Amplitude analysis of resonant production in three pions

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The Joint Physics Analysis Center (JPAC) formed in October 2013

We support physics analysis of experimental data for accelerator facilities (JLab12, COMPASS, ...)

http://www.indiana.edu/~jpac/

JPAC Talks

- Vladislav Pauk (Today 17:55 in Parallel B)
- Adam Szczepaniak (Friday 9:00 Plenary)
- Emilie Passemar (Friday 15:25 in Parallel A)
- Alessandro Pilloni (Monday 17:15 in Parallel B)
- Vincent Mathieu (Poster Session)
3π at COMPASS

- Study peripheral resonance production of 3π systems at COMPASS.
  - High statistics, high purity data allows for detailed analysis
  - JPAC affiliated with COMPASS to perform analysis on data
- Construct analytic amplitudes to extract resonance information
  - Amplitude satisfy S-matrix principles
  - Emphasize production process and unitarization of amplitude

[ C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]
Peripheral production is advantageous - Effective $2 \rightarrow 2$, $2 \rightarrow 3$, etc. meson scattering

By effective we mean particle-reggeon scattering

Production mechanisms dictate physics

Expect exchange mechanism dominated by pomeron at high-energies

Effective $2 \rightarrow 2$, $2 \rightarrow 3$, etc. meson scattering production by particle exchange
PWA of $3\pi$ final state

- Develop method of analysis satisfying S-matrix principles, study $J^{PC}$ resonances in $3\pi$
- In this presentation, we focus on $2^{-+}$,
  - long standing puzzle about $\pi_2(1670) - \pi_2(1880)$ interplay,
  - 17 waves out of 88 have $J^{PC} = 2^{-+}$,

[C. Adolph et al. [COMPASS Collaboration], arXiv:1509.00992]
The Model

- Partial wave analysis of $3\pi$ system in $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$
- Use isobar model, with first approximation of stable isobars in $(\pi^- \pi^+)$
- Pomeron phenomenologically approximated by vector particle, $\alpha_P \approx 1$
  - Factorize $N \rightarrow P N$ vertex from rest of amplitude

- For $J^{PC} = 2^{-+}$, focus on high event intensities
  - e.g. $\rho \pi$ F-wave, $f_2(1270)\pi$
  - $S$- and $D$-waves, ...

- Coupled channel analysis for partial wave amplitudes $F_i(s)$, with channel index $i = \{\rho \pi (F), f_2 \pi (S), f_2 \pi (D), \ldots\}$
Unitarity and Analyticity

- Partial wave unitarity of $\pi^- P \rightarrow (\pi^- \pi^+) \pi^-$ amplitude

$$\text{Disc } F_i(s) = 2i \sum_j t_{ij}^*(s) \rho_j(s) F_j(s)$$

- Rescattering amplitude satisfies its own unitarity equation

$$\text{Im } t_{ij}(s) = \sum_k t_{ik}^*(s) \rho_k(s) t_{kj}(s)$$

- One can separate $F_i$ into LHC and RHC terms, and write dispersive integral equation for $F_i$, with solution given by Omnes

$$F_i(s) = b_i(s) + \sum_j t_{ij}(s) c_j + \frac{1}{\pi} \sum_j t_{ij}(s) \int_{s_j}^{\infty} ds' \frac{\rho_j(s') b_j(s')}{s' - s}$$
K-Matrix Parameterization

- To preserve unitarity, rescattering amplitude $t_{ij}(s)$ is parameterized by $K$-matrix
  \[
  [t^{-1}]_{ij}(s) = [K^{-1}]_{ij}(s) - l_i(s)\delta_{ij}
  \]
  where $l_i(s)$ is Chew-Mandelstam phase space factor, with $\text{Im} l_i(s) = \rho_i(s)$

- The real $K$-matrix is parameterized by resonant and non-resonant contributions
  \[
  K_{ij}(s) = \sum_r g_i^r g_j^r \frac{1}{m_r^2 - s} + \sum_n \gamma_{ij}^n s^n
  \]

- Fit $K$-matrix parameters to data and extract resonance information
For the production amplitude $b_i(s)$, we model with Deck amplitude.

Consider $\pi$ exchange:
- Closest LHC to physics region $\Rightarrow$ Expected to be significant contribution.
- Ignoring subtleties of $\pi$-exchange (May need absorption corrections).

