A Progress on Formulating Bethe-Salpeter Kernels

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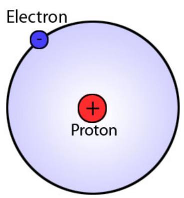
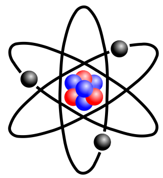
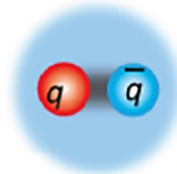
Fundamental Forces versus **Bound States**

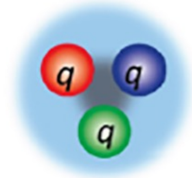
QED





QCD





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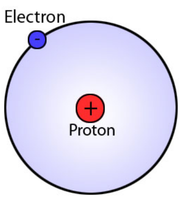
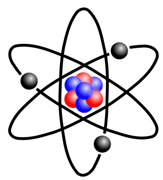
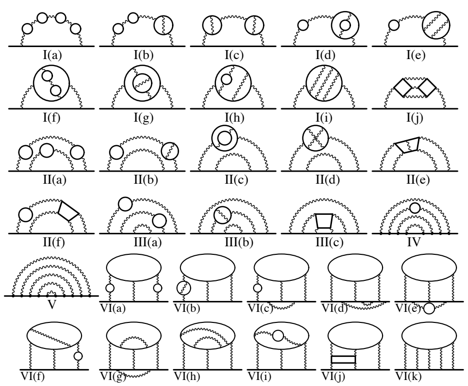




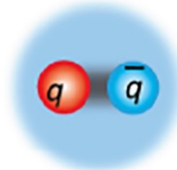
Perturbative

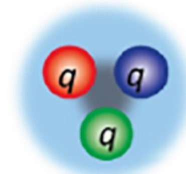


$$\alpha^{-1} = 137.035\ 999\ 174\ (35)$$

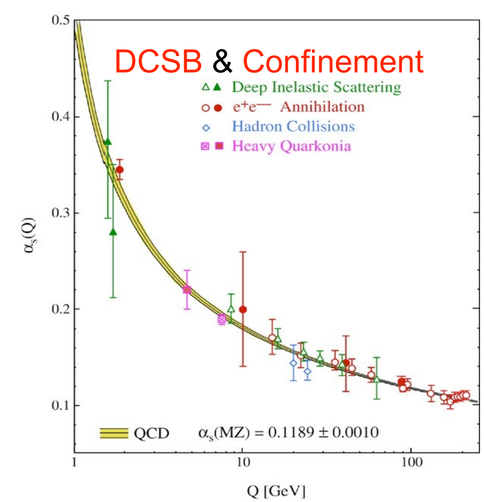
QED fine-structure constant

QCD





Non-perturbative



QCD running coupling constant

Dyson-Schwinger Equations: Equation of motion of Green functions

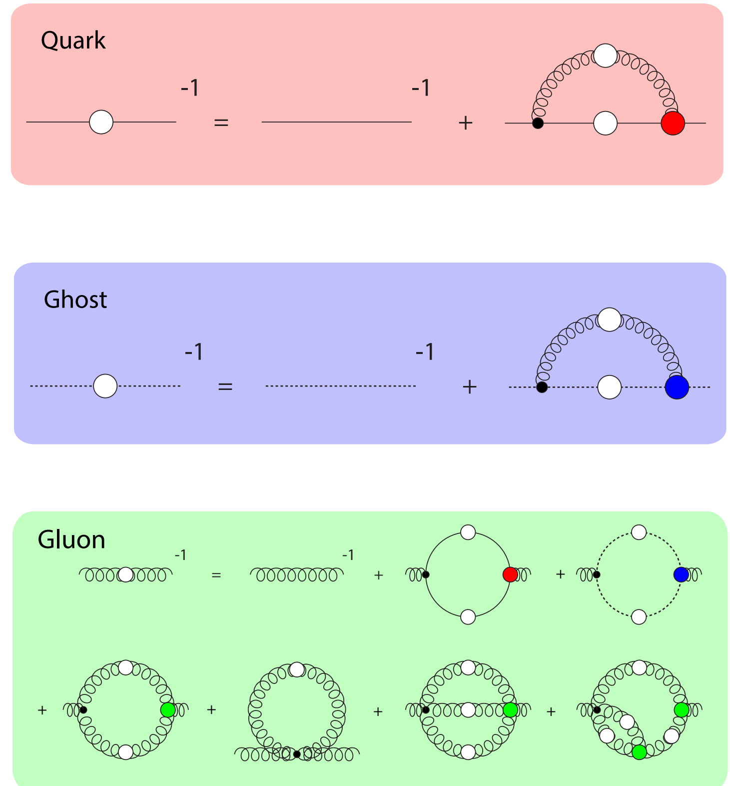
Classical Mechanics | Quantum Field Theory

Principle of Least Action

$$\frac{\delta S[q]}{\delta q} = 0 \qquad \left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

