# Amplitude analysis of resonant production in three pions

A. Jackura
with M. Mikhasenko & A. Szczepaniak

Indiana University, Joint Physics Analysis Center

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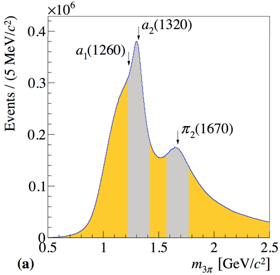
# Joint Physics Analysis Center (JPAC)

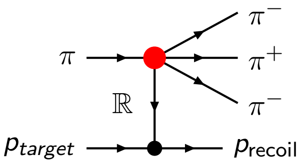
- The Joint Physics Analysis Center (JPAC) formed in October 2013
- We support physics analysis of experimental data for accelerator facilities (JLab12, COMPASS, . . . )
- http://www.indiana.edu/~jpac/
- JPAC Talks
    - Vladiszlav Pauk (Today 17:55 in Parallel B)
    - Adam Szczepaniak (Friday 9:00 Plenary)
    - Emilie Passemar (Friday 15:25 in Parallel A)
    - Alessandro Pilloni (Monday 17:15 in Parallel B)
    - Vincent Mathieu (Poster Session)

# $3\pi$ at COMPASS

- Study peripheral resonance production of $3\pi$ systems at COMPASS.
  - High statistics, high purity data allows for detailed analysis
  - JPAC affiliated with COMPASS to perform analysis on data
- Construct analytic amplitudes to extract resonance information
  - Amplitude satisfy S-matrix principles
  - Emphasize production process and unitarization of amplitude

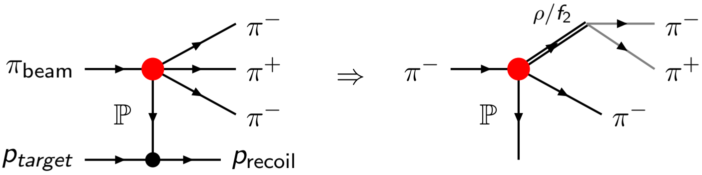



[ C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]



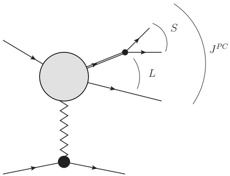
# $3\pi$ Production Mechanisms

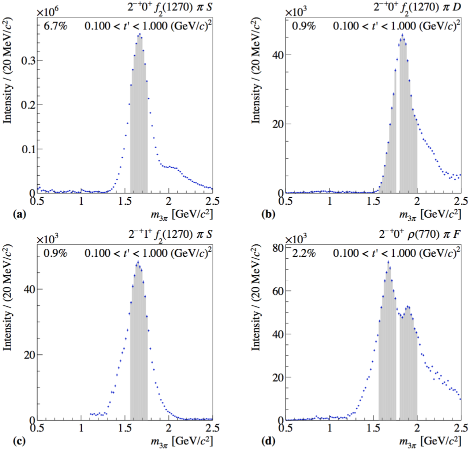
- Peripheral production is advantageous - Effective $2 \to 2$, $2 \to 3$, etc. meson scattering
  - By effective we mean particle-reggeon scattering
- Production mechanisms dictate physics
  - Expect exchange mechanism dominated by pomeron at high-energies
  - Effective $2 \to 2$, $2 \to 3$, etc. meson scattering production by particle exchange



# PWA of $3\pi$ final state

- Develop method of analysis satisfying S-matrix principles, study $J^{PC}$ resonances in $3\pi$
- In this presentation, we focus on $2^{-+}$,
  - long standing puzzle about $\pi_2(1670)$–$\pi_2(1880)$ interplay,
  - 17 waves out of 88 have $J^{PC} = 2^{-+}$,



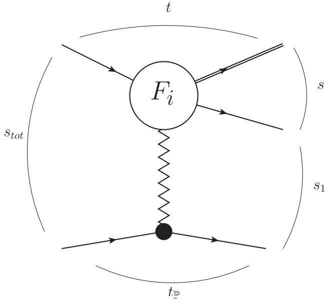




[C. Adolph *et al.* [COMPASS Collaboration], arXiv:1509.00992]

