Status of the ChPT calculations in the light meson sector

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Outline

1. Introduction and Motivation
2. Success of ChPT: $\pi\pi$ scattering
3. $\eta \rightarrow 3\pi$ and light quark masses
4. K_{l4} decays and determination of some LECs
5. Conclusion and Outlook

1. Introduction and Motivation

1.1 Hadronic Physics

- New era of precision experiments

  ⇨ Build amplitudes to look for exotics, hybrid mesons

- Require building blocks:
  - $\pi\pi$
  - $K\pi$

  ⇨ ChPT + dispersion relations

- Precise tests of the Standard Model

- Look for physics beyond the Standard Model: High precision at low energy as a key to new physics?

1.2 Chiral Symmetry

- Limit $m_k \to 0$

$$\mathcal{L}_{QCD} \to \boxed{\mathcal{L}^0_{QCD} = -\frac{1}{4} G_{\mu\nu} G^{\mu\nu} + \bar{q}_L i \gamma^\mu D_\mu q_L + \bar{q}_R i \gamma^\mu D_\mu q_R}, \quad q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

with $q_{L/R} \equiv \frac{1}{2}\left(1 \mp \gamma_5\right) q$

Symmetry: $\boxed{G \equiv SU(3)_L \otimes SU(3)_R \to SU(3)_V}$

- Chiral Perturbation Theory: dynamics of the Goldstone bosons (kaons, pions, eta)

- Goldstone bosons interact weakly at low energy and $m_u, m_d \ll m_s < \Lambda_{QCD}$
Expansion organized in external momenta and quark masses

Weinberg's power counting rule

$$\boxed{\mathcal{L}_{eff} = \sum_{d \geq 2} \mathcal{L}_d \ , \mathcal{L}_d = \mathcal{O}\left(p^d\right) \ , p \equiv \left\{q, m_q\right\}}$$

$$\boxed{\mathrm{p} << \Lambda_H = 4\pi F_\pi \sim 1 \text{ GeV}}$$

1.3 Chiral expansion

- $$\mathcal{L}_{ChPT} = \underbrace{\mathcal{L}_2}_{\text{LO}:\ \mathcal{O}(p^2)} + \underbrace{\mathcal{L}_4}_{\text{NLO}:\ \mathcal{O}(p^4)} + \underbrace{\mathcal{L}_6}_{\text{NNLO}:\ \mathcal{O}(p^6)} + \ldots$$

- The structure of the lagrangian is fixed by chiral symmetry but not the coupling constants → LECs appearing at each order

- The method has been rigorously established and can be formulated as a set of calculational rules:

LO : tree level diagrams with $\mathcal{L}_2$

$\mathcal{L}_2 :\ F_0, B_0$

NLO: tree level diagrams with $\mathcal{L}_4$
1-loop diagrams with $\mathcal{L}_2$

$$\mathcal{L}_4 = \sum_{i=1}^{10} L_i\, O_4^i,$$

NNLO: tree level diagrams with $\mathcal{L}_6$
2-loop diagrams with $\mathcal{L}_2$
1-loop diagrams with one vertex from $\mathcal{L}_4$

$$\mathcal{L}_6 = \sum_{i=1}^{90} C_i\, O_6^i$$

- Renormalizable and unitary order by order in the expansion

1.4 ChPT in the meson sector: precision calculations

- Today's standard in the meson sector: 2-loop calculations

- Main obstacle to reaching high precision: determination of the LECs: $\mathcal{O}(p^6)$ LECs proliferation makes the program to pin down/ estimate all of them prohibitive

- In a specific process, only a limited number of LECs appear

- The LECs calculable if QCD solvable, instead
  - Determined from experimental measurement
  - Estimated with models: Resonances, large N_C
  - Computed on the lattice

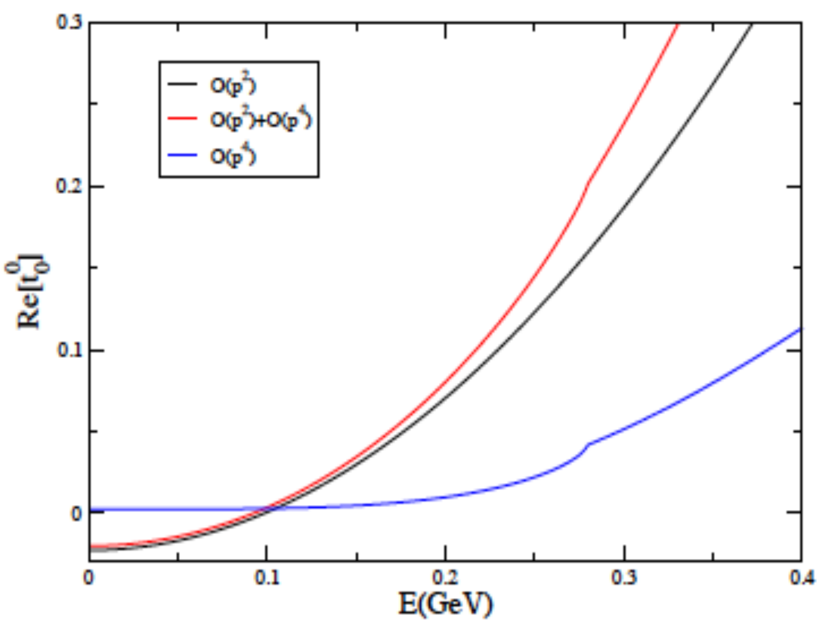
2. Success: ππ scattering

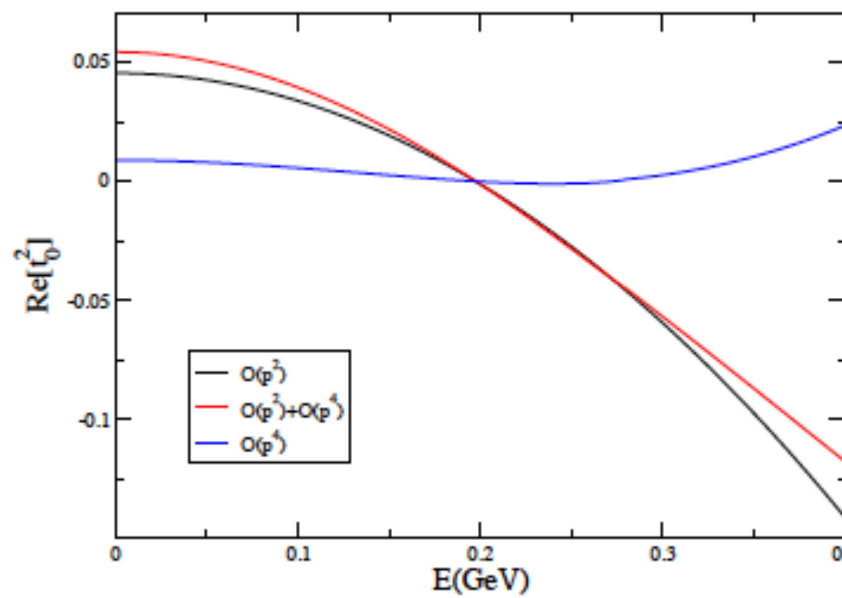
2.1 $\pi\pi$ scattering lengths

- $\pi\pi$ scattering computed early on, one of the first applications of SU(2) x SU(2)
  ⇒ converge better

- The scattering lengths : computed at NLO by *Gasser & Leutwyler'83*

G. Colangelo





- The momentum dependence is reflected in a chiral log in a^I_0 $\quad \ell_\chi = \frac{M_\pi^2}{16\pi^2 F_\pi^2} \ln \frac{\mu^2}{M_\pi^2}$

⇒ $$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{9}{2}\ell_\chi + \ldots\right]$$

How large are the higher orders?

2.2 Roy equations

- Unitarity effects can be calculated *exactly* using dispersive methods

- Unitarity, analyticity and crossing symmetry ≡ *Roy equations*

- Input: imaginary parts above a matching point (e.g. s_{match}~ 0.8 GeV)
  two subtraction constants, e.g. a_0^0 and a_2^0

- Output: the full ππ scattering amplitude below s_{match}
  ⇨ extended recently up to s_{match} = 1.15 GeV *Caprini, Colangelo, Leutwyler'11*

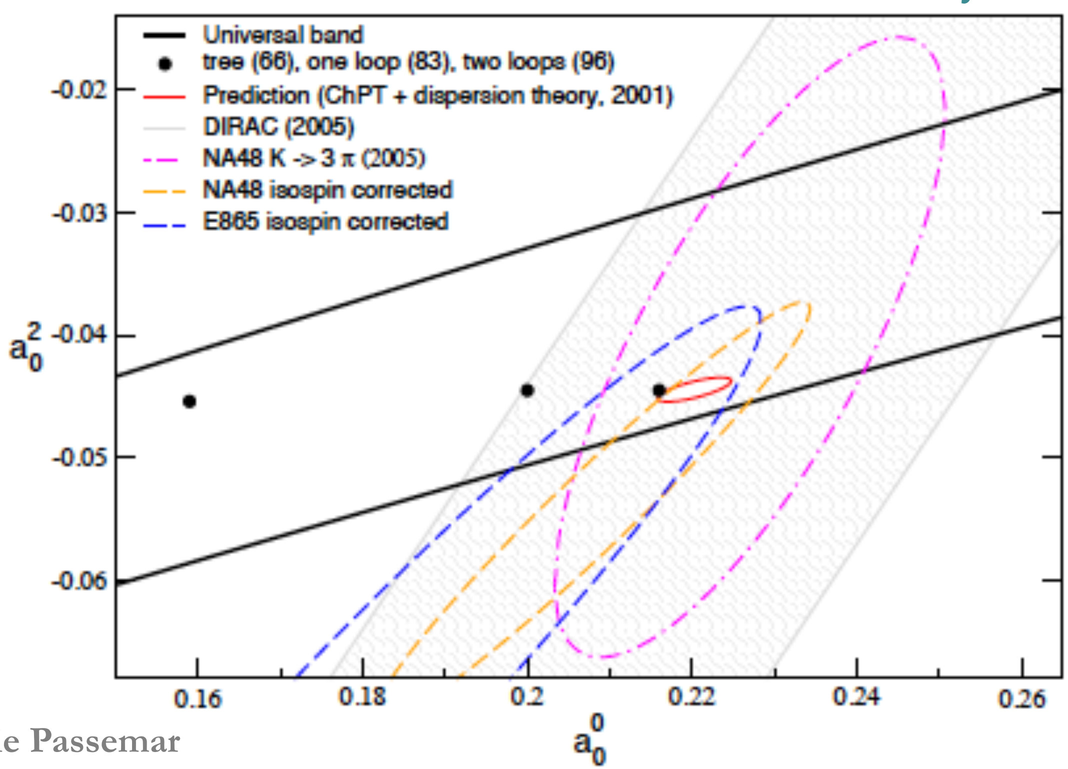
- Numerical solutions of the Roy equations
  Pennington-Protopopescu, Basdevant-Froggatt-Petersen (70s)
  Bern group: Ananthanarayan, Colangelo, Gasser and Leutwyler'00
  Caprini, Colangelo, Leutwyler'11
  Orsay group: Descotes-Genon, Fuchs, Girlanda and Stern'01
  Madrid-Cracow group: Garcia-Martin, Kamiński Pelaez, Ruiz de Elvira,Yndurain'11

