



Hadronic molecules with hidden charm and bottom

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Based on:

FKG, C. Hidalgo-Duque, J. Nieves, M. Pavón Valderrama, PRD88(2013)054007

FKG, C. Hidalgo-Duque, J. Nieves, A. Ozpineci, M. Pavón Valderrama, EPJC74(2014)2885

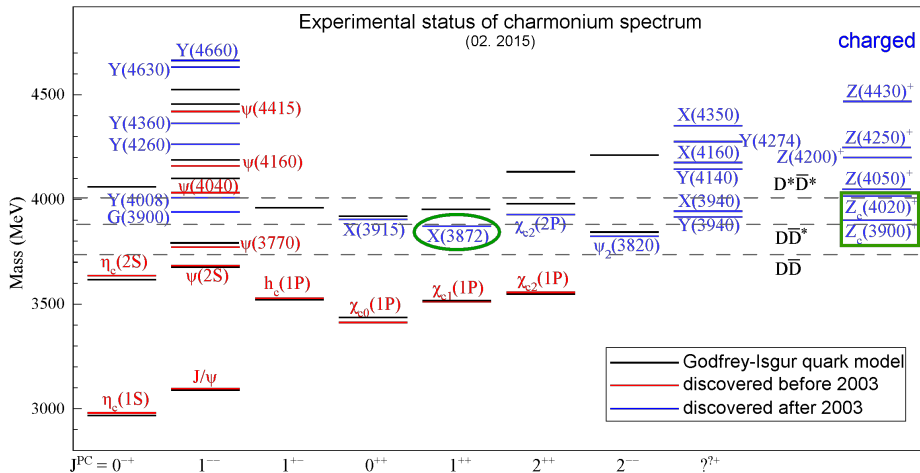
FKG, C. Hanhart, Y. Kalashnikova, U.-G. Meißner, A. Nefediev, PLB742(2015)394

M. Albaladejo, FKG, C. Hidalgo-Duque, J. Nieves, M. Pavón Valderrama, EPJC75(2015)547

M. Albaladejo, FKG, C. Hidalgo-Duque, J. Nieves, PLB755(2016)337

Y.-H. Chen, J. T. Daub, FKG, B. Kubis, U.-G. Meißner, B.-S. Zou, PRD93(2016)034030

Charmonium spectrum



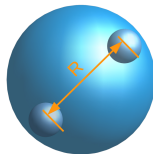
Note: $X(3915)$ is probably just the $\chi_{c2}(2P)$ with 2^{++} Z.-Y. Zhou et al., PRL115(2015)022001

Hadronic molecules (I)

- Hadronic molecule:
dominant component is a composite state of 2 or more hadrons
- **Concept at large distances**, so that can be approximated by system of multi-hadrons **at low energies**

Consider a 2-body bound state with a mass $M = m_1 + m_2 - E_B$

size:
$$R \sim \frac{1}{\sqrt{2\mu E_B}} \gg r_{\text{hadron}}$$



- Only **narrow** hadrons can be considered as components of hadronic molecules, $\Gamma_h \ll 1/r$, r : range of forces

Filin *et al.*, PRL105(2010)019101; FKG, Meißner, PRD84(2011)014013

- Why are hadronic molecules interesting?
 - ☞ one realization of color-neutral objects, analogue of light nuclei
 - ☞ important information for hadron-hadron interaction
 - ☞ **model-independent** statements can be madefor S -wave, **compositeness** $(1 - Z)$ related to measurable quantities

Weinberg, PR137(1965); Baru *et al.*, PLB586(2004); Hyodo, IJMPA28(2013)1330045; ...

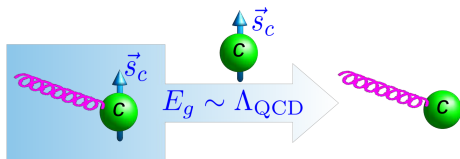
see also, e.g., Weinberg's books: QFT Vol.I, Lectures on QM

$$g_{\text{NR}}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$$
$$a \approx -\frac{2(1 - Z)}{(2 - Z)\sqrt{2\mu E_B}}, \quad r_e \approx \frac{Z}{(1 - Z)\sqrt{2\mu E_B}}$$

scale separation: EFT can be applied

- ☞ understanding the XYZ states, nice objects to apply **heavy quark symmetries**

Heavy quark spin symmetry (I)



- Heavy quark spin symmetry (HQSS):

- ☞ define $\vec{s}_\ell \equiv \vec{J} - \vec{s}_Q$: total angular momentum of the light quark system

- \vec{J} : total angular momentum, \vec{s}_Q : heavy quark spin

- E.g., for D and D^* : $s_\ell^P = \frac{1}{2}^-$

- ☞ s_ℓ is a good quantum number

- ☞ spin multiplet: $(D, D^*), (\eta_c, J/\psi)$

- For hadronic molecules of $Q\bar{Q}$ +light hadron

FKG et al., PRL102(2009)242004

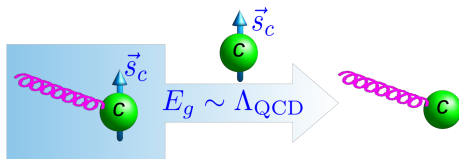
- ☞ exchange at least two gluons, LO: chromo-electric

