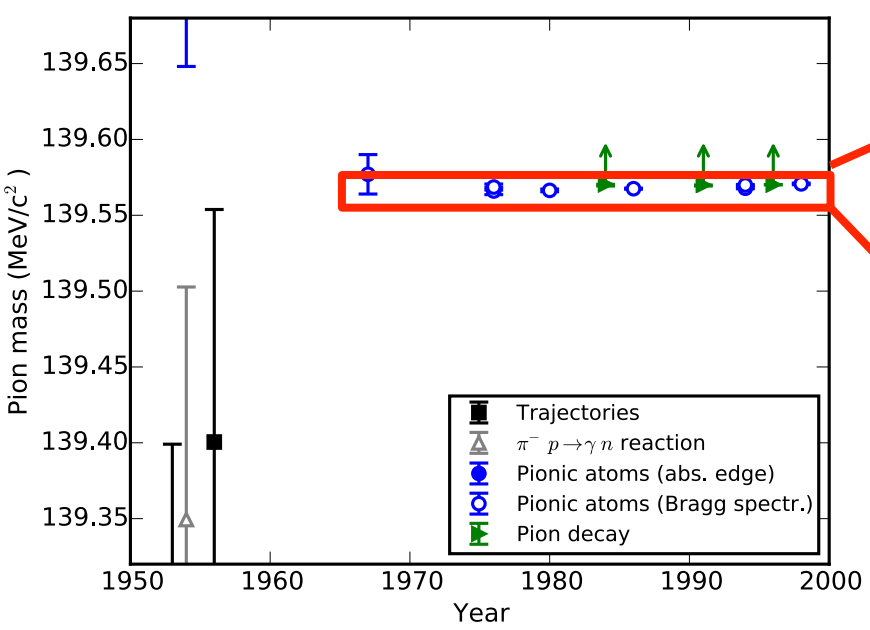
$$\frac{hc}{E} = 2d \sin \Theta_{Bragg}$$

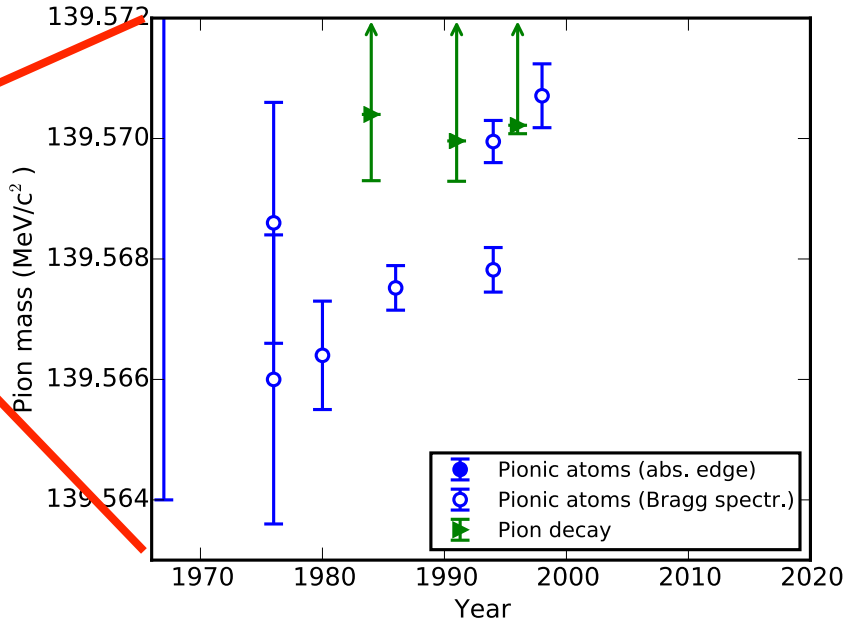
Pionic spectroscopy and pion decay measurements



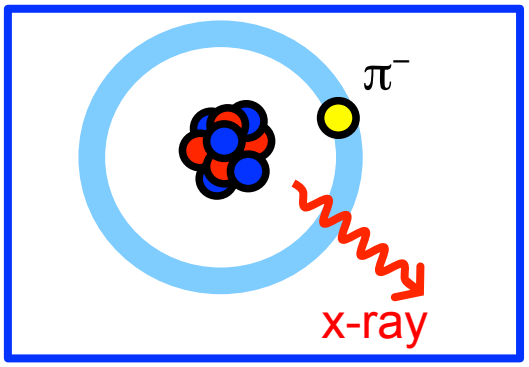
New **pionic atom spectroscopy** and **pion decay** measurements

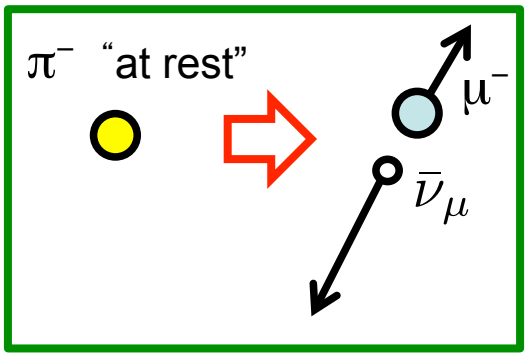
Pionic spectroscopy and pion decay measurements





New **pionic atom spectroscopy** and **pion decay** measurements

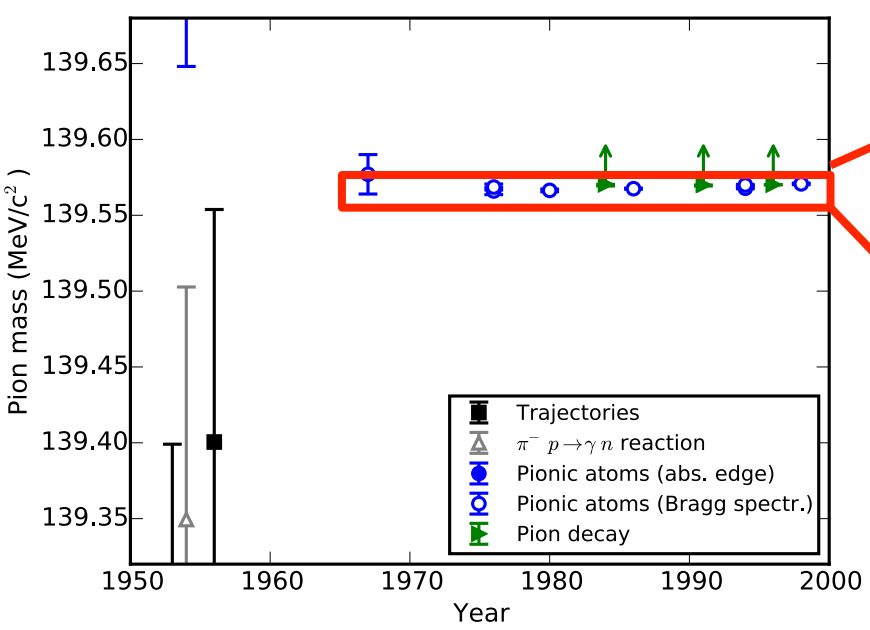


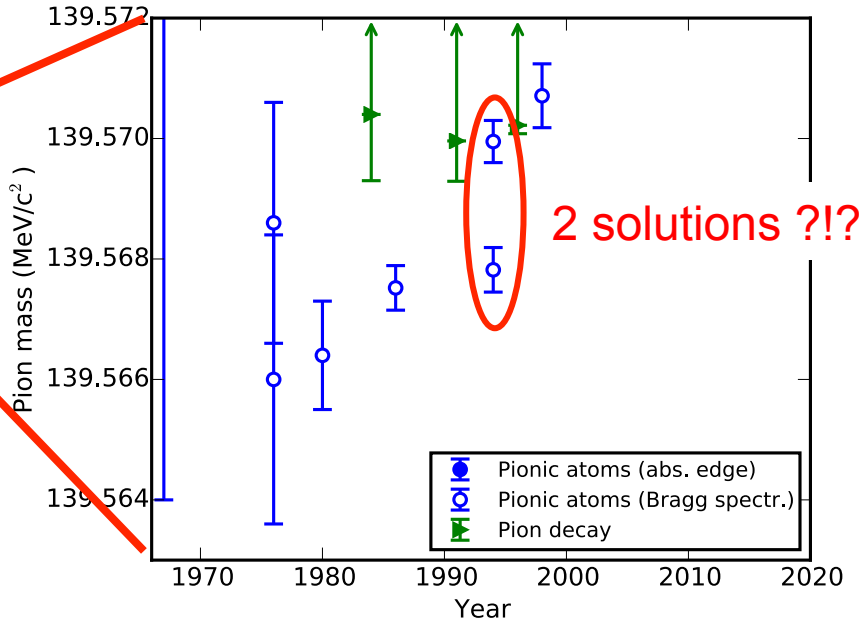


Measurement of
$\mathbf{p}_\mu$ (, m_μ)

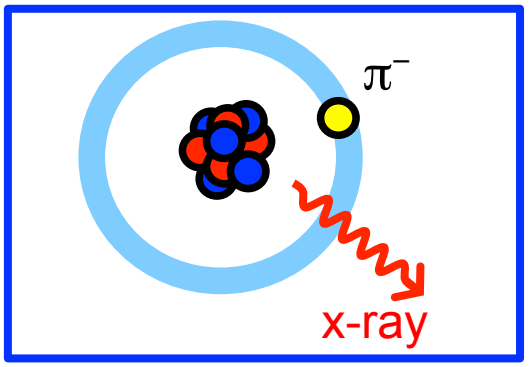
→ pion mass
low boundary if no assumptions on the neutrino mass are made

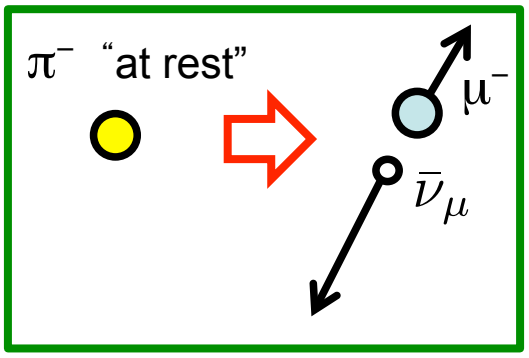
Pionic spectroscopy and pion decay measurements





New **pionic atom spectroscopy** and **pion decay** measurements

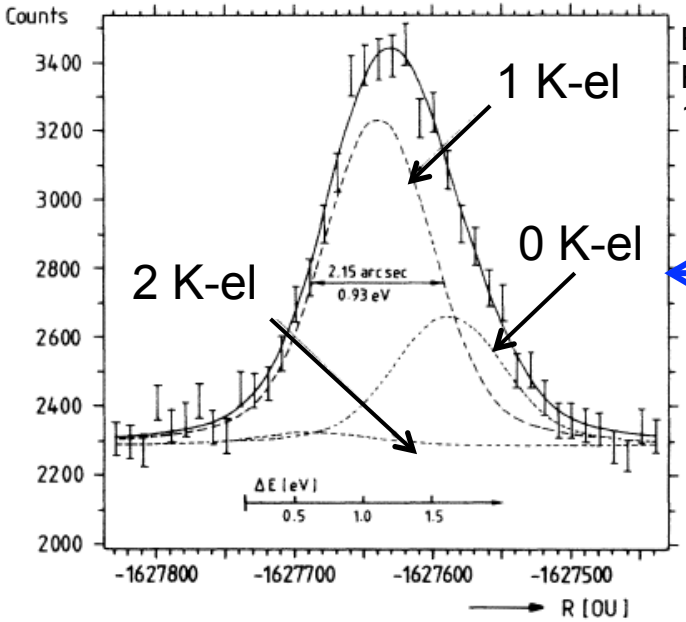




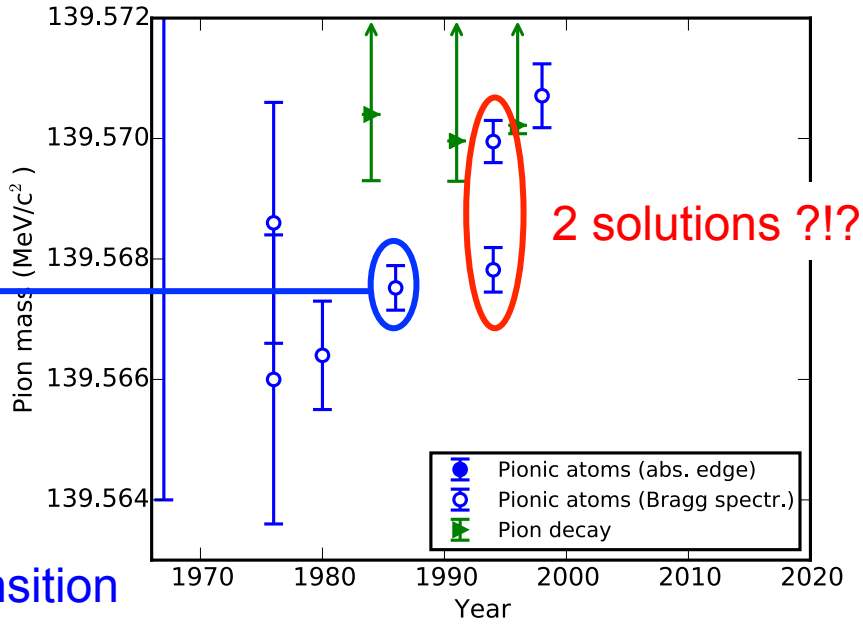
Measurement of
$\mathbf{p}_\mu$ (, m_μ)

