
Preserving local gauge invariance for single-meson production currents with t -channel Regge exchanges

PRC92, 055503 (2015) & PRC93, 045204 (2016)

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Collaborators: Xiao-Yun Wang and Jun He (CAS, Lanzhou & Beijing)



... or: *How to marry Regge with Feynman*

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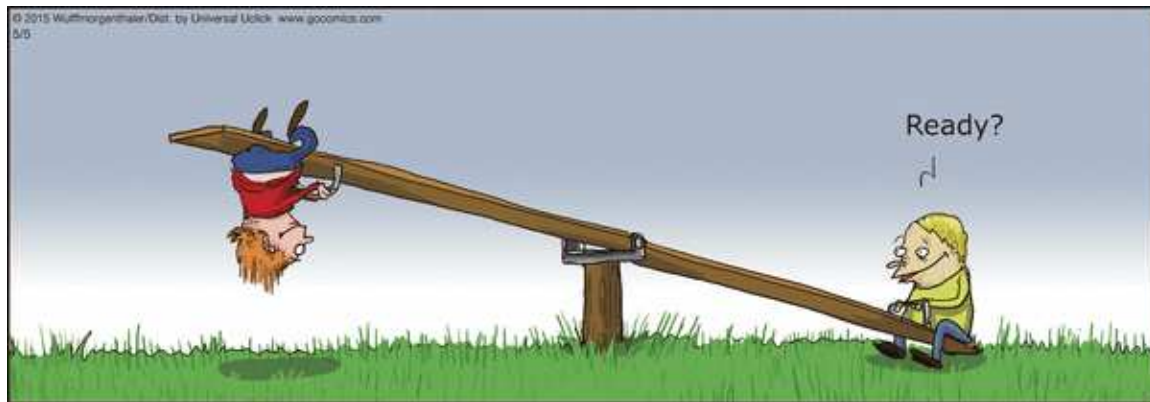
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How to marry Regge with Feynman . . .



Outline

- Outlining the problem
- Regge-trajectory basics
- How *not* to cure the problem
- The origin of the problem
- Gauge-invariance basics
- Implementation of **local** gauge invariance
- The cure
- Application: $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$
- Summary

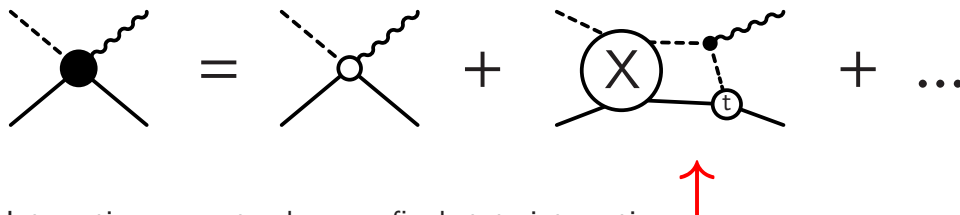


Outlining the Problem

← time →

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram (s-channel)}}_{s\text{-channel}} + \underbrace{\text{diagram (u-channel)}}_{u\text{-channel}} + \underbrace{\text{diagram (t-channel)}}_{t\text{-channel}} + \underbrace{\text{diagram (interaction current)}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

Interaction current:



Outlining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four terms in the sum:

- s-channel:** A vertex labeled 's' with incoming lines q (dashed) and p' (solid), and outgoing lines k (wavy) and p (solid).
- u-channel:** A vertex labeled 'u' with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid).
- t-channel:** A vertex labeled 't' with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid). This vertex is highlighted with a grey background.
- interaction current:** A single vertex with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid).

Replace t -channel single-meson exchange by **Regge-trajectory exchange**: $\Delta_m F_t \rightarrow \mathcal{P}_m$



Outlining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
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- s-channel:** A vertex labeled 's' with incoming lines q (dashed) and p' , and outgoing lines k (wavy) and p .
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- t-channel:** A vertex labeled 't' with incoming lines q (dashed) and p , and outgoing lines k (wavy) and p' .
- interaction current:** A single vertex with incoming lines q (dashed) and p , and outgoing lines k (wavy) and p' .

Replace t -channel single-meson exchange by **Regge-trajectory exchange**: $\Delta_m F_t \rightarrow \mathcal{P}_m$

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \mathcal{P}_m + M_{\text{int}}^\mu
 \end{aligned}$$

In the t -channel diagram, the internal line is now a vertical pink arrow labeled \mathcal{P}_m , representing Regge-trajectory exchange.

\mathcal{P}_m : Regge-trajectory propagator



Outlining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
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The diagrams show four Feynman-like diagrams for the amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines are solid lines with momenta p' and p .

