Construction of the pion scalar form factor from few poles and zero

Robert Kamiński
IFJ PAN, Kraków

Stanislav Dubnicka, Andrej Liptaj
Institute of Physics, Slovak Academy of Sciences, Bratislava

Anna Zuzana Dubnickova
Department of Theoretical Physics, Comenius University, Bratislava

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Pion scalar form factor

$$< \pi^i(p_2) \mid \widehat{m}(\bar{u}u + \bar{d}d) \mid \pi^j(p_1) >= \delta^{ij}\Gamma_\pi(t)$$

$$\text{where} \quad t = (p_2 - p_1)^2 \text{ and } \quad \widehat{m} = \frac{1}{2}(m_u + m_d).$$

It posses all known properties of the pion vector electromagnetic FF $F_\pi(t)$ like

- analyticity in t-plane beside cuts on the positive real axis from two-pion threshold $t = 4m_\pi^2$ to $+\infty$
- elastic unitarity condition $Im\Gamma_\pi(t) = \Gamma_\pi(t)e^{-i\delta_0^0} \sin \delta_0^0$, where $\delta_0^0(t)$ is the S-wave isoscalar $\pi\pi$ phase shift
- asymptotic behavior $\Gamma_\pi(t)_{|t|\to\infty} \sim \frac{1}{t}$
- reality condition $\Gamma_\pi^*(t) = \Gamma_\pi(t^*)$
- normalization, however, now to the pion sigma term value $\Gamma(0) = (0.99 \pm 0.02)m_\pi^2$ to be predicted by the χPT,
  $\Gamma_\pi(0) = m_u \frac{\partial M_\pi^2}{\partial m_u} + m_d \frac{\partial M_\pi^2}{\partial m_d}$

Earlier analyses

- T. N. Truong and R. S. Willey, Phys. Rev. D40 (1989) 3635,
- J. F. Donoghue, J. Gasser and H. Leutwyler, Nucl. Phys. B343, 341 (1990),
- B. Moussallam, Eur. Phys. J. C14 (2000) 111,
- G. Colangelo, J. Gasser and H. Leutwyler, Nucl. Phys. B603 (2001) 125,
- *"Scalar form factors of light mesons"*; B. Ananthanaryan, I. Caprini, G. Colangelo, J. Gasser, H. Leutwyler, PLB'2004,
- *"The Quadratic scalar radius of the pion and the mixed $\pi - K$ radius"*, F. J. Yndurain, Phys. Lett. B578, 99 (2004)

Our (Bratislava-Kraków) analysis -the method

$$\Gamma_\pi(t) = P_n(t) exp\Big[\frac{t}{\pi}\int_{4m_\pi^2}^{\infty}\frac{\delta_0^0(t')}{t'(t'-t)}dt'\Big], \qquad (1)$$

$$\tan\delta_\Gamma(t) = \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + ...}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + ...} \qquad (2)$$

$$\Gamma_\pi(t) = P_n(t) exp\left[\frac{(q^2+1)}{2\pi i}\times\int_{-\infty}^{\infty}\frac{q' \, ln\frac{(1+A_2 q'^2 + A_4 q'^4)+i(A_1 q' + A_3 q'^3 + A_5 q'^5)}{(1+A_2 q'^2 + A_4 q'^4)-i(A_1 q' + A_3 q'^3 + A_5 q'^5)}}{(q'^2+1)(q'^2-q^2)}dq'\right], \qquad (3)$$

$$\Gamma_\pi(q) = \frac{\sum_{n=0}^{M} a_n q^n}{\prod_{i=1}^{N}(q-q_i)}, \qquad \oint \phi(q')dq' = 2\pi i \sum_{n=1} Res_n \qquad (4)$$

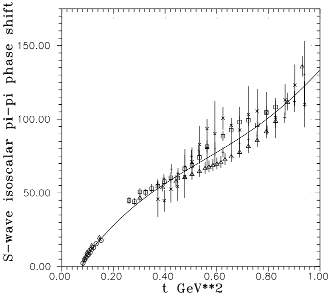
explicit form of the pion scalar FF:

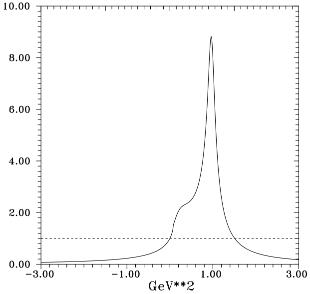
$$\Gamma_\pi(t) = P_n(t)\frac{(q-q_1)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)}\times\frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)}{(i-q_1)}$$

where $P_n(t)$ is any polynomial normalised at $t=0$ to one.