- ☞ spin multiplet with approximately the same splitting as that for $Q\bar{Q}$

- ☞ the same for hadro-quarkonium

Cleven et al., PRD92(2015)014005

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- Consider S -wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

$$0^{++} : D\bar{D}, \quad D^*\bar{D}^*$$

$$1^{+-} : \frac{1}{\sqrt{2}} (D\bar{D}^* + D^*\bar{D}), \quad D^*\bar{D}^*$$

$$1^{++} : \frac{1}{\sqrt{2}} (D\bar{D}^* - D^*\bar{D})$$

$$2^{++} : D^*\bar{D}^*$$

- Heavy quark spin irrelevant \Rightarrow interaction matrix elements:

$$\left\langle s_{\ell 1}, s_{\ell 2}, s_L \left| \hat{\mathcal{H}} \right| s'_{\ell 1}, s'_{\ell 2}, s_L \right\rangle$$

For each isospin, 2 independent terms

$$\left\langle \frac{1}{2}, \frac{1}{2}, 0 \left| \hat{\mathcal{H}} \right| \frac{1}{2}, \frac{1}{2}, 0 \right\rangle, \quad \left\langle \frac{1}{2}, \frac{1}{2}, 1 \left| \hat{\mathcal{H}} \right| \frac{1}{2}, \frac{1}{2}, 1 \right\rangle$$

\Rightarrow 6 pairs grouped in 2 multiplets with $s_L = 0$ and 1, respectively

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\Rightarrow **6** pairs grouped in **2 multiplets** with $s_L = 0$ and **1**, respectively

- In the limit $m_c \rightarrow \infty$, D and D^* degenerated, convenient to use the basis of states: $s_L^{PC} \otimes s_{c\bar{c}}^{PC}$

☞ S -wave: $s_L^{PC}, s_{c\bar{c}}^{PC} = 0^{-+}$ or 1^{--}

☞ multiplet with $s_L = 0$:

$$0_L^{-+} \otimes 0_{c\bar{c}}^{-+} = 0^{++}, \quad 0_L^{-+} \otimes 1_{c\bar{c}}^{--} = 1^{+-}$$

☞ multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \quad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus 1^{++} \oplus 2^{++}$$

⇒ if $X(3872)$ is a $1^{++} D\bar{D}^*$ molecule, then its $s_L = 1$ Voloshin, PLB604(2004)69

- Multiplets in strict heavy quark limit:

☞ might be 6 molecules: Z_b, Z'_b and W_{b0}, W'_{b0}, W_{b1} and W_{b2} (for $I = 1$)

Bondar et al., PRD84(2011)054010; Voloshin, PRD84(2011)031502;

Mehen, Powell, PRD84(2011)114013

☞ $X(3872)$ has three partners with $0^{++}, 2^{++}$ and 1^{+-}

Hidalgo-Duque et al., PLB727(2013)432; Baru et al., arXiv:1605.09649

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- Calculations using physical D and D^* masses
 - the effective Lagrangian for the LO interaction between spin multiplets,

AlFiky et al., PLB640(2006)238 ...

$$\begin{aligned}
 \mathcal{L}_{4H} = & C_A \text{Tr} \left[\bar{H}_a^{(Q)} H_a^{(Q)} \gamma_\mu \right] \text{Tr} \left[H_b^{(\bar{Q})} \bar{H}_b^{(\bar{Q})} \gamma^\mu \right] \\
 & + C_A^{(\tau)} \text{Tr} \left[\bar{H}_a^{(Q)} \vec{\tau}_{ab} H_b^{(Q)} \gamma_\mu \right] \text{Tr} \left[H_c^{(\bar{Q})} \vec{\tau}_{cd} \bar{H}_d^{(\bar{Q})} \gamma^\mu \right] \\
 & + C_B \text{Tr} \left[\bar{H}_a^{(Q)} H_a^{(Q)} \gamma_\mu \gamma_5 \right] \text{Tr} \left[H_b^{(\bar{Q})} \bar{H}_b^{(\bar{Q})} \gamma^\mu \gamma_5 \right] \\
 & + C_B^{(\tau)} \text{Tr} \left[\bar{H}_a^{(Q)} \vec{\tau}_{ab} H_b^{(Q)} \gamma_\mu \gamma_5 \right] \text{Tr} \left[H_c^{(\bar{Q})} \vec{\tau}_{cd} \bar{H}_d^{(\bar{Q})} \gamma^\mu \gamma_5 \right]
 \end{aligned}$$

$\vec{\tau}$: Pauli matrices in isospin space; $H_a^{(Q)}$: D, D^* ; $H_a^{(\bar{Q})}$: \bar{D}, \bar{D}^*

Isospin $I = 0$ or $1 \Rightarrow$ 4 independent terms:

$C_{0A}, C_{0B}; C_{1A}, C_{1B}$: linear combinations of $C_{A,B}^{(\tau)}$

$$C_{0\phi} = C_\phi + 3C_\phi^{(\tau)}, \quad C_{1\phi} = C_\phi - C_\phi^{(\tau)}, \quad \text{for } \phi = A, B$$

- Some channels have the same linear combination of contact terms

$$V(D\bar{D}^*, 1^{++}) = V(D^*\bar{D}^*, 2^{++}) = C_{IA} + C_{IB}$$

$$V(D\bar{D}^*, 1^{+-}) = V(D^*\bar{D}^*, 1^{+-}) = C_{IA} - C_{IB}$$

$D\bar{D}$ does not have the same: $V(D\bar{D}, 0^{++}) = C_{IA}$

- This would suggest spin multiplets. Good candidates:

☞ $X(3872)$ and $X_2(4013)$ (not observed yet!); $Z_c(3900)$ and $Z_c(4020)$

Nieves, Valderrama, PRD86(2012)056004; ...