# The Model

- Partial wave analysis of $3\pi$ system in $\pi^- p \to \pi^-\pi^-\pi^+ p$
- Use isobar model, with first approximation of stable isobars in $(\pi^-\pi^+)$
- Pomeron phenomenologically approximated by vector particle, $\alpha_{\mathbb{P}} \approx 1$
  - Factorize $N \to \mathbb{P}N$ vertex from rest of amplitude

- For $J^{PC} = 2^{-+}$, focus on high event intensities
  - *e.g.* $\rho\pi$ $F$-wave, $f_2(1270)\pi$ $S$- and $D$-waves, ...
- Coupled channel analysis for partial wave amplitudes $F_i(s)$, with channel index $i = \{\rho\pi(F), f_2\pi(S), f_2\pi(D), \dots\}$



# Unitarity and Analyticity

- Partial wave unitarity of $\pi^- \mathbb{P} \to (\pi^- \pi^+)\pi^-$ amplitude

$$\mathrm{Disc}\, F_i(s) = 2i \sum_j t^*_{ij}(s)\rho_j(s)F_j(s)$$

- Rescattering amplitude satisfies its own unitarity equation

$$\mathrm{Im}\, t_{ij}(s) = \sum_k t^*_{ik}(s)\rho_k(s)t_{kj}(s)$$

- One can separate $F_i$ into LHC and RHC terms, and write dispersive integral equation for $F_i$, with solution given by Omnes

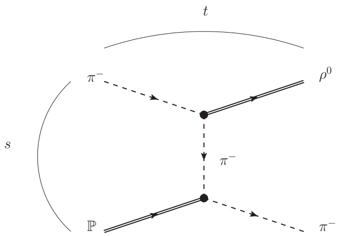
$$F_i(s) = b_i(s) + \sum_j t_{ij}(s)c_j + \frac{1}{\pi}\sum_j t_{ij}(s)\int_{s_j}^{\infty} ds' \frac{\rho_j(s')b_j(s')}{s'-s}$$

# K-Matrix Parameterization

- To preserve unitarity, rescattering amplitude $t_{ij}(s)$ is parameterized by $K$-matrix
$$[t^{-1}]_{ij}(s) = [K^{-1}]_{ij}(s) - I_i(s)\delta_{ij}$$
where $I_i(s)$ is Chew-Mandelstam phase space factor, with $\mathrm{Im}\, I_i(s) = \rho_i(s)$
- The real $K$-matrix is parameterized by resonant and non-resonant contributions
$$K_{ij}(s) = \sum_r \frac{g_i^r g_j^r}{m_r^2 - s} + \sum_n \gamma_{ij}^n s^n$$
- Fit $K$-matrix parameters to data and extract resonance information

# Production Amplitude

- For the production amplitude $b_i(s)$, we model with Deck amplitude
- Consider $\pi$ exchange
  - Closest LHC to physics region $\Longrightarrow$ Expected to be significant contribution
  - Ignoring subtleties of $\pi$-exchange (May need absorption corrections)




- Model:

$$A_{\text{Deck}}(s,\Omega) = \frac{g_{\rho\pi\pi} g_{\mathbb{P}\pi\pi}}{t(s,\theta) - m_\pi^2} \epsilon_\lambda \cdot p_2 \, \epsilon_{\lambda'}^{\sigma *} \cdot \{p_a\}$$

- $b_i(s)$ is partial wave projection of $A_{\text{Deck}}$ in definite $J$, $M$, and $L$ states

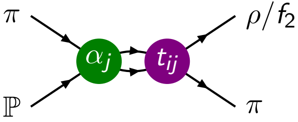
# Fit Attempts

- As first attempt, we consider a more simplified model, where the production amplitude is conformal expansion