→ pion mass
low boundary if no assumptions on the neutrino mass are made

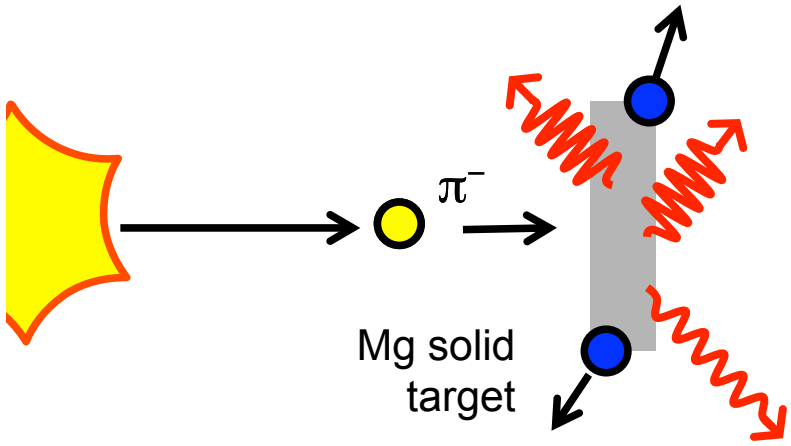
Pion mass measurement problems



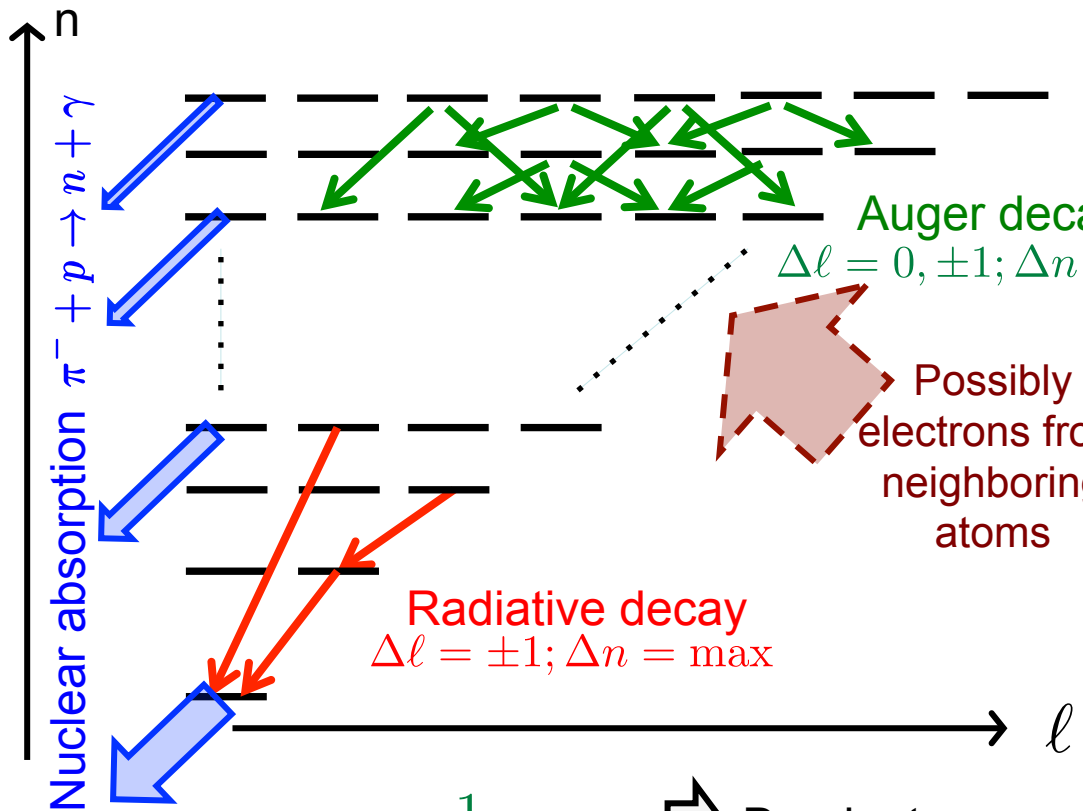
B. Jeckelmann *et al.*, Phys. Rev. Lett. **56**, 1444 (1986)



4f–3d πMg transition



Pionic atoms formation and atomic cascade



Capture at the radii of outmost electrons

$$n_\pi \sim n_{el}\sqrt{\frac{m_\pi}{m_{el}}} \sim n_{el} \times 16$$

De-excitation via Auger (electron emission) and radiative (X-ray emission) decay

$$\Gamma_{\text{Auger}} \propto \frac{1}{\sqrt{2\Delta E + 38\,eV}}$$

⇨ Dominates for large n [1]

$$\Gamma_{\text{Auger}} \gg \Gamma_{\text{X-ray}} \text{ for } n \gg 1$$

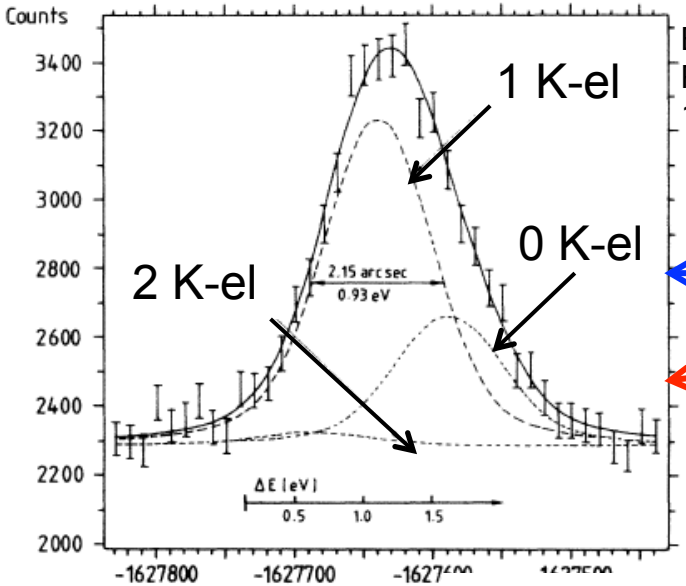
$$\Gamma_{\text{X-ray}} \propto \Delta E^3 \quad [2]$$

⇨ Dominates for small n

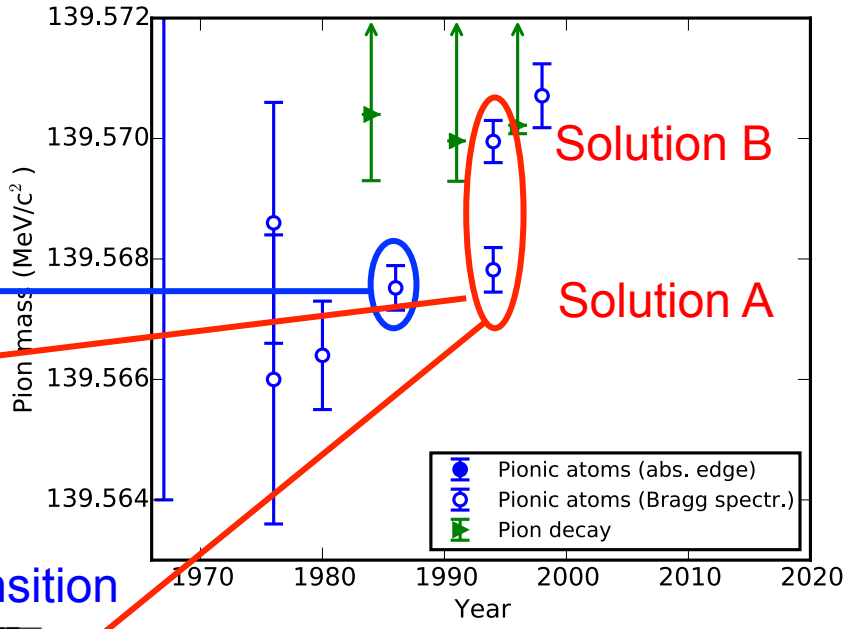
⇨
- Low Z + dilute targets → **no electrons remaining**
- Medium-high Z + dense targets → **remaining electrons**

[1] M. Leon *et al.*, Phys. Rev. **127**, 636 (1962)
[2] H.B. Bethe *et al.*, *Quantum Mechanics of One- and Two-Electron Atoms*, 1957, Springer-Verlag.

Pion mass measurement problems



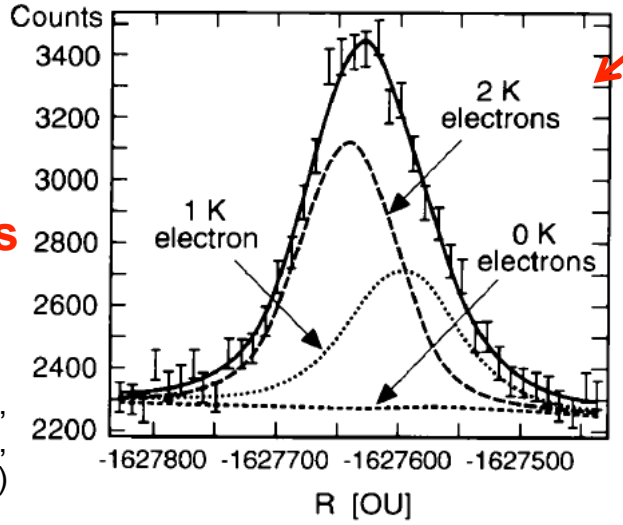
B. Jeckelmann *et al.*, Phys. Rev. Lett. **56**, 1444 (1986)



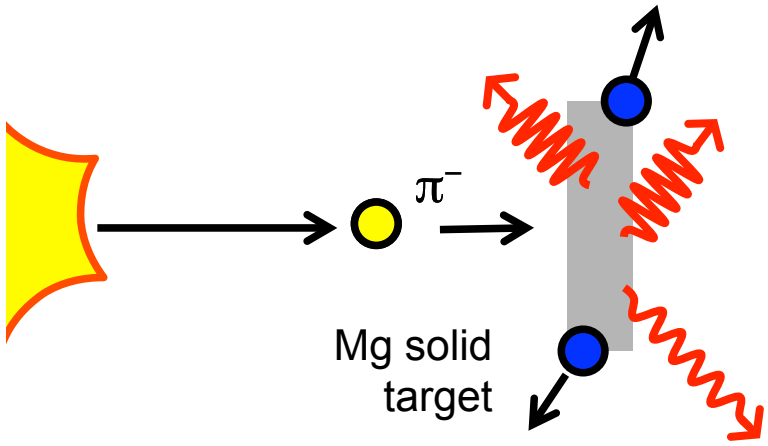
4f–3d πMg transition

Refilling from the neighboring atoms

→ **Two possibilities**

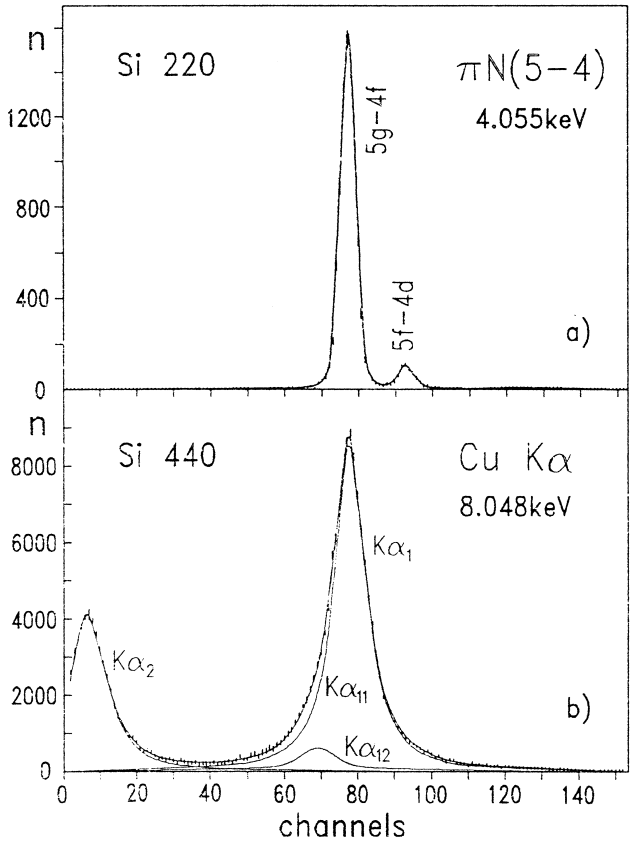


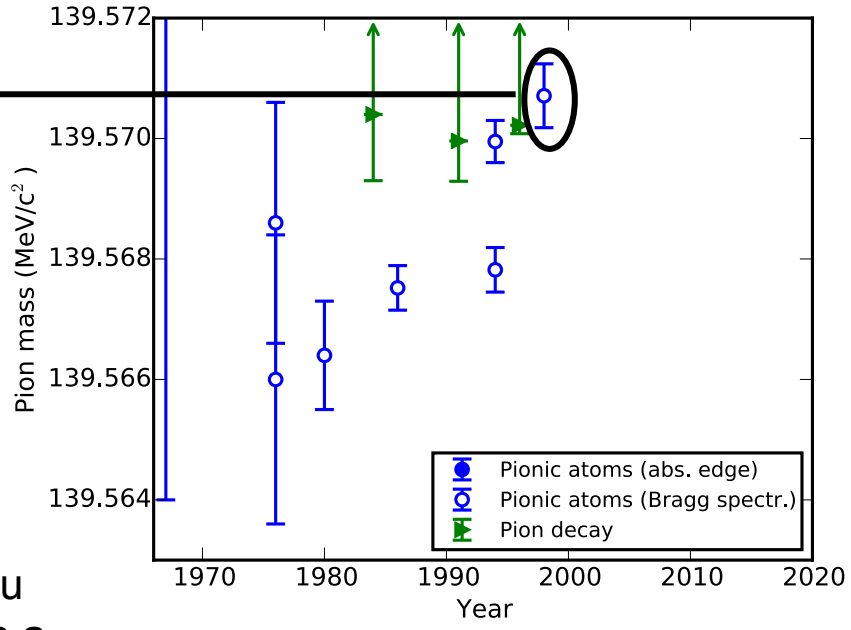
B. Jeckelmann *et al.*, Phys. Lett. B **335**, 326 (1994)



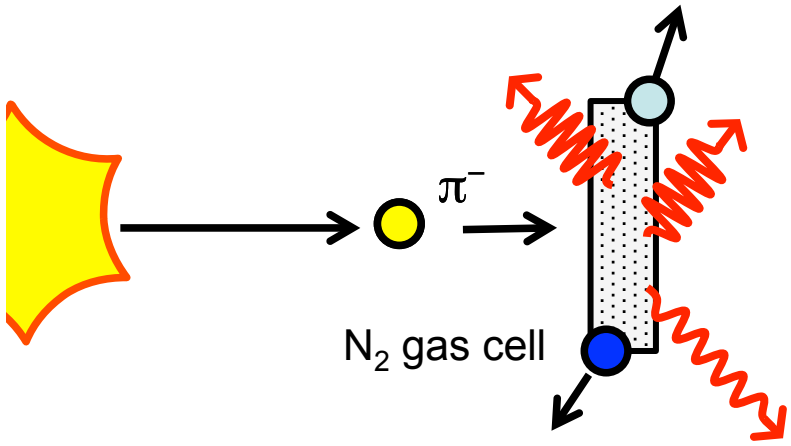
New measurement of the pion mass

New measurement of the pion mass with a gaseous target [1]





Calibration from Cu Kα fluorescence in a different diffraction order



[1] S. Lenz *et al.*, Phys. Lett. B **416**, 50 (1998)