$$M_{X_2(4013)} - M_{X(3872)} \approx M_{Z_c(4020)} - M_{Z_c(3900)} \approx M_{D^*} - M_D$$

☞ $Z_b(10610)$ and $Z_b(10650)$:

Bondar et al., PRD84(2011)054010; ...

$$M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B$$

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$$M_{Z_b(10650)} - M_{Z_b(10610)} \approx M_{B^*} - M_B$$

Inputs and predictions (I)

- Solve Lippmann–Schwinger equation regularized with a Gaussian form factor, bound states appear as poles in the first Riemann sheet below threshold

- Inputs:

☞ Mass of $X(3872) \Rightarrow C_{0A} + C_{0B}$

☞ Mass of $Z_b(10610) \Rightarrow C_{1A} - C_{1B}$

- Predicted many partners of $X(3872)$ and $Z_b(10610)$

FKG et al., PRD88(2013)

- ☞ Partners of $X(3872) [1^{++}]$:

$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)
$0(1^{++})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* - D^*\bar{D})$	3875.87	3871.68 (input)
$0(2^{++})$	$D^*\bar{D}^*$	4017.3	4012^{+4}_{-5}
$0(1^{++})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* - B^*\bar{B})$	10604.4	10580^{+9}_{-8}
$0(2^{++})$	$B^*\bar{B}^*$	10650.2	10626^{+8}_{-9}
$0(2^+)$	D^*B^*	7333.7	7322^{+6}_{-7}

Here $V(D\bar{D}^*) = V(\bar{B}B^*) \left[1 + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_c}\right) \right], \dots$ assumed

Inputs and predictions (II)

Partners of $Z_b(10610) [1^{+-}]$:

$I(J^{PC})$	States	Thresholds	Masses ($\Lambda = 0.5$ GeV)
$1(1^{+-})$	$\frac{1}{\sqrt{2}}(B\bar{B}^* + B^*\bar{B})$	10604.4	10602.4 ± 2.0 (input)
$1(1^{+-})$	$B^*\bar{B}^*$	10650.2	10648.1 ± 2.1
$1(1^{+-})$	$\frac{1}{\sqrt{2}}(D\bar{D}^* + D^*\bar{D})$	3875.87	3871_{-12}^{+4} (V)
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Two **virtual states** in charm sector, could correspond to $Z_c(3900)$ and $Z_c(4020)$

- So far, the assignments of the predicted states to the observed ones only based on masses, \Rightarrow **decays and productions**

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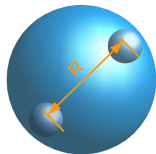
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What can we say about productions and decays (I)

- Essential point of in the spirit of **effective field theory**: **scale separation**
 - ☞ to include **all relevant d.o.f.** at the given scale
 - ☞ to study **near-threshold** structures, one has to take into account the corresponding channel, unless the coupling is weak,
no matter what structure was used as the starting point
⇒ for $X(3872)$: has to consider $D\bar{D}^*$
- Hadronic molecular structure: a long-distance concept
⇒ **not all processes** are sensitive to it !



What can we say about the productions and decays (II)

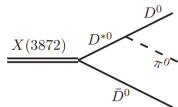
- For processes dominated by long-distance physics:

calculable with controlled uncertainties using low-energy EFT

Examples:

$$X(3872) \rightarrow D^0 \bar{D}^0 \pi^0, X(3872) \rightarrow D^0 \bar{D}^0 \gamma$$

Voloshin (2004); ...



- For processes dominated by short-distance physics (unknown in low-energy EFT): order-of-magnitude estimate at best

Examples:

$$\Rightarrow X(3872) \rightarrow e^+ e^-$$

Denig, FKG, Hanhart, Nefediev, PLB736(2014)221

$$\text{estimate: } \Gamma(X \rightarrow e^+ e^-) \gtrsim 0.03 \text{ eV}$$

$$\text{BESIII: } \Gamma(X \rightarrow e^+ e^-)_{\text{exp}} < 4.3 \text{ eV}$$

BESIII: PLB749(2015)414

\Rightarrow production of $X(3872)$ in B decays, at hadron colliders with large p_T

$$\Rightarrow X(3872) \rightarrow J/\psi \gamma$$

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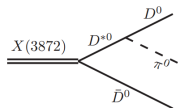
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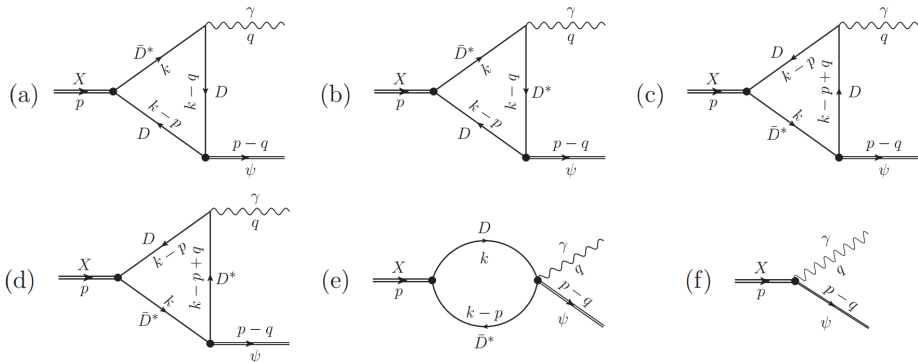
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Decays: $X(3872) \rightarrow \psi\gamma$

FKG, Hanhart, Kalashnikova, Meißner, Nefediev, PLB742(2015)394



The ratio
$$\frac{\mathcal{B}(X(3872) \rightarrow \psi'\gamma)}{\mathcal{B}(X(3872) \rightarrow J/\psi\gamma)} = 2.46 \pm 0.64 \pm 0.29$$

