

# The $B_c \rightarrow J/\psi K D$ weak decay and its relation with the $D_{s0}^*(2317)$ resonance

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Krakow, Poland

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- ▶ V. Summary

Based on:

" $D_{s0}^*(2317)^+$  IN THE DECAY OF  $B_c$  INTO  $J/\Psi DK$ ,"

Z. F. SUN, M. BAYAR, P. FERNANDEZ-SOLER AND E. OSET, PHYS. REV. D **93** (2016) NO.5, 054028

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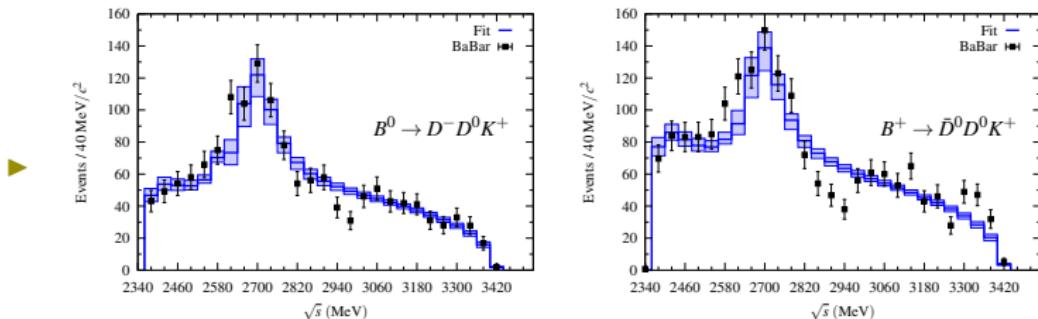
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2. Several theoretical interpretations:  $c\bar{s}$  state, molecular meson-meson,  $K - D$ -mixing, two meson and four quark state, four quark state
3. Lattice simulations have found the  $D_{s0}^*(2317)$  state with a  $KD$  component of  $\approx 70\%$

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- ▶ An experimental test of the molecular nature:  $B_s \rightarrow \pi^+ \bar{D}^0 K^-$ ,  
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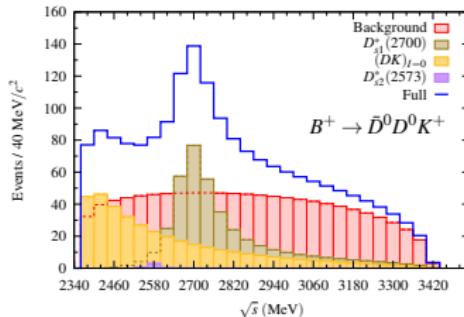
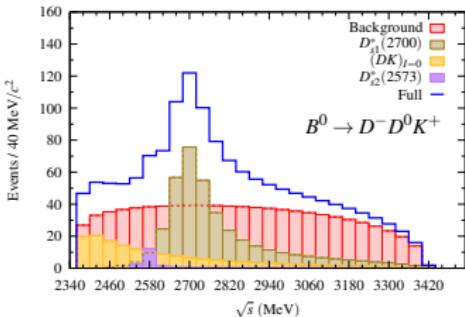
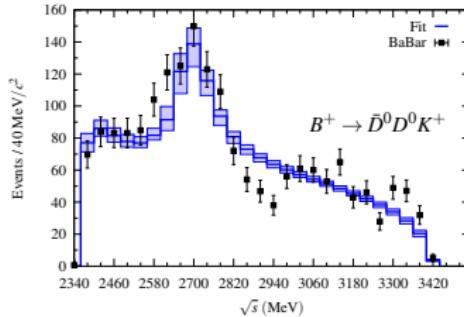
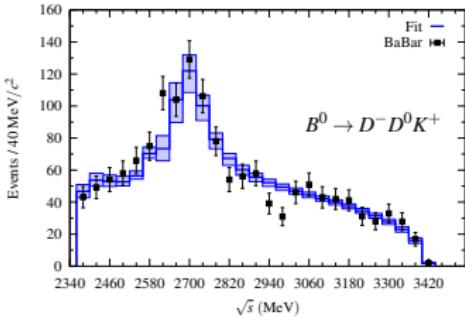
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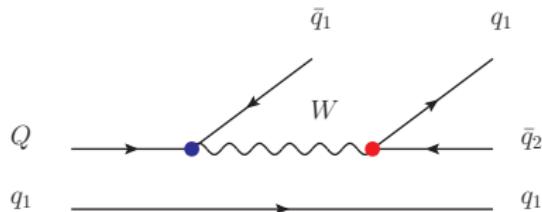
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## **II. Statement of the problem: weak decay of a heavy hadron**