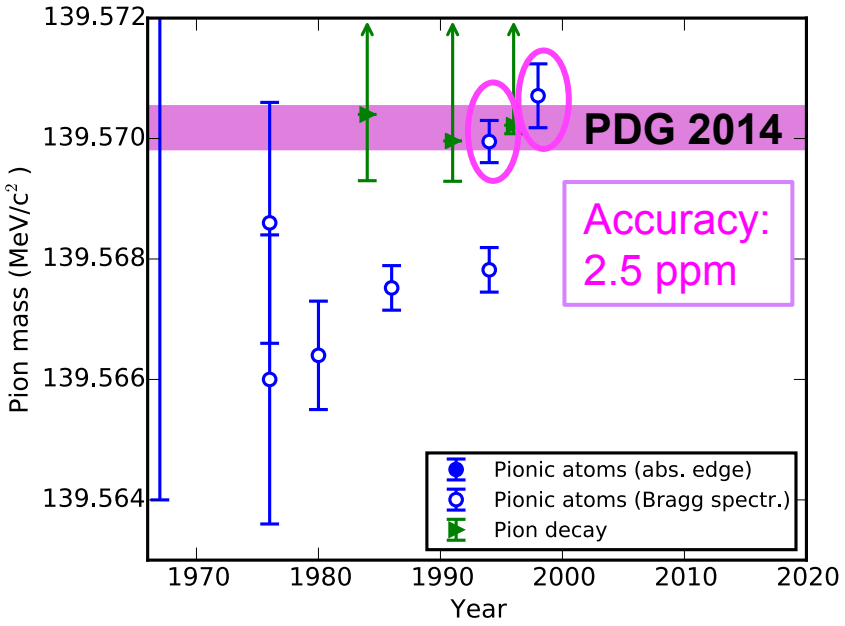
Present official value and new proposal

Present official value

Average between :

- Jeckelmann 1994 solution B (**solid target**)
- Lenz 1998 (**gaseous target**)

Particle Data Group. Chinese Phys. C **38**, 090001 (2014)



PSI proposal R-97-02

NEW PRECISION DETERMINATION OF THE CHARGED PION MASS

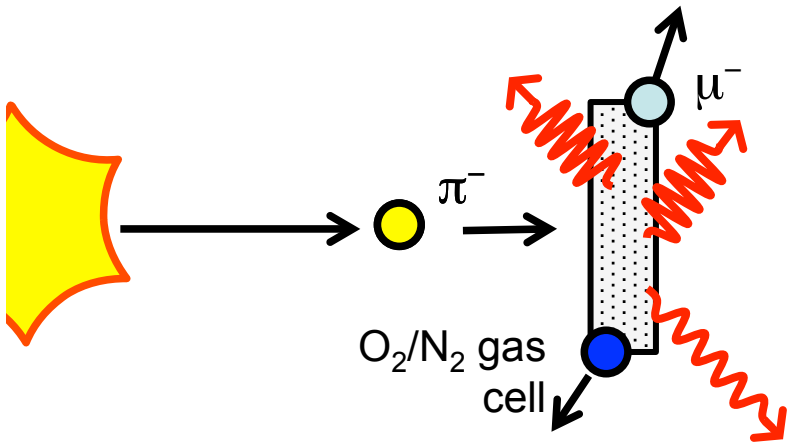
D. Anagnostopoulos[1], M. Augsburger[2], G. Borchert[1], D. Chatellard[2], M. Daum[3], J.-P. Egger[2], D. Gotta[1], P. Hauser[3], P. Indelicato[4], E. Jeannet[2], K. Kirch[3], O. W. B. Schult[1], Th. Siems[1], L. M. Simons[3]

- **Gaseous target**

→ no remaining el. contamination

- **Muonic Oxygen** transition as **calibration**

→ high accuracy of the energy reference



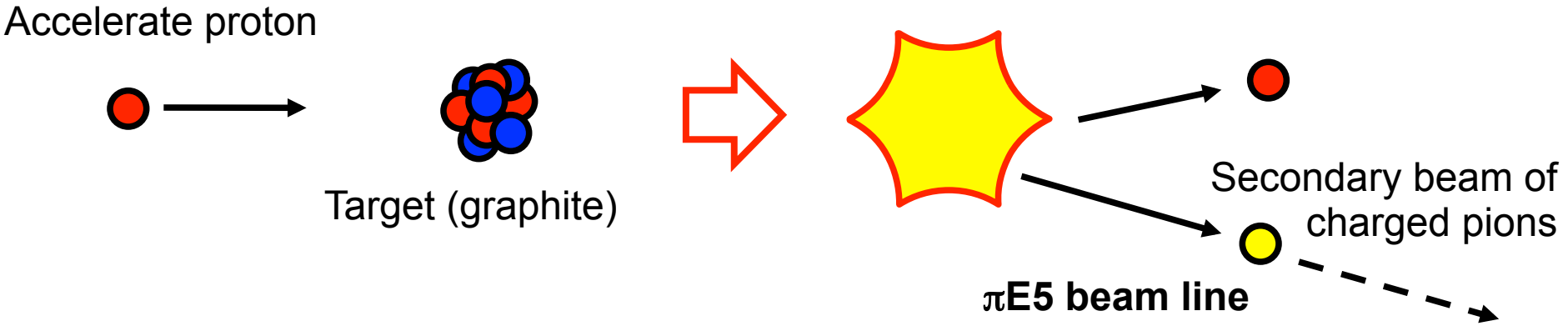
Pion beam production



Production at the Paul Scherrer Institut (Villigen, Switzerland)

- Proton beam :E_{kin}=590 MeV, I=1.9 mA
- Graphite target

- 10^8 pions/s, E_{kin}=110 MeV

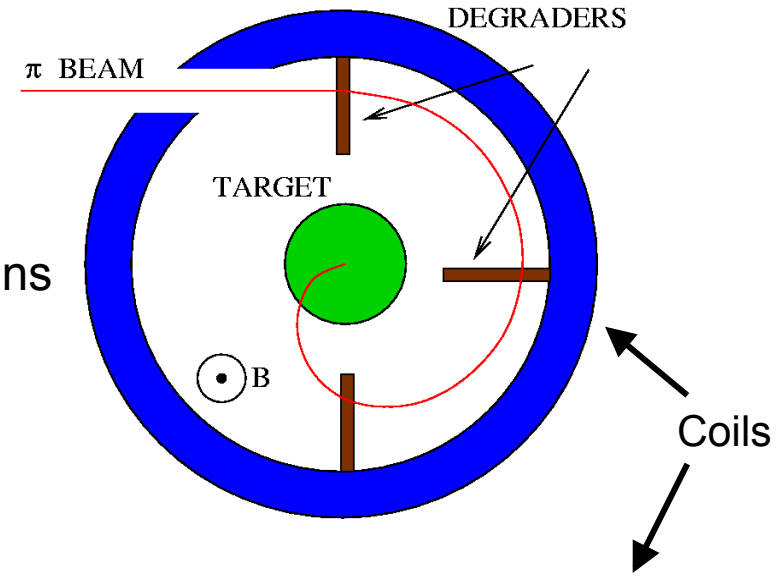


Pionic and muonic atoms production

- Cyclotron trap to stop the pions:
  - strong magnetic field (B_{max}=3.5 Tesla)
  - plastic degraders (energy loss)

- Gaseous target:
  - N_2/O_2 gas mixture of 10%/90%
  - Room temperature and P = 1.4 bar

τ_{π^-}= 26 ns



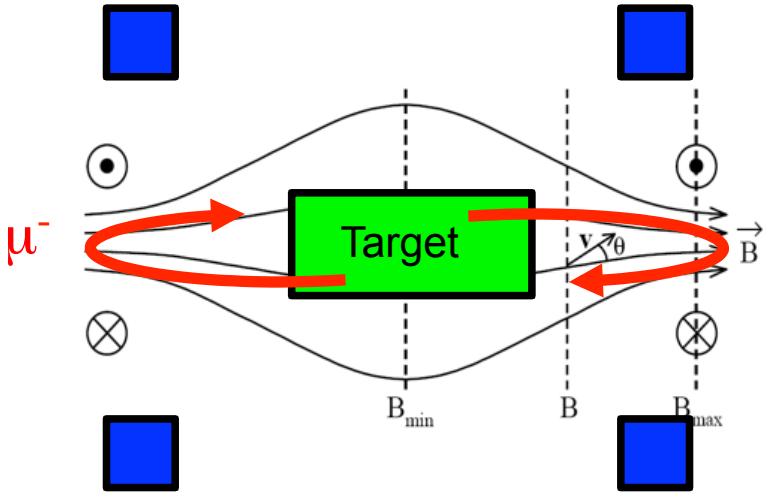
1-5% of the incoming pions are stopped inside the target

- Production and trapping of the muons

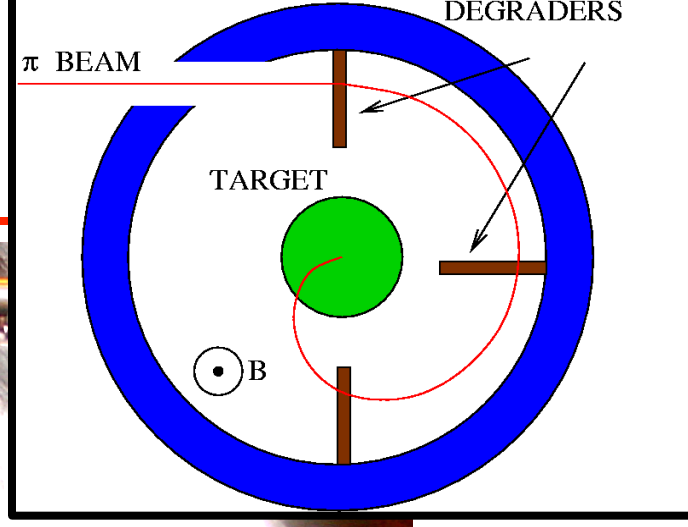
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

Formation of muonic and pionic atoms

N. of πN atoms = 10 x N. of μO atoms



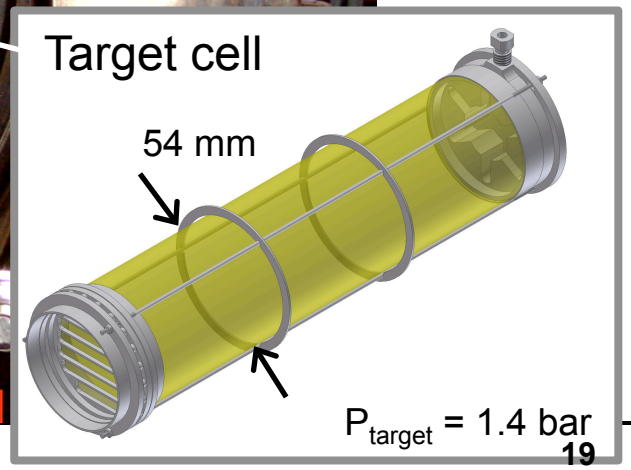
Cyclotron trap



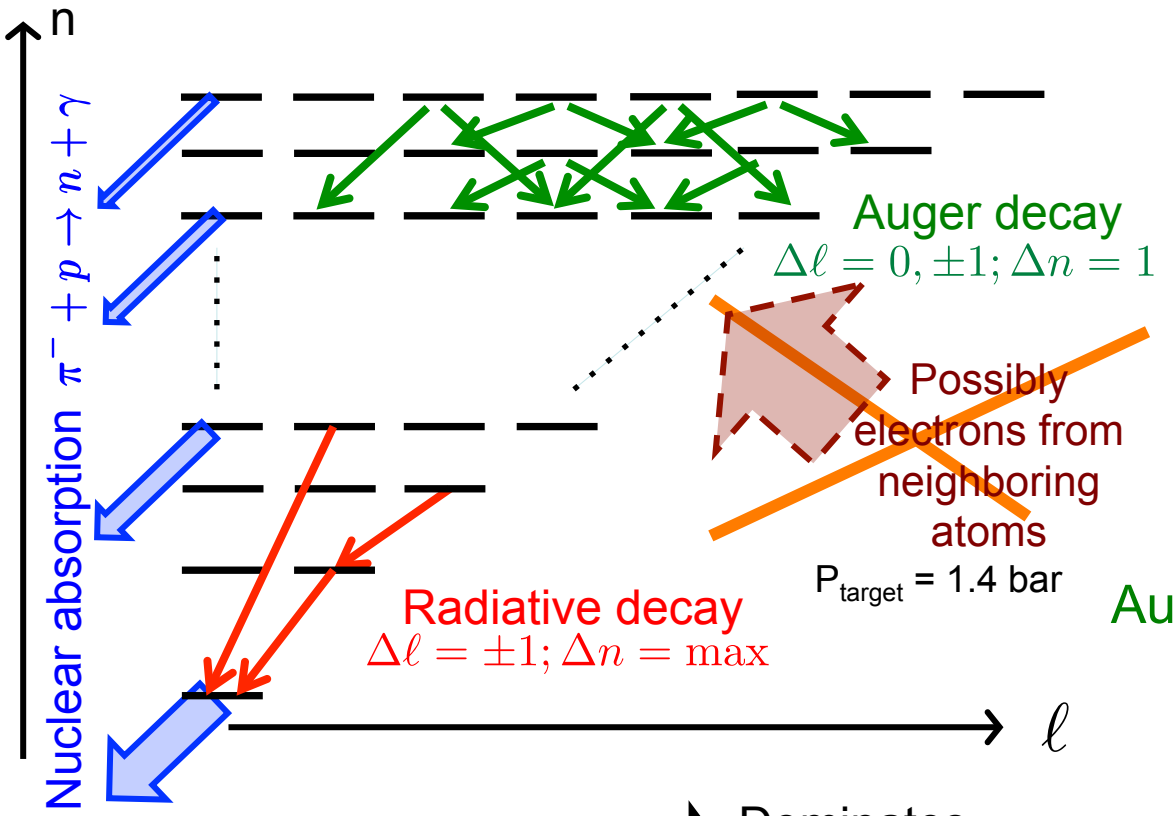
Incoming pion beam

Polyethylene degraders

Scintillator/degrader



Pionic atoms formation and atomic cascade with N_2 gaseous target



$$\Gamma_{\text{Auger}} \propto \frac{1}{\sqrt{2\Delta E + 38\,eV}}$$

Dominates for n > 6-7

$$\Gamma_{\text{Auger}} \gg \Gamma_{\text{X-ray}} \text{ for } n \gg 1$$

$$\Gamma_{\text{X-ray}} \propto \Delta E^3$$

Dominates for n < 6-7

Capture at the radii of outmost electrons

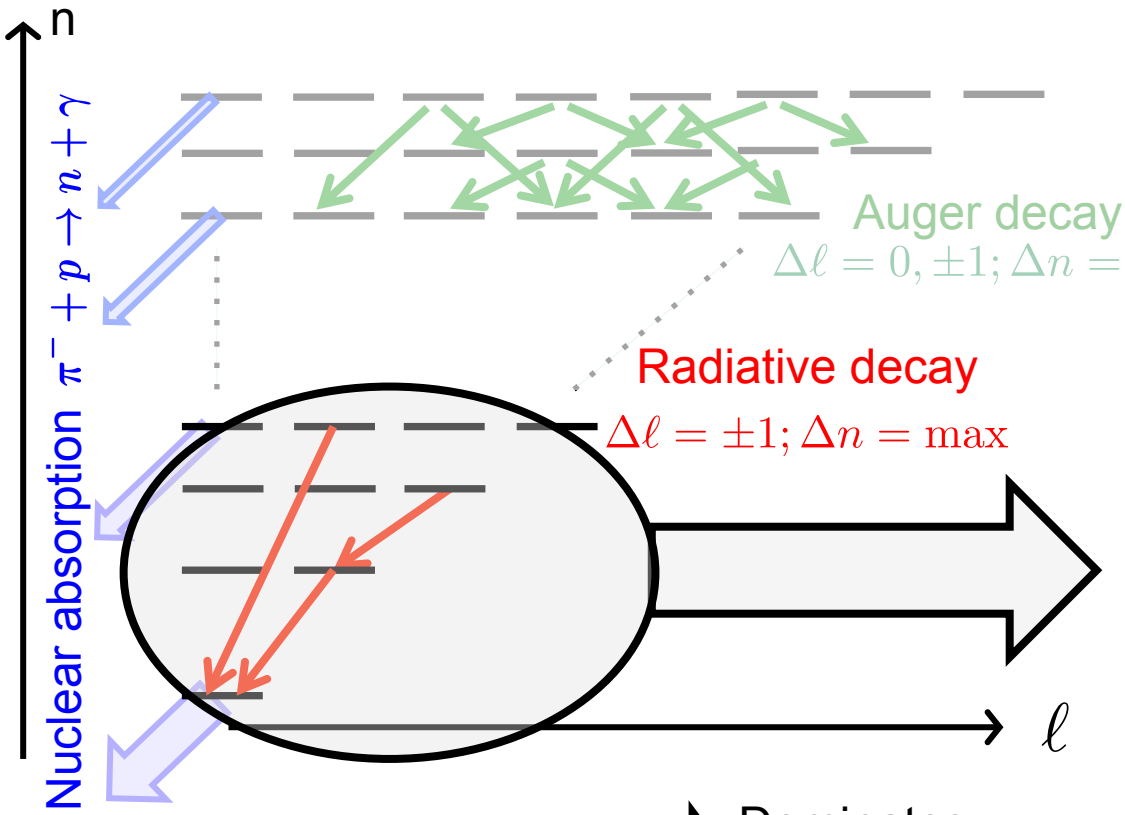
$$n_\pi \sim n_{el}\sqrt{\frac{m_\pi}{m_{el}}} \sim n_{el} \times 16$$