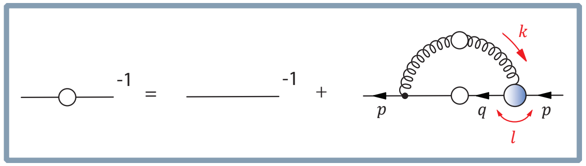
Equations of Motion (EoM)

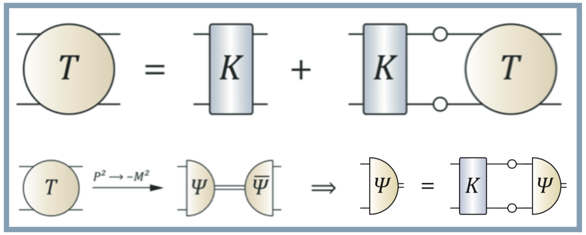
Euler-Lagrange Equation | Dyson-Schwinger Equation



✦ Complicated integral equations;
✦ Coupled tower of all equations.

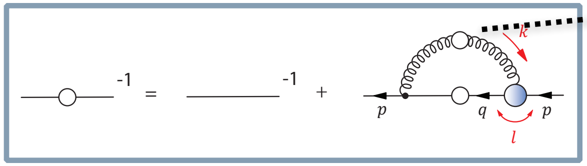
Dyson-Schwinger Equations: Equations for mesons

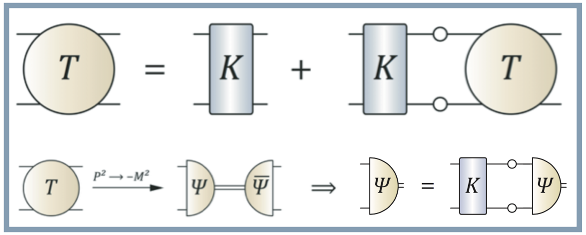




Dyson-Schwinger Equations: Equations for mesons

Gluon propagator





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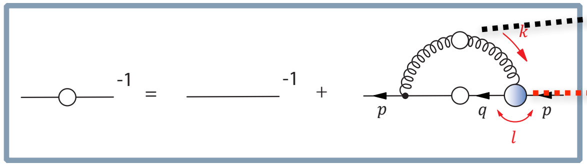
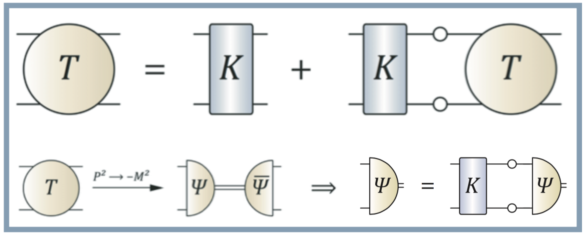
Gluon propagator

Quark-gluon vertex





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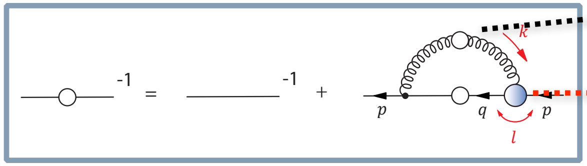

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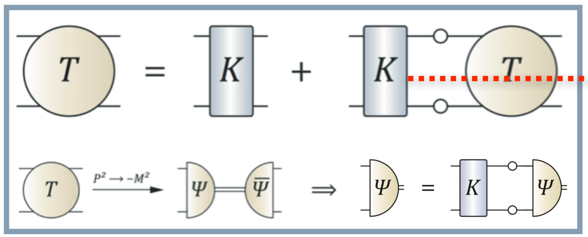
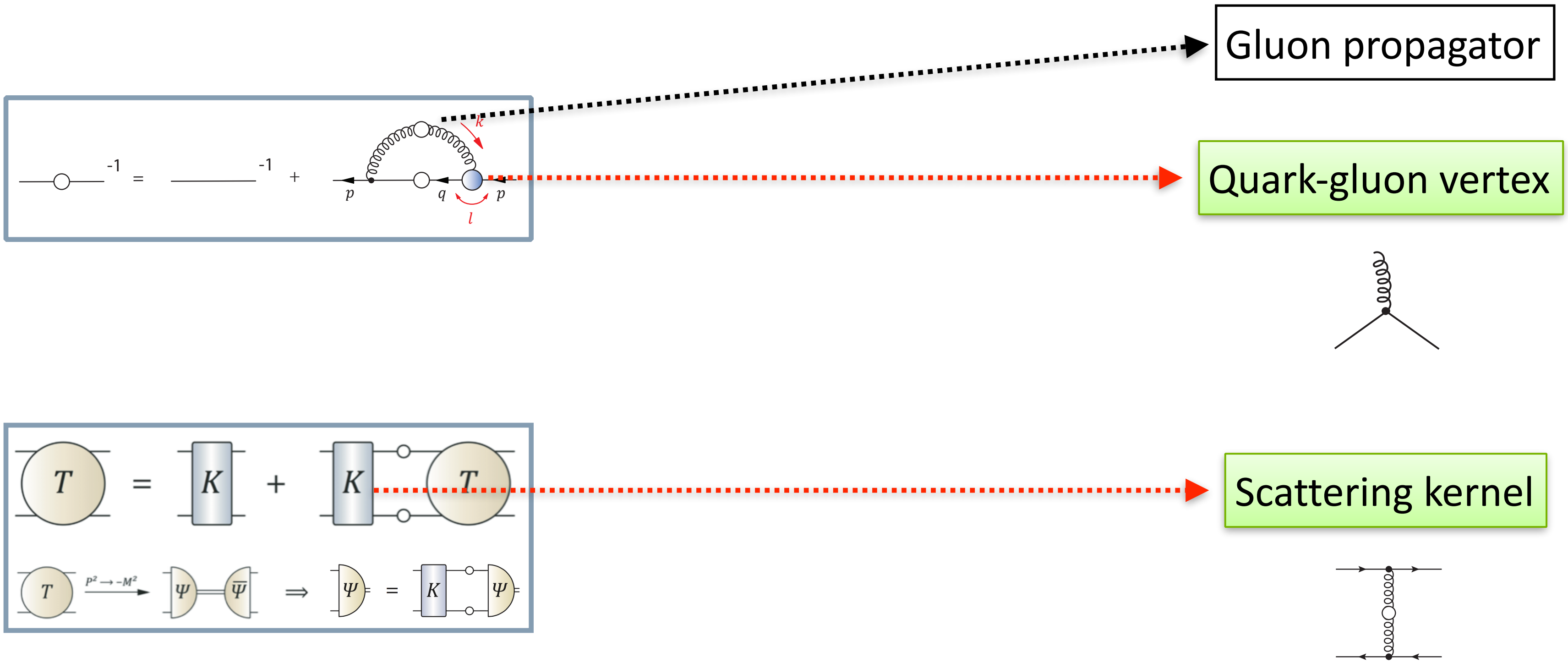