- s-channel:** A solid line with a circle containing 's' connects the p' and p lines. A solid line connects the q and k lines.
- u-channel:** A solid line with a circle containing 'u' connects the p' and p lines. A solid line connects the q and k lines.
- t-channel:** A solid line with a circle containing 't' connects the p' and p lines. A solid line connects the q and k lines.
- interaction current:** A solid line connects the q and k lines. A solid line connects the p' and p lines.

$$k_\mu M^\mu = 0$$

current conserved



Outlining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

Diagram 1: s-channel exchange with vertices S and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 2: u-channel exchange with vertices U and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 3: t-channel exchange with vertex T and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 4: Interaction current with a black dot vertex. Incoming lines p' and q, outgoing lines p and k.

$$k_\mu M^\mu = 0$$

current conserved

With Reggeized *t*-channel:

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{diagram 1} + \text{diagram 2} + \underbrace{\text{diagram 3}}_{\text{Reggeized } t\text{-channel}} + \text{diagram 4} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \mathcal{P}_m + M_{\text{int}}^\mu
 \end{aligned}$$

Diagram 3: Reggeized *t*-channel exchange with vertex T and a black dot. A pink arrow points to the vertex T. Incoming lines p' and q, outgoing lines p and k.

$$k_\mu \mathcal{M}^\mu \neq 0$$

current not conserved



Outlining the Problem

No problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{[s-channel diagram]}}_{s\text{-channel}} + \underbrace{\text{[u-channel diagram]}}_{u\text{-channel}} + \underbrace{\text{[t-channel diagram]}}_{t\text{-channel}} + \underbrace{\text{[interaction current diagram]}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

$$k_\mu M^\mu = 0$$

current conserved

With Reggeized t -channel:

Problem!

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{[s-channel diagram]} + \text{[u-channel diagram]} + \text{[Reggeized t-channel diagram]} + \text{[interaction current diagram]} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \mathcal{P}_m + M_{\text{int}}^\mu
 \end{aligned}$$

$$k_\mu \mathcal{M}^\mu \neq 0$$

current not conserved

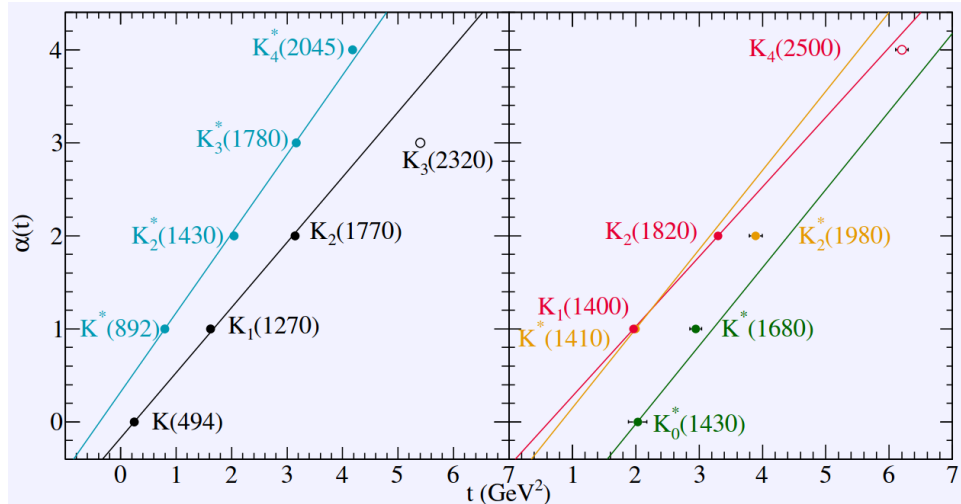


Let's recap Regge . . .



Regge trajectories

Graph stolen from P. Vancraeyveld, PhD Thesis, U. Gent, 2011



Trajectories:

pseudoscalar:

$$\alpha_+(t) = \alpha_0(t),$$

$$\alpha_0(t) = \alpha'_0(t - M_0^2)$$

vector:

$$\alpha_-(t) = 1 + \alpha_1(t),$$

$$\alpha_1(t) = \alpha'_1(t - M_1^2)$$

Regge exchange for t -channel:

$m = 0, 1$

$$\Delta_m F_t \rightarrow \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t),$$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{sc}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma(1 + \alpha_m(t))} \frac{\pi \alpha_m(t)}{\sin(\pi \alpha_m(t))}$$

residual Regge function

$s_{sc} = 1 \text{ GeV}^2$



Regge trajectories

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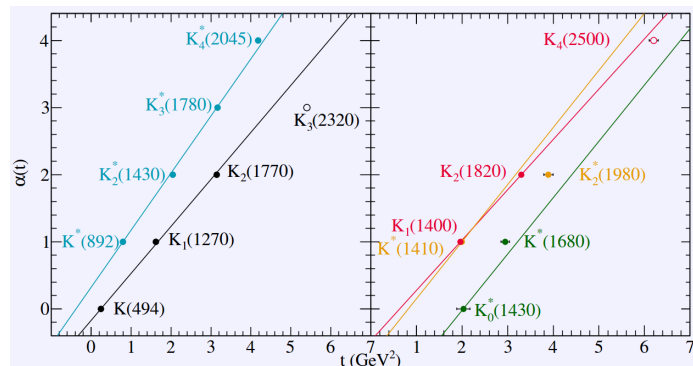
residual Regge function