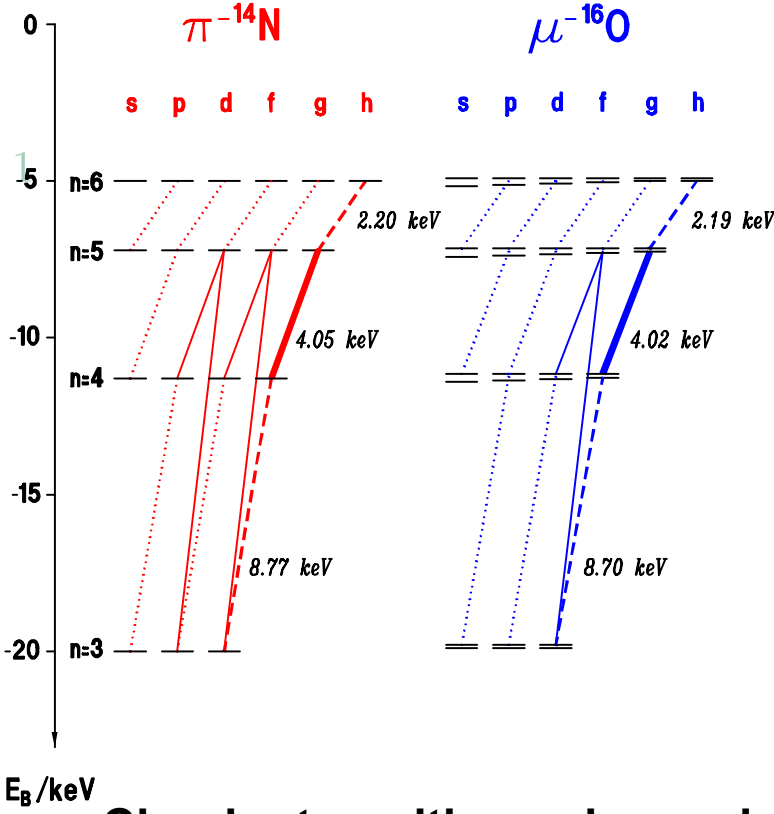
De-excitation via Auger (electron emission) decay

Excited hydrogen-like pionic nitrogen [1,2]

[1] R. Bacher *et al.*, Phys. Rev. A **39**, 1610 (1989)
[2] K. Kirch *et al.*, Phys. Rev. A **59**, 3375 (1999)

Pionic atoms formation and atomic cascade with N_2 gaseous target





$$\Gamma_{\text{Auger}} \propto \frac{1}{\sqrt{2\Delta E + 38\,eV}}$$

⇨ Dominates for n > 6-7

$$\Gamma_{\text{Auger}} \gg \Gamma_{\text{X-ray}} \text{ for } n \gg 1$$

$$\Gamma_{\text{X-ray}} \propto \Delta E^3$$

⇨ Dominates for n < 6-7

Circular transition enhanced

$$|n, \ell = n\rangle \rightarrow |n-1, \ell = n-1\rangle$$

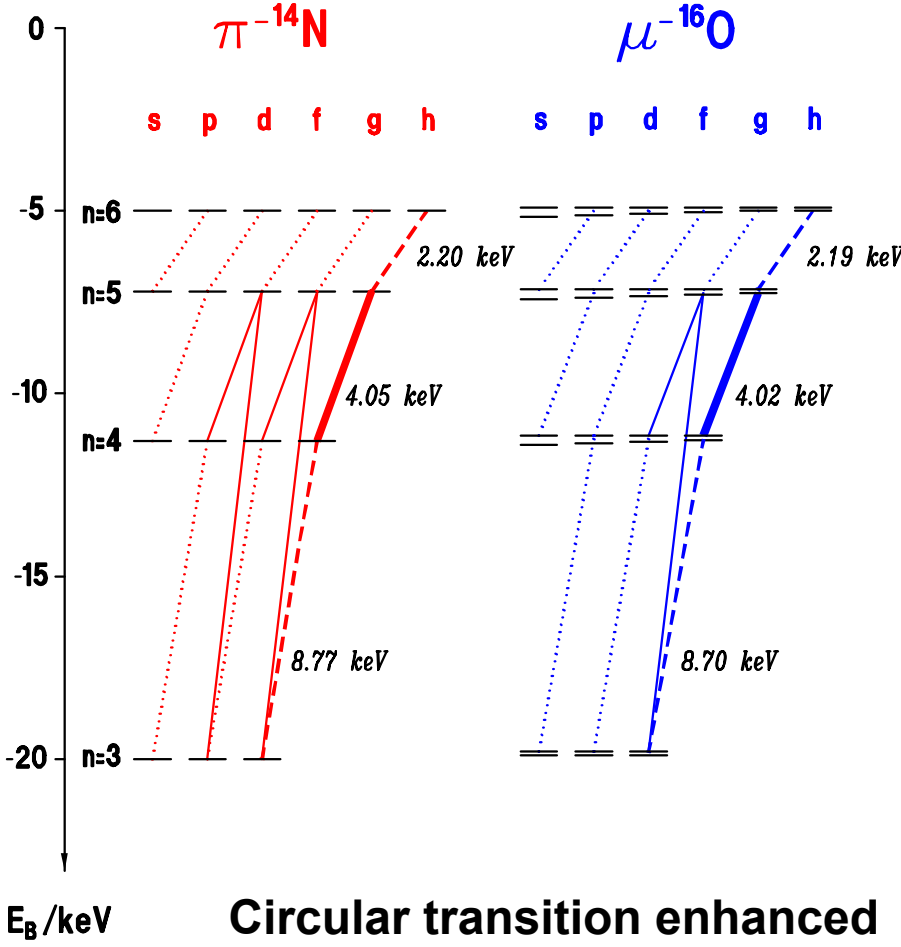
Strong interaction effects minimized

Pion mass measurement from πN and μO X-ray spectroscopy

Pion mass (unknown) or muon mass (reference)

$$E_{\text{X-ray}} = mc^2 \frac{(Z\alpha)^2}{2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) + \mathcal{O}(Z^4\alpha^4)$$

QED calculation only



Measurement of $E_{X\text{-ray}}$

⇨ X-ray diffraction spectroscopy

Circular transition enhanced

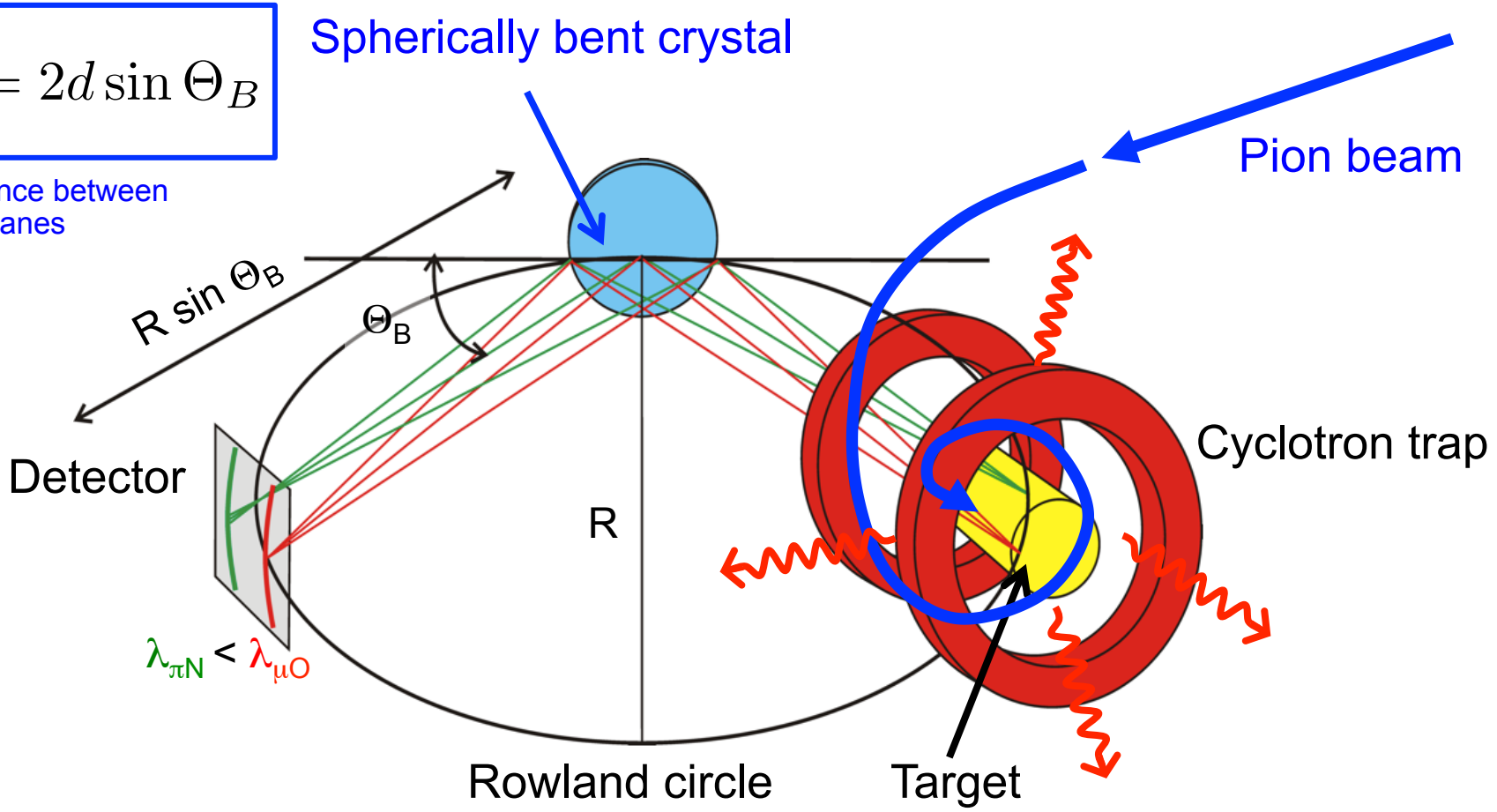
$$|n, \ell = n\rangle \rightarrow |n-1, \ell = n-1\rangle$$

Strong interaction effects minimized

X-ray Bragg spectroscopy with a Johann-type spectrometer [1-4]