LHCb, NPB886(2014)665

is **insensitive to the molecular component** of the $X(3872)$:

- ☞ loops are sensitive to **unknown** couplings $g_{\psi DD}/g_{\psi' DD}$
- ☞ loops are divergent, needs a counterterm (**short-distance** physics)!

see also Mehen, Springer, PRD83(2011)094009; Molnar et al., arXiv:1601.03366

Decays: $X(3872) \rightarrow D^0 \bar{D}^0 \pi^0$

- A long-distance process, thus can be studied in nonrelativistic EFT
- Already studied by many authors

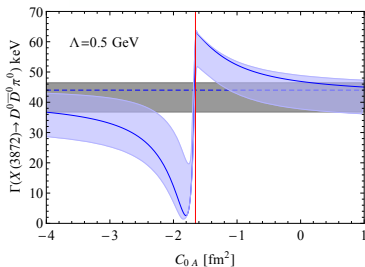
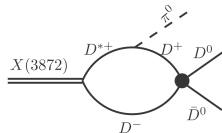
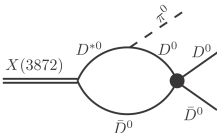
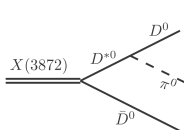
Voloshin (2004); Fleming et al (2007); Braaten, Lu (2007); Hanhart et al (2007); ...

Our new insight:

FKG et al., EPJC74(2014)2885

If there is a **near-threshold** $D\bar{D}$ hadronic molecule \Rightarrow a large impact

Problem: one unknown contact term C_{0A}



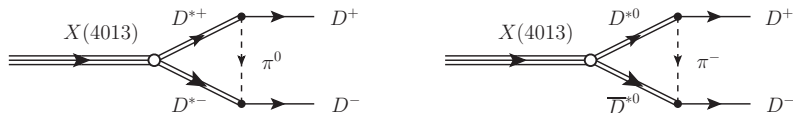
grey band: tree-level result (consistent with Fleming et al., PRD76(2007))

vertical line: a $D\bar{D}$ bound state at threshold

Phenomenology: $X_2(4013) \rightarrow D\bar{D}/D\bar{D}^*$

Albaladejo, FKG, Hidalgo-Duque, Nieves, Pavon Valderrama, EPJC75(2015)547

- $X_2(4013)$: 2^{++} , above $D\bar{D}$, $D\bar{D}^*$ thresholds
dominant decay modes: $D\bar{D}$ and $D\bar{D}^* + c.c.$ in D -wave



- Order-of-magnitude estimate:** $\Gamma(X_2) \sim$ a few MeV–tens of MeV

[MeV]	without pion-exchange FF		with pion-exchange FF	
	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1 \text{ GeV}$	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1 \text{ GeV}$
$\Gamma(D^+ D^-)$	$3.3^{+3.4}_{-1.4}$	$7.3^{+7.9}_{-2.1}$	$0.5^{+0.5}_{-0.2}$	$0.8^{+0.7}_{-0.2}$
$\Gamma(D^0 \bar{D}^0)$	$2.7^{+3.1}_{-1.2}$	$5.7^{+7.8}_{-1.8}$	$0.4^{+0.5}_{-0.2}$	$0.6^{+0.7}_{-0.2}$

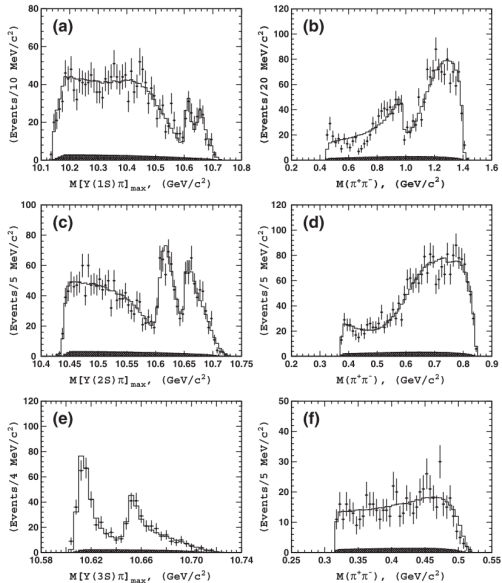
- Fresh calculation with nonperturbative pion: $\Gamma(X_2) \sim 50 \text{ MeV}$

Baru et al., arXiv:1605.09649

Phenomenology: Z_b (I)

Observation of two Z_b structures at around 10.61 GeV and 10.65 GeV

Belle (2011)



Phenomenology: Z_b (II)

- Such a Z_b was first proposed by Voloshin in 1982 to explain the transition $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$

Possible four-quark isovector resonance in the family of Υ particles

M. B. Voloshin

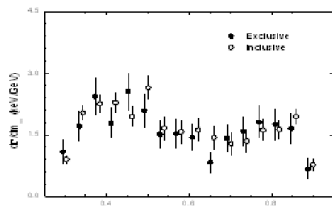
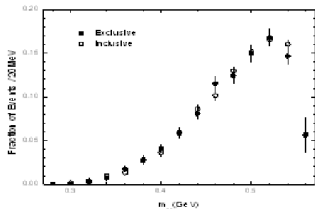
Institute of Theoretical and Experimental Physics

(Submitted 15 November 1982)

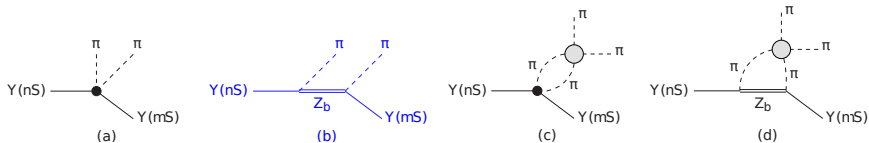
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 1, 58–60 (5 January 1983)

It is suggested, on the basis of data on the pion spectrum in the decay $\Upsilon^n \rightarrow \Upsilon\pi^+\pi^-$, that there is an isovector resonance with a mass near the Υ^n mass.