$s_{\text{sc}} = 1 \text{ GeV}^2$

Signature function:

$$N[\alpha_m(t); \eta] = \eta + (1 - \eta)e^{-i\pi\alpha_m(t)}$$

$$\eta = \begin{cases} \frac{1}{2}, & \text{pure-signature trajectory} \\ 0, & \text{add trajectories: rotating phase} \\ 1, & \text{subtract trajectories: constant phase} \end{cases}$$



Regge trajectories

Regge exchange for t -channel:

$$\Delta_m F_t \rightarrow \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t),$$

$m = 0, 1$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{\text{sc}}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma(1 + \alpha_m(t))} \frac{\pi \alpha_m(t)}{\sin(\pi \alpha_m(t))}$$

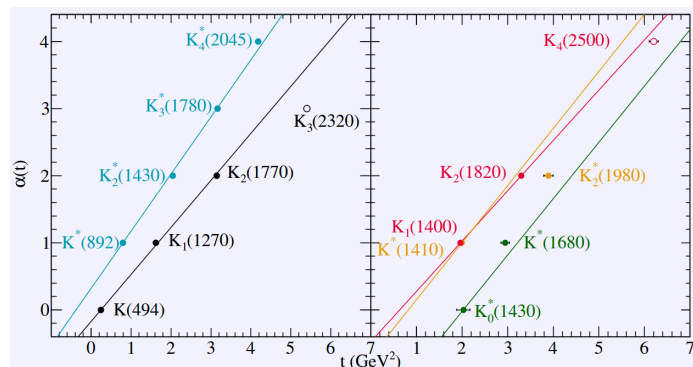
residual Regge function

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Normalization:

$$\mathcal{F}_m(M_m^2) = 1$$

independent of η



Recipe: Take gauge-invariant amplitude M^μ and multiply by *residual function* $\mathcal{F}_m(t)$

$$M_{\text{GLV}}^\mu = M^\mu \times \mathcal{F}_m = \left[\begin{array}{c} q \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ p' \quad \text{---} \text{---} \text{---} \\ p \end{array} \text{S} + \begin{array}{c} q \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ p' \quad \text{---} \text{---} \text{---} \\ p \end{array} \text{U} + \begin{array}{c} q \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ p' \quad \text{---} \text{---} \text{---} \\ p \end{array} \text{t} + \begin{array}{c} q \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ p' \quad \text{---} \text{---} \text{---} \\ p \end{array} \right] \times \mathcal{F}_m(t)$$

t-channel okay

$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$



Recipe: Take gauge-invariant amplitude M^μ and multiply by *residual function* $\mathcal{F}_m(t)$

$$M_{\text{GLV}}^\mu = M^\mu \times \mathcal{F}_m = \left[\begin{array}{c} q \\ \diagdown \\ \textcircled{S} \\ \diagup \\ p' \end{array} \text{---} \text{---} \text{---} \begin{array}{c} k \\ \diagup \\ \bullet \\ \diagdown \\ p \end{array} + \begin{array}{c} q \\ \diagdown \\ \bullet \\ \diagup \\ p' \end{array} \text{---} \text{---} \text{---} \begin{array}{c} k \\ \diagdown \\ \textcircled{U} \\ \diagup \\ p \end{array} + \begin{array}{c} q \\ \diagdown \\ \bullet \\ \diagup \\ p' \end{array} \text{---} \text{---} \text{---} \begin{array}{c} k \\ \diagdown \\ \textcircled{t} \\ \diagup \\ p \end{array} + \begin{array}{c} q \\ \diagdown \\ \bullet \\ \diagup \\ p' \end{array} \text{---} \text{---} \text{---} \begin{array}{c} k \\ \diagup \\ \bullet \\ \diagdown \\ p \end{array} \right] \times \mathcal{F}_m(t)$$

t-channel okay

$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$

- Very popular
- Quite successful in providing good descriptions of data for many applications



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$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$

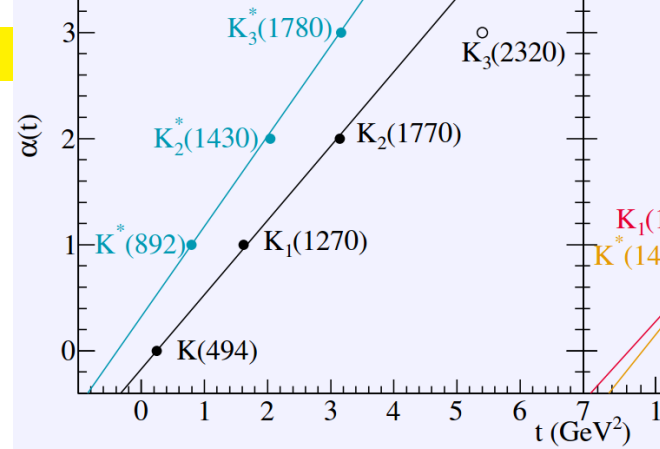
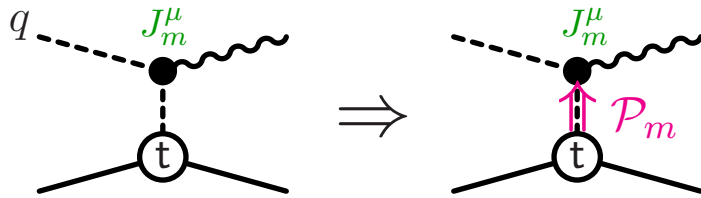
- Very popular
- Quite successful in providing good descriptions of data for many applications
- **Without any dynamical foundation**