2.3 Combining ChPT and dispersion relations: A happy marriage

G. Colangelo

- In ChPT the two subtraction constants are predicted
- Subtracting the amplitude at threshold (a_0^0, a_2^0) is not mandatory
- The freedom in the choice of the subtraction point can be exploited to use the chiral expansion where it converges best, i.e. below threshold

H. Leutwyler



- *Perfect agreement with data*
- Isospin breaking corrections for K_{l4} data

Colangelo, Gasser, Rusetsky'09
Bernard, Descotes-Genon, Knecht '13 '14

2.4 $\pi\pi$ as a building block

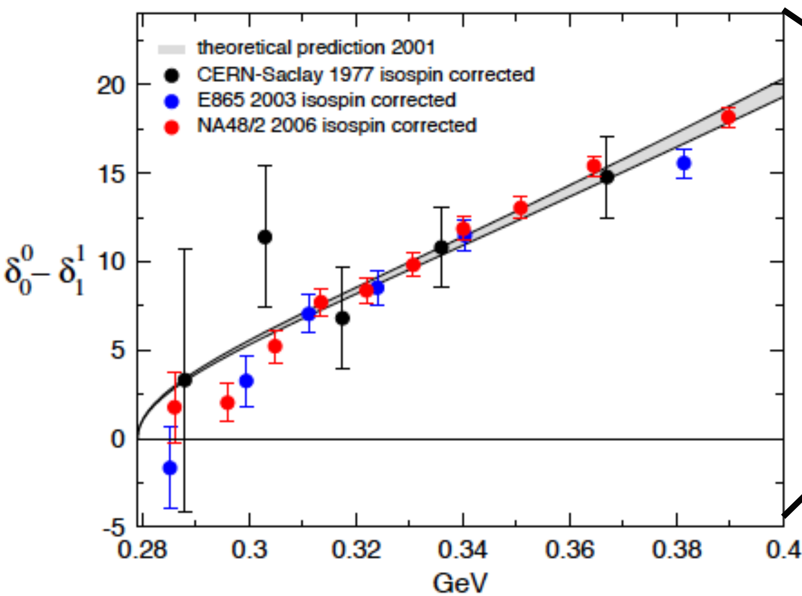
- Extremely precise extraction of $\pi\pi$ scattering using ChPT and dispersion relations
- Similar works done solving Roy-Steiner equations for
  - $K\pi$: *Buettiker, Descotes-Genon, Moussallam'07*
  - πN : *Hoferichter, Ruiz de Elvira, Kubis, Meißner'15*

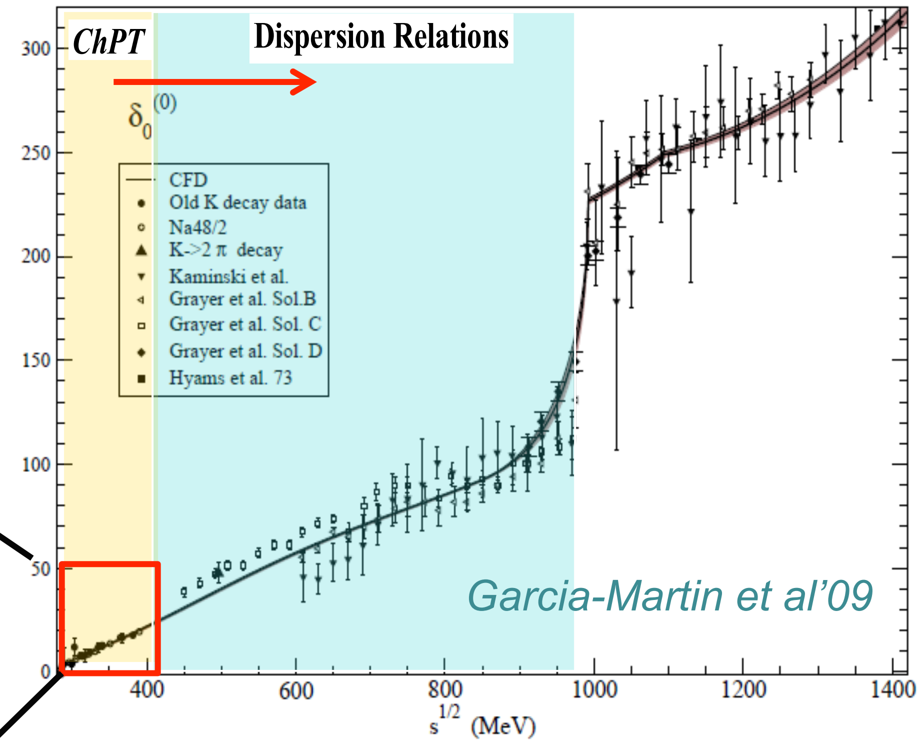
    ⇨ See talk by *B. Kubis*
- Compare to lattice results
- Use these as building blocks for phenomenology:
  - $\pi\pi$ rescattering: e.g., π form factors, $e^+e^- \to \pi\pi$, $\gamma\gamma \to \pi\pi$, $\omega/\varphi/\eta \to 3\pi$, $\tau \to 3\pi\nu_\tau$, $J/\Psi \to \gamma\pi^0\pi^0$. $B \to 3\pi$, $B \to J/\Psi\pi\pi$, etc..

    ⇨ See e.g. talks by *A. Pilloni*, *B. Moussallam*, *R. Kamiński*
  - $K\pi$ rescattering: e.g., $K\pi$ form factors, $K \to \pi\pi e\nu_e$, $\tau \to K\pi\nu_\tau$, $\tau \to K\pi\pi\nu_\tau$, $D \to K\pi\pi$, $B \to K\pi$

2.4 ππ as a building block

H. Leutwyler

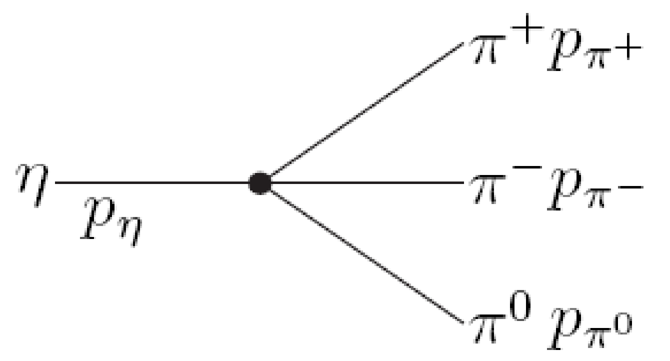




Garcia-Martin et al'09

3. $\eta \to 3\pi$ and light quark masses

3.1 $\eta \rightarrow \pi^+ \pi^- \pi^0$



- Decay forbidden by isospin symmetry

$$\Rightarrow \boxed{A = (m_u - m_d) A_1 + \alpha_{em} A_2}$$

- α_{em} effects are small   *Sutherland'66, Bell & Sutherland'68*
  Baur, Kambor, Wyler'96, Ditsche, Kubis, Meissner'09

- Decay rate measures the size of isospin breaking ($m_u - m_d$) in the SM:

$$L_{QCD} \rightarrow \boxed{L_{IB} = -\frac{m_u - m_d}{2}\left(\bar{u}u - \bar{d}d\right)}$$

$\Rightarrow$ Clean access to ($m_u - m_d$)

- $\boxed{\Gamma_{\eta \to 3\pi} \propto \int \left| A(s,t,u) \right|^2 \propto Q^{-4}}$ $\Rightarrow$ $\boxed{Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}}$ $\left[\hat{m} \equiv \frac{m_d + m_u}{2} \right]$

3.2 ChPT

- Slow convergence of the chiral series (SU(3) ChPT)

$$\boxed{\Gamma_{\eta\to 3\pi} = (\underset{\text{LO}}{66} + \underset{\text{NLO}}{94} + \underset{\text{NNLO}}{100} + \ldots)\,\text{eV} = (300 \pm 12)\,\text{eV}}$$

PDG'14

LO: *Osborn, Wallace'70*

NLO: *Gasser & Leutwyler'85*

NNLO: *Bijnens & Ghorbani'07*

- CHPT amplitudes have problems with measured Dalitz plot distributions

- Main deficiency: strong $\pi\pi$ rescattering included only perturbatively

- Large $\pi\pi$ final state interactions

  ⇨ call for a dispersive treatment :
  - analyticity, unitarity and crossing symmetry
  - Take into account all the rescattering effects

- Match to CHPT amplitude to obtain Q from rates

3.3 Dispersive method

G. Colangelo, S. Lanz,
H. Leutwyler , E.P., in progress

- Decomposition of the amplitude as a function of $\pi\pi$ isospin states

$$\boxed{M(s,t,u) = M_0(s) + (s-u)M_1(t) + (s-t)M_1(u) + M_2(t) + M_2(u) - \frac{2}{3}M_2(s)}$$

Fuchs, Sazdjian & Stern'93

Anisovich & Leutwyler'96

  - M_I isospin *I* rescattering in two particles
  - Amplitude in terms of S and P waves ⇨ exact up to NNLO ($\mathcal{O}(p^6)$)
  - Main two body rescattering corrections inside M_I

- Unitary relation for $M_I(s)$: $disc\left[M_I(s)\right] \equiv disc\left[f_\ell^I(s)\right] \propto t_\ell^*(s) f_\ell^I(s)$

⇨ $\boxed{f_1^I(s) = M_I(s) + \hat{M}_I(s)}$

$$\boxed{disc\ M_I(s) = 2i\ \left(M_I(s) + \hat{M}_I(s)\right)\ \sin\delta_I(s) e^{-i\delta_I(s)}\theta\left(s - 4M_\pi^2\right)}$$

right-hand cut left-hand cut

3.3 Dispersive method

G. Colangelo, S. Lanz, H. Leutwyler , E.P.

- Unitary relation for $M_I(s)$:

$$\boxed{disc\ M_I(s) = 2i\ \left(M_I(s) + \hat{M}_I(s)\right)\ \sin\delta_I(s) e^{-i\delta_I(s)} \theta\left(s - 4M_\pi^2\right)}$$

right-hand cut   left-hand cut

- Dispersion relation for the M_I's

$$\boxed{M_I(s) = \Omega_I(s)\left(P_I(s) + \frac{s^n}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'^n}\frac{\sin\delta_I(s')\ \hat{M}_I(s')}{|\Omega_I(s')|\left(s'-s-i\varepsilon\right)}\right)}$$

$$\left[\Omega_I(s) = \exp\left(\frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_I(s')}{s'(s'-s-i\varepsilon)}\right)\right]$$