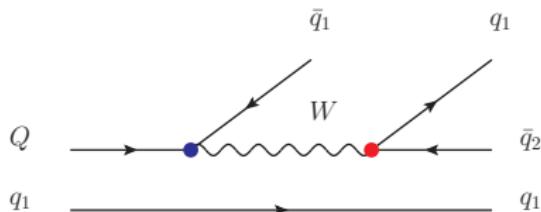
## I. Weak decay of a heavy hadron. 1.



### 1. The dominant weak mechanism

$$H_W = \frac{G_F}{\sqrt{2}} V_{Qq_1} V_{q_1 q_2} \bar{q}_1 \gamma_\mu (1 - \gamma_5) Q \bar{q}_2 \gamma^\mu (1 - \gamma_5) q_1$$

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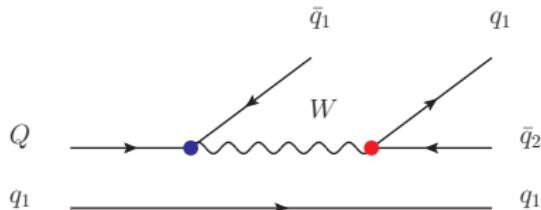


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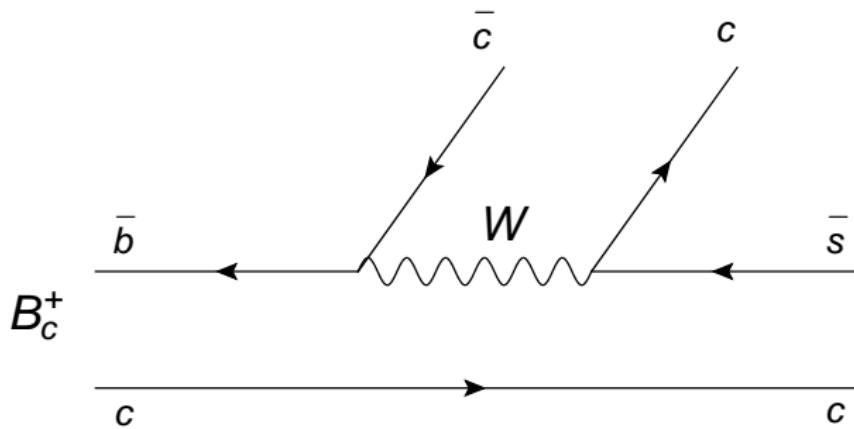


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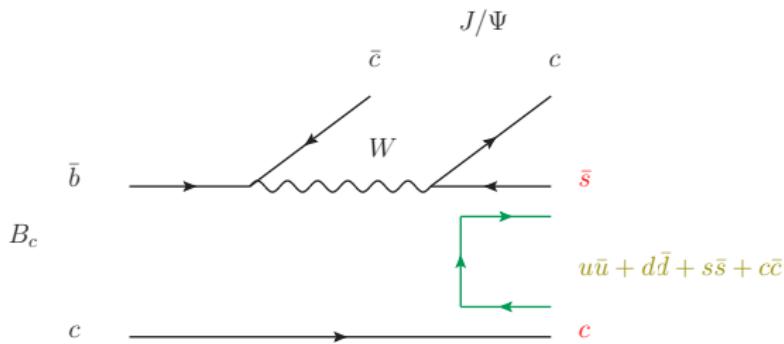
2. We look to a small energy range: smooth energy dependence
3. Amplitude considered as constant:  $V_p$