Cannot be obtained from field theory in **any** approximation



Origin of Problem

Reason #1:



Every state in the Regge trajectory appears with **same** current.

Ward-Takahashi identity:

$$k_\mu J_m^\mu = (q^2 - M_0^2)Q_m - Q_m(t - M_0^2)$$

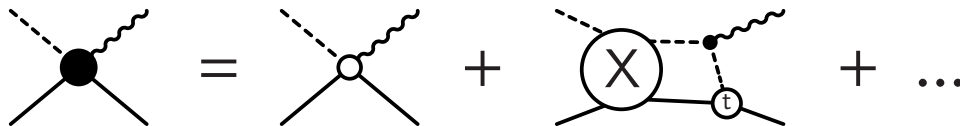


Violates Ward-Takahashi identity for intermediate higher-mass states



Origin of Problem

Reason #2:



t -Channel exchanges inside interaction current **not Reggeized**.

Needed: Consistent treatment



Interaction-current contribution must be Reggeized as well



Gauge-invariance recap . . .



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four terms in the sum:

- s-channel:** A vertex 's' (circle) with incoming lines 'q' (dashed) and 'p'' (solid), and outgoing lines 'k' (wavy) and 'p' (solid).
- u-channel:** A vertex 'u' (circle) with incoming lines 'q' (dashed) and 'p' (solid), and outgoing lines 'k' (wavy) and 'p'' (solid).
- t-channel:** A vertex 't' (circle) with incoming lines 'q' (dashed) and 'p' (solid), and outgoing lines 'k' (wavy) and 'p'' (solid).
- interaction current:** A single vertex with incoming lines 'q' (dashed) and 'p' (solid), and outgoing lines 'k' (wavy) and 'p'' (solid).

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$

conserved current \Rightarrow implies charge conservation



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
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 \end{aligned}$$

The diagrams show four terms in the sum for M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines are solid lines with momenta p' and p .
 - **s-channel**: A white circle labeled 's' is connected to a black circle. The dashed line enters the white circle, and the wavy line exits the black circle.
 - **u-channel**: A black circle is connected to a white circle labeled 'u'. The dashed line enters the black circle, and the wavy line exits the white circle.
 - **t-channel**: A white circle labeled 't' is connected to a black circle. The dashed line enters the white circle, and the wavy line exits the black circle.
 - **interaction current**: A single black circle with the dashed line entering and the wavy line exiting.

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$

conserved current \Rightarrow implies charge conservation

Fixing global gauge invariance does **not** mean internal damage is fixed as well



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
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 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the transition amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines have momenta p' and p .
 - **s-channel**: A solid line connects a vertex labeled 's' to a vertex labeled 't'.
 - **u-channel**: A solid line connects a vertex labeled 'u' to a vertex labeled 't'.
 - **t-channel**: A solid line connects a vertex labeled 't' to a vertex labeled 't'.
 - **interaction current**: A solid line connects a vertex labeled 't' to a vertex labeled 't'.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$

Generalized Ward-Takahashi identities (gWTI)

$$k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m (t - M_m^2)$$

$$k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$

off-shell relations



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
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The diagrams show four Feynman-like diagrams for the transition amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines have momenta p' and p .
 1. s-channel: A solid line connects a vertex labeled 's' to a vertex labeled '●'.
 2. u-channel: A solid line connects a vertex labeled '●' to a vertex labeled 'u'.
 3. t-channel: A solid line connects a vertex labeled '●' to a vertex labeled 't'.
 4. interaction current: A solid line connects a vertex labeled '●' to a vertex labeled '●'.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$

Generalized Ward-Takahashi identities (gWTI)

$$\begin{aligned}
 k_\mu M^\mu &= (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p) \\
 k_\mu J_m^\mu &= (q^2 - M_m^2) Q_m - Q_m (t - M_m^2) \\
 k_\mu M_{\text{int}}^\mu &= Q_m F_t + Q_f F_u - F_s Q_i
 \end{aligned}$$