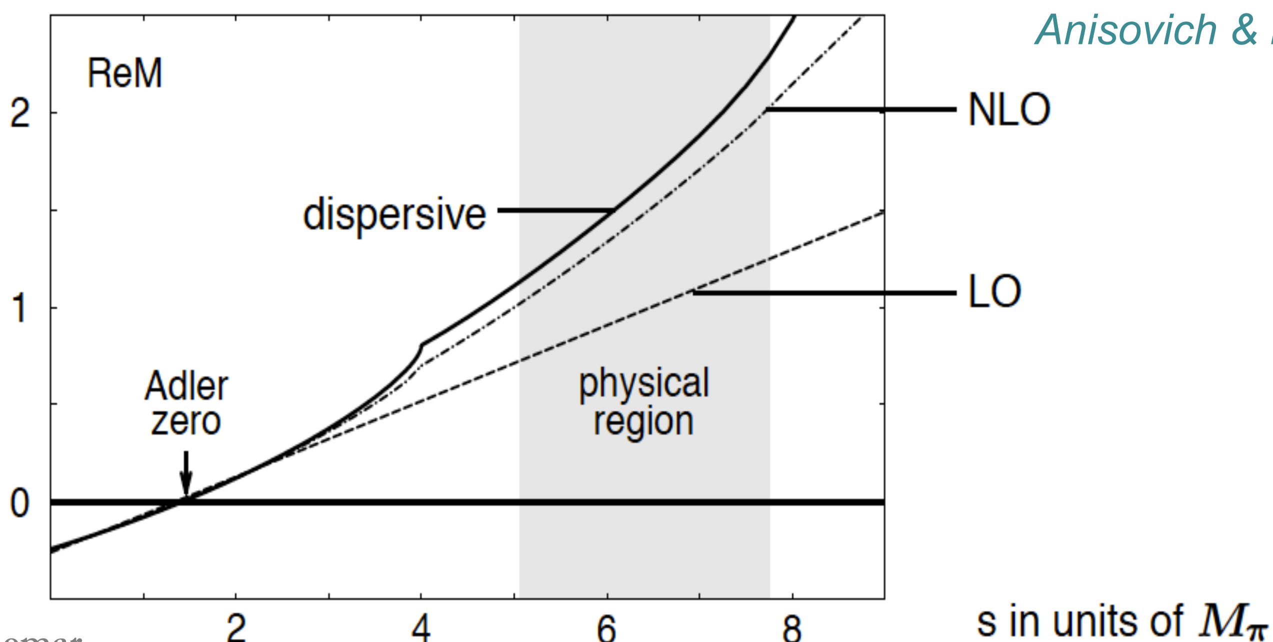
Omnès function

- $\hat{M}_I(s)$: singularities in the t and u channels, depend on the other $M_I(s)$
  subtract $M_I(s)$ from the partial wave projection of $M(s,t,u)$
  ⇨ Angular averages of the other functions ⇨ Coupled equations

3.4 Combining ChPT and dispersion relations

- As for $\pi\pi$, combine dispersion relations with ChPT where it works the best
- Use representation holding up to and including NNLO
  $\pi\pi$ partial-wave discontinuities for ℓ = 0,1 only and I=0,1,2
- Interesting matching point: Adler zero
  The real part of the amplitude along the line s=u has a zero
  Chiral SU(2) prediction ⇨ small higher order corrections

Anisovich & Leutwyler'96



3.5 Different recent analyses

1. *Schneider, Kubis, Ditsche 2011*: 2-loop NREFT approach
   Allows investigation of isospin-violating corrections

2. *Kampf, Knecht, Novotny, Zdrahal 2011*: Analytic dispersive approach
   Match to absorptive part at NNLO along t=u ⇨ R (Q)
   Problem: do not reproduce the Adler's zero

3. *Guo et al. 2015: JPAC* analysis, Khuri-Treiman equations
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Match to NLO ChPT near Adler zero

4. *Albaladejo, Moussallam 2015:* solve Khuri-Treiman equations
   - Matching at NLO
   - Include coupled channel KK and ηπ

   ⇨ See talk by *B. Moussallam*

5. *Colangelo, Lanz, Leutwyler, E.P. in progress:* dispersive approach following *Anisovich, Leutwyler*
   (*Ditsche, Kubis, Meißner'09*)
   - Electromagnetic effects to NLO fully taken into account
   - Matching to one loop ChPT : Taylor expand the partial wave around s=0

3.6 Dalitz plot parameters: Charged channel

- Dalitz plot measurement of $\eta \to \pi^+ \pi^- \pi^0$

  Amplitude expanded in X and Y around X=Y=0

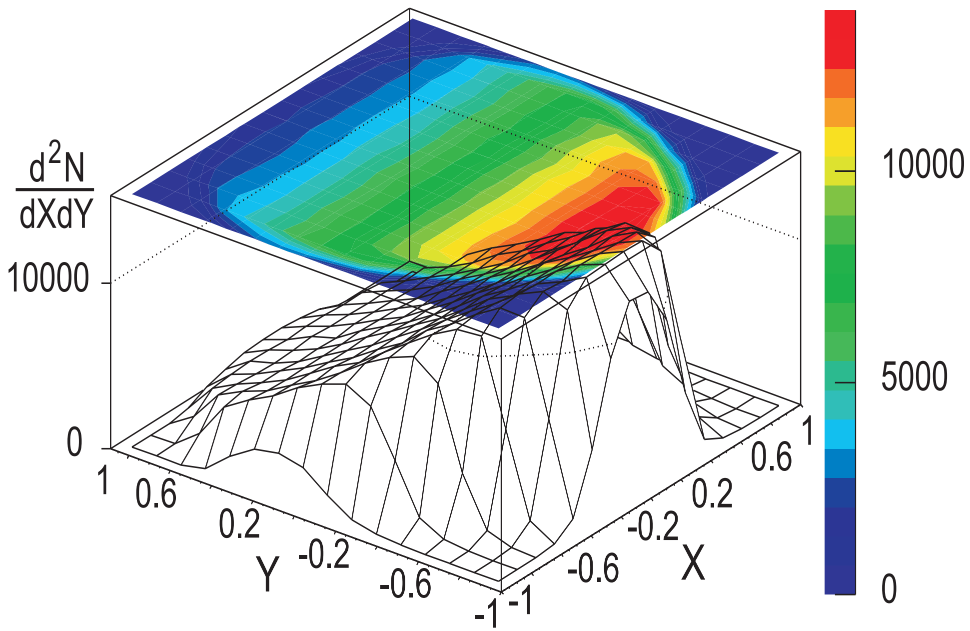
$$Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$\left|A_c(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3\right)$$

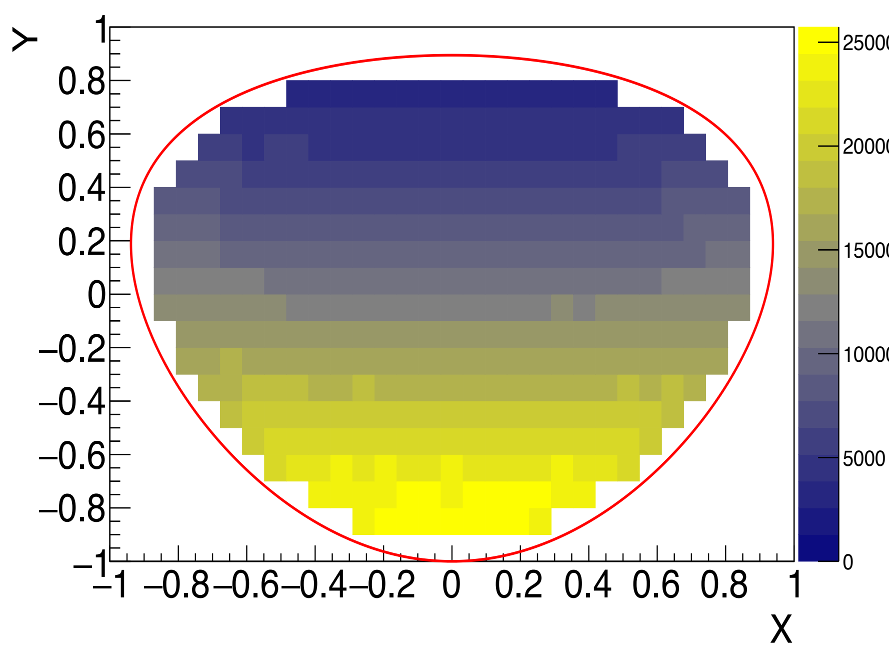
$$X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

KLOE'08



KLOE'16



3.6 Dalitz plot parameters: Charged channel

- Dalitz plot measurement of $\eta \to \pi^+ \pi^- \pi^0$

  Amplitude expanded in X and Y around X=Y=0

$$Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1$$

$$\left|A_c(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3\right)$$

$$X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

	$-a$	b	d	f
KLOE 2008	$1.090\left(^{+20}_{-9}\right)$	0.124(11)	$0.057\left(^{+9}_{-17}\right)$	0.140(20)
WASA/COSY 2014	1.144(18)	0.219(51)	0.086(23)	0.115(37)
BESIII 2015	1.128(17)	0.153(17)	0.085(18)	0.173(35)
KLOE 2016	1.095(4)	0.145(6)	0.081(7)	0.141(10)
NNLO CHPT	1.271(75)	0.394(102)	0.055(57)	0.025(160)
NREFT	1.213(14)	0.308(23)	0.050(3)	0.083(19)
JPAC	1.117(35)	0.188(14)	0.079(3)	0.090(3)
KT-elastic (prel.)	1.154	0.202	0.088	0.107
KT-coupled (prel.)	1.146	0.181	0.090	0.116

Future measurements: *CLAS*, *GlueX*, *JEF*

3.7 Comparison of results for α: neutral decay

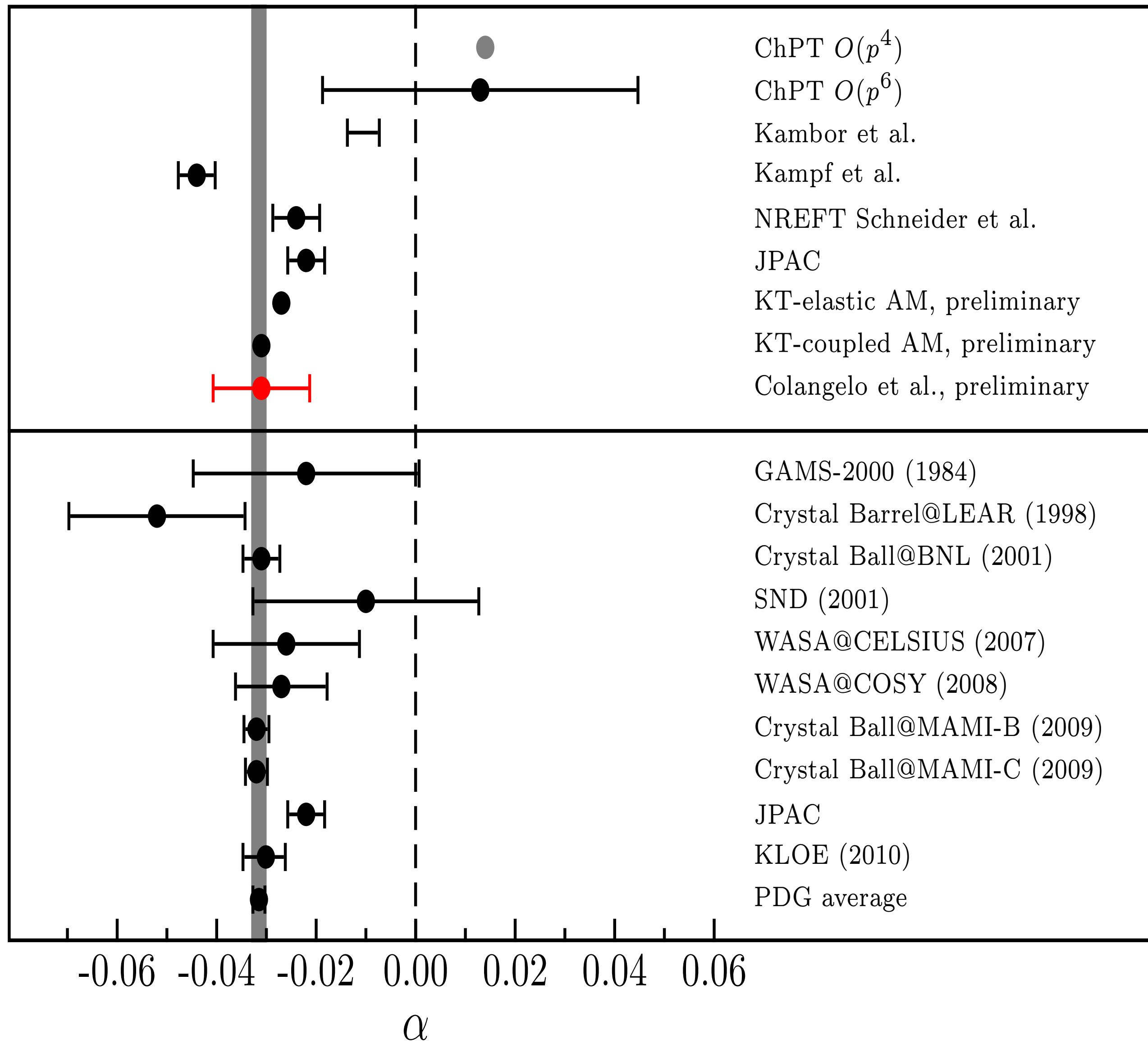
- Dalitz plot measurement of $\eta \to 3\pi^0$