Scattering kernel

Dyson-Schwinger Equations: Equations for mesons



The simplest rainbow-ladder truncation:

Rainbow-Ladder truncation: Successes

✦ **T & mu = 0** — global properties of hadrons: mass spectra, decay constants, radii …; hadron structures: FF, PDF, PDA, GPD…

Summary of light meson results

$m_{u=d} = 5.5$ MeV, $m_s = 125$ MeV at $\mu = 1$ GeV

Pseudoscalar (PM, Roberts, PRC56, 3369)

	expt.	calc.
$-\langle \bar{q}q \rangle^0_\mu$	(0.236 GeV)^3	(0.241†)^3
m_π	0.1385 GeV	0.138†
f_π	0.0924 GeV	0.093†
m_K	0.496 GeV	0.497†
f_K	0.113 GeV	0.109

Charge radii (PM, Tandy, PRC62, 055204)

r^2_π	0.44 fm^2	0.45
$r^2_{K^+}$	0.34 fm^2	0.38
$r^2_{K^0}$	-0.054 fm^2	-0.086

$\gamma\pi\gamma$ transition (PM, Tandy, PRC65, 045211)

$g_{\pi\gamma\gamma}$	0.50	0.50
$r^2_{\pi\gamma\gamma}$	0.42 fm^2	0.41

Weak K_{l3} decay (PM, Ji, PRD64, 014032)

$\lambda_+(e3)$	0.028	0.027
$\Gamma(K_{e3})$	7.6 $\cdot 10^6$ s^{-1}	7.38
$\Gamma(K_{\mu 3})$	5.2 $\cdot 10^6$ s^{-1}	4.90

Vector mesons (PM, Tandy, PRC60, 055214)

$m_{\rho/\omega}$	0.770 GeV	0.742
$f_{\rho/\omega}$	0.216 GeV	0.207
m_{K^*}	0.892 GeV	0.936
f_{K^*}	0.225 GeV	0.241
m_ϕ	1.020 GeV	1.072
f_ϕ	0.236 GeV	0.259

Strong decay (Jarecke, PM, Tandy, PRC67, 035202)

$g_{\rho\pi\pi}$	6.02	5.4
$g_{\phi KK}$	4.64	4.3
$g_{K^*K\pi}$	4.60	4.1

Radiative decay (PM, nucl-th/0112022)

$g_{\rho\pi\gamma}/m_\rho$	0.74	0.69
$g_{\omega\pi\gamma}/m_\omega$	2.31	2.07
$(g_{K^*K\gamma}/m_K)^+$	0.83	0.99
$(g_{K^*K\gamma}/m_K)^0$	1.28	1.19

Scattering length (PM, Cotanch, PRD66, 116010)

a^0_0	0.220	0.170
a^2_0	0.044	0.045
a^1_1	0.038	0.036

Tandy @ Beijing Lectures 2010

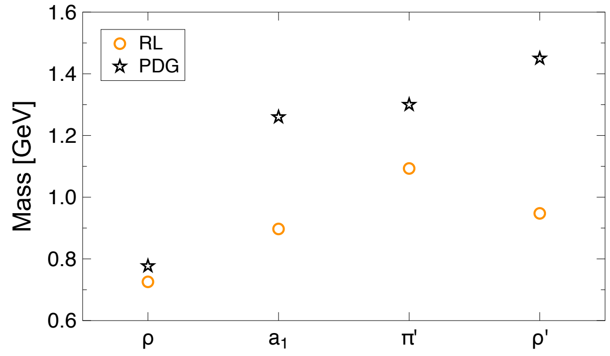
✦ **T & mu > 0** — phase diagram: critical lines, CEP…; properties of QGP: excitation modes, electrical conductivity, shear viscosity to entropy density ratio…