- Anisovich et al. revisited it and suggested its mass to be within [10.4, 10.8] GeV PRD51(1995)378(R)
- Data of $M_{\pi\pi}$ distributions for $\Upsilon(2S, 3S) \rightarrow \Upsilon(1S)\pi\pi$ CLEO (1998, 1994)



- Reexamine **with measured Z_b properties**



- Use dispersion relation to account for the $\pi\pi$ FSI [Anisovich, Leutwyler, PLB375\(1996\)335](#)

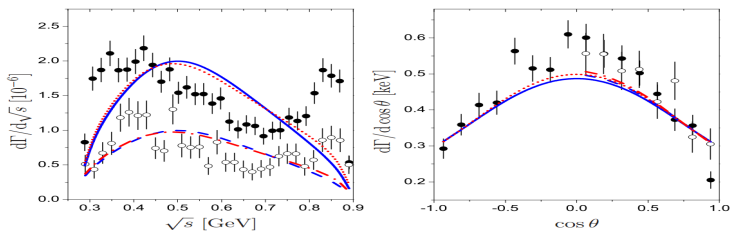
$$\mathcal{M}^{\text{full}}(s, \cos \theta) = \epsilon_{Y(nS)} \cdot \epsilon_{Y(mS)} \sum_{l=0}^{\infty} \left[M_l(s) + \hat{M}_l(s) \right] P_l(\cos \theta),$$

$$M_l(s) = \Omega_l^0(s) \left\{ P_l^{n-1}(s) + \frac{s^n}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dx}{x^n} \frac{\hat{M}_l(x) \sin \delta_l^0(x)}{|\Omega_l^0(x)|(x-s)} \right\}$$

Omnès function:
$$\Omega_l^I(s) = \exp \left\{ \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} \frac{dx}{x} \frac{\delta_l^I(x)}{x-s} \right\}$$

Phenomenology: Z_b (IV)

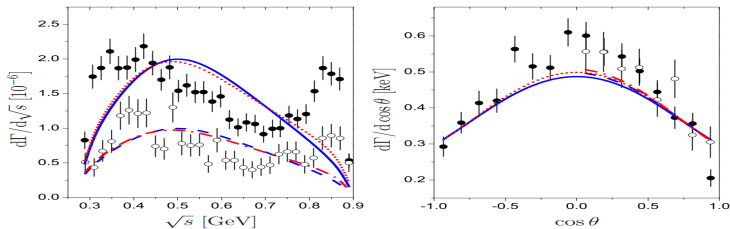
- w/o Z_b or w/ Z_b but using the $Z_b \Upsilon \pi$ coupling constants fixed from the branching fractions reported by Belle in [arXiv:1209.6450 \[hep-ex\]](https://arxiv.org/abs/1209.6450)



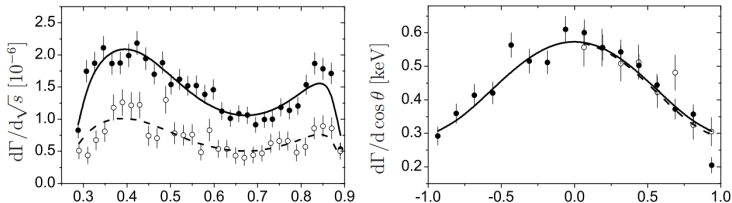
- But best fit gives a **much larger $Z_b \Upsilon \pi$ coupling**

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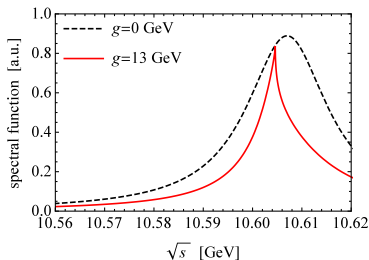


Phenomenology: Z_b (V)

- Belle used BW parameterization in fit/extraction of branching fractions
- $Z_b^{(\prime)}$ located at the $B^{(*)}\bar{B}^*$ threshold \Rightarrow use Flatté parameterization

$$\frac{1}{\left|s - m_{Z_b}^2 + im_{Z_b} [\Gamma_1 + \Gamma_{B\bar{B}^*}(s)]\right|^2}$$

here $\Gamma_{B\bar{B}^*}(s) = \frac{g^2}{8\pi m_{Z_b}^2} [k\theta(\sqrt{s} - m_B - m_{B^*}) + i\kappa\theta(-\sqrt{s} + m_B + m_{B^*})]$



using $m_{Z_b} = 10.607$ GeV and $\Gamma_1 = 20$ MeV

- Γ_1 larger than the nominal peak width \Rightarrow data need be reanalyzed using Flatté

- hadronic molecule structure should be detected in processes sensitive to **long-distance physics**
- heavy quark symmetry can provide interesting predictions/insights to hadronic molecules
- should go beyond the mass spectrum because **important structure information is contained in the coupling**
⇒ **decays and productions**
- decay width of $X_2(4013)$ **estimated** to be narrow
- effects of **virtual Z_b** in $\Upsilon(3S)$ decays; Z_b data should be reanalyzed to get the correct partial widths

Thank you !