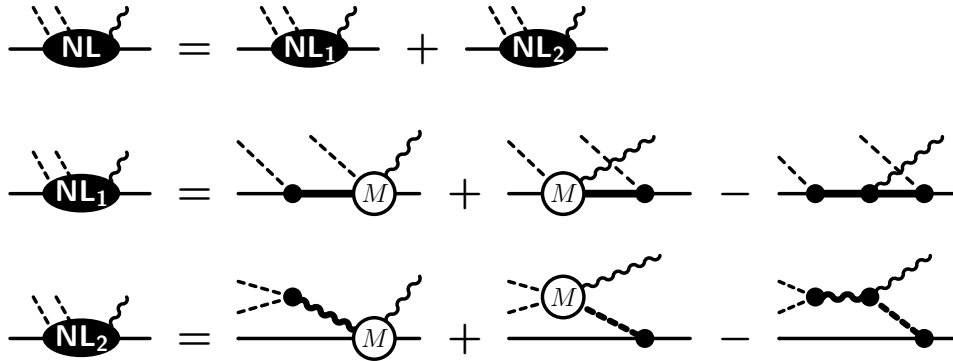
off-shell relations

local gauge invariance \Rightarrow implies existence of e.m. field

Without gWTI underlying e.m. field is damaged

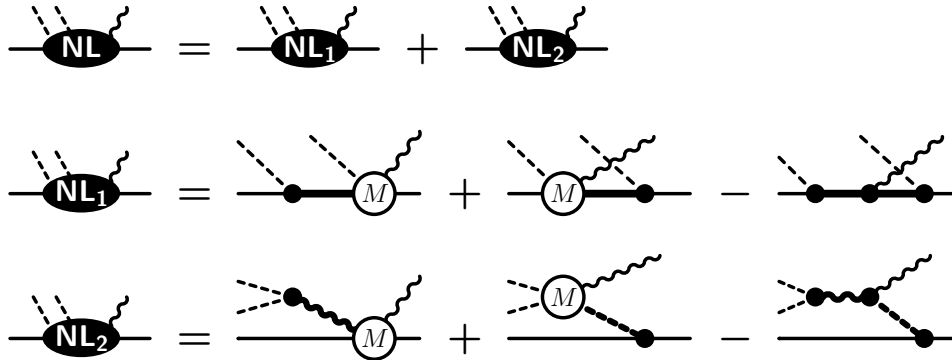


Example: Two-pion production at the no-loop level



Practical Relevance of Local Gauge Invariance

Example: Two-pion production at the no-loop level:



Without gWTI, this amplitude will not be gauge invariant



Generalized Ward-Takahashi Identities

$$(1) \quad k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$(2) \quad k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2) \quad \text{trivial}$$

$$(3) \quad k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$

Only two relations are independent \Rightarrow Use (2) & (3)

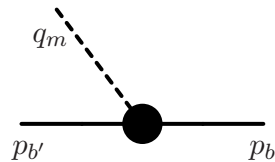


Generalized Ward-Takahashi Identities

- (1) $k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$
- (2) $k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$ trivial
- (3) $k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$

Only two relations are independent \Rightarrow Use (2) & (3)

Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

Interaction-current Ansatz:

$$M_{\text{int}}^\mu = m_c^\mu f_t(t) + \mathbf{G}(q) \mathbf{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

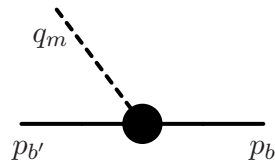


Generalized Ward-Takahashi Identities

- (1) $k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$
- (2) $k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$ trivial
- (3) $k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$

Only two relations are independent \Rightarrow Use (2) & (3)

Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

Interaction-current Ansatz:

$$M_{\text{int}}^\mu = m_c^\mu f_t(t) + \mathbf{G}(q) C^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

\Rightarrow Determine C^μ such that (3) is true



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$

where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \quad x = s, u, t$$

Charge conservation: $Q_m \tau + Q_f \tau - \tau Q_i = e_m + e_f - e_i = 0$



Non-singular

$$\begin{aligned}
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Four-divergence: $k_\mu C^\mu = e_m f_t + e_f f_u - e_i f_s$

ensures correct four-divergence for M_{int}^μ

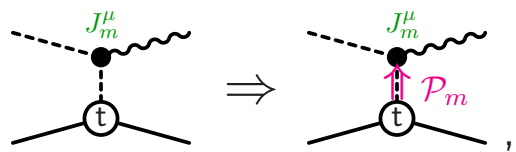


Back to Regge . . .