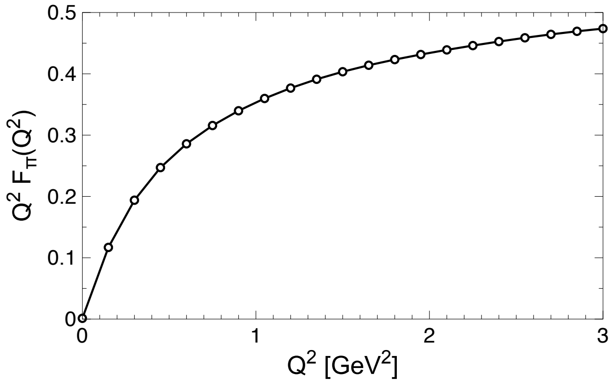
Rainbow-Ladder truncation: Drawbacks

- **Ground state > 1GeV**: too small rho-a1 mass splitting;

- **Radial excitation states**: wrong ordering and magnitudes;

- **Structures**: rho monopole form pion EM form factor;





RL truncation fails to describe quantities which are sensitive to details of interaction.

Is there a **systematic** way to truncate the DSEs in order to approach the full QCD?

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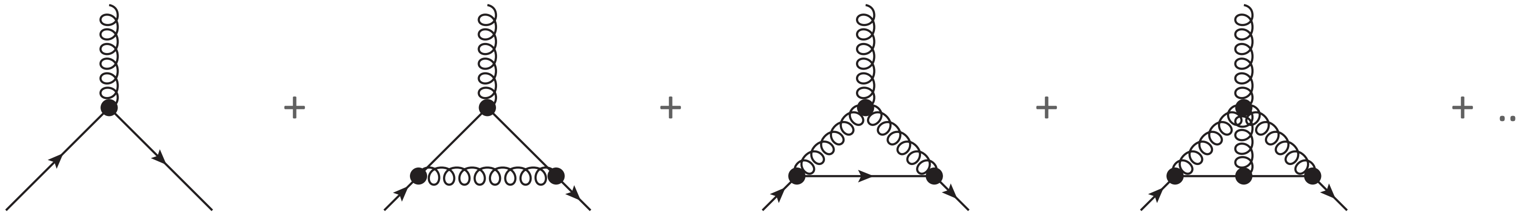
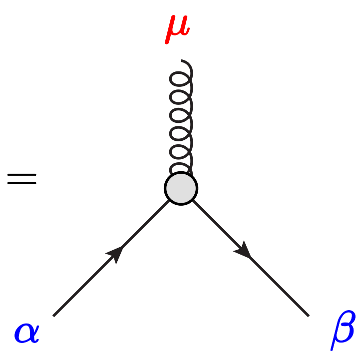
I. Quark-gluon vertex

II. Scattering kernel

I. Quark-gluon vertex: General structure

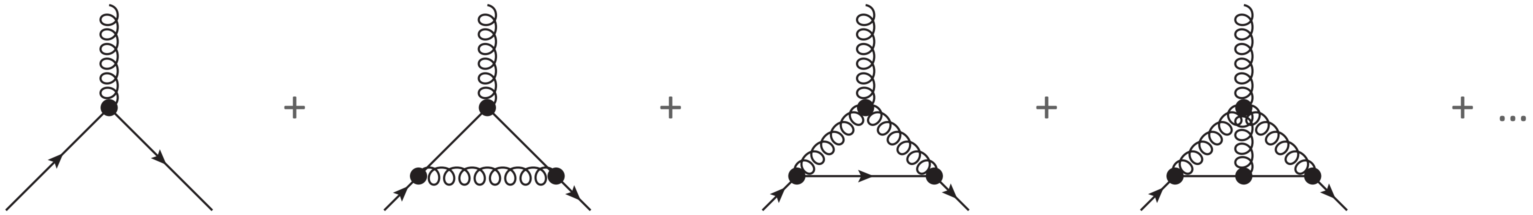


$$[\,\Gamma_{\mu}(p,q)\,]_{\alpha\beta} =$$



$$\{\gamma_{\mu}, p_{\mu}, q_{\mu}\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$$

I. Quark-gluon vertex: General structure



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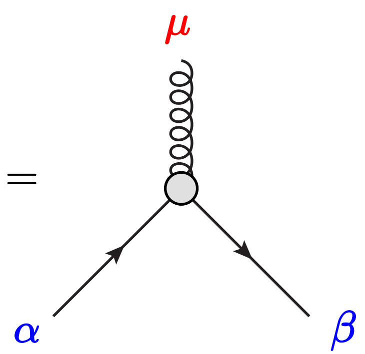

$\{\gamma_{\mu}, p_{\mu}, q_{\mu}\} \times \{\mathbf{1},\ \gamma\cdot p,\ \gamma\cdot q,\ \sigma_{p,q}\}$

- ✦ The vertex has **3 x 4 = 12** independent Lorentz structures.
- ✦ The appearance may be **modified** in nonperturbative QCD.