$$\boxed{\left|A_n(s,t,u)\right|^2 = N(1+2\alpha Z)}$$

$$\boxed{Z = X^2 + Y^2}$$

$$Z = \frac{2}{3}\sum_{i=1}^{3}\left(\frac{3T_i}{Q_n} - 1\right)^2$$

$$Q_n \equiv M_\eta - 3M_{\pi^0}$$



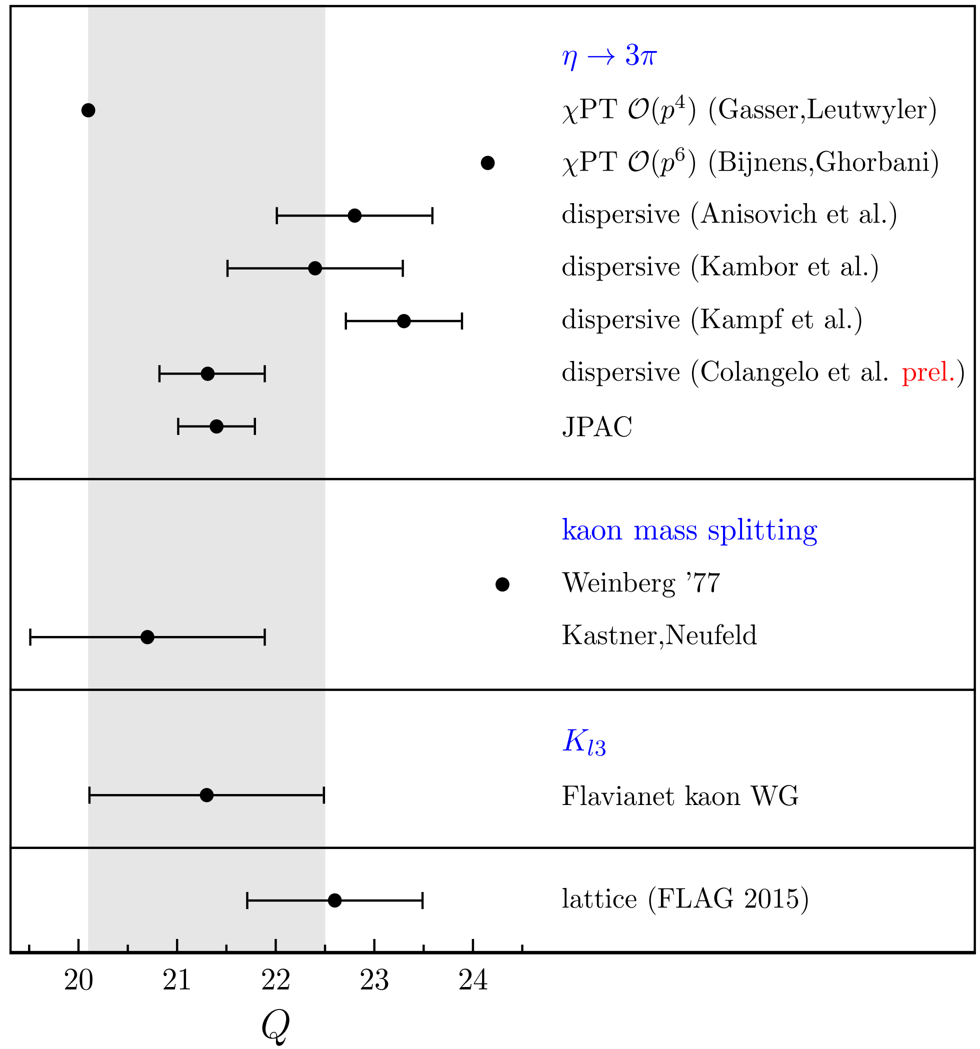
3.8 Quark mass ratio

$$\boxed{\Gamma_{\eta\to 3\pi} \propto \int \left|A(s,t,u)\right|^2 \propto Q^{-4}}$$

$$\boxed{Q^2 \equiv \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}} \qquad \left[\hat{m} \equiv \frac{m_d + m_u}{2}\right]$$

$$\boxed{A(s,t,u) = \frac{N}{Q^2} M(s,t,u)}$$

- M(s,t,u) determined through the dispersive analysis of the data but for N one has to rely on ChPT

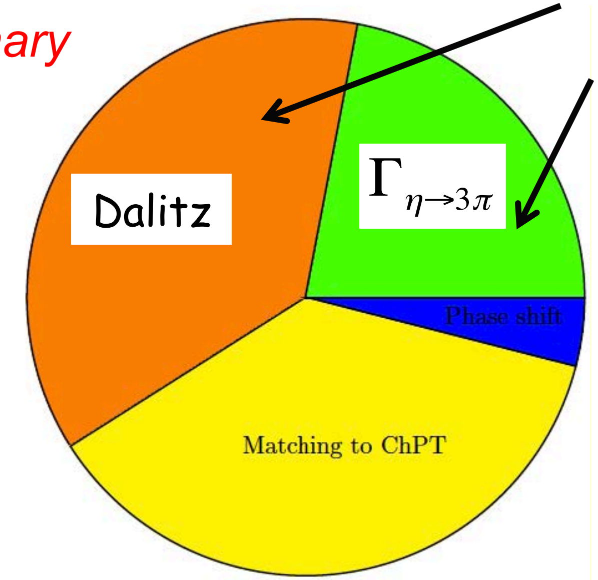


3.8 Quark mass ratio

Colangelo, Lanz, Leutwyler, E.P., in preparation

- Uncertainties in the quark mass ratio (rough attempt) based on fit to KLOE’08

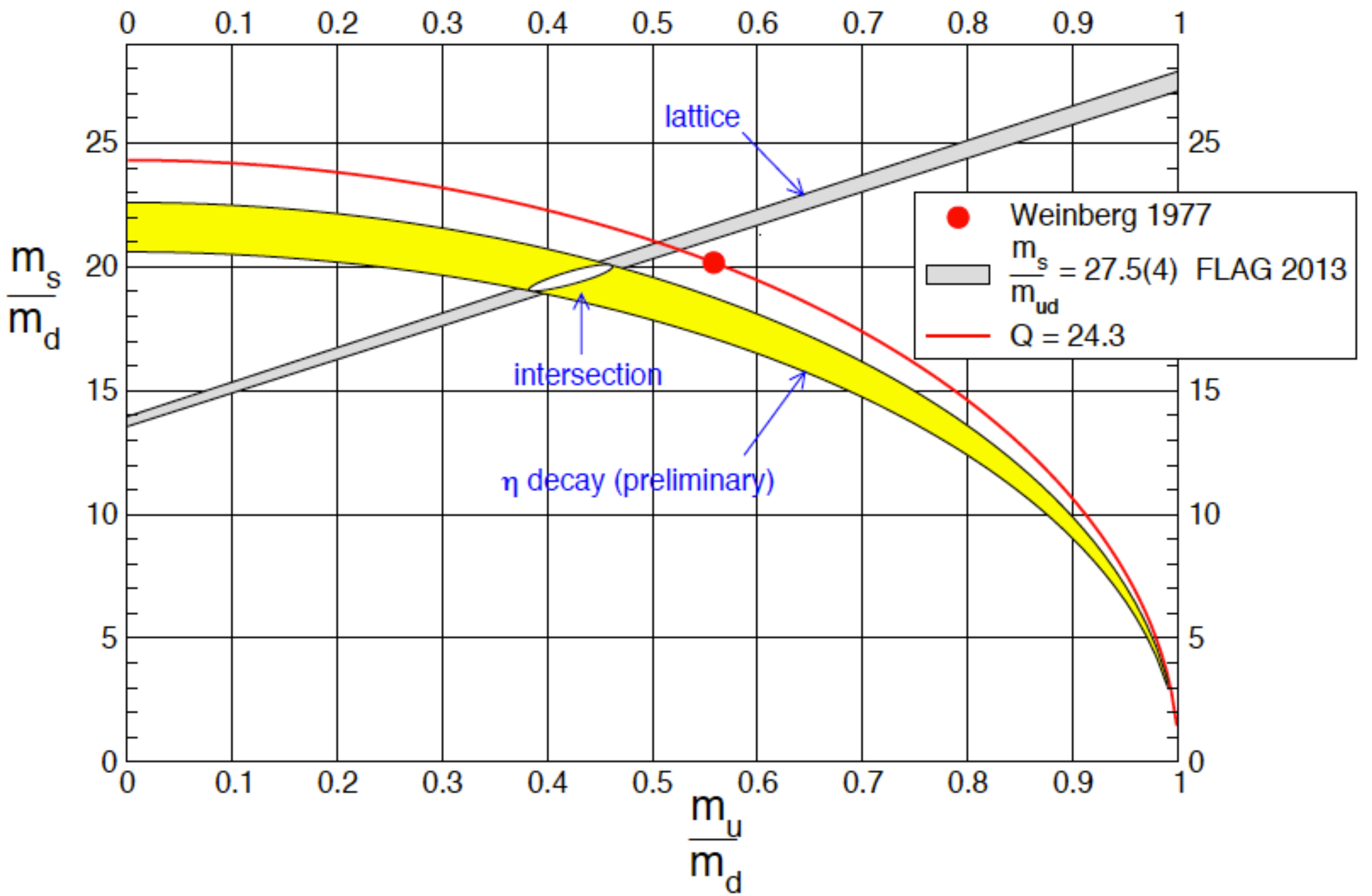
Preliminary



Can be investigated and reduced by future experiments, e.g. *JEF*

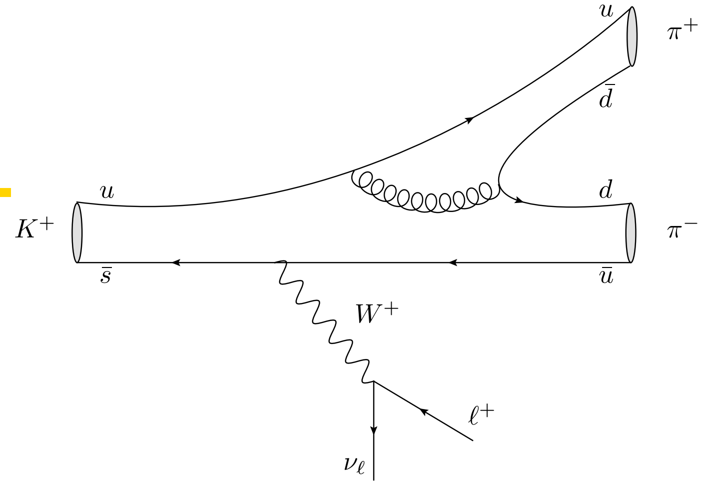
3.9 $\eta \to 3\pi$ and light quark masses

H. Leutwyler

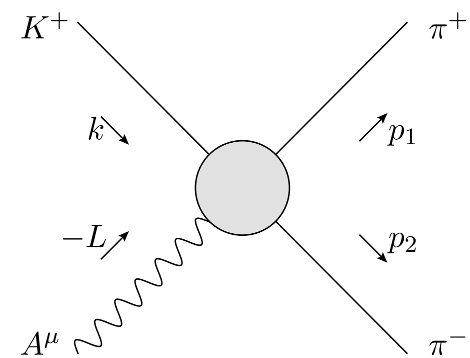


4. K_{l4} decays and determination of some LECs

4.1 K_{l4} decays



- Main interest:
  - access to ππ threshold region ⇨ ππ scattering lengths
  - form factors, LECs, . . .