## I. Weak decay of a heavy hadron. 2.



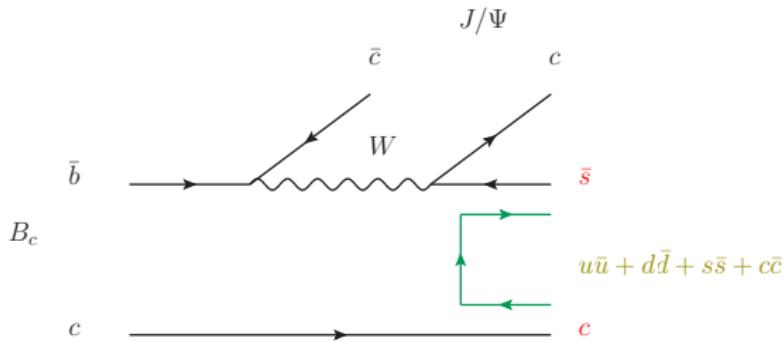
## II. Theoretical approach: hadronization and rescattering.

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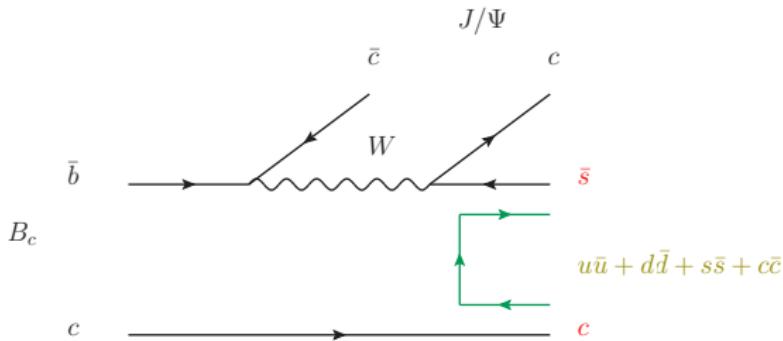
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- ▶ An extra  $q\bar{q}$  pair with the quantum numbers of the vacuum
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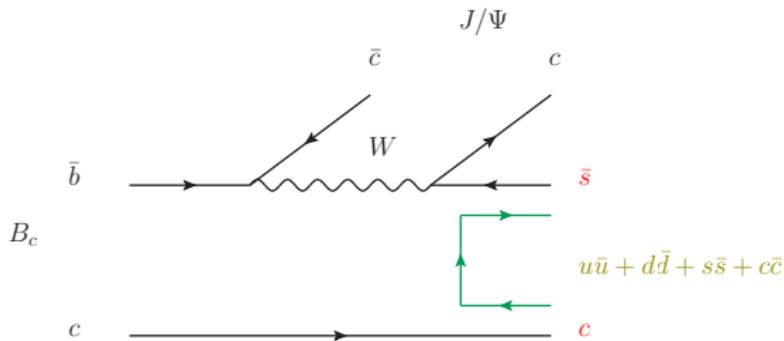
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- An extra  $q\bar{q}$  pair with the quantum numbers of the vacuum
- $c\bar{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})$
- Projection into meson-meson states is done with the  $M$  matrix

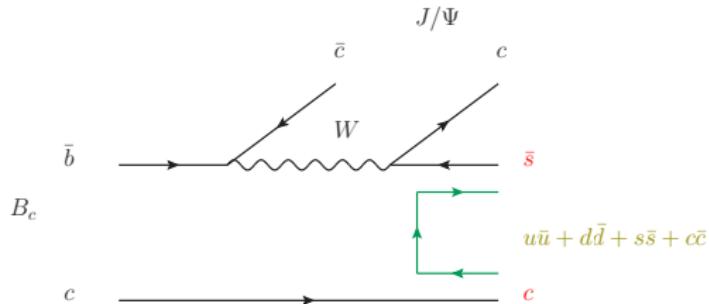
$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} & u\bar{c} \\ d\bar{u} & d\bar{d} & d\bar{s} & d\bar{c} \\ s\bar{u} & s\bar{d} & s\bar{s} & s\bar{c} \\ c\bar{u} & c\bar{d} & c\bar{s} & c\bar{c} \end{pmatrix} = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \cdot (\bar{u} \quad \bar{d} \quad \bar{s} \quad \bar{c})$$