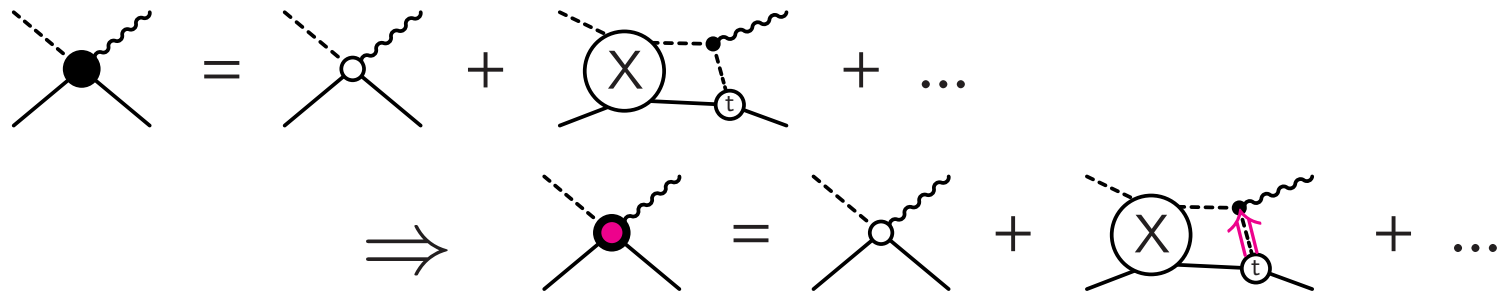


Reggeizing Final-state Interaction

Reggeize both t -channel,

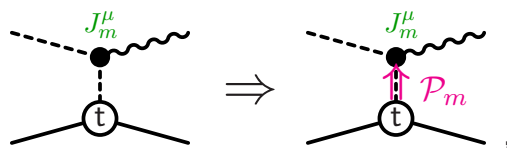


and FSI contribution,

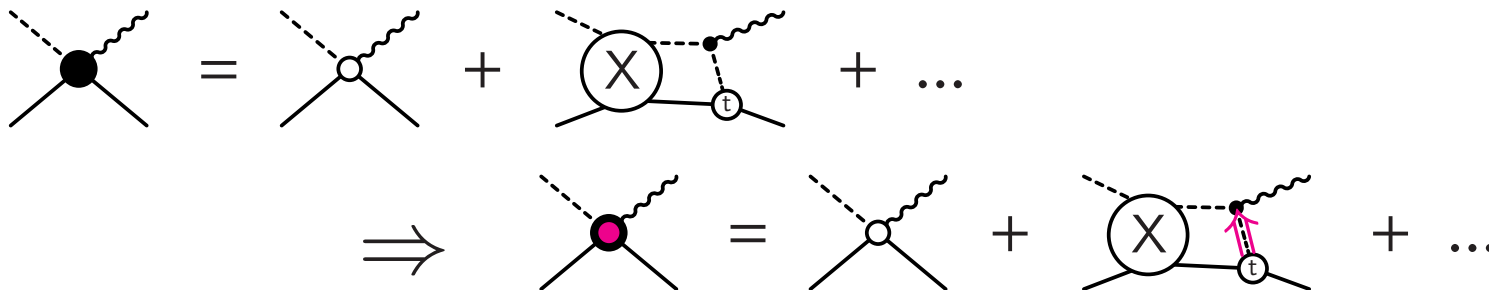


Reggeizing Final-state Interaction

Reggeize both t -channel,



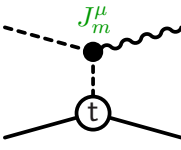
and FSI contribution,



Not necessary to calculate FSI loops \Rightarrow modify C^μ instead



Before Reggeization

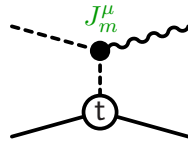
$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t =$$




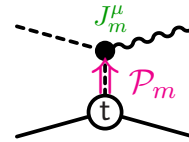
Reggeizing t -Channel

Before Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t =$$



\Rightarrow



$=$

After Reggeization

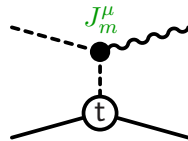
$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t$$



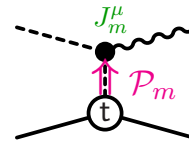
Reggeizing t -Channel

Before Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t \quad \uparrow$$



\Rightarrow



$=$

After Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t \quad \uparrow$$

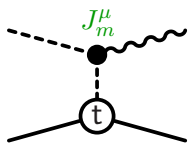
Reggeization corresponds to an effective prescription for hadronic form factor



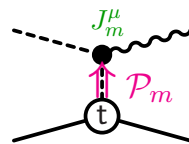
Reggeizing t -Channel

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\Rightarrow



$=$

After Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t$$

Reggeization corresponds to an effective prescription for hadronic form factor

To preserve local gauge invariance,
replace f_t by Regge residual function \mathcal{F}_t everywhere



The Cure: Modified Auxiliary Contact Current \mathcal{C}^μ

$$\mathcal{M}_{\text{int}}^\mu = m_c^\mu \mathcal{F}_t + \mathbf{G}(q) \mathcal{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

Non-singular

$$\begin{aligned} \mathcal{C}^\mu = & -e_m(2q - k)^\mu \frac{\mathcal{F}_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\ & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t \mathcal{F}_t + \delta_s f_s - \delta_t \delta_s \mathcal{F}_t f_s) \\ & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t \mathcal{F}_t - \delta_u \delta_t f_u \mathcal{F}_t) \end{aligned}$$



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Provides correct generalized Ward-Takahashi identity:

$$k_\mu \mathcal{M}^\mu = (q^2 - M_m^2) Q_m \hat{\mathcal{F}}_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$