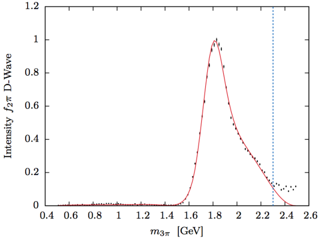
$$F_i(s) = \sum_j t_{ij}(s)\alpha_j(s)$$

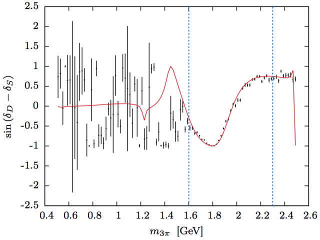
- $\alpha_i$ contains no RHCs and has free parameters
- Also, consider only $f_2\pi$ in $S$- and $D$-wave

$$F_i = \sum_j$$



# Simple Production Model Fit

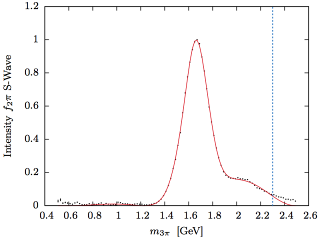



$$\chi^2/dof = 12.2$$

$$m_{R_1} = 1.820\ GeV$$
$$\Gamma_{R_1} = 0.214\ GeV$$

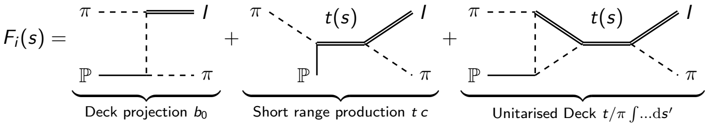
$$m_{R_2} = 1.612\ GeV$$
$$\Gamma_{R_2} = 0.194\ GeV$$



# Unitarized Deck Fits

- The fits for a general production term $\alpha_i$ seem too flexible in the current approach
- Now use unitarized Deck amplitude developed for this analysis

$$F_i(s) = b_i(s) + \sum_j t_{ij}(s) c_j + \frac{1}{\pi} \sum_j t_{ij}(s) \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_j(s')}{s' - s}$$




- Fit Intensities and phase differences of three channel case

# Unitarized Deck Fits

Data: three main waves at low $|t'|$ ($0.1\,\mathrm{GeV}^2$-$0.113\mathrm{GeV}^2$):

$$2^{-+}0^{+}\, f_2\pi\, S, \qquad 2^{-+}0^{+}\, f_2\pi\, D, \qquad 2^{-+}0^{+}\, (\pi\pi)_s\pi D.$$

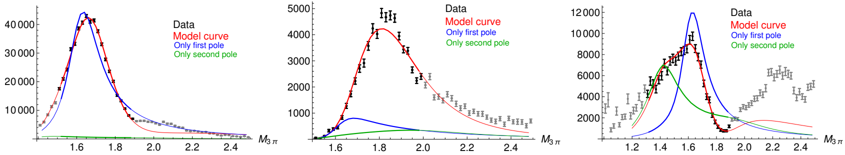



Figure: Fit model: 3 channel K-matrix with two poles and unitarized "Deck".

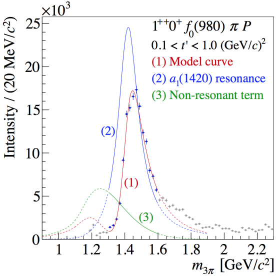
- K-matrix assumes elasticity, so simultaneous fit of all decay channels are needed (all $3\pi$ waves),
- data for 11 $|t'|$ intervals are available. $|t'|$-dependence of non-resonance component is fixed by "Deck" model.