First results (2014)

PHYSICAL REVIEW D 90, 114003 (2014)
"Pion scalar form factor and another confirmation of the existence of the $f_0(500)$ meson"

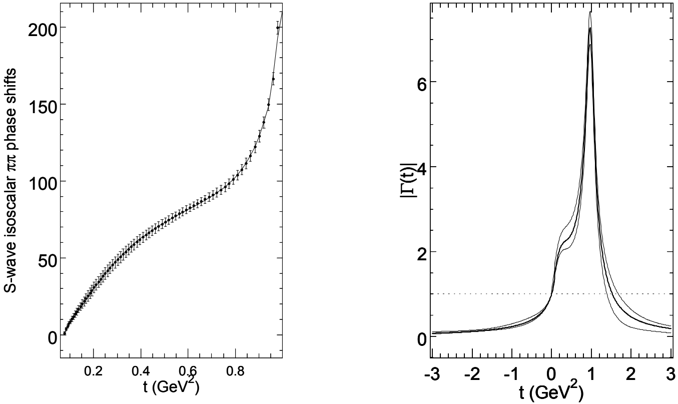




Results:
$f_0(500)$: $388 \pm 23 - i(301 \pm 33)$ MeV
$f_0(980)$: $1066 \pm 142 - i(110 \pm 97)$ MeV

PDG:
$400 - 550 - i(200 - 350)$ MeV
$990 \pm 20 - i(25 - 50)$ MeV

New results (2015/2016); new data for $\delta_{\pi\pi}$



New results (2015/2016); poles

$$\tan \delta_\Gamma(t) = \frac{A_1 q + A_3 q^3 + A_5 q^5 + A_7 q^7 + \ldots}{1 + A_2 q^2 + A_4 q^4 + A_6 q^6 + \ldots}$$

	fit to the output	fit to the input
A_1	0.220 fixed	0.220 fixed
A_3	0.151 ± 0.020	0.128 ± 0.016
A_5	-0.0144 ± 0.0016	-0.0125 ± 0.0013
A_2	-0.055 ± 0.038	-0.085 ± 0.023
A_4	-0.0089 ± 0.0048	-0.0051 ± 0.0029
$f_0(500)$	$451 \pm 14 - i260 \pm 33$	$467 \pm 14 - i261 \pm 29$
$f_0(980)$	$988 \pm 81 - i52 \pm 32$	$987 \pm 76 - i47 \pm 25$

$$\Gamma_\pi(t) = P_n(t) \frac{(q - q_1)}{(q + q_2)(q + q_3)(q + q_4)(q + q_5)} \times \frac{(i + q_2)(i + q_3)(i + q_4)(i + q_5)}{(i - q_1)}$$

$$\begin{array}{lclr}
q_1 & = & 0.00 & -i1.943 \pm 0.20 \\
q_2 & = & 3.397 \pm 0.40 & +i0.196 \pm 0.04 \\
q_3 & = & -3.397 \pm 0.40 & +i0.196 \pm 0.04 \\
q_4 & = & 1.385 \pm 0.10 & +i1.085 \pm 0.12 \\
q_5 & = & -1.385 \pm 0.10 & +i1.085 \pm 0.12
\end{array}$$

New results (2015/2016); analytical structure of the $\Gamma(s)$

$$
\begin{array}{lccc}
q_1 & = & 0.00 & -i1.943 \pm 0.20 \\
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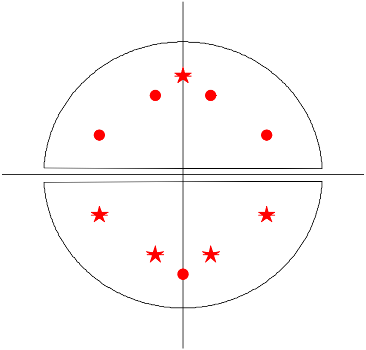