## II. Theoretical approach: hadronization. 2.



$$\begin{aligned}
 M^2 &= \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \cdot \left[ (\bar{u} \quad \bar{d} \quad \bar{s} \quad \bar{c}) \cdot \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \right] \cdot (\bar{u} \quad \bar{d} \quad \bar{s} \quad \bar{c}) \\
 &= M(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}) \rightarrow (M^2)_{4,3} = \cancel{c}\cancel{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})
 \end{aligned}$$

## II. Theoretical approach: hadronization. 2.

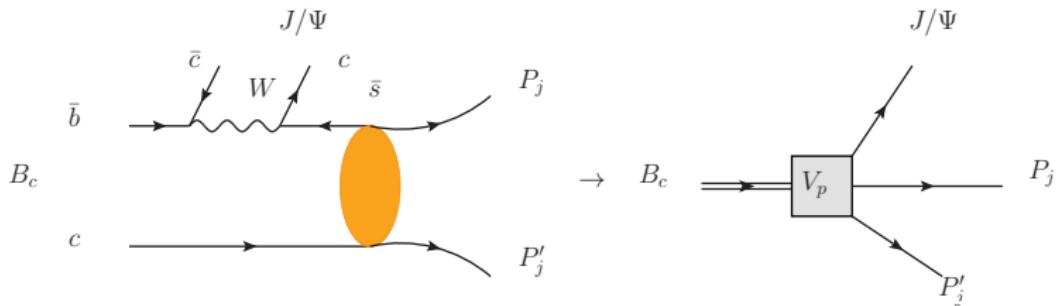


$$\phi = \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & \frac{\eta}{\sqrt{3}} - \frac{\pi^0}{\sqrt{2}} + \frac{\eta'}{\sqrt{6}} & K^0 & D^- \\ K^- & \bar{K}^0 & \frac{2\eta'}{\sqrt{6}} - \frac{\eta}{\sqrt{3}} & D_s^- \\ D^0 & D^+ & D_s^+ & \eta_c \end{pmatrix}$$

$$(M^2)_{4,3} = c\bar{s}(u\bar{u} + d\bar{d} + c\bar{c} + s\bar{s})$$

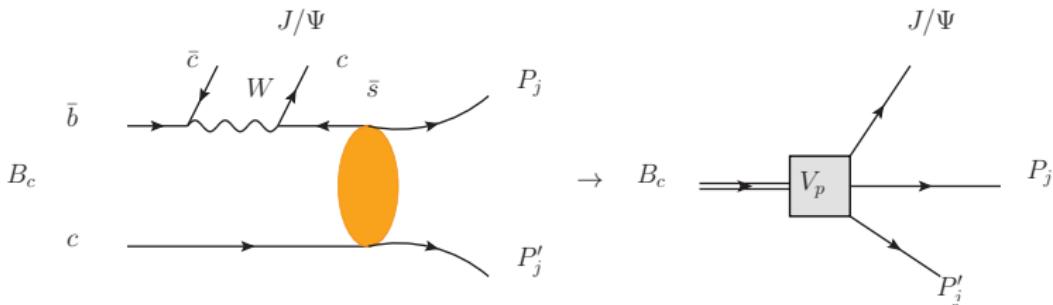
$$(\phi^2)_{4,3} = D^0 K^+ + D^+ K^0 - \frac{1}{\sqrt{3}} \eta D_s^+ + \sqrt{\frac{2}{3}} D_s^+ \eta' + \eta_c D_s^+ \equiv \sum_j P_j P'_j h_j$$