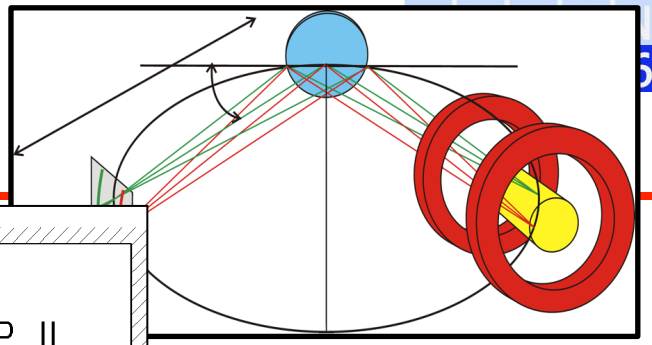
$$\frac{hc}{E} = 2d \sin \Theta_B$$

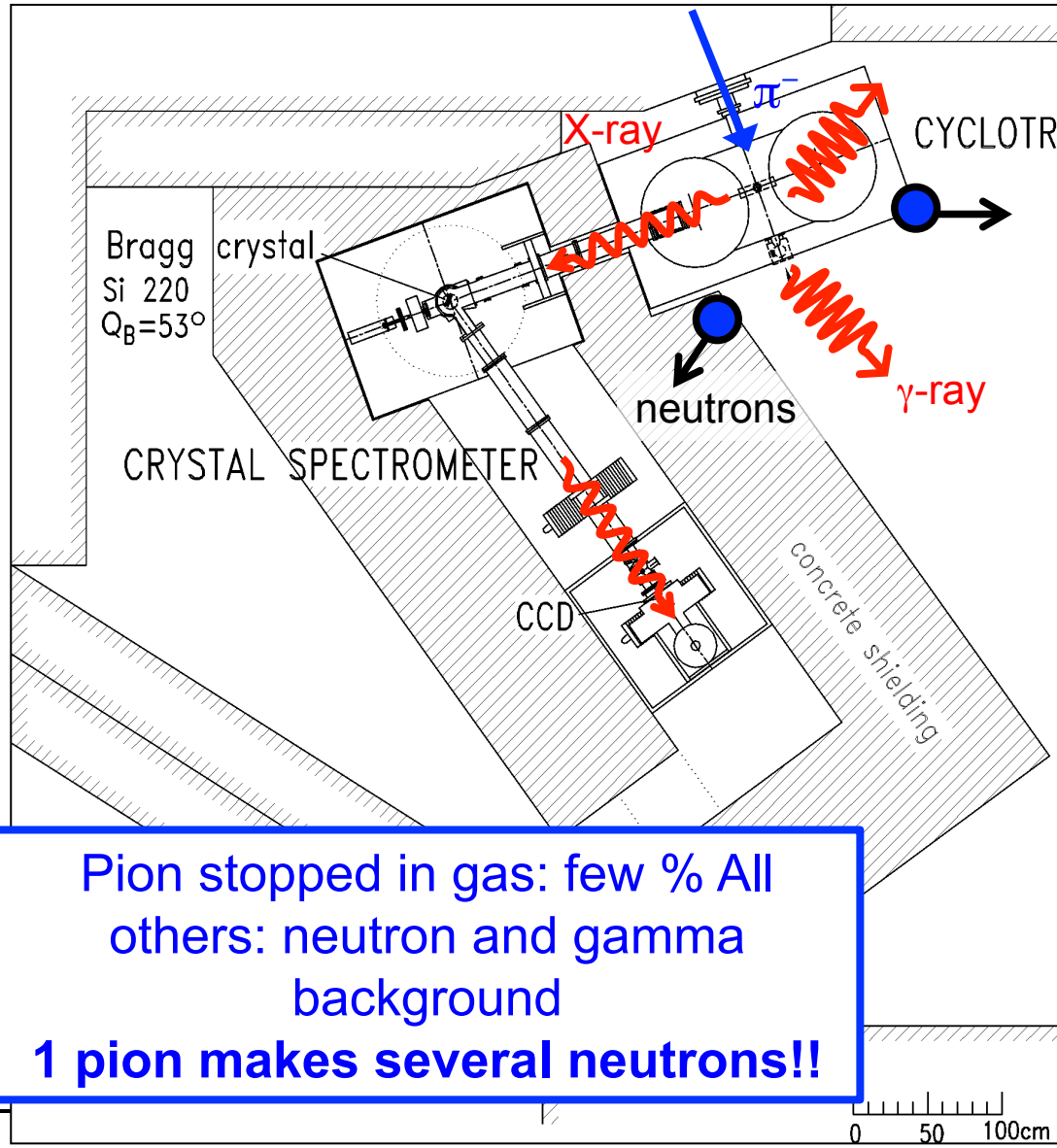
d = distance between crystal planes



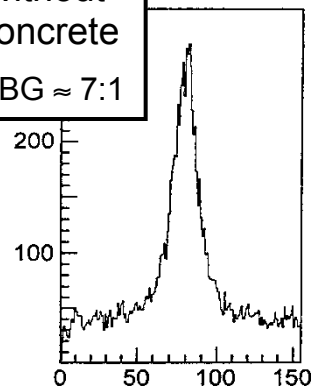
[1] H.H. Johann, Zeitschrift für Physik **69**, 185 (1931)
[2] J. Eggs *et al.*, Zeitschrift für angewandte Physik **20**, 118 (1965)
[3] D. Gotta, Progress in Particle and Nuclear Physics **52**, 133 (2004)
[4] D. Gotta *et al.*, Spectrochim. Acta, Part B **120**, 9 (2016)

Set-up at PSI





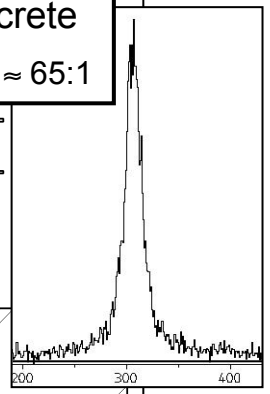
without concrete
P/BG ≈ 7:1



Huge background on the CCD

Heavy concrete

with concrete
P/BG ≈ 65:1

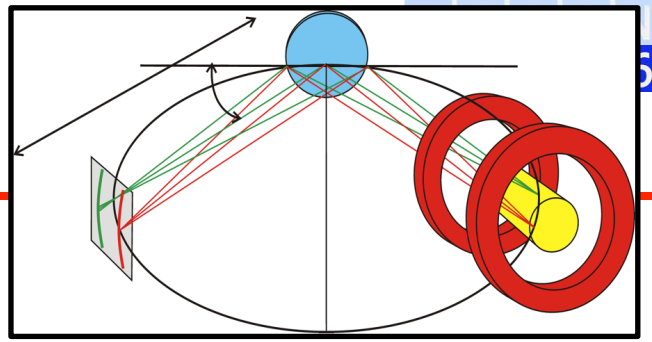


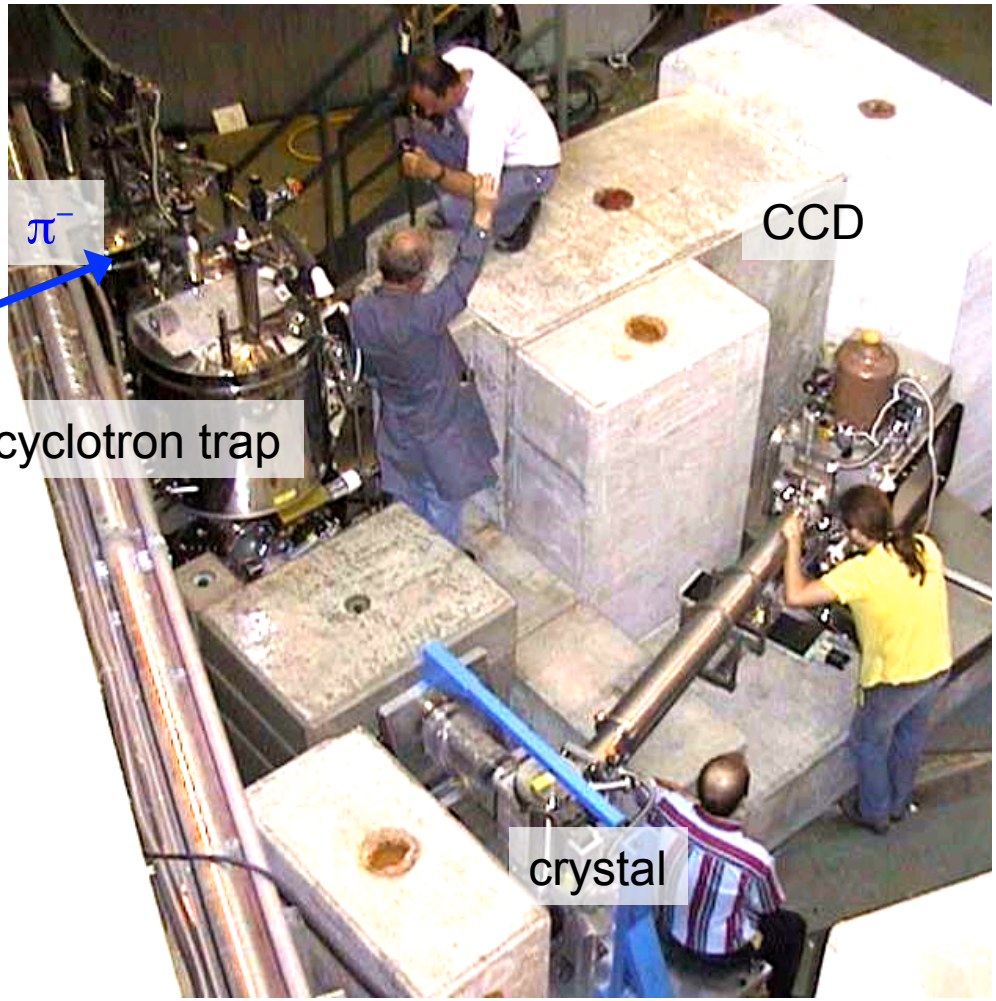
Peak-to-background increased by a factor of 10

Pion stopped in gas: few % All others: neutron and gamma background

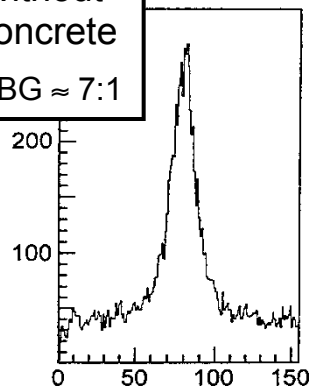
1 pion makes several neutrons!!

Set-up at PSI





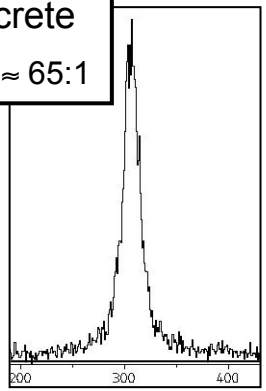
without concrete
P/BG ≈ 7:1



Huge background on the CCD

Heavy concrete

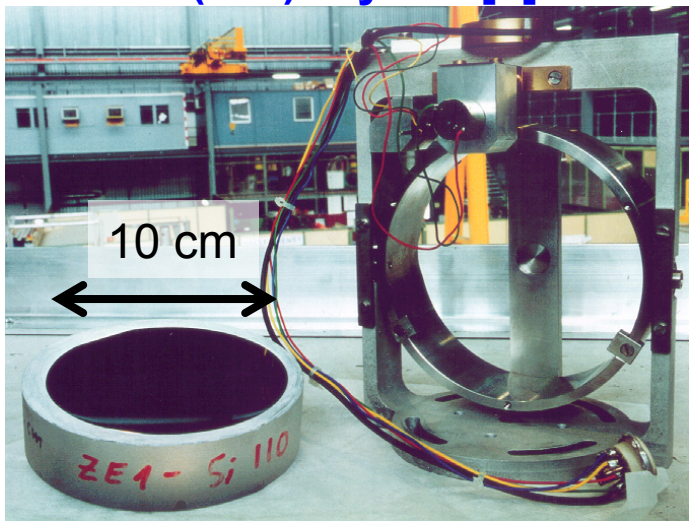
with concrete
P/BG ≈ 65:1



Peak-to-background increased by a factor of 10

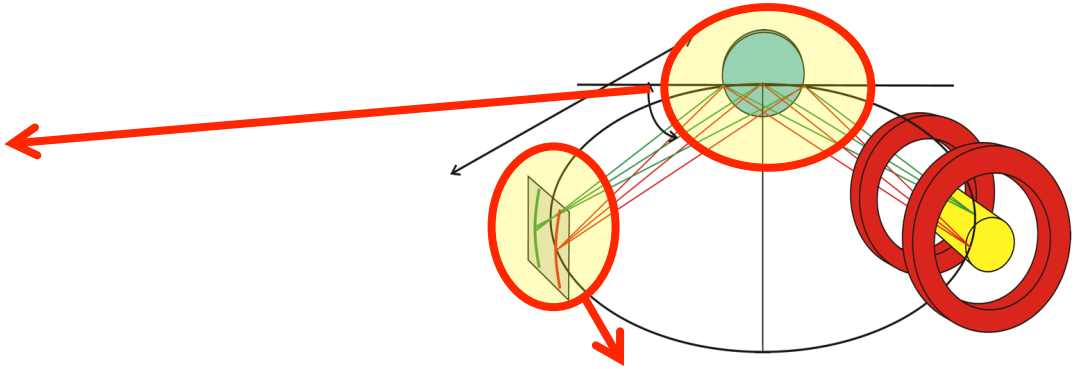
Diffraction crystal and position sensitive detector

Si(110) crystal [1]

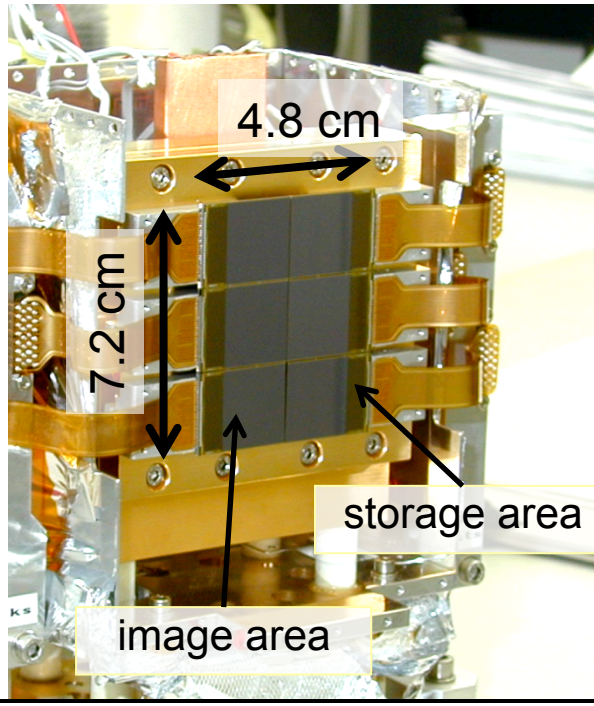


Radius of curvature: ~ 3 m
Diameter: 10 cm
Thickness: 290 μm
Support: polished quartz lens

Produced by Zeiss (Oberkochen, Germany)



2 × 3 X-ray CCD array with frame buffer [2,3]



pixel size 40 μm × 40 μm
600 × 600 pixels per chip
frame transfer ≈ 10 ms

data processing 2.4 s
operates at – 100°C

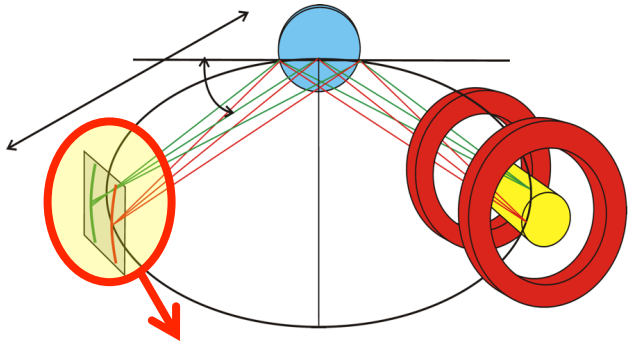
ΔE ≈ 150 eV @ 4 keV
Efficiency ≈ 90%

[1] D.S. Covita *et al.*, Rev. Sci. Instum. **79**, 033102-3 (2008)
[2] N. Nelms *et al.*, Nucl. Instrum. Methods A **484**, 419 (2002)
[3] P. Indelicato *et al.*, Rev. Sci. Instum. **77**, 043107 (2006)