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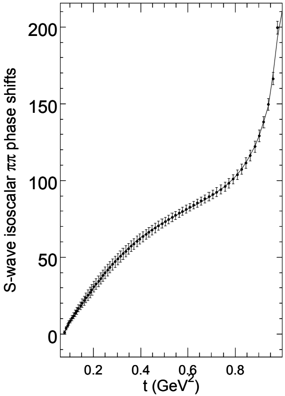

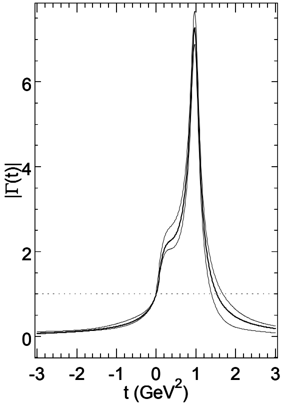

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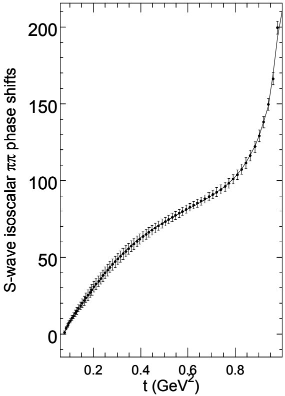

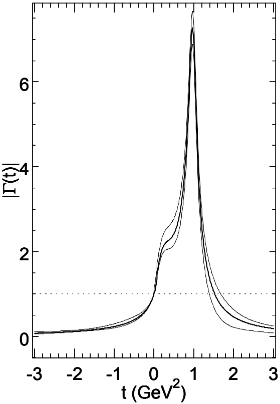

Results:
$f_0(500)$: $467 \pm 14 - i(261 \pm 29)$ MeV
$f_0(980)$: $987 \pm 76 - i(47 \pm 25)$ MeV

GKPY eqs:
$457 \pm 14 - i(279 \pm 11)$ MeV
$996 \pm 7 - i(25 \pm 10)$ MeV

Correction caused by squared scalar radius

Scalar radius: χPT: $r^2 = 0.61 \pm 0.04$ fm^2

$< r^2 >= \frac{6}{\pi} \int_{4m_\pi^2}^{\infty} ds \frac{\delta_{\Gamma(s)}}{s^2}$

and from

$\Gamma_\pi(s)/\Gamma(_\pi(0) = 1 + \frac{1}{6} < r^2 >_s^\pi \; s + O(s^2)$

we have:

$< r^2 >= 6 \frac{\partial \Gamma_\pi(s)}{\partial s}$ at $s = 0$

Our result:

five-parameter (two for $f_0(600)$ + two for $f_0(980)$ + one for lhc) $\longrightarrow 0.76 \pm 0.12$ fm^2
Therefore at least two poles (two parameters) more are needed

Then explicit form of the corrected pion scalar FF:

$$\Gamma_\pi(t) = P_n(t) \frac{(q - q_1)}{(q+q_2)(q+q_3)(q+q_4)(q+q_5)(q+q_6)(q+q_7)} \times \frac{(i+q_2)(i+q_3)(i+q_4)(i+q_5)(i+q_6)(i+q_7)}{(i - q_1)}$$

Final results

Additional constrain in the fit: $r^2 = 0.61$ gives: $r^2 = 0.61 \pm 0.08$

	fit to the output	fit to the input
A_1	0.220 fixed	0.220 fixed
A_3	0.150 ± 0.018	0.123 ± 0.014
A_5	-0.0123 ± 0.0013	-0.0135 ± 0.0013
A_7	0.0034 ± 0.0012	0.0041 ± 0.0011
A_2	-0.045 ± 0.033	-0.065 ± 0.022
A_4	-0.0089 ± 0.0048	-0.0051 ± 0.0029
A_6	-0.0023 ± 0.0018	-0.0031 ± 0.0015
$f_0(500)$	$461 \pm 14 - i259 \pm 32$	$468 \pm 14 - i261 \pm 29$
$f_0(980)$	$991 \pm 79 - i51 \pm 30$	$990 \pm 74 - i46 \pm 25$

$$
\begin{aligned}
q_1 &= 0.00 & &- i1.824 \pm 0.18 \\
q_2 &= 3.452 \pm 0.40 & &+ i0.186 \pm 0.04 \\
q_3 &= -3.452 \pm 0.40 & &+ i0.186 \pm 0.04 \\
q_4 &= 1.376 \pm 0.10 & &+ i1.091 \pm 0.12 \\
q_5 &= -1.376 \pm 0.10 & &+ i1.091 \pm 0.12 \\
q_6 &= 3.985 \pm 0.09 & &- i0.085 \pm 0.09 \\
q_7 &= -3.985 \pm 0.09 & &- i0.085 \pm 0.09
\end{aligned}
$$