## II. Theoretical approach: *hadronization*. 3.



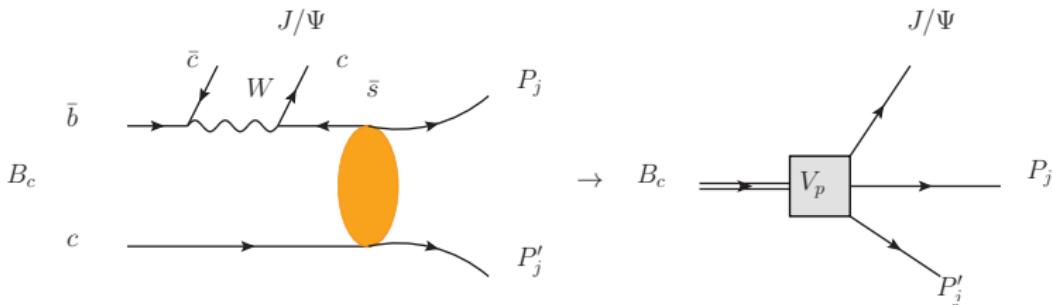
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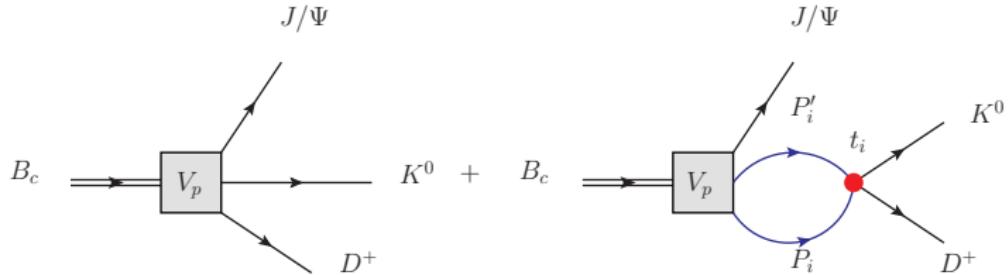
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- There are more contributions to the final  $J/\Psi P_j P'_j$  state

## II. Theoretical approach: *hadronization*. 4.



- ▶  $t_{\text{Decay}}(B_c \rightarrow J/\Psi K^0 D^+) = V_p \times \left( h_{K^0 D^+} + \sum_j \textcolor{blue}{G}_j \textcolor{red}{t}_{j, K^0 D^+} h_j \right)$

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- ▶  $G$  is a diagonal matrix whose elements are the loop functions of the two mesons  $P_i, P'_i$ , in each channel

$$G_{i,i}(s = P^2) = \int_{\mathbb{R}^4} \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - m_i^2 + i\epsilon} \frac{1}{q^2 - (m'_i)^2 + i\epsilon}$$

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- ▶  $V_{i,j}$  is the kernel potential, L.O. S-wave amplitudes (GAMERMANN, OSET, STROTTMAN, VICENTE VACAS. PHYSICAL REVIEW D 76, 074016 (2007)),  
 $\chi$ -heavy-meson lagrangian

$$V_{i,j}(s) = \frac{1}{f_\pi} \left( A_{i,j}(m, m') + B_{i,j}(m, m') s + C_{i,j}(m, m') \frac{1}{s} \right)$$

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- ▶ Fit the scattering free parameter  $\alpha$  in order to have the  $D_{s0}^*(2317)$  as a bound state. A pole in the  $t$  matrix,  
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- ▶ The differential decay width  $d\Gamma(B_c \rightarrow J/\Psi K D)/dM_{inv}$ ,