- Standard problem of the NNLO treatment
  ⇨ strong final state rescattering

  Amoros, Bijnens, Talavera'00

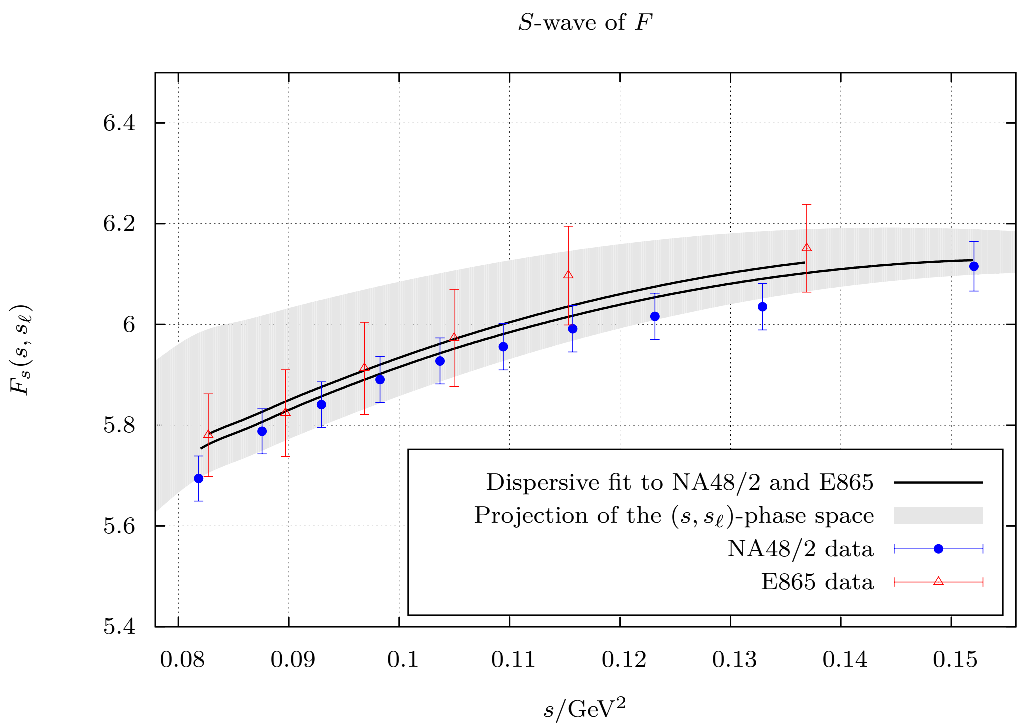
- Use dispersion relations:
  ⇨ matching to CHPT at both one- and two-loop levels: LECs

- Isospin breaking and radiative corrections have been computed

  Stoffer'13,
  Descotes-Genon, Bernard, Knecht'13, '14

4.2 Determination of some LECs

Colangelo, E.P., Stoffer'15



Contrary to ChPT, the dispersive measurement allows to take into account for the curvature in the form factors

	NLO	NNLO	Bijnens, Ecker (2014)
$10^3 \cdot L_1^r$	$0.51(2)(6)$	$0.69(16)(8)$	$0.53(6)$
$10^3 \cdot L_2^r$	$0.89(5)(7)$	$0.63(9)(10)$	$0.81(4)$
$10^3 \cdot L_3^r$	$-2.82(10)(7)$	$-2.63(39)(24)$	$-3.07(20)$
χ^2/dof	$141/116 = 1.2$	$124/122 = 1.0$	

5. Conclusion and outlook

5. Conclusion and Outlook

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme

  ⇨ precise predictions in the meson sector

- But when one wants to go to higher energy or more precise prediction

  ⇨ nicely complement by dispersion relation: analyticity, unitarity, crossing

  Ex: ππ scattering, $\eta \to 3\pi$, K_{l4} decays
  - $\eta \to 3\pi$: extraction of Q, fundamental parameter of the SM
  - K_{l4} : improve knowledge of meson dynamics at low energy

    ⇨ LECs

- Challenge: higher energies for B, D decays etc
  - Coupled channels : see talk by *B. Moussallam* for an example in $\eta \to 3\pi$
  - Finite energy sum-rules, e.g., see talk by *V. Pauk*

5. Back-up

2.1 Low energy constants

- Recent fit by Ecker and Bijnens of NLO LECs Li (i = 1,…,10) and NNLO LECs Ci (i = 1,…,90)
  Update and extension of Bijnens, Jemos 2012

- New ingredients:
  - relations li (Li ; Ci) (j = 1,…, 4) Gasser, Haefeli, Ivanov, Schmid 2007
    ⇨ altogether 17 input data
  - penalize bad convergence of meson masses
  - intelligent guesses (priors) for 34 (combinations of the) Ci
  - renormalization scale μ = 0.77 GeV

- Fitting procedure:
  - minimization/random walk in restricted Ci –space
  - iterate after possible modication of Ci –space
  - normal $\chi 2$ fit for Li for (fixed) “best" values of the Ci
    ⇨ “best" values for Li

2.1 Low energy constants

	NNLO free fit	NNLO BE14	NLO 2014	GL 1985
$10^3 L_A^r$	0.68(11)	0.24(11)	0.4(2)	
$10^3 L_1^r$	0.64(06)	0.53(06)	1.0(1)	0.7(3)
$10^3 L_2^r$	0.59(04)	0.81(04)	1.6(2)	1.3(7)
$10^3 L_3^r$	−2.80(20)	−3.07(20)	−3.8(3)	−4.4(2.5)
$10^3 L_4^r$	0.76(18)	0.3	0.0(3)	−0.3(5)
$10^3 L_5^r$	0.50(07)	1.01(06)	1.2(1)	1.4(5)
$10^3 L_6^r$	0.49(25)	0.14(05)	0.0(4)	−0.2(3)
$10^3 L_7^r$	−0.19(08)	−0.34(09)	−0.3(2)	−0.4(2)
$10^3 L_8^r$	0.17(11)	0.47(10)	0.5(2)	0.9(3)
F_0 [MeV]	64	71		

- Strong sensitivity to (large-Nc) suppressed L_4
  ⇨ enforce small L4 (supported by lattice), $10^3 L^r_4 = 0.3$;
  NLO: $0.3 \leq 10^3 L^r_4 \leq 0.3$ fixed → $L_A = 2 L_1 - L_2$ and L_6 automatically suppressed

- NNLO only makes sense with certain set of Cr

- Except for last column: no estimate of higher-order uncertainties

2.1 Low energy constants

	NNLO free fit	NNLO BE14	NLO 2014	GL 1985
$10^3 L_A^r$	0.68(11)	0.24(11)	0.4(2)	
$10^3 L_1^r$	0.64(06)	0.53(06)	1.0(1)	0.7(3)
$10^3 L_2^r$	0.59(04)	0.81(04)	1.6(2)	1.3(7)
$10^3 L_3^r$	−2.80(20)	−3.07(20)	−3.8(3)	−4.4(2.5)
$10^3 L_4^r$	0.76(18)	0.3	0.0(3)	−0.3(5)
$10^3 L_5^r$	0.50(07)	1.01(06)	1.2(1)	1.4(5)
$10^3 L_6^r$	0.49(25)	0.14(05)	0.0(4)	−0.2(3)
$10^3 L_7^r$	−0.19(08)	−0.34(09)	−0.3(2)	−0.4(2)
$10^3 L_8^r$	0.17(11)	0.47(10)	0.5(2)	0.9(3)
F_0 [MeV]	64	71		

- Reasonable convergence of observables (enforced for masses)
- Qualitative evidence for resonance saturation, even for scalars
- Last 3 columns: good stability

3.3 Different recent analyses

1. *Schneider, Kubis, Ditsche 2011*: 2-loop NREFT approach
   - allows investigation of isospin-violating corrections
   - relations between charged and neutral Dalitz plots

2. *Kampf, Knecht, Novotny, Zdrahal 2011*: Analytic dispersive approach
   - Amplitudes involve 6 parameters (subtraction constants)
   - Fit to Dalitz plot distribution (KLOE 2008: $\eta \rightarrow \pi^+\pi^-\pi^0$)
   - Predict Dalitz plot parameter α (neutral decay mode)
   - Match to absorptive part of NNLO chiral amplitude where differences between NLO and NNLO are small $\Rightarrow$ R (Q)

   Problem: do not reproduce the Adler's zero

3.3 Different recent analyses

3. *Guo et al. 2015: JPAC* analysis, Khuri Treiman equations solved numerically using Pasquier inversion techniques
   - Madrid/Cracow ππ phase shifts, 3 subtraction constants
   - Fit experimental Dalitz plot (*WASA/COSY* 2014: $\eta \to \pi^+\pi^-\pi^0$)
     ⇨ predict Dalitz plot parameter α
   - Match to NLO ChPT near Adler zero ⇨ Q

4. *Colangelo, Lanz, Leutwyler, E.P. in progress:* dispersive approach following *Anisovich, Leutwyler*
   - Electromagnetic effects to NLO fully taken into account (*Ditsche, Kubis, Meißner'09*)
   - Dispersive amplitudes: Bern ππ phase shifts, 6 subtraction constants
   - Fit similtanously Charged (*WASA*, *KLOE*) and Neutral Dalitz plots (*MAMI*)
   - Matching to one loop ChPT : Taylor expand the partial wave around s=0

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3.4 Dalitz plot parameters

- Dalitz plot measurement : Amplitude expanded in X and Y around X=Y=0

$$\boxed{\left|A_c(s,t,u)\right|^2 = N\left(1 + aY + bY^2 + dX^2 + fY^3\right)}$$

$$\boxed{X = \frac{\sqrt{3}}{2M_\eta Q_c}(u-t)}$$

$$\boxed{Y = \frac{3}{2M_\eta Q_c}\left(\left(M_\eta - M_{\pi^0}\right)^2 - s\right) - 1}$$

$$\boxed{\left|A_n(s,t,u)\right|^2 = N\left(1 + 2\alpha Z + 6\beta Y\left(X^2 - \frac{Y^2}{3}\right) + 2\gamma Z^2\right)}$$