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Thank you !

Backup slides

Compositeness (I)

Weinberg, PR137(1965); Baru *et al.*, PLB586(2004); ...

see also, e.g., Weinberg's books: QFT Vol.I, Lectures on QM

Model-independent result for *S*-wave loosely bound composite states:

Consider a system with Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + V$$

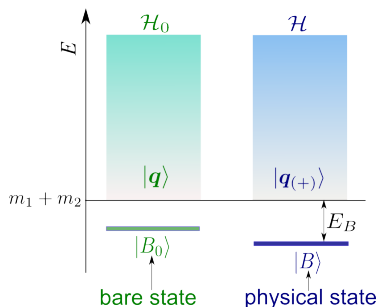
\mathcal{H}_0 : free Hamiltonian, V : interaction potential

- Compositeness:**

the probability of finding the physical state $|B\rangle$ in the 2-body continuum $|q\rangle$

$$1 - Z = \int \frac{d^3\mathbf{q}}{(2\pi)^3} |\langle \mathbf{q} | B \rangle|^2$$

- $Z = |\langle B_0 | B \rangle|^2$, $0 \leq (1 - Z) \leq 1$
 - $Z = 0$: pure bound (composite) state
 - $Z = 1$: pure elementary state



Compositeness (II)

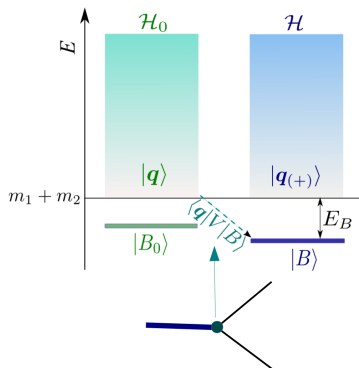
Compositeness : $1 - Z = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} |\langle \mathbf{q} | B \rangle|^2$

- Schrödinger equation

$$(\mathcal{H}_0 + V)|B\rangle = -E_B|B\rangle$$

multiplying by $\langle \mathbf{q} |$ and using $\mathcal{H}_0|\mathbf{q}\rangle = \frac{\mathbf{q}^2}{2\mu}|\mathbf{q}\rangle$

$$\langle \mathbf{q} | B \rangle = -\frac{\langle \mathbf{q} | V | B \rangle}{E_B + \mathbf{q}^2/(2\mu)}$$



- S*-wave, small binding energy so that $R = 1/\sqrt{2\mu E_B} \gg r$, r : range of forces

$$\langle \mathbf{q} | V | B \rangle = g_{\text{NR}} [1 + \mathcal{O}(r/R)]$$

- Compositeness:

$$1 - Z = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{g_{\text{NR}}^2}{[E_B + \mathbf{q}^2/(2\mu)]^2} \left[1 + \mathcal{O}\left(\frac{r}{R}\right) \right] = \frac{\mu^2 g_{\text{NR}}^2}{2\pi\sqrt{2\mu E_B}} \left[1 + \mathcal{O}\left(\frac{r}{R}\right) \right]$$

- **Coupling constant measures the compositeness** for an S -wave bound state with a small binding energy (model-independent)

$$g_{\text{NR}}^2 \approx (1 - Z) \frac{2\pi}{\mu^2} \sqrt{2\mu E_B} \leq \frac{2\pi}{\mu^2} \sqrt{2\mu E_B}$$

- Z can be measured directly from observables, such as scattering length a and effective range r_e Weinberg (1965)

$$a = -\frac{2R(1-Z)}{2-Z} \left[1 + \mathcal{O}\left(\frac{r}{R}\right) \right], \quad r_e = \frac{RZ}{1-Z} \left[1 + \mathcal{O}\left(\frac{r}{R}\right) \right]$$

- Example: deuteron as pn bound state. Exp.: $E_B = 2.2$ MeV, $a_{3S_1} = -5.4$ fm

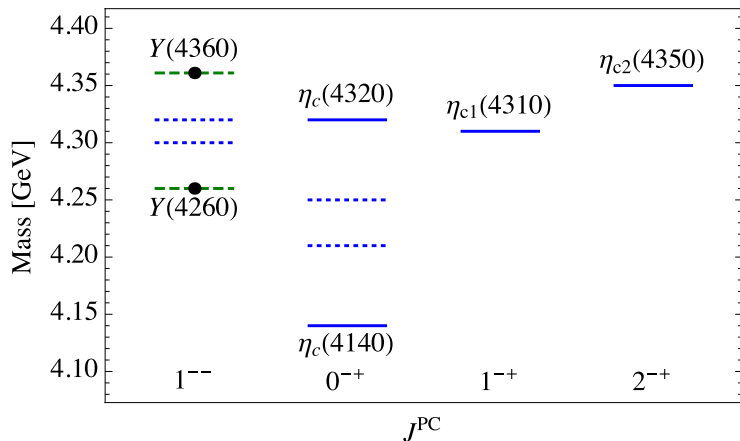
$$a_{Z=1} = 0 \text{ fm}, \quad a_{Z=0} = (-4.3 \pm 1.4) \text{ fm}$$

If the $Y(4260)$ and $Y(4360)$ are mixed hadro-charmonia with h_c and ψ' core,

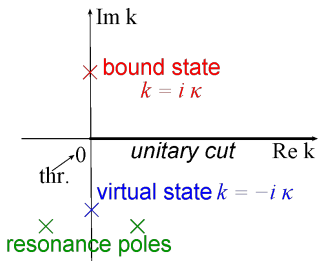
Li, Voloshin, MPLA29(2014)1450060

implications of HQSS for hadro-charmonia:

Cleven et al., PRD92(2015)014005



Bound state and virtual state (I)



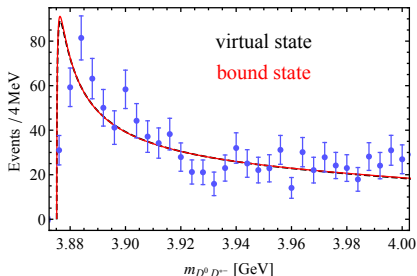
Suppose the scattering length is very large, the S -wave scattering amplitude

$$f_0(k) = \frac{1}{k \cot \delta_0(k) - ik} \simeq \frac{1}{-1/a - ik}$$

- 👉 bound state pole: $1/a = \kappa \equiv \sqrt{2\mu b}$
- 👉 virtual state pole: $1/a = -\kappa$

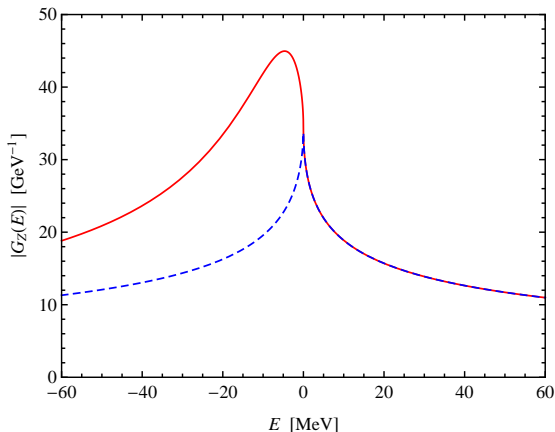
- If the same “binding” energy, **cannot** be distinguished **above threshold** (k is real):

$$|f_0(k)|^2 \sim \frac{1}{\kappa^2 + k^2}$$



Bound state and virtual state (II)

- Different line shapes below threshold \Rightarrow in **inelastic** channel



A **bound state** and **virtual state** with a 5 MeV binding energy, a width to the **inelastic** channel is allowed.


Cleven et al., EPJA47(2011)120

- Two P -wave vertices: divergent loop integral?

$$\epsilon_{ij} \int \frac{d^4l}{(2\pi)^4} \frac{l^i l^j}{[(l+q)^2 - M^2 + i\epsilon][(k-l)^2 - M^2 + i\epsilon](l^2 - m^2 + i\epsilon)}$$

here ϵ_{ij} : polarization tensor, **symmetric, traceless**

only **convergent part** contributes! reason: D -wave decay

- so it seems straightforward to calculate the widths, however, **a problem**:
 a large dependence on the cutoff using the same Gaussian form factor

[MeV]	$\Lambda = 0.5 \text{ GeV}$	$\Lambda = 1 \text{ GeV}$
$\Gamma(D^+D^-)$	$3.3^{+3.4}_{-1.4}$	$7.3^{+7.9}_{-2.1}$
$\Gamma(D^0\bar{D}^0)$	$2.7^{+3.1}_{-1.2}$	$5.7^{+7.8}_{-1.8}$

Errors in the table: HQSS breaking (20% in potential) + $X(3872)$ input

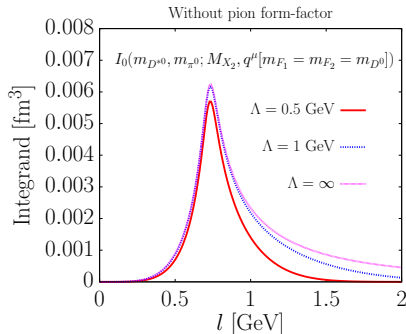
$X_2(4013) \rightarrow D\bar{D}/D\bar{D}^*$ (backup-II)

- Reason for the large cutoff dependence: **large external momenta**

$$q_D \simeq 730 \text{ MeV} \quad [q_D \simeq 510 \text{ MeV for } D\bar{D}^*]$$

\Rightarrow momentum of the exchanged pion can be large

- a large contribution from $l \gtrsim 1 \text{ GeV}$, (l : pion momentum)

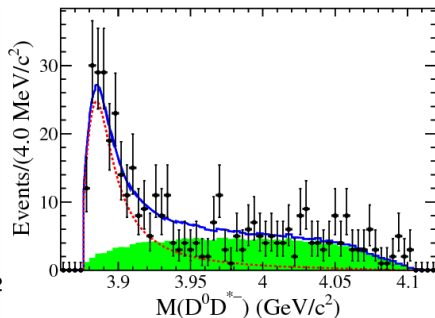
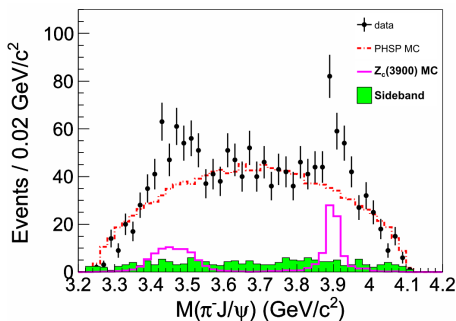


reaches the valid/invalid edge of EFT, **NO** reliable/precise prediction can be made

Phenomenology: $Z_c(3900)$ (I)

- structure around 3.9 MeV seen in both $J/\psi\pi$ and $D\bar{D}^*$, discovered by BESIII and Belle