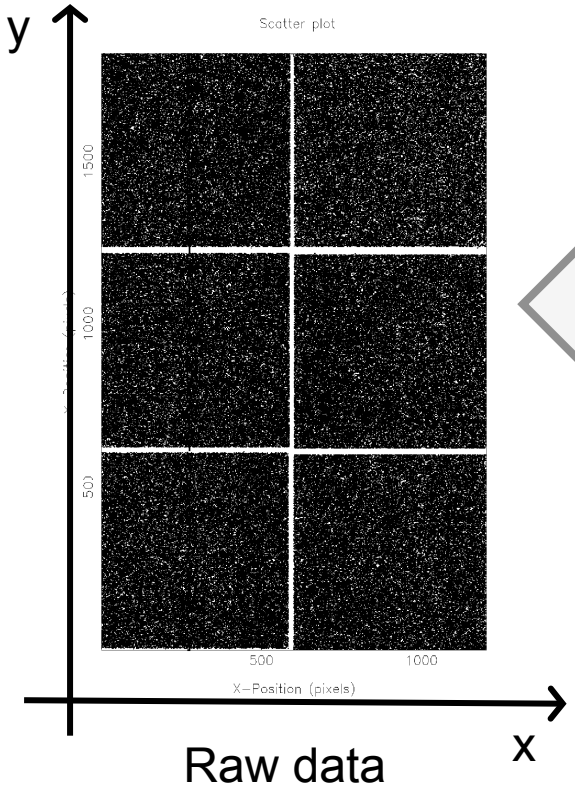
Data acquisition and pre-analysis

… after 5 weeks of data collection …

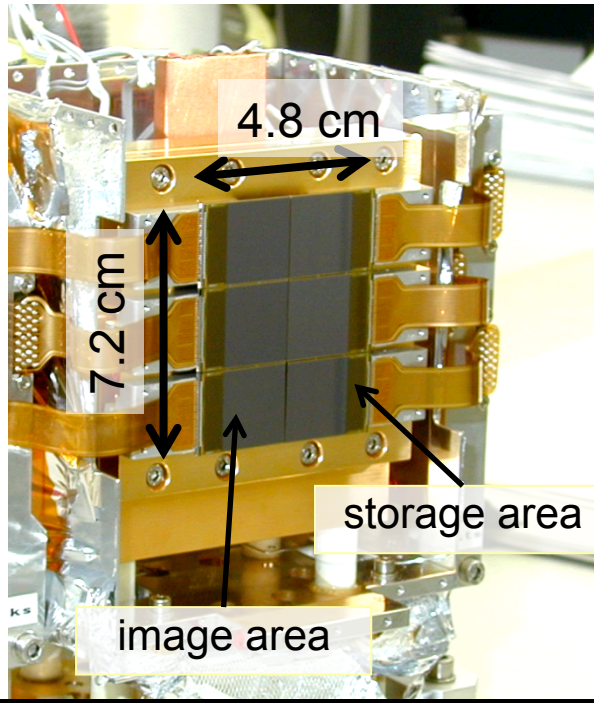
6000 counts in each line
Spectrometer transmission: 5×10^{-8}
Stability of the set-up monitored by 8 keV Cu Kα fluorescence line (fourth order reflection)



2 × 3 X-ray CCD array with frame buffer [2,3]



Raw data



pixel size 40 μm × 40 μm
600 × 600 pixels per chip
frame transfer ≈ 10 ms

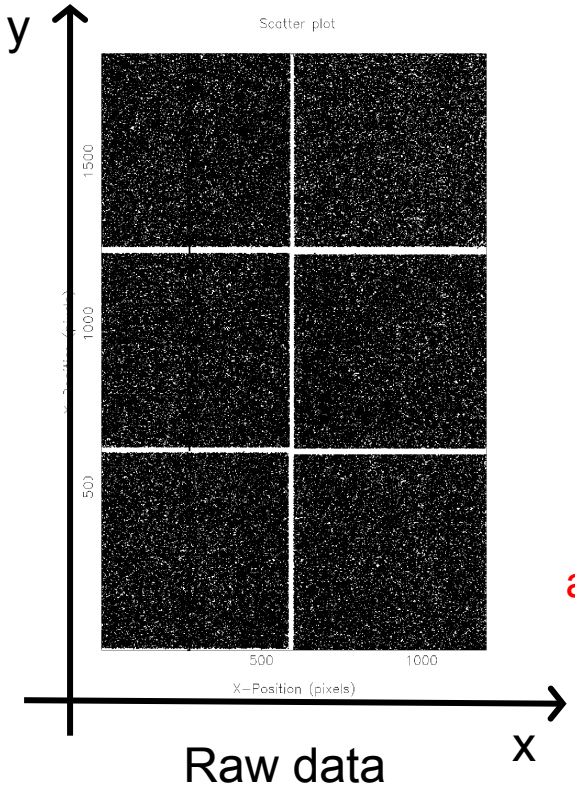
data processing 2.4 s
operates at – 100°C

ΔE ≈ 150 eV @ 4 keV
Efficiency ≈ 90%

Data acquisition and pre-analysis

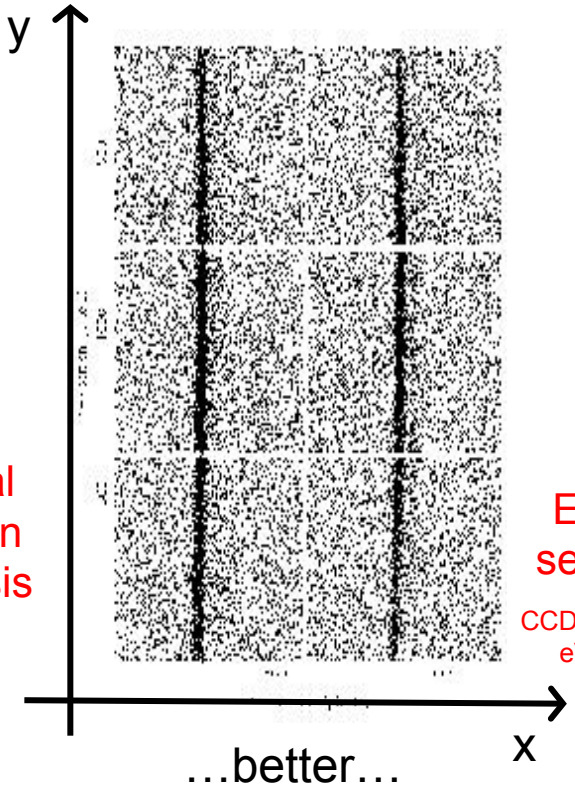
… after 5 weeks of data collection …

6000 counts in each line
Spectrometer transmission: 5×10^{-8}
Stability of the set-up monitored by 8 keV Cu Kα fluorescence line (fourth order reflection)



Raw data

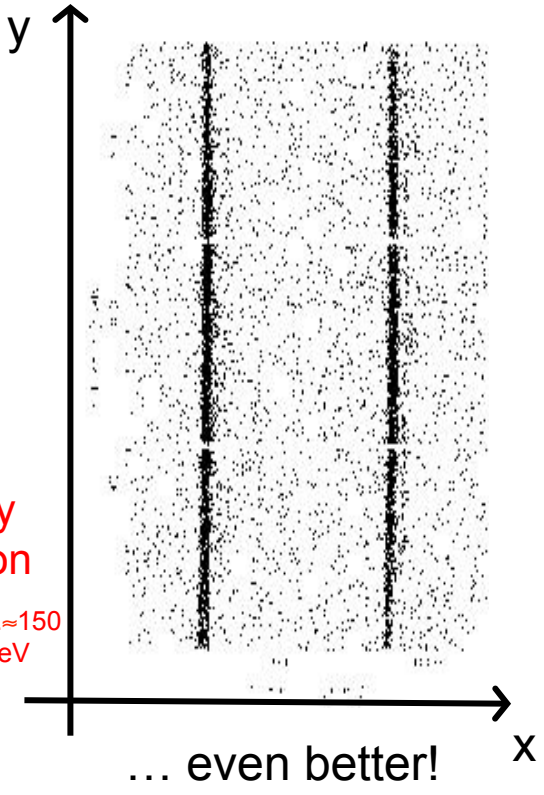
Signal pattern analysis



…better…

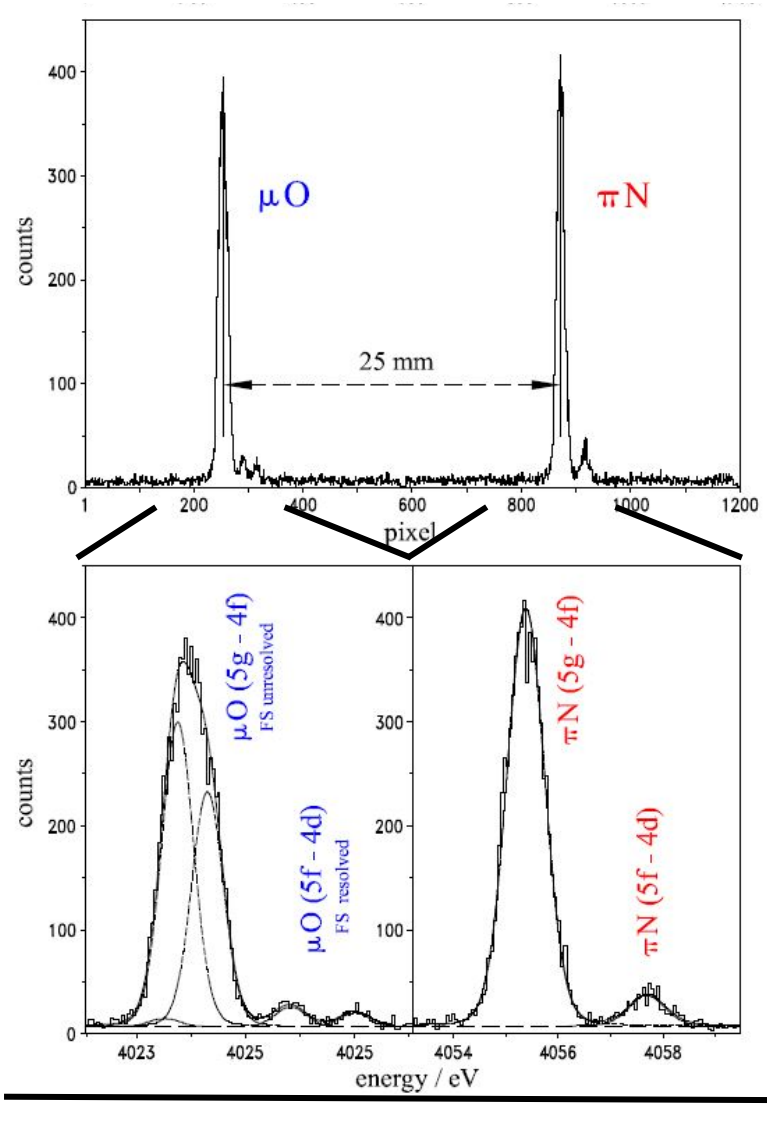
Energy selection

CCD en. res.≈150 eV @ 4 keV

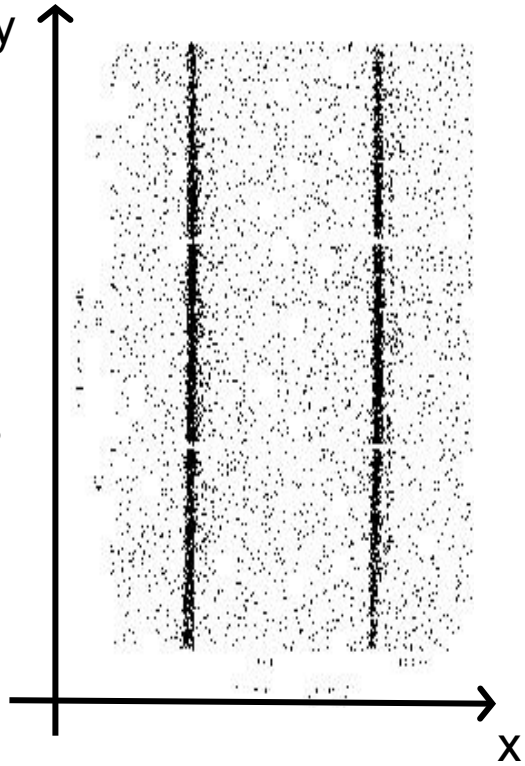


… even better!