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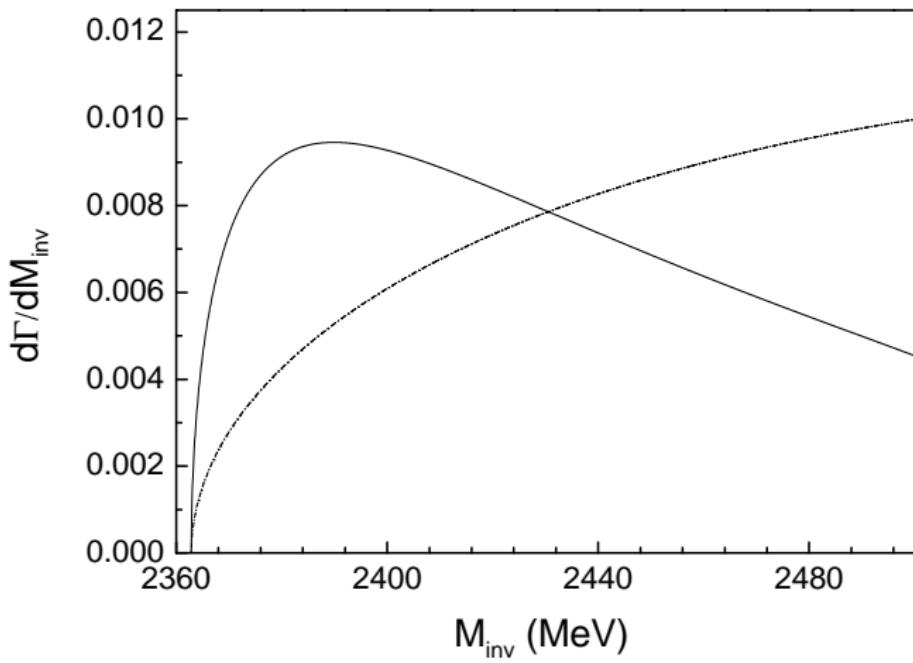
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$$\frac{d\Gamma}{dM_{inv}} = A^2 \frac{p_{J/\Psi}^3 p_{DK}}{(2\pi)^3 4m_{B_c}^2} \left| \frac{t_{\text{Decay}}(B_c \rightarrow J/\Psi D^+ K^0)}{V_p} \right|^2$$

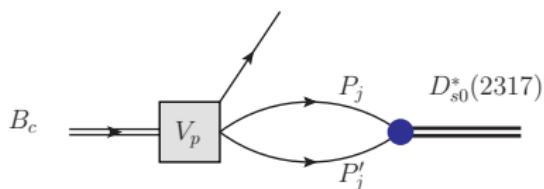
### III. Results and predictions for observables. 1.



### III. Results and predictions for observables. 2.

- ▶  $\Gamma(B_c \rightarrow J/\Psi D_{s0}^*(2317))$  Coalescence production of the  $D_{s0}^*(2317)^+$ :  $t(B_c \rightarrow J/\Psi R) = V_p \sum_j h_j G_j \Big|_{E=M_R}$

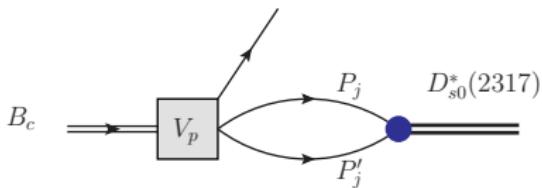
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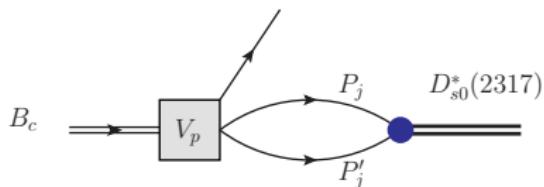


- ▶ 
$$\frac{d\tilde{\Gamma}(B_c \rightarrow J/\Psi KD)}{dM_{inv}} = \frac{d\Gamma(B_c \rightarrow J/\Psi KD)}{dM_{inv}} \frac{1}{p_{J/\Psi}^3 p_{DK}} \dots$$

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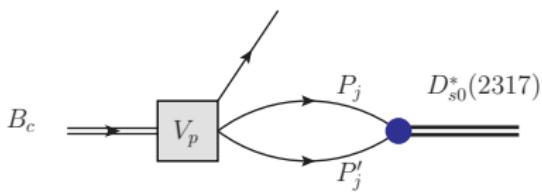


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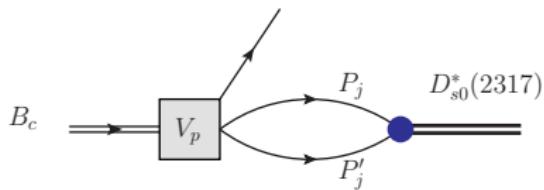
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- ▶  $\frac{d\tilde{\Gamma}(B_c \rightarrow J/\Psi KD)}{dM_{inv}} =$