Model:

$$A_{\text{Deck}}(s, \Omega) = \frac{g_{\rho\pi\pi} g_{\rho\pi\pi}}{t(s, \theta) - m^2_{\pi}} \epsilon \lambda \cdot p_2 \epsilon_{\lambda'} \cdot \{p_a\}$$

$b_i(s)$ is partial wave projection of $A_{\text{Deck}}$ in definite $J$, $M$, and $L$ states.
As first attempt, we consider a more simplified model, where the production amplitude is conformal expansion

$$F_i(s) = \sum_j t_{ij}(s) \alpha_j(s)$$

- $\alpha_i$ contains no RHCs and has free parameters
- Also, consider only $f_2\pi$ in $S$- and $D$-wave
Current Results

Simple Production Model Fit

\[ \chi^2 / dof = 12.2 \]

\[ m_{R_1} = 1.820 \text{ GeV} \]
\[ \Gamma_{R_1} = 0.214 \text{ GeV} \]

\[ m_{R_2} = 1.612 \text{ GeV} \]
\[ \Gamma_{R_2} = 0.194 \text{ GeV} \]
The fits for a general production term $\alpha_i$ seem too flexible in the current approach.

Now use unitarized Deck amplitude developed for this analysis

$$F_i(s) = b_i(s) + \sum_j t_{ij}(s) c_j + \frac{1}{\pi} \sum_j t_{ij}(s) \int_{s_j}^{\infty} ds' \rho_j(s') b_j(s') \frac{s' - s}{s'}$$

- **Deck projection $b_0$**
- **Short range production $t\ c$**
- **Unitarised Deck $t/\pi \int ...ds'$**

**Fit Intensities and phase differences of three channel case**
Unitarized Deck Fits

Data: three main waves at low $|t'|$ (0.1 GeV$^2$-0.113 GeV$^2$):

$$2^{-+0^+} f_2 \pi S, \quad 2^{-+0^+} f_2 \pi D, \quad 2^{-+0^+} (\pi \pi)_S \pi D.$$ 

Figure: Fit model: 3 channel K-matrix with two poles and unitarized "Deck".

- K-matrix assumes elasticity, so simultaneous fit of all decay channels are needed (all $3\pi$ waves),
- data for 11 $|t'|$ intervals are available. $|t'|$-dependence of non-resonance component is fixed by “Deck” model.
Future Developments for COMPASS Analysis

- Develop Framework to analyze $3\pi$ resonances satisfying S-matrix principles
- Will investigate Finite Energy Sum Rules to constrain amplitudes
- We are fitting data based on COMPASS model. Will extend to 4-vectors and for GlueX at JLab

- Want to describe entire $3\pi$ spectrum, but some interesting cases along the way ($2^{-+}$ and $1^{++}$)
  - Will continue the work on COMPASS in $2^{-+}$ sector
  - Perform analysis on $1^{++}$ sector, $a_1(1420)$ puzzle

We have developed the analysis formalism to analyze $3\pi$ systems for peripheral reactions.

Formalism satisfies S-matrix principles.

Applying formalism to COMPASS and extracting resonances:
- Focus on $J^{PC} = 2^{-+}$ first, then apply to all $3\pi\ J^{PC}$.

Extend formalism for photon beams (JLab12 physics).
Phase Space Factors

- In stable isobar limit, phase space factor is 2-body: \( \rho_i \sim \sqrt{(s - s_i)/s} \)
- Decaying isobar introduces \( \pi^+\pi^- \) scattering amplitude \( f(s) \)
- Phase space factors change to quasi-two body phase space factors
  \[ \rho_{\text{Quasi}}(s) \sim \int_{4m_{\pi}^2}^{\sqrt{s-m_{\pi}}} ds' \rho_{\text{Isobar}-\pi}(s') \text{Im} f(s') \]
- Affects how we continue to unphysical sheets, new (Woolly) cut introduced
Resonance Extraction

- Analytically continue amplitudes to unphysical sheets to search for poles
- Stable isobars involve only two-body phase space factors (simple square-roots)
- For decaying isobars, Woolly cut may hide pole onto a deeper sheet
Summary of the project

1. Unitarity condition:
   - two body unitarity and quasi-two-body, isobar+pion
   - consideration of various solutions, \( \bar{N}/D \) (deadlock), \( K \)-matrix
   - generalisation for multi-channel case,
   - incorporation of threshold behaviour.

2. Analytical continuation of amplitude:
   - additional isobar structure
     - “Woolly” cut [Aitchison]
   - pole search

3. Production mechanism
   - \( P \)-vector solution (deadlock),
   - short-long range approximation,
     - explicit incorporation of “Deck” amplitude [Basdevant-Berger]
   - PW projection of scalar “Deck”, threshold behaviour check
   - PW projection of spin-“Deck”, threshold behaviour check [Ascoli, Griss-Fox]

4. Fit and systematics
   - Implementation of the fit procedure, C++, Mathematica, Fortran
   - MC studies of \( \chi^2 \)-function