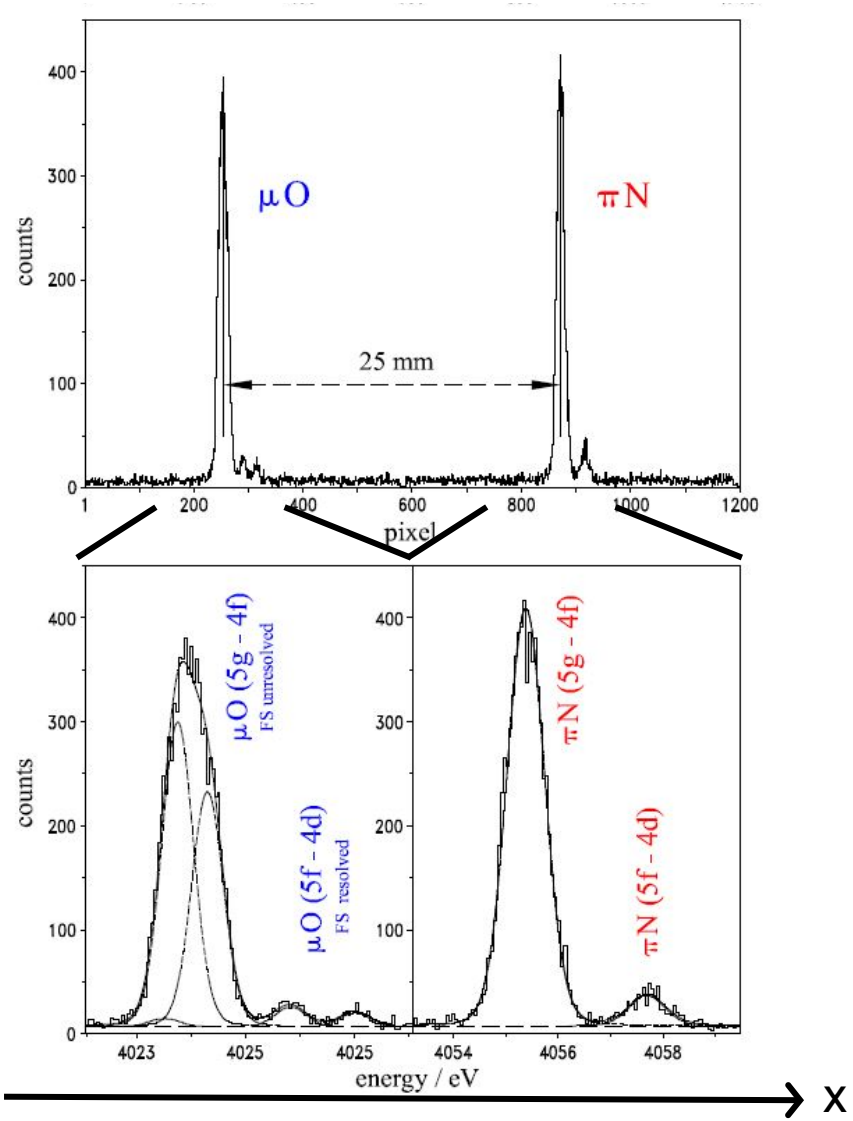
5g-4f transitions in πN and μO

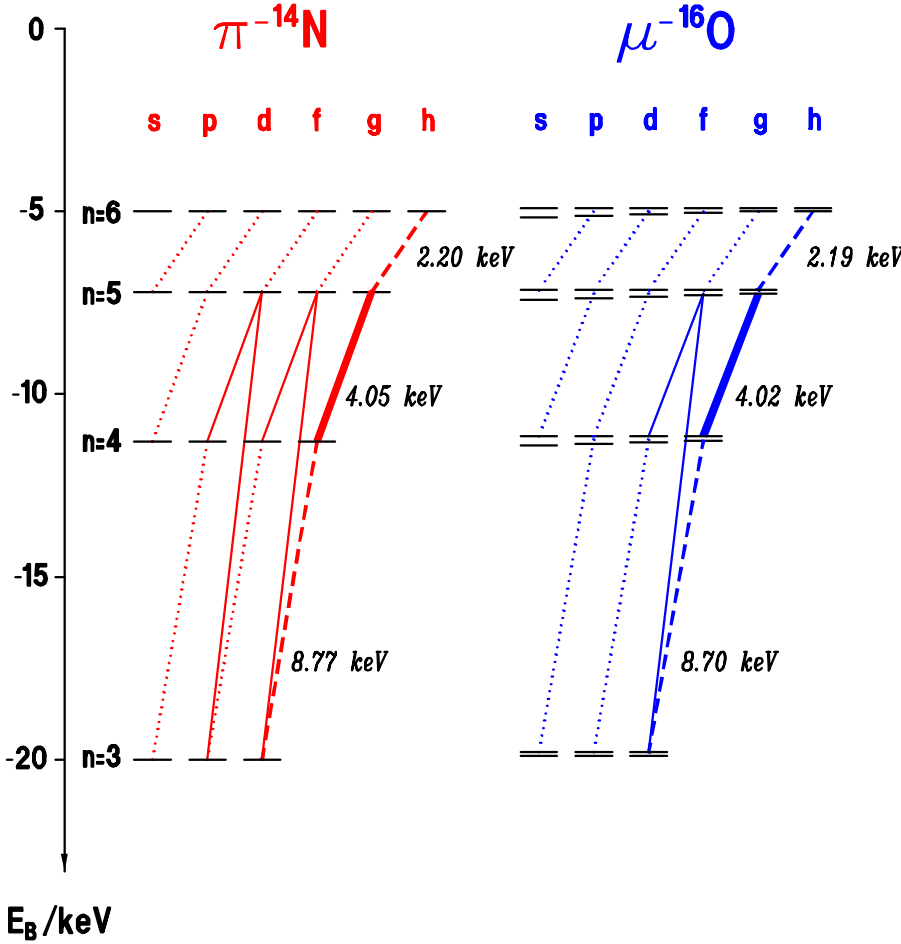


Projection on the x-axis (spectrometer energy dispersion axis)



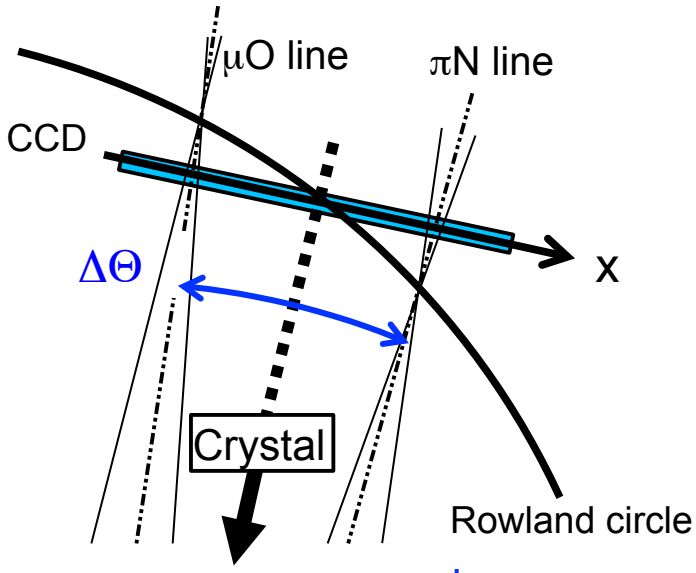
5g-4f transitions in πN and μO

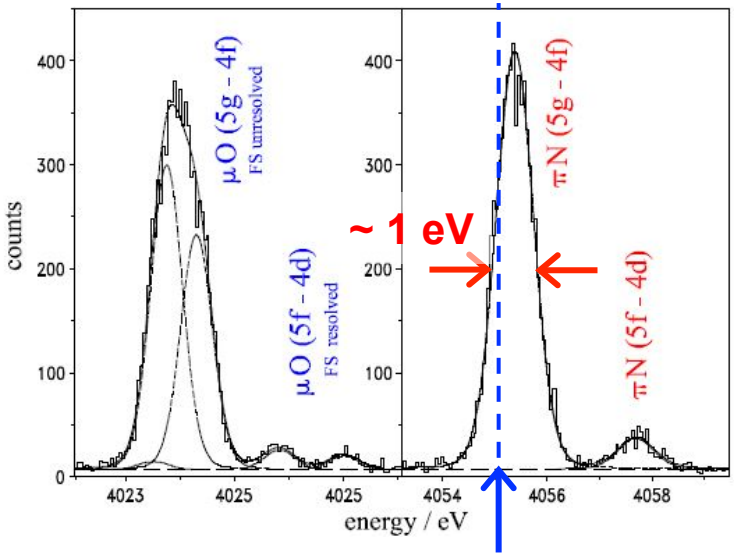




Muon = ½-spin particle
→ fine structure in μO

Lines profiles and positions





Eventual position of the satellite line

Lines profile contributions:

- Doppler broadening from Coulomb explosion of O_2 and N_2 molecules
- Rocking curve of the crystal
- Defocussing

⇦ Monte Carlo simulations and spectrometer characterization [1-5]

Spectra modeling:

- Remaining electrons contribution (satellite line)

⇨ **Possible relative intensity: < 3×10^{-6}**
Obtained by hypothesis test via Bayesian evidence calculation

- Distance and amplitude between parallel and fine structure transitions

⇦ Fixed by the theory

- Different width of the lines

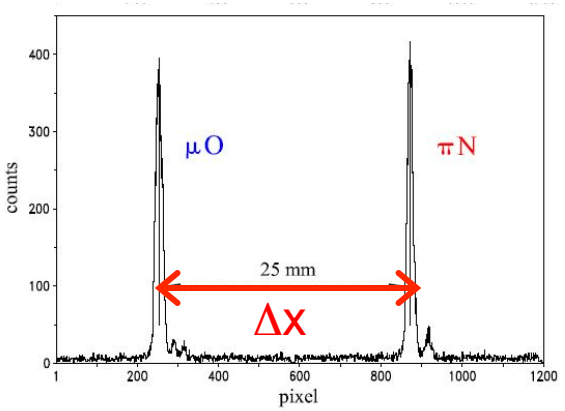
⇨ Information on the atomic de-excitation

[1] D.F. Anagnostopoulos *et al.*, Nucl. Instrum. Methods B **205**, 9 (2003)
[2] D.F. Anagnostopoulos *et al.*, Nucl. Instrum. Methods A **545**, 217 (2005)
[3] D.S. Covita, Ph.D. thesis University of Coimbra (2008)
[4] M. Theisen, Diplomarbeit thesis, University of Aachen (2013)
[5] D.E. Gotta *et al.*, Spectrochim. Acta, Part B **120**, 9 (2016)

From the line position to the pion mass

From the spatial diff. to the angular position diff.

$$\Delta\Theta = -2\arctan\left(\frac{\Delta x}{2D}\right)$$



Statistical uncertainty:
line positions: ± 0.045 pixel

Systematics uncertainty:
crystal–detector distance,
CCD positions and pixel size [1], line modeling,
spectrometer response function [2-6],
Bragg law corrections [7], …

$$\frac{hc}{E} = 2d\sin\Theta_B$$

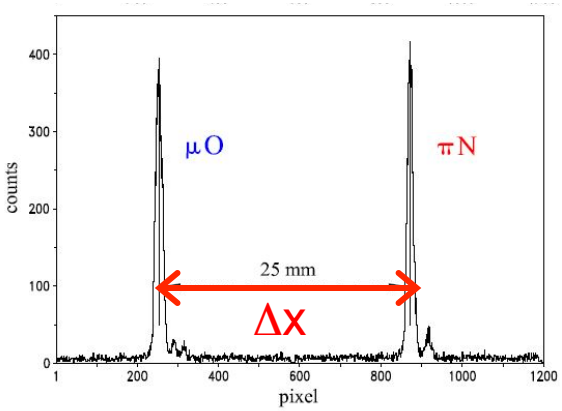
Bragg law

[1] P. Indelicato *et al.*, Rev. Sci. Instum. **77**, 043107 (2006)
[2] D.F. Anagnostopoulos *et al.*, Nucl. Instrum. Methods B **205**, 9 (2003)
[3] D.F. Anagnostopoulos *et al.*, Nucl. Instrum. Methods A **545**, 217 (2005)
[4] D.S. Covita, Ph.D. thesis University of Coimbra (2008)
[5] M. Theisen, Diplomarbeit thesis, University of Aachen (2013)
[6] D.E. Gotta *et al.*, Spectrochim. Acta, Part B **120**, 9 (2016)
[7] M.T., Ph.D. thesis Pierre et Marie Curie University (2005)

From the line position to the pion mass

From the spatial diff. to the angular position diff.

$$\Delta\Theta = -2\arctan\left(\frac{\Delta x}{2D}\right)$$



From the angular position diff. to the transition energy

$$E_{\pi N} = E_{\mu O}\frac{1}{\cos\Delta\Theta - \dfrac{\sqrt{1-[hc/(2dE_{\mu O})]^2}}{hc/(2dE_{\mu O})}\sin\Delta\Theta}$$

$$\frac{hc}{E} = 2d\sin\Theta_B$$

Bragg law

where

$$E_{\mu O} = f^{\rm QED}_{\rm Dirac}(m_\mu) = \tilde{m}_\mu c^2\frac{(Z\alpha)^2}{2}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) + \mathcal{O}^{\rm QED}_{\rm Dirac}(Z^4\alpha^4)$$

E(5g–4f πN) = 4055.397 ± 0.005 eV

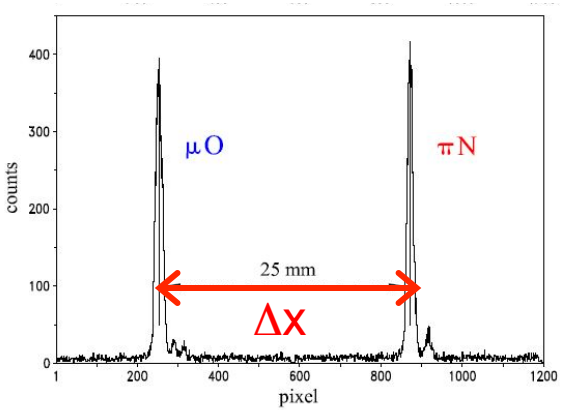
Crystal spacing, conversion constant, QED calculation (for μO) [1-3], ...

[1] P. Indelicato, private comunication
[2] J. Desclaux et al., *Computational Approaches of Relativistic Models in Quantum Chemistry*, vol. 10, Elsevier, 2003.
[3] J. P. Santos et al., Phys. Rev. A 71 (2005) 032501

From the line position to the pion mass

From the spatial diff. to the angular position diff.

$$\Delta\Theta = -2\arctan\left(\frac{\Delta x}{2D}\right)$$



From the angular position diff. to the transition energy

$$E_{\pi N} = E_{\mu O}\frac{1}{\cos\Delta\Theta - \dfrac{\sqrt{1-[hc/(2dE_{\mu O})]^2}}{hc/(2dE_{\mu O})}\sin\Delta\Theta}$$

$$\boxed{\frac{hc}{E} = 2d\sin\Theta_B}$$

Bragg law

where

$$E_{\mu O} = f^{\rm QED}_{\rm Dirac}(m_\mu) = \tilde{m}_\mu c^2\frac{(Z\alpha)^2}{2}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) + \mathcal{O}^{\rm QED}_{\rm Dirac}(Z^4\alpha^4)$$

From the transition energy to the **pion mass**

$$m_\pi = f^{-1}_{\substack{\rm QED\\ \rm Klein\text{-}Gordon}}(E_{\pi N})$$

QED calculation (for πN) [1], ...

[1] M. Trassinelli *et al.*, Phys. Rev. A **76**, 012510(2007)

πN transition energy calculations

The Klein-Gordon equation

$$\left(\frac{1}{c^2}[E+eV_0(r)]^2+\hbar^2\nabla^2-m^2c^2-W(\mathbf{r})\right)\varphi(\mathbf{r})=0$$

$$E^{nl}_{(0)}=\frac{mc^2}{\sqrt{1+\dfrac{(Z\alpha)^2}{\left[n-l-1/2+\sqrt{(l+1/2)^2-(Z\alpha)^2}\right]^2}}}$$

for Coulomb potential with point-like and infinite mass nucleus

5→4 πN QED transition energies details (in eV) [1]

	$5g$-$4f$	$5f$-$4d$
Coulomb	4054.1180	4054.7189
Finite size	0.0000	0.0000
Self-energy	−0.0001	−0.0003
Vacuum polarization (Uehling)	1.2485	2.9470
Vacuum polarization (Wichman-Kroll)	−0.0007	−0.0010
Vacuum polarization (loop after loop)	0.0008	0.0038
Vacuum polarization (Källén-Sabry)	0.0116	0.0225
Relativistic recoil	0.0028	0.0028
HFS shift	−0.0008	−0.0023
Total	4055.3801	4057.6914
Error	±0.0011	±0.0011