I. Quark-gluon vertex: (Abelian) Ward-Green-Takahashi Identities

- **Gauge symmetry (vector current cons.):** vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x)$$

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry (axial-vector current cons.):** axial-vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\gamma^5\psi(x),$$
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- **Lorentz symmetry + (axial-)vector current conservation:** transverse WGTIs

$$\delta_T \phi^a(x) = \delta_{\text{Lorentz}}(\delta\phi^a(x)) = -\frac{i}{2}\epsilon^{\mu\nu} S^{(\delta\phi^a)}_{\mu\nu}(\delta\phi^a(x)).$$
$$S^{(\text{spinor})}_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu}, \qquad (S^{(\text{vector})}_{\mu\nu})^\alpha_\beta = i(\delta^\alpha_\mu g_{\nu\beta} - \delta^\alpha_\nu g_{\mu\beta});$$

He, PRD, 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p), \\ q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ & + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p), \qquad \sigma^5_{\mu\nu} = \gamma_5 \sigma_{\mu\nu} \end{aligned}$$

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$$\nabla \cdot \Phi$$

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$$\nabla \times \Phi$$

- The WGTIs express the divergences and curls of the vertices.
- The WGTIs of the vertices in different channels couple together.
- The WGTIs involve contributions from high-order Green functions.

I. Quark-gluon vertex: Solution of WGTIs

Qin et. al., PLB 722, 384 (2013)

Define two projection tensors and contract them with the transverse WGTIs,

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_\mathrm{D}, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta.$$

one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

$$\begin{aligned} q\cdot tt\cdot\Gamma(k,p) &= T^1_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + t^2 q\cdot\Gamma(k,p) + T^1_{\mu\nu}V^A_{\mu\nu}(k,p), \\ q\cdot t\gamma\cdot\Gamma(k,p) &= T^2_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p). \end{aligned}$$

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It is a group of full-determinant linear equations.
A unique solution for the vector vertex is exposed:

$$\Gamma^{\mathrm{Full}}_\mu(k,p) = \Gamma^{\mathrm{BC}}_\mu(k,p) + \Gamma^{\mathrm{T}}_\mu(k,p) + \Gamma^{\mathrm{FP}}_\mu(k,p).$$

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❖ The quark propagator contributes to the longitudinal and transverse parts. The DCSB-related terms are highlighted.

$$\Gamma^{\mathrm{BC}}_\mu(k,p) = \gamma_\mu \Sigma_A + t_\mu \not{t} \frac{\Delta_A}{2} - i t_\mu \Delta_B,$$

$$\Gamma^{\mathrm{T}}_\mu(k,p) = -\sigma_{\mu\nu} q_\nu \Delta_B + \gamma^T_\mu q^2 \frac{\Delta_A}{2} - \left(\gamma^T_\mu [\not{q}, \not{t}] - 2 t^T_\mu \not{q}\right) \frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x) + \phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x) - \phi(y)}{x - y}.$$

$$X^T_\mu = X_\mu - \frac{q \cdot X q_\mu}{q^2}$$

❖ The unknown high-order terms only contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

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$$q\cdot t\gamma\cdot\Gamma(k,p) = T^2_{\mu\nu}[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)] + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p).$$

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❖ The quark propagator contributes to the longitudinal and transverse parts. The DCSB-related terms are highlighted.

$$\Gamma^{\rm BC}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B,$$

$$\Gamma^{\rm T}_\mu(k,p) = -\sigma_{\mu\nu}q_\nu\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

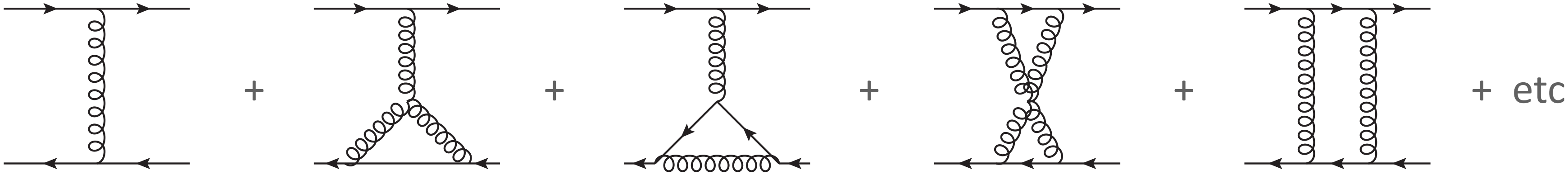
$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x)+\phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x)-\phi(y)}{x-y}.$$

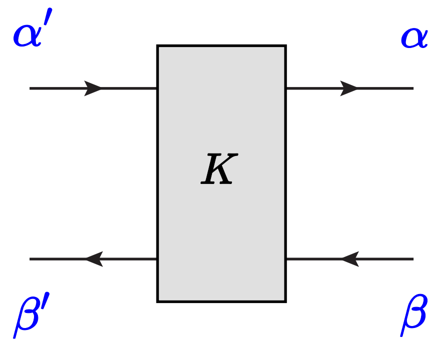
$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