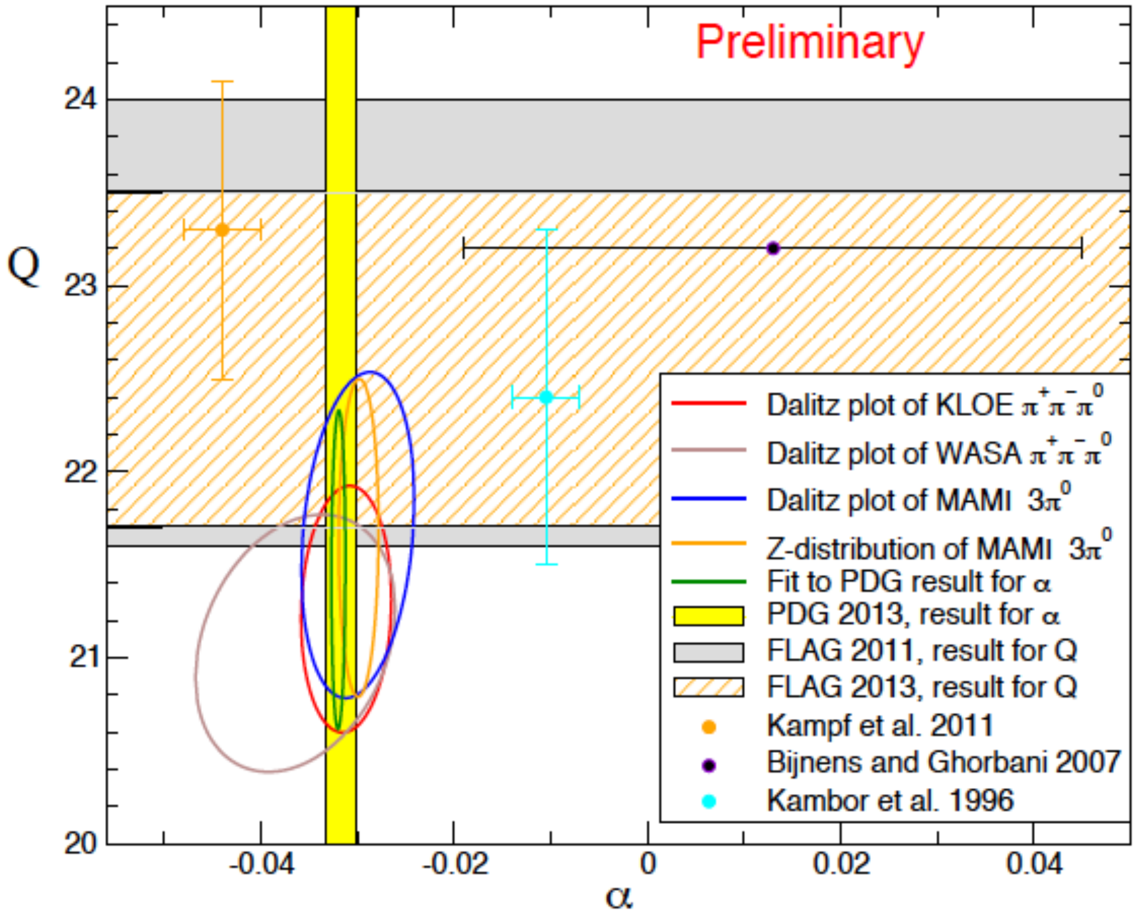
$$\boxed{Z = X^2 + Y^2}$$

$$Q_c \equiv M_\eta - 2M_{\pi^+} - M_{\pi^0}$$

	$-a$	b	d	α
KLOE 2015	1.095(4)	0.145(6)	0.081(7)	
BESIII 2015	1.128(17)	0.153(17)	0.085(18)	−0.055(15)
WASA/COSY 2014	1.144(18)	0.219(51)	0.086(23)	
NNLO CHPT	1.271(75)	0.394(102)	0.055(57)	0.013(32)
KKNZ				−0.044(4)
NREFT	1.213(14)	0.308(23)	0.050(3)	−0.025(5)
JPAC	1.116(32)	0.188(12)	0.063(4)	−0.022(4)
PDG 2014				−0.0315(15)

3.3 Qualitative results of our analysis

- Plot of Q versus α :



NB: Isospin breaking has not been accounted for

From kaon mass spliting :

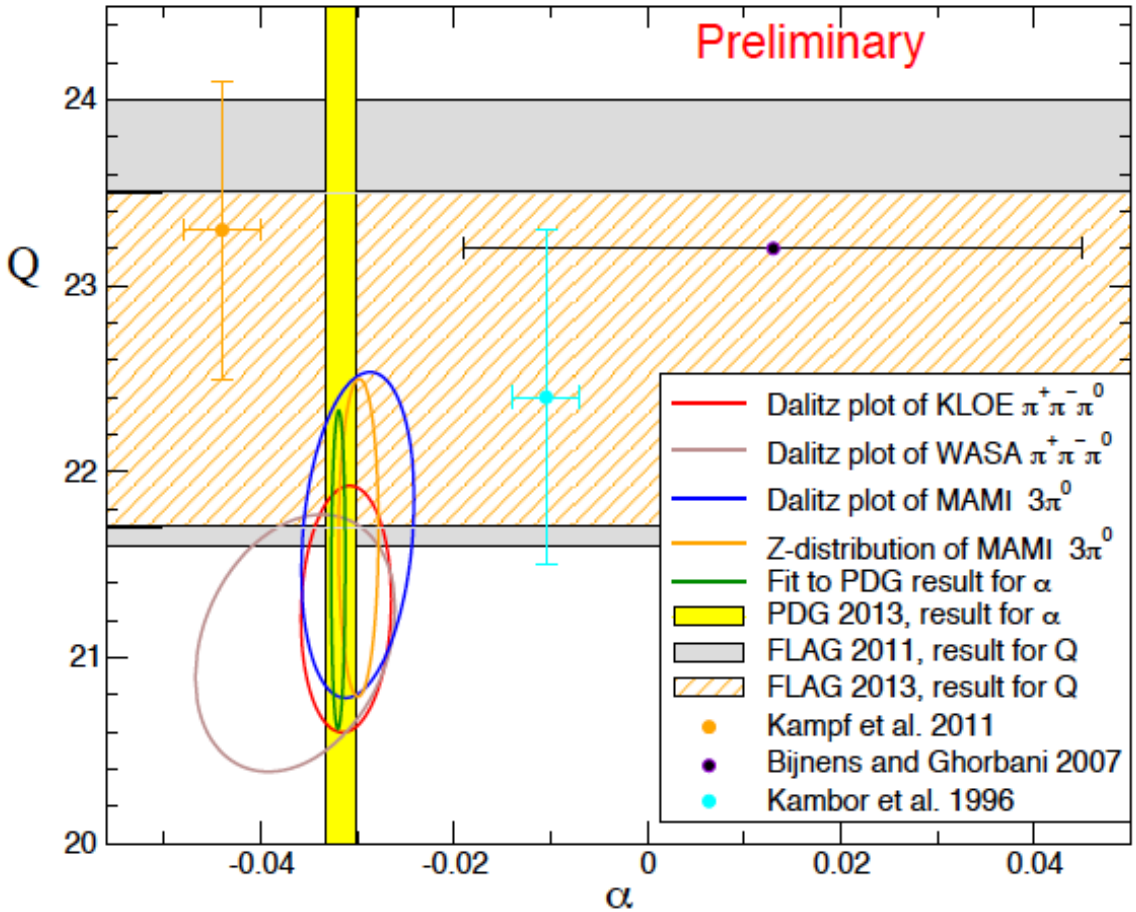
$$\boxed{Q = 20.7 \pm 1.2}$$

Kastner & Neufeld'08

- All the data give consistent results. The preliminary outcome for Q is intermediate between the lattice result and the one of Kastner and Neufeld.

3.3 Qualitative results of our analysis

- Plot of Q versus α :



NB: Isospin breaking has not been accounted for

- All our preliminary results give a negative value for α. In particular the result using KLOE data for $\eta \rightarrow \pi^+ \pi^- \pi^0$ is in perfect agreement with the PDG value!

2.1 ππ scattering lengths

- ππ scattering computed early on, one of the first applications of SU(2) x SU(2) ⇨ converge better
  ⇨ the scattering lengths

Gasser & Leutwyler'83

- At NLO:

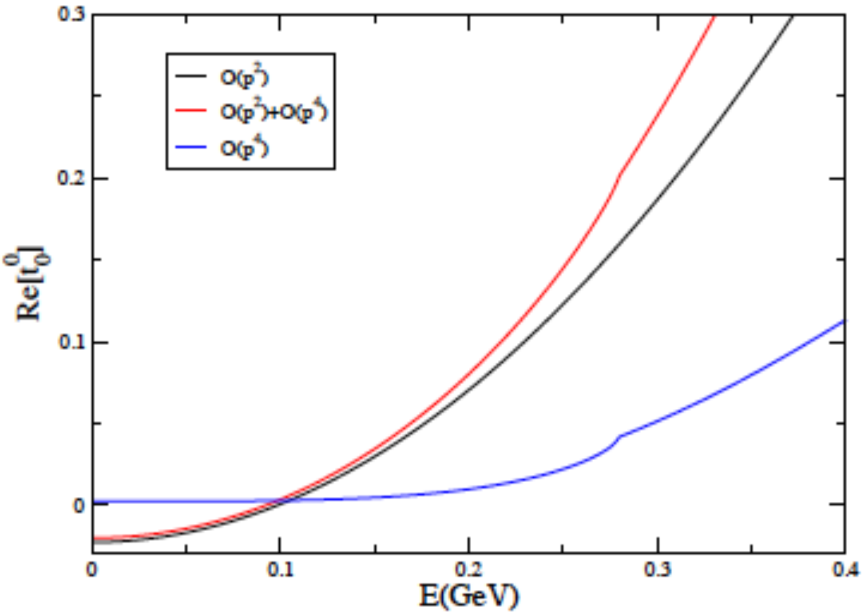
$$
\begin{aligned}
a_0^0 &= \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1+\frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi+\frac{200\pi F_\pi^2 M_\pi^2}{7}(a_2^0+2a_2^2)\right.\\
&\quad \left.-\frac{M_\pi^2}{672\pi^2F_\pi^2}(15\bar{\ell}_3-353)\right]\\
&= 0.16\cdot 1.25 = 0.20
\end{aligned}
$$