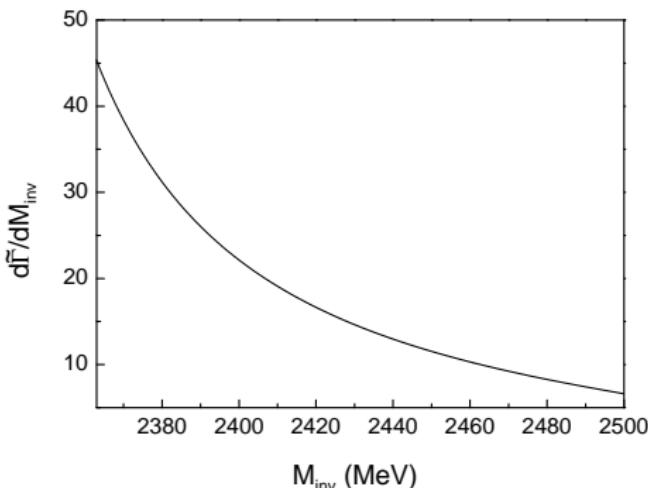
$$\frac{d\Gamma(B_c \rightarrow J/\Psi KD)}{dM_{inv}} \frac{1}{p_{J/\Psi}^3 p_{DK}} \times \left[ \frac{1}{\Gamma(B_c \rightarrow J/\Psi R)} \right] \times p_{J/\Psi}^3 \Big|_{E=M_R} M_R^2$$

$$= \frac{M_R^2}{4\pi^2} \frac{\left| h_{D^+ K^0} + \sum_i h_i G_i t_{i,D^+ K^0} \right|^2}{\left| \sum_i h_i G_i g_i \right|^2}_{\text{pole}},$$

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### III. Results and predictions for observables. 3.

Possible  $q\bar{q}$  component to the  $D_{s0}^*(2317)$  generation (A. MARTÍNEZ TORRES, E. OSET, S. PRELOVSEK AND A. RAMOS, JHEP **1505** (2015) 153):

- ▶ The amount of  $KD$  from the lattice data

$$P(KD) = (72 \pm 14)\% \leftrightarrow P(KD) = - \sum_{i=K^+D^0, K^0D^+} g_i^2 \frac{\partial G_i}{\partial s} \Big|_{\text{pole}}$$

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- ▶ We add a *Castillejo-Dalitz-Dyson* pole to the potential

$$V_{i,j} \rightarrow V_{i,j} + \delta V, \quad \delta V = \frac{\gamma}{M_{inv} - M_{q\bar{q}}}$$