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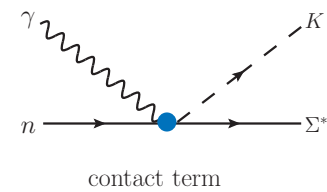
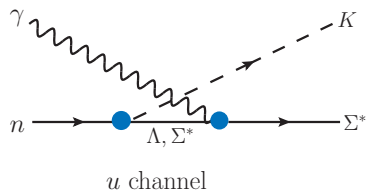
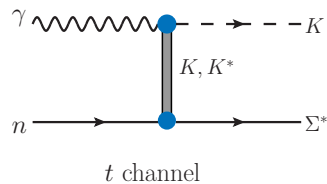
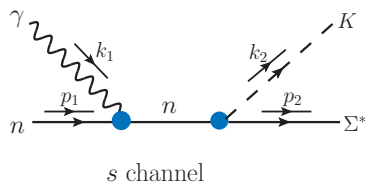
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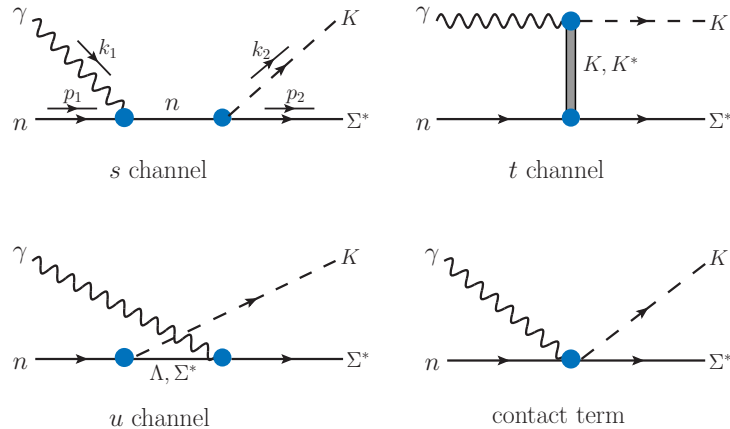
⇒ **Production current locally gauge invariant**





Compared with data from
 CLAS: P. Mattione (CLAS Collaboration),
 Int. J. Phys. Conf. Series **26**, 1460101, (2014);
 LEPS: K. Hicks et al. (LEPS Collaboration),
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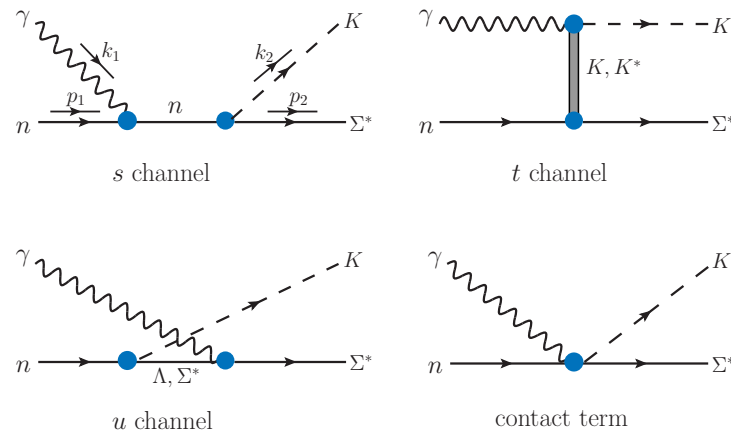
For gauge-invariance considerations, the s -channel is irrelevant.

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$$\tilde{f}_t(t) = \mathcal{F}_t(t) R_s + f_t(t) (1 - R_s)$$

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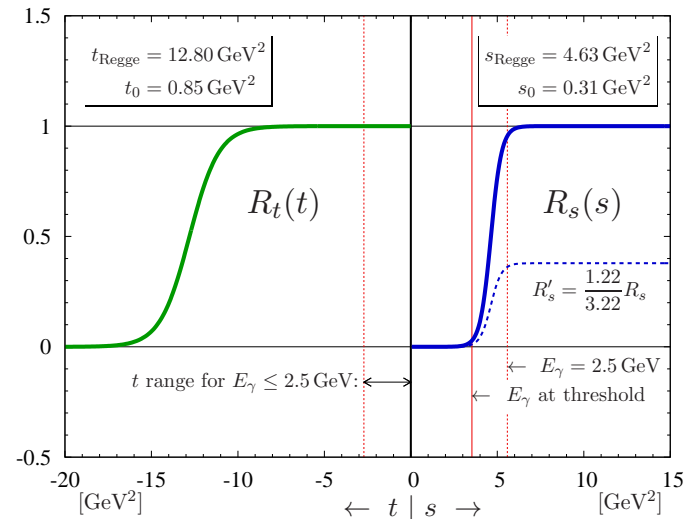