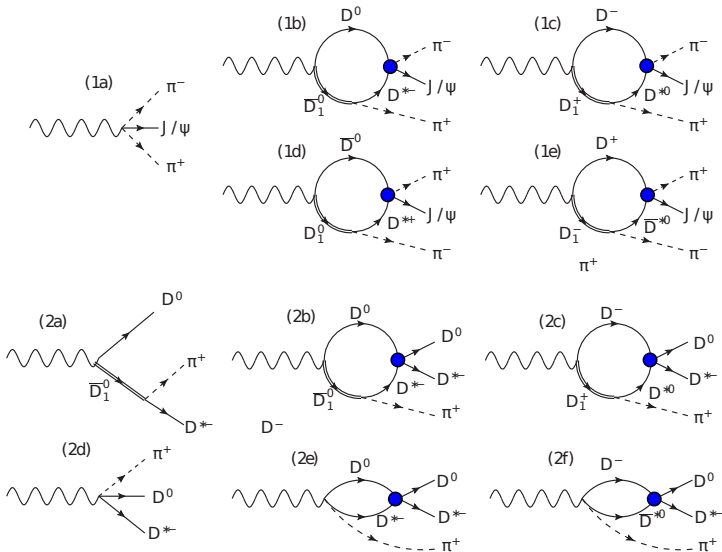
see talks by Z.-Y. Wang and C.-P. Shen



BESIII, PRL110(2013)252001; PRD92(2015)092006

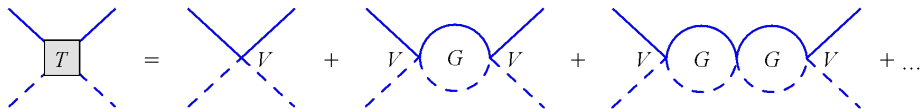
Phenomenology: $Z_c(3900)$ (II)

M. Albaladejo, FKG, C. Hidalgo-Duque, J. Nieves, PLB755(2016)337



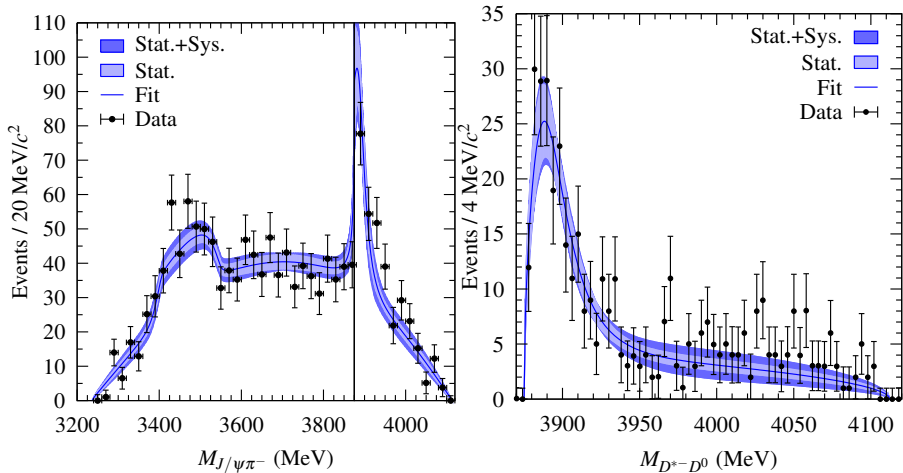
Phenomenology: $Z_c(3900)$ (III)

Blobs in the figure:



- Two channels: 1: $J/\psi\pi$, 2: $D\bar{D}^* + c.c.$
- $V_{ij} = 4\sqrt{m_{i1}m_{i2}}\sqrt{m_{j1}m_{j2}} e^{-q_i^2/\Lambda_i^2} e^{-q_j^2/\Lambda_j^2} C_{ij}$
- $J/\psi\pi$ interaction very weak: $|a_{J/\psi\pi}| \lesssim 0.02 \text{ fm}$ Liu, FKG, Epelbaum, EPJC73(2013)2284
 \Rightarrow taking $C_{11} = 0$
- $C_{22} = C_{1Z} + b(E - m_D - m_{D^*})$
- 3 parameters: C_{1Z} , b and C_{12}

Phenomenology: $Z_c(3900)$ (IV)



Phenomenology: $Z_c(3900)$ (V)

	M_{Z_c} (MeV)	$\Gamma_{Z_c}/2$ (MeV)	Ref.	Final state
	3899 ± 6	23 ± 11	BESIII, PRL110	$J/\psi \pi$
	3895 ± 8	32 ± 18	Belle, PRL110	$J/\psi \pi$
	3886 ± 5	19 ± 5	Xiao et al., PLB727	$J/\psi \pi$
	3884 ± 5	12 ± 6	BESIII, PRL112	$\bar{D}^* D$
	3882 ± 3	13 ± 5	BESIII, PRD92	$\bar{D}^* D$
$b \neq 0$	$3894 \pm 6 \pm 1$	$30 \pm 12 \pm 6$	$\Lambda_2 = 1.0$ GeV	$J/\psi \pi, \bar{D}^* D$
	$3886 \pm 4 \pm 1$	$22 \pm 6 \pm 4$	$\Lambda_2 = 0.5$ GeV	$J/\psi \pi, \bar{D}^* D$
$b = 0$	$3831 \pm 26_{-28}^{+7}$	virtual state	$\Lambda_2 = 1.0$ GeV	$J/\psi \pi, \bar{D}^* D$
	$3844 \pm 19_{-21}^{+12}$	virtual state	$\Lambda_2 = 0.5$ GeV	$J/\psi \pi, \bar{D}^* D$

Phenomenology: $Z_c(3900)$ (VI)