Analytical structure of the $\Gamma(s)$

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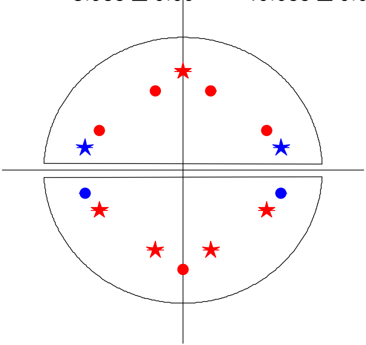
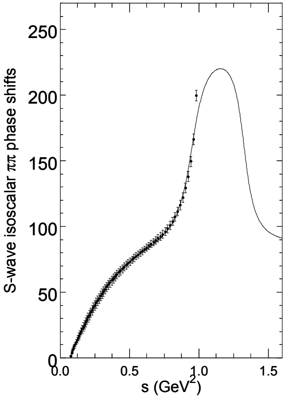
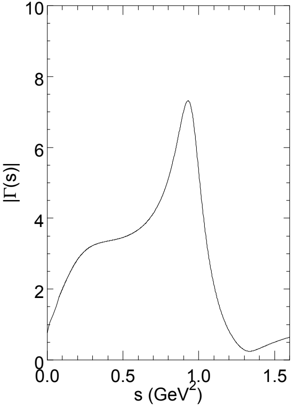


Final results





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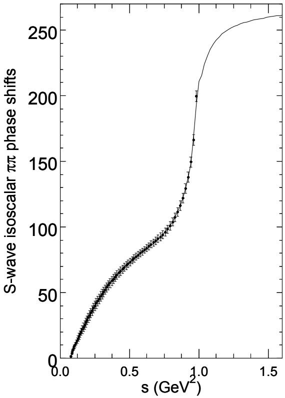
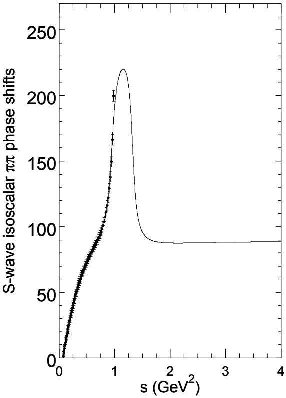
Final results

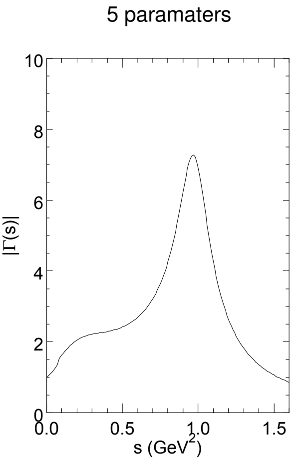
5 paramaters

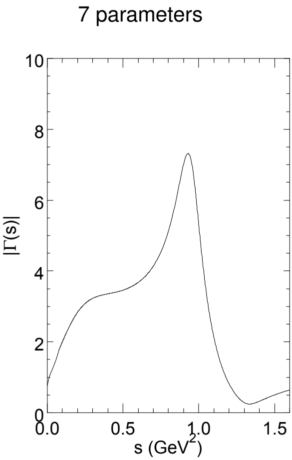


7 parameters



Final results





Conclusions

- good fit to the data + DR (GKPY egs): $\chi^2 = 0.88$ pdf,
- $\pi\pi$ scalar scattering length = 0.22,
- two lowest scalar resonances $f_0(500)$ and $f_0(980)$ with very good parameters are found,
- scalar radius $r^2 = 0.61 \pm 0.8$ fm^2,
- first explicit form of the $\pi\pi$ scalar form factor is found