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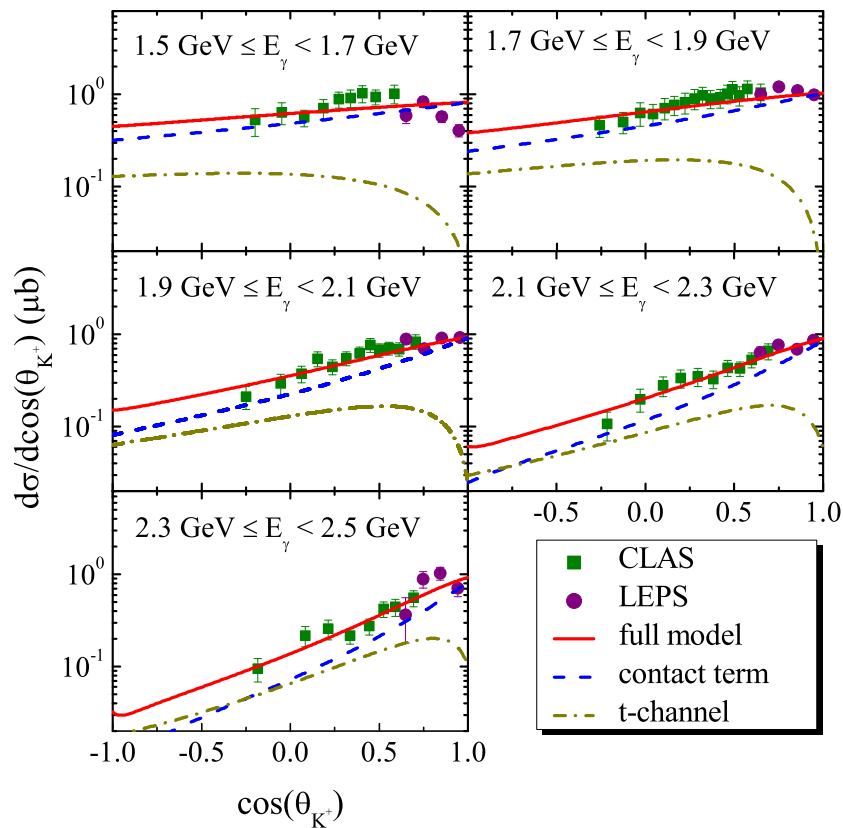
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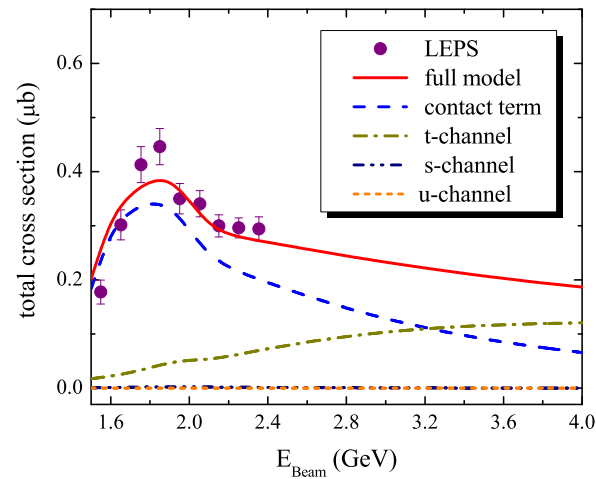
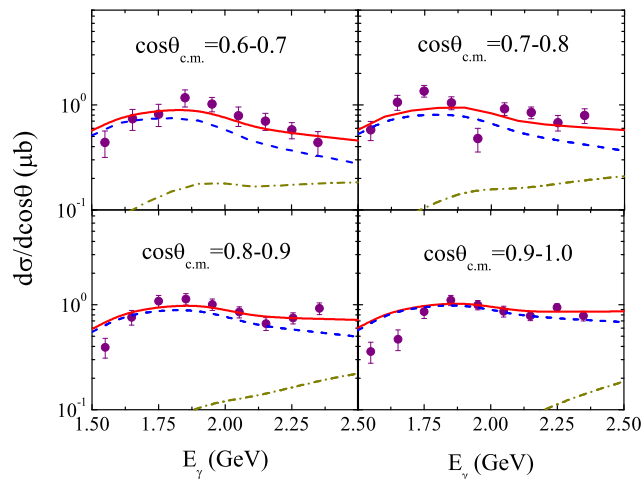
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Reggeized contact term
provides sizeable contribution





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Summary

- Implementation of Regge phenomenology for t -channel exchange corresponds to replacing usual phenomenological form factor f_t by Regge residual function \mathcal{F}_t
- Reggeization of t -channel leads to violation of gauge invariance due to *inconsistent* implementation of Reggeization
- Interaction-current M_{int}^μ needs to be Reggeized as well
- Global gauge invariance doesn't constrain dynamics: not a good starting point ~~[GLV]~~
- Correct dynamical basis provided by **generalized Ward-Takahashi identities** as they follow from **local** gauge invariance
- The cure: Modify auxiliary contact current C^μ
- Application to $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$ at energies up to 2.5 GeV requires mixing of conventional and Reggeized t -channel to provide acceptable χ^2
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Thank you!