❖ The unknown high-order terms only contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

II. Scattering kernel: General structure



$$K(p_f, k_f; q_i, k_i)_{\alpha\alpha',\beta'\beta} =$$



$$K^{L}_{\alpha\alpha'} \otimes K^{R}_{\beta\beta'}$$

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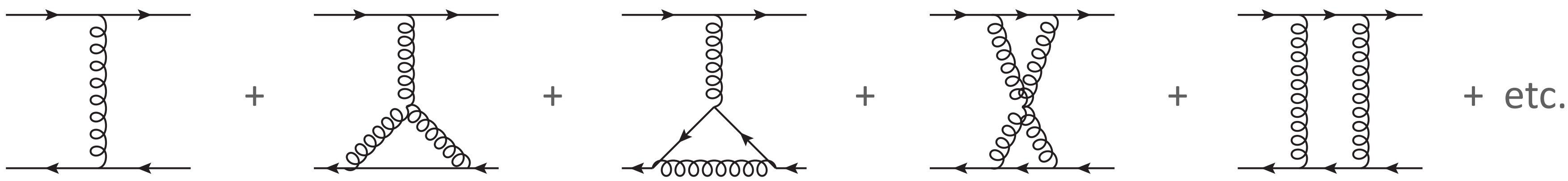

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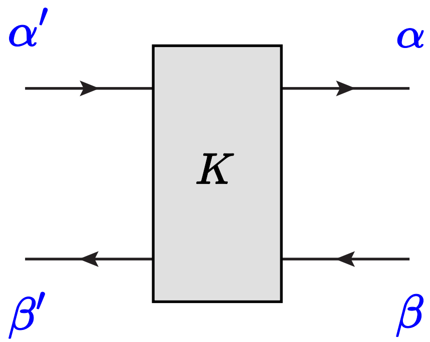

$$K^{L}_{\alpha\alpha'} \otimes K^{R}_{\beta\beta'}$$

- ✦ The kernel has **4 x 4 x 4 x 4 = 256** independent Lorentz structures.
- ✦ It is extremely complicated and must be constrained by symmetries.

II. Scattering kernel: symmetries—color-singlet WGTIs

The color-singlet axial-vector and vector WGTIs are written as

$$P_\mu \Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),$$
$$iP_\mu \Gamma_\mu(k,P) = S^{-1}(k_+) - S^{-1}(k_-).$$

The Bethe-Salpeter equation and the quark gap equation are written as

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$
$$S^{-1}(k) = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

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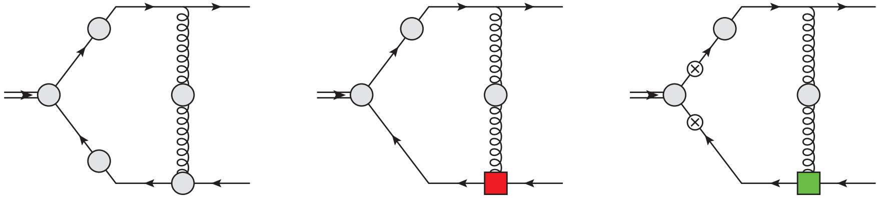
$$\boxed{S^{-1}(k)} = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

The kernel satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\boxed{\begin{aligned} \int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} &= \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)], \\ \int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} &= \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)]. \end{aligned}}$$

II. Scattering kernel: Elements of quark gap equation

Assuming the scattering kernel has the following structure:



Ladder-like term

Symmetry-rescuing term

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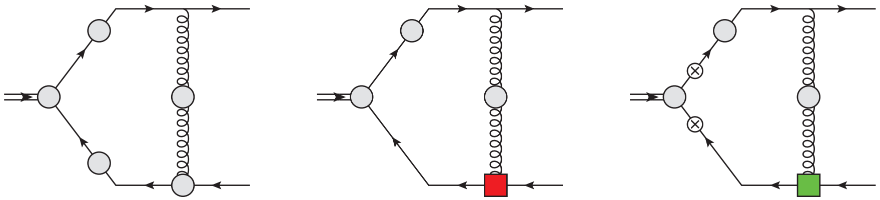

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Symmetry-rescuing term

Algebraic version of the WGTIs, which the scattering kernel satisfy, are written as

$$\boxed{\begin{aligned}\Gamma_\nu^+ - \Gamma_\nu^- &= (S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \gamma_5 (S_+^{-1} - S_-^{-1})\gamma_5 \mathcal{K}_\nu^- \,,\\ \Gamma_\nu^+\gamma_5 + \gamma_5\Gamma_\nu^- &= (S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}_\nu^+ + (\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}_\nu^- \,.\end{aligned}}$$

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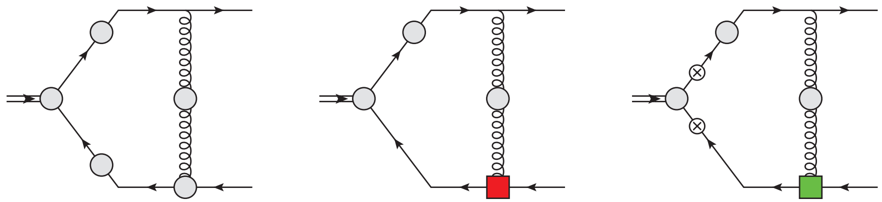