Strong interaction effects: 44 μeV 7 meV

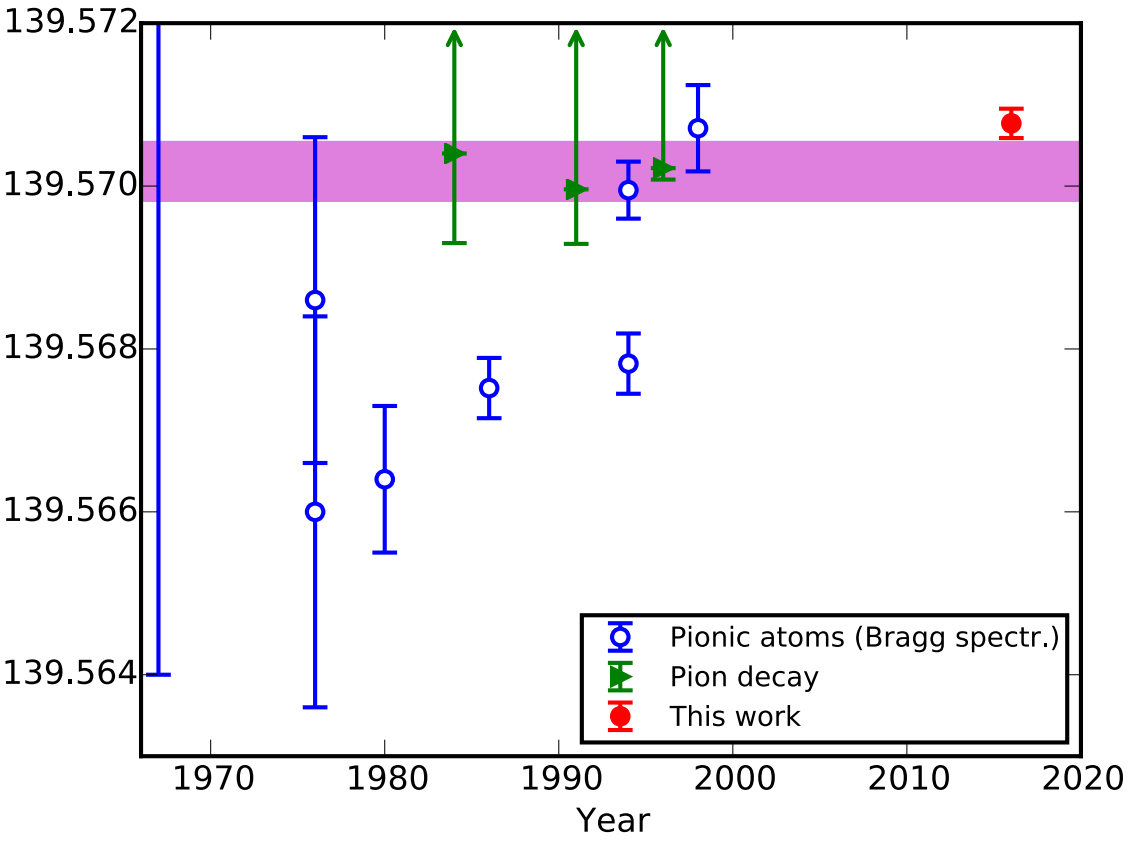
[1] M. Trassinelli *et al.*, Phys. Rev. A **76**, 012510(2007)

List of systematic effects

type of uncertainty	μO / arcsec	πN / arcsec	total / arcsec	uncertainty / ppb
index of refraction shift	13.22	12.94	-0.28	± 20
silicon lattice constant				± 2
bending correction	14.01	13.71	0.30	± 20
penetration depth correction	-0.07	-0.07	0	± 4
focal length				± 670
CCD alignment				± 340
pixel distance				± 120
alignment of detector normal				+ 0 − 30
detector height offset				+ 0 − 35
shape of target window				± 100
shape of reflection				± 225
individual curvature correction				± 150
temperature correction				± 30
response function and Doppler broadening				+ 290 − 350
line pattern modelling				+ 190 − 290
fit interval				± 15
QED energy				± 350
conversion constant hc				± 2
$4f$ strong interaction 45 μeV		0.003	-0.003	± 10
$5g$ strong interaction 0.2 μeV		0.000	0.000	± 0
K electron screening				± 0
total systematic error				+ 950 − 1000
statistical error				± 820

Total uncertainty: 1.3×10^{-6}
Statistical: 0.8×10^{-6}
Systematics: $\sim 1 \times 10^{-6}$

The new measurement of the pion mass



Particle data group [1]:
139.57018 ± 0.00035 MeV/c^2
→ **2.5x10^{-6}** accuracy

Our work [2]:
139.57077 ± 0.00018 MeV/c^2
→ **1.3x10^{-6}** accuracy

- No effect of eventual remaining K-shell electrons (<10^{-9})
- High accuracy calibration line (0.25x10^{-6} from theory calc.)

[1] Particle Data Group. Chinese Phys. C **38**, 090001 (2014)
[2] M. Trassinelli *et al.*, arXiv:1605.03300 (2016)

Additional results

Muonic neutrino mass

Pion decay measurement [1]

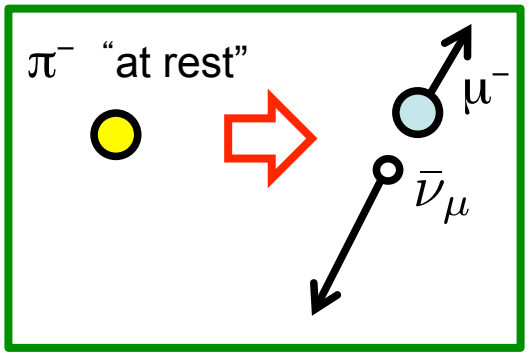
p_μ = 29.792006 ± 0.00011 MeV/c

Pion mass measurement [2]

m_π = 139.57077 ± 0.00018 MeV/c^2

$$m_{\nu_\mu} \in [100, 244]\ keV/c^2 \quad \text{(90\% c.l.)}$$

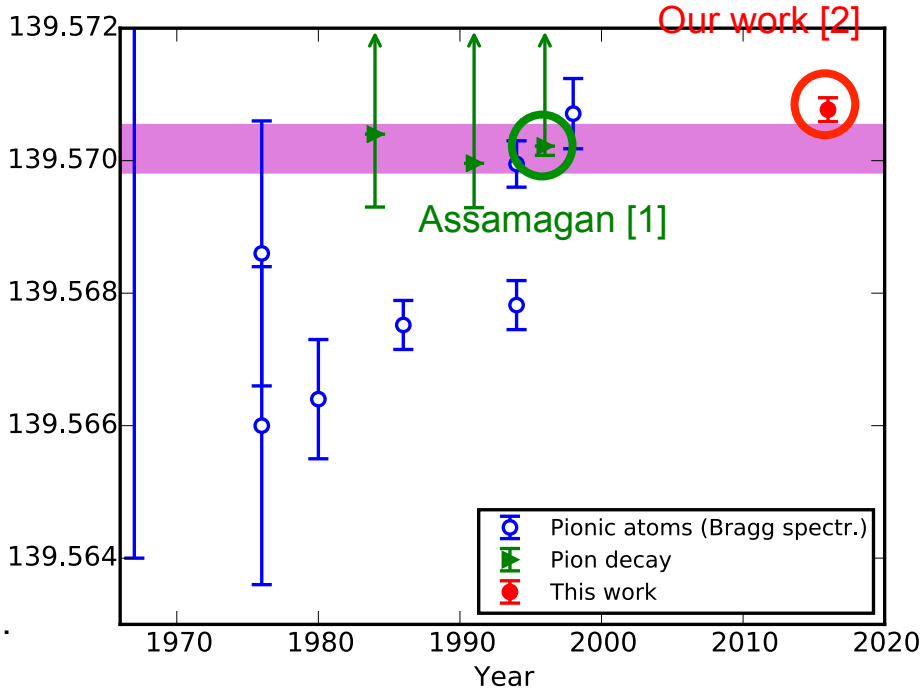
in disagreement with astrophysical boundaries (WMAP and Planck) and ^{3}H decay + ν oscillations **<0.6–2.3 eV**



Measurement of $\mathbf{p}_\mu$ (, m_μ)

→ pion mass

low boundary if no assumptions on the neutrino mass are made



[1] K. Assamagan *et al.*, Phys. Rev. D **53**, 6065–6077 (1996).

[2] M. Trassinelli et al., arXiv:1605.03300, (2016)

Additional results

Muonic neutrino mass

Pion decay measurement

p_μ = 29.792006 ± 0.00011 MeV/c

Pion mass measurement

m_π = 139.57077 ± 0.00018 MeV/c^2

$$m_{\nu_\mu} \in [100, 244]\ keV/c^2 \quad \text{(90\% c.l.)}$$

Klein-Gordon equation test

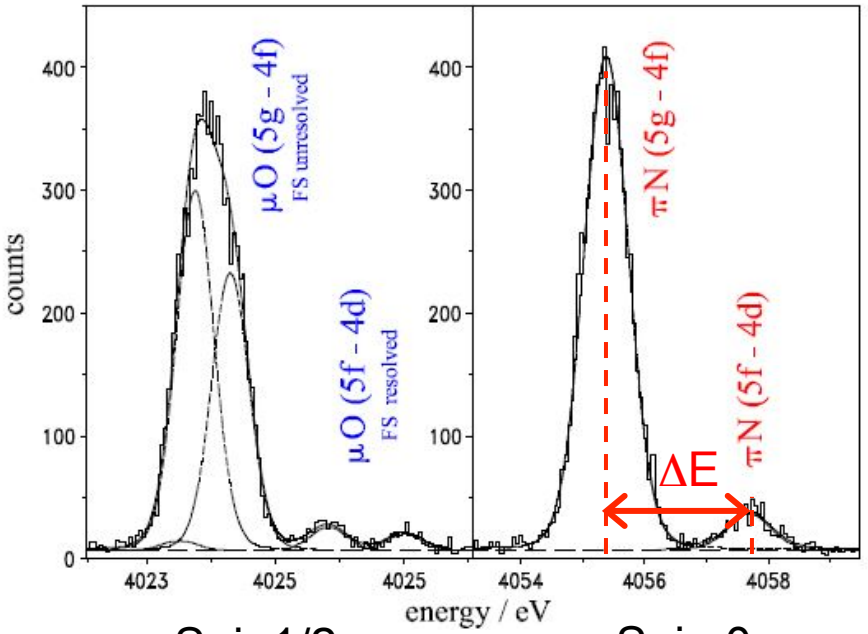
Relativistic quantum mechanic equation for spin-0 particles (+ QED corrections)

ΔE_{theory} = 2.318 ± 0.007 eV

error due to the strong int. corr.

ΔE_{exp} = **2.306 ± 0.015 eV** 95% c.l.

0.4% accuracy test



Spin1/2 → Dirac eq.

Spin 0 → Klein-Gordon eq.

Additional results

Muonic neutrino mass

Pion decay measurement

p_μ = 29.792006 ± 0.00011 MeV/c

Pion mass measurement

m_π = 139.57077 ± 0.00018 MeV/c^2

$$m_{\nu_\mu} \in [100, 244]\ keV/c^2 \quad \text{(90\% c.l.)}$$

Klein-Gordon equation test

Relativistic quantum mechanic equation for spin-0 particles (+ QED corrections)

ΔE_{theory} = 2.318 ± 0.007 eV

error due to the strong int. corr.

0.4% accuracy test

ΔE_{exp} **= 2.306 ± 0.015 eV** 95% c.l.

X-ray standard from pionic atoms [1]

Accuracy of X-ray energies limited by the pion mass: good standard for the few keV regime [1]

[1] D.F. Anagnostopoulos *et al.*, Phys. Rev. Lett. **91**, 240801 (2003).

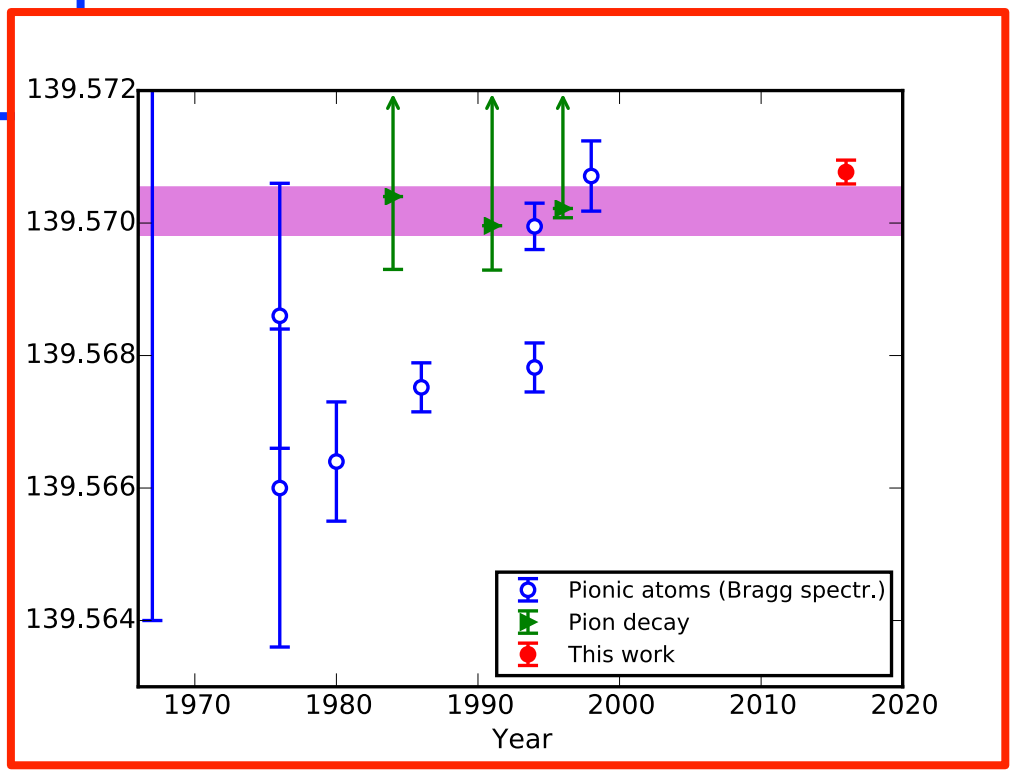
Conclusions

PSI proposal R-97-02

NEW PRECISION DETERMINATION OF THE CHARGED PION MASS

D. Anagnostopoulos[1], M. Augsburger[2], G. Borchert[1], D. Chatellard[2], M. Daum[3], J.-P. Egger[2], D. Gotta[1], P. Hauser[3], P. Indelicato[4], E. Jeannet[2], K. Kirch[3], O. W. B. Schult[1], Th. Siems[1], L. M. Simons[3]

Almost 20 year long story…



New measurement of the charged pion mass using X-ray spectroscopy exotic atoms

m_π = 139.57077 ± 0.00018 MeV/c^2

1.3×10^{-6} rel. accuracy,

PDG present acc. = 2.5×10^{-6}

- No remaining electron contamination
- High accuracy reference energy

M. Trassinelli et al., arXiv:1605.03300 (**2016**)

Thank you!

PION MASS collaboration

Dept. of Material Science, University of Ioannina, Greece
D. F. Anagnostopoulos,

Forschungszentrum Jülich, IKP, JCHP, ZEL, Jülich, Germany
G. Borchert, **D. Gotta**, M. Hennebach,

Paul Scherrer Institut, Lab. for Part. Physics, Villigen, Switzerland
A. Dax, Y.-W. Liu, **L. M. Simons**

Institut de Physique de l'Univerisité de Neuchâtel, Switzerland
J.-P. Egger

Laboratoire Kastler-Brossel (LKB) UPMC ENS CNRS, Paris, France
B. Manil (PhD. in 2001), P. Indelicato,

Dept. of Physics and Astronomy, University of Leicester, England
N. Nelms (PhD. in 2002), A. Wells

Institute des NanoSciences de Paris, France
M.T. (PhD. in 2005 @ LKB)

Cascade theory

V. E. Markushin (PSI), Th. Jensen (ETHZ,PSI,LKB,FZJ,SMI), V. Pomerantsev,
V. Popov (MSU)