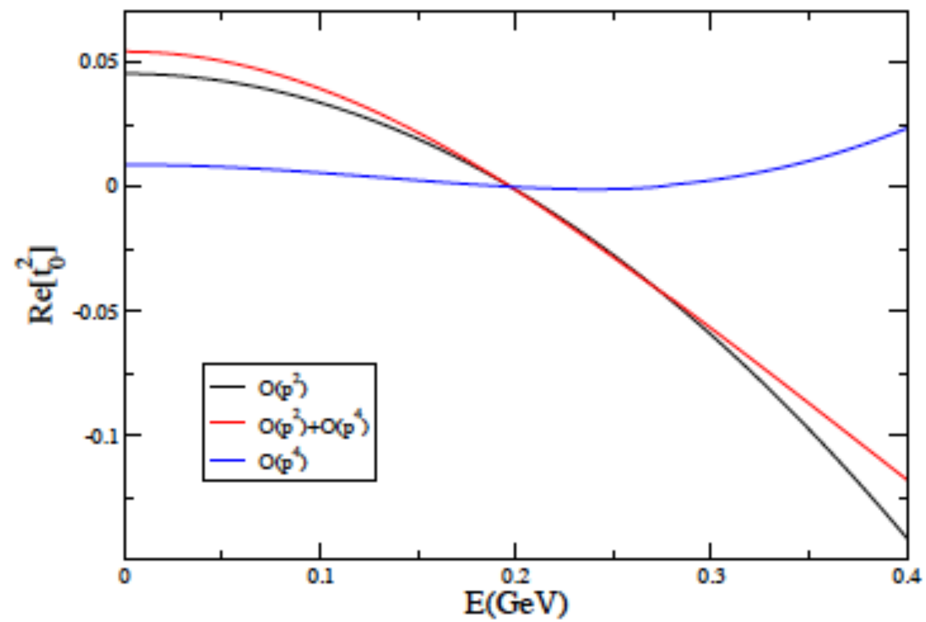
$$
2a_0^0-5a_0^2 = \frac{3M_\pi^2}{4\pi F_\pi^2}\left[1+\frac{M_\pi^2}{3}\langle r^2\rangle_S^\pi+\frac{41M_\pi^2}{192\pi^2F_\pi^2}\right] = 0.547\cdot 1.14 = 0.624
$$

- Higher order corrections are suppressed by $\mathcal{O}(m/\Lambda)$, $\Lambda = \mathcal{O}(1\text{GeV})$
  ⇨ expected to be a few percent

2.1 ππ scattering lengths

G. Colangelo





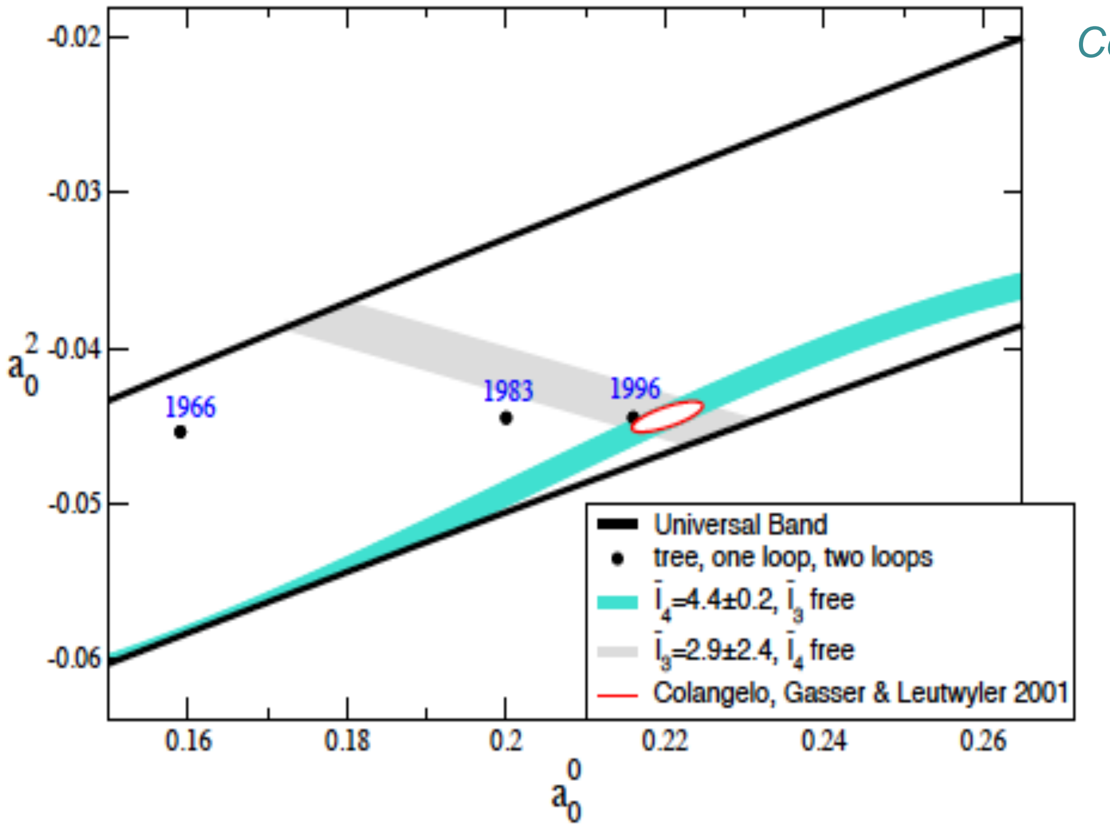
- This momentum dependence is reflected in a chiral log in a^I_0

$$a_0^0 = \frac{7M_\pi^2}{32\pi F_\pi^2}\left[1 + \frac{9}{2}\ell_\chi + \ldots\right] \qquad a_0^2 = -\frac{M_\pi^2}{16\pi F_\pi^2}\left[1 - \frac{3}{2}\ell_\chi + \ldots\right]$$

$$\ell_\chi = \frac{M_\pi^2}{16\pi^2 F_\pi^2}\ln\frac{\mu^2}{M_\pi^2}$$

How large are yet higher orders?
Is it at all possible to make a precise prediction?

2.4 Chiral Predictions for a_0^0 and a_0^2

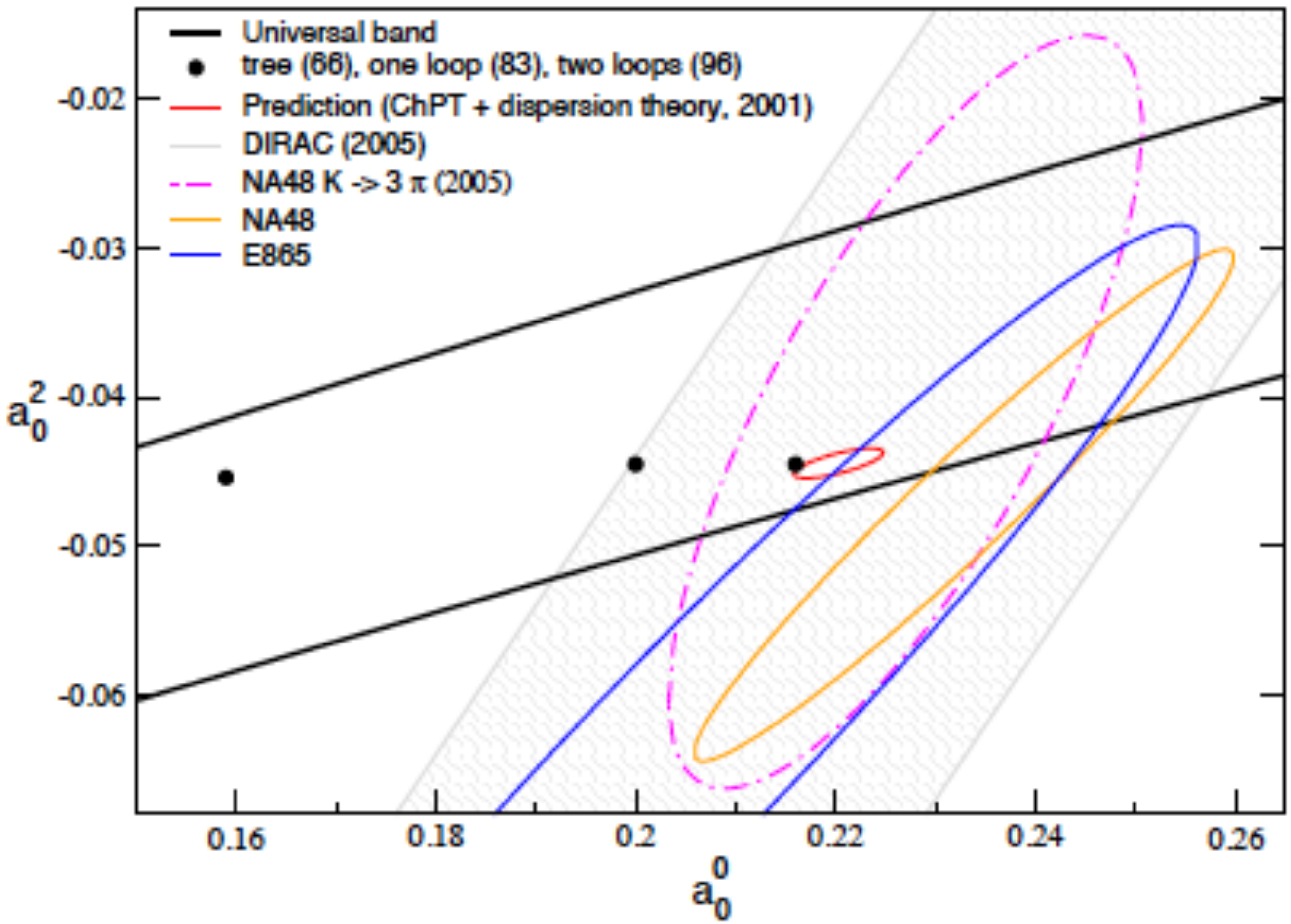


Colangelo, Gasser & Leutwyler'01

Where can we test these predictions?

- Production experiments πN → ππN, ψ → ππω, B → Dππ, . . .
- Extraction of ππ scattering amplitude is not simple
- Best accuracy in K_{l4} data, K → 3π, ππ atoms