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Eventually, the solution is straightforward:

$$\boxed{\mathcal{K}_\nu^\pm = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma_\nu^\Sigma \pm B_\Sigma \Gamma_\nu^\Delta].}$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

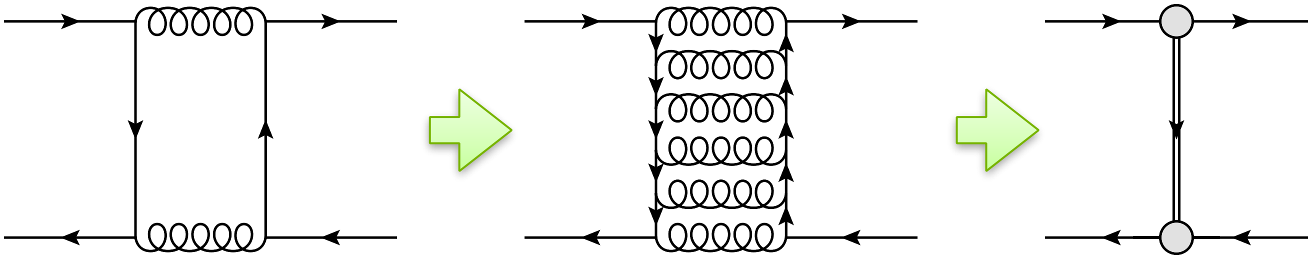
$$\Gamma_\nu^\Sigma = \Gamma_\nu^+ + \gamma_5\Gamma_\nu^+\gamma_5 \qquad \Gamma_\nu^\Delta = \Gamma_\nu^+ - \Gamma_\nu^-$$
$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$
$$A_\Delta = i(\gamma\cdot q_+)A_+ - i(\gamma\cdot q_-)A_-$$

- ✦ The form of scattering kernel is simple.
- ✦ The kernel has no kinetic singularities.
- ✦ All channels share the same kernel.

II. Scattering kernel: Meson cloud — higher-order correction

In Quantum Field theory (infinitely many degrees of freedom), high-order Green functions **cannot** be completely truncated by low-order ones (unclosed).

For example, meson cloud, e.g., pion cloud, can enter the scattering kernel:



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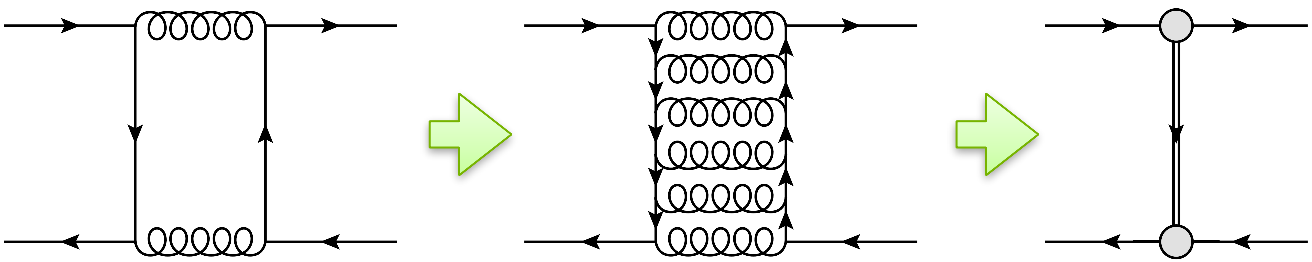

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Accordingly, the **WGTIs** require that meson cloud must modify the quark propagator:

$$\boxed{\begin{cases} \dfrac{\partial |k| A(k^2)}{\partial |k|} = 1 + \dfrac{1}{4}\displaystyle\int_q \left[k_\mu^{\parallel}\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta} \left[\dfrac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'}, \\ B(k^2) = m + \dfrac{1}{4}\displaystyle\int_q \left[\gamma_5\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta} \left[\gamma_5 \sigma_B(q^2)\right]_{\alpha'\beta'}, \end{cases}}$$

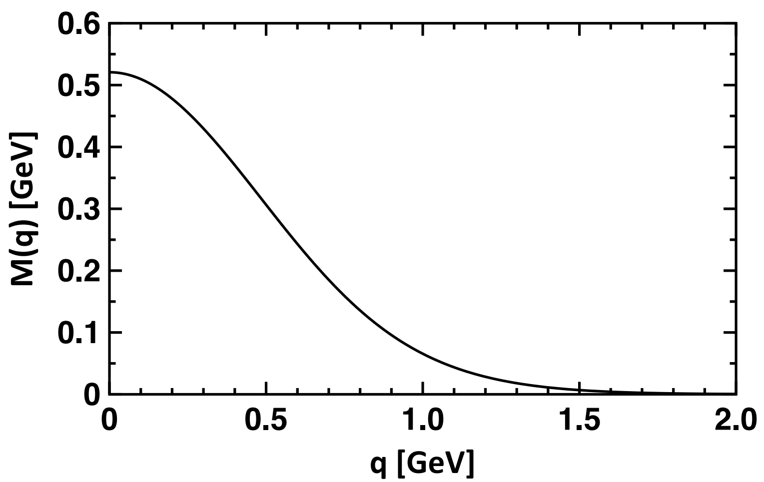
Ansatz and application: ground and radially excited mesons