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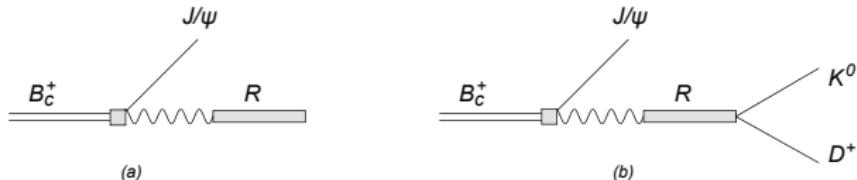
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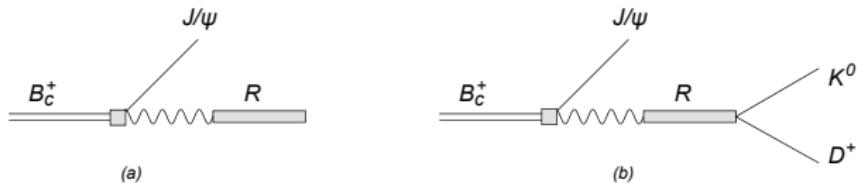
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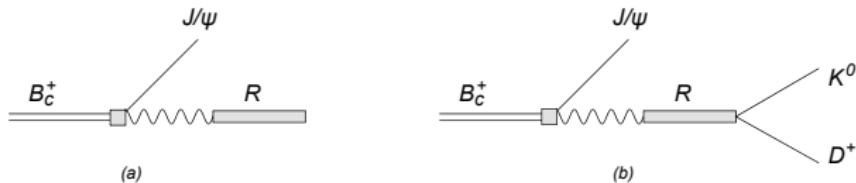
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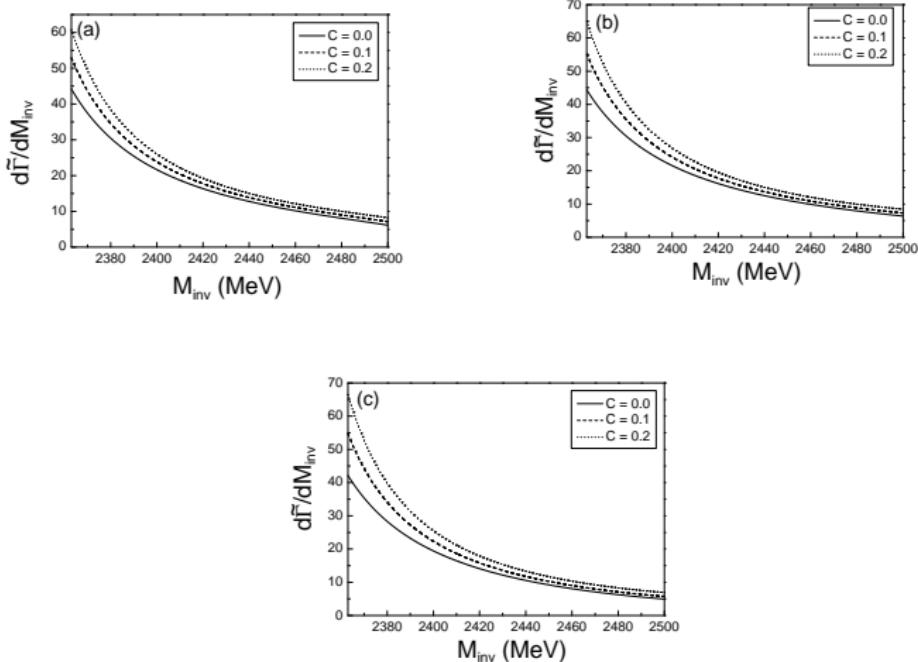
### III. Results and predictions for observables. 3.

- ▶ We consider direct coupling to this component in the  $B_c$  decay



- ▶ For different cases  $P(KD) = 58, 72, 86\%$ , we consider a  $q\bar{q}$  coupling to the resonance such that  $\Rightarrow 0, 21, 44\%$  of increase in  $\Gamma(B_c \rightarrow J/\psi D_{s0}^*(2317))$

### III. Results and predictions for observables. 3.



(a)  $P(KD) = 0.68$ . (b)  $P(KD) = 0.72$ . (c)  $P(KD) = 0.86$ .

## V. Summary

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- ▶ The enhancement of events seen close to the  $KD$  threshold in  $B$  decays can be an experimental test of the molecular nature of the  $D_{s0}^*(2317)$

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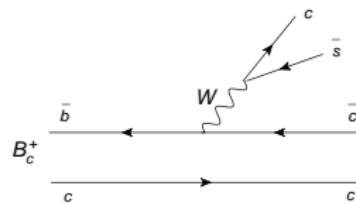
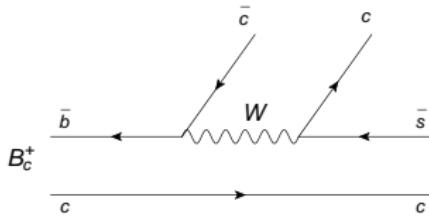
- ▶ The enhancement of events seen close to the  $KD$  threshold in  $B$  decays can be an experimental test of the molecular nature of the  $D_{s0}^*(2317)$
- ▶ We have proposed an unmeasured weak decay process where, with the molecular  $KD$  hypothesis, this feature is also observed and due to the generation of the  $D_{s0}^*(2317)$

# Thank you for your attention

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## I. Weak decay of a heavy hadron. 2.



$$V_p + V'_p$$

Global constant factor