2.5 Experimental tests



H. Leutwyler

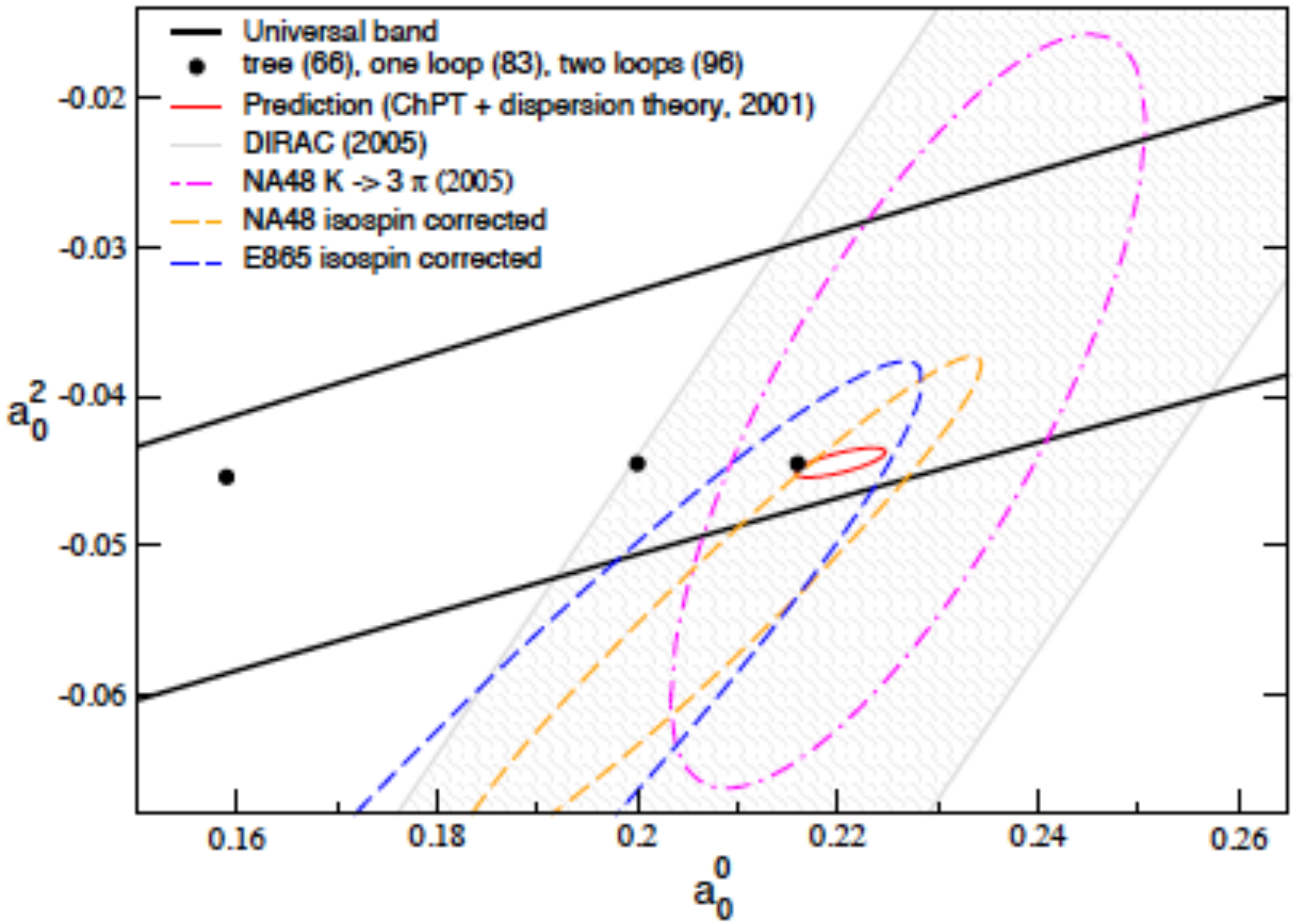
- With the new much more precise NA48 data it seemed that there was a disagreement ⇨ isospin breaking corrections

2.6 On the importance of isospin breaking corrections

- Isospin breaking computed recently
  ⇨ Perfect agreement!

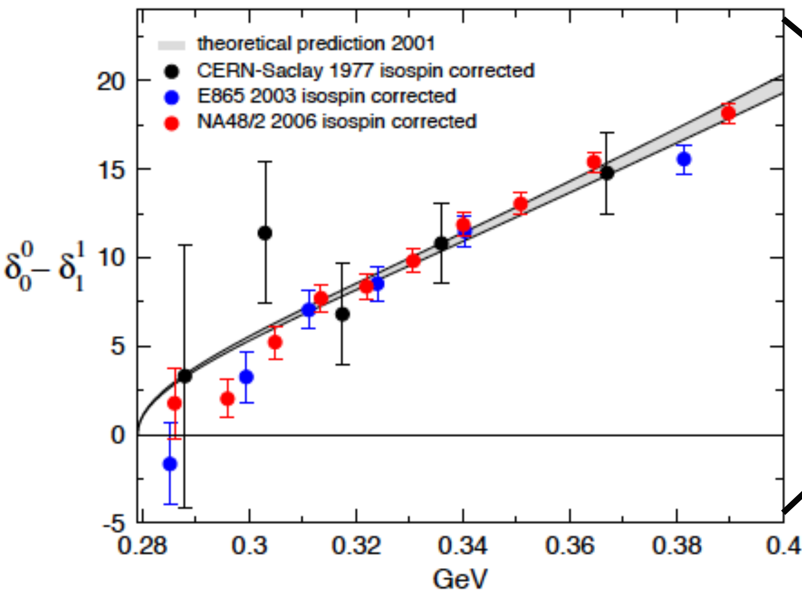
Colangelo, Gasser, Rusetsky'09
Bernard, Descotes-Genon, Knecht '13

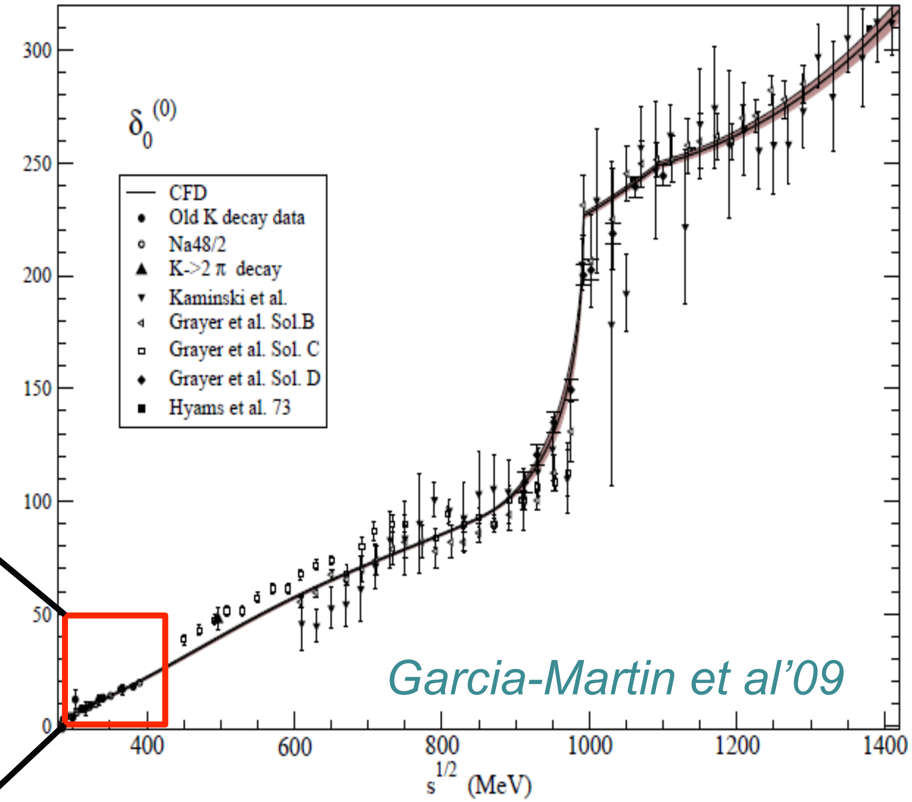
H. Leutwyler



2.7 ππ as a building block

H. Leutwyler



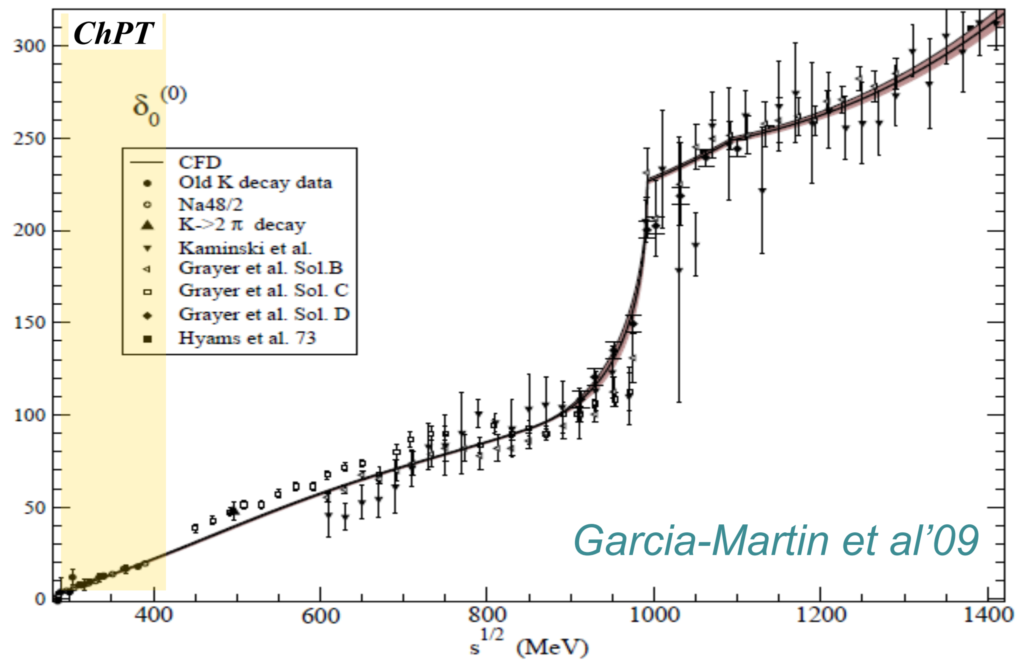


Garcia-Martin et al'09

5.1 Conclusion

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme

⇨ precise predictions in the meson sector



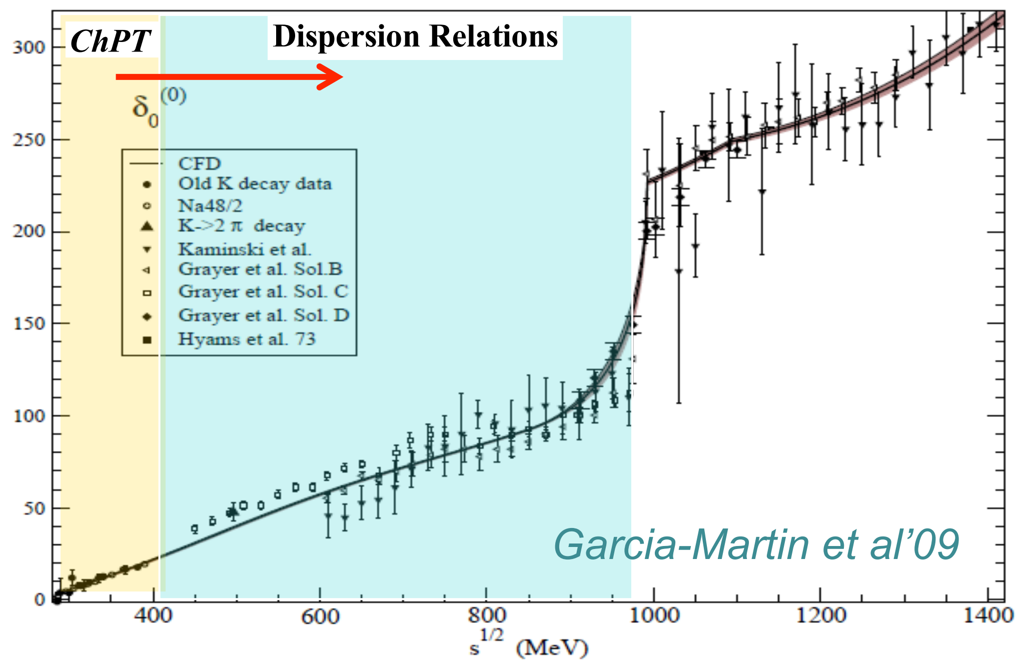
5.1 Conclusion

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme

  ⇨ precise predictions in the meson sector

- But when one wants to go to higher energy or more precise prediction

  ⇨ nicely complement by dispersion relation: analyticity, unitarity, crossing Ex: ππ scattering, η → 3π



5.2 Outlook: Challenges for the future

- ChPT is a very interesting tool at low energy
  - Model independent
  - Build amplitude using a power counting scheme

  ⇨ precise predictions in the meson sector

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