# Future Developments for COMPASS Analysis

- Develop Framework to analyze $3\pi$ resonances satisfying S-matrix principles
- Will investigate Finite Energy Sum Rules to constrain amplitudes
- We are fitting data based on COMPASS model. Will extend to 4-vectors and for GlueX at JLab

- Want to describe entire $3\pi$ spectrum, but some interesting cases along the way ($2^{-+}$ and $1^{++}$)
  - Will continue the work on COMPASS in $2^{-+}$ sector
  - Perform analysis on $1^{++}$ sector, $a_1(1420)$ puzzle

[C. Adolph *et al.* [COMPASS Collaboration], Phys. Rev. Lett. 115, 082001 (2015)]



# Summary

- We have developed the analysis formalism to analyze $3\pi$ systems for peripheral reactions
- Formalism satisfies S-matrix principles
- Applying formalism to COMPASS and extracting resonances
    - Focus on $J^{PC} = 2^{-+}$ first, then apply to all $3\pi$ $J^{PC}$
- Extend formalism for photon beams (JLab12 physics)

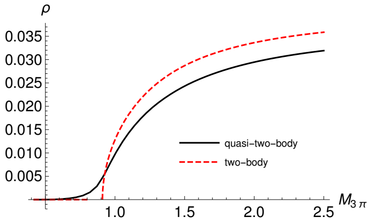
# Backup

# Phase Space Factors

- In stable isobar limit, phase space factor is 2-body: $\rho_i \sim \sqrt{(s - s_i)/s}$
- Decaying isobar introduces $\pi^+\pi^-$ scattering amplitude $f(s)$
- Phase space factors change to quasi-two body phase space factors

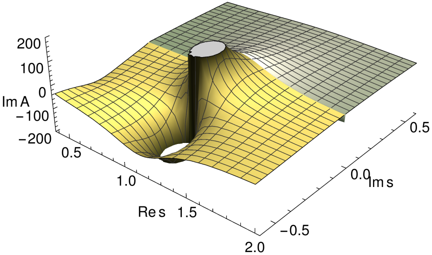
$$\rho_{\text{Quasi}}(s) \sim \int_{4m_\pi^2}^{\sqrt{s}-m_\pi} ds' \, \rho_{Isobar-\pi}(s') \text{Im}\, f(s')$$

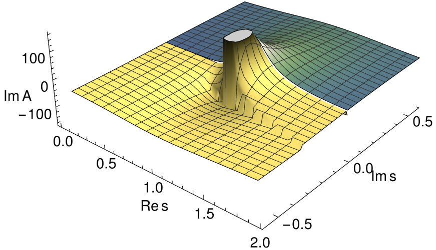
- Affects how we continue to unphysical sheets, new (Woolly) cut introduced



# Resonance Extraction

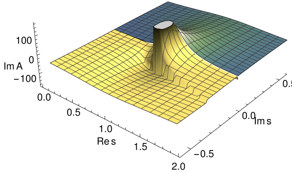
- Analytically continue amplitudes to unphysical sheets to search for poles
- Stable isobars involve only two-body phase space factors (simple square-roots)
- For decaying isobars, Woolly cut may hide pole onto a deeper sheet

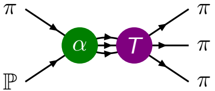


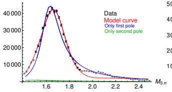



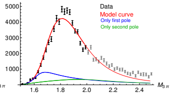
# Summary of the project

$$\begin{cases} \text{Disc}\, t = 2i\, t^* \rho\, t \\ \text{Disc}\, F = 2i\, t^* \rho\, F \end{cases}$$




1. Unitarity condition:
   - ✓ two body unitarity and quasi-two-body, isobar+pion
   - ✓ consideration of various solutions, ✗ $N/D$ (deadlock), ✓ $K$-matrix
   - ✓ generalisation for multi-channel case,
   - ✓ incorporation of threshold behaviour.
2. Analytical continuation of amplitude:
   - ✓ additional isobar strucure "Woolly" cut [Aitchison]
   - ✓ pole search
3. Production mechanism
   - ✓ $P$-vector solution(deadlock),
   - ✓ short-long range approximation, explicit incorporation of "Deck" amplitude [Basdevant-Berger]
   - ✓ PW projection of scalar "Deck", threshold behaviour check
   - ✏ PW projection of spin-"Deck", threshold behaviour check [Ascoli, Griss-Fox]
4. Fit and systematics
   - ✏ Implementation of the fit procedure, C++, Mathematica, Fortran
   - ✏ MC studies of $\chi^2$-function