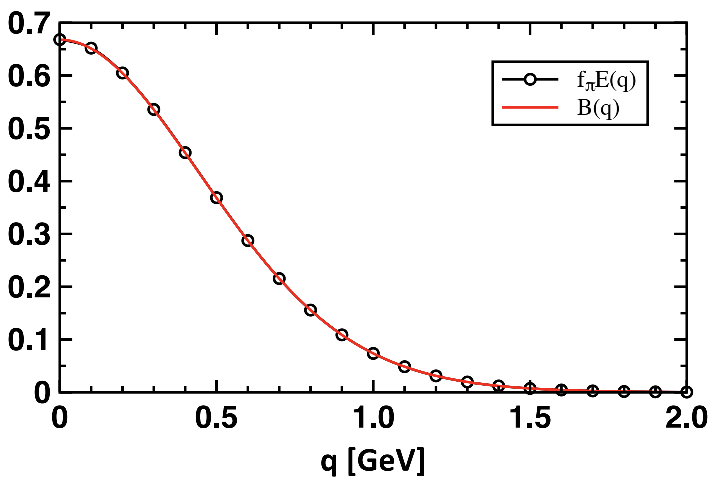
Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\rm BC}(p,q) + \Gamma_\mu^{\rm T}(p,q) \qquad \Gamma_\mu^{\rm T}(p,q) = \eta\Delta_B\tau_\mu^5 + \xi\Delta_B\tau_\mu^8 + 4(\eta+\xi)\Delta_A\tau_\mu^4$$

$$\boxed{\begin{aligned}\tau_\mu^4 &= l_\mu^{\rm T}\gamma\cdot k + i\gamma_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho,\\ \tau_\mu^5 &= \sigma_{\mu\nu}k_\nu,\\ \tau_\mu^8 &= 3\,l_\mu^{\rm T}\sigma_{\nu\rho}l_\nu k_\rho/(l^{\rm T}\cdot l^{\rm T}).\end{aligned}}$$

- ✦ The longitudinal part is the Ball-Chiu vertex—an exact piece from symmetries.
- ✦ The transverse part is the Anomalous Chromomagnetic Moment (ACM) vertex.

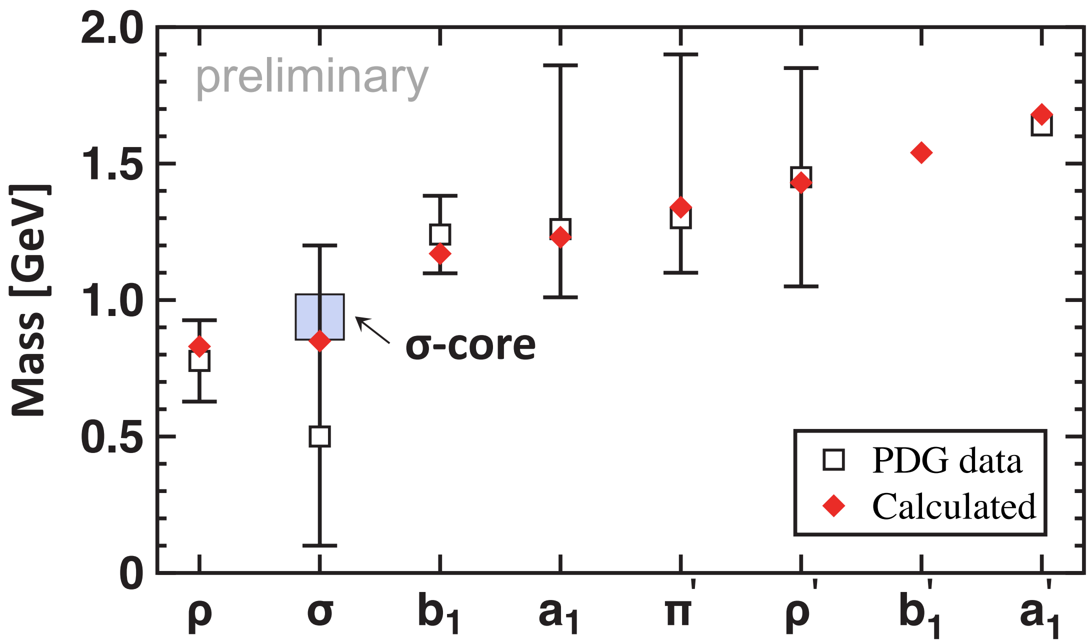




To generate the quark mass scale which is comparable to that of LQCD, the coupling strength can be so small that the rainbow-ladder approximation has NO DCSB at all.

Ansatz and application: ground and radially excited mesons

The correct mass ordering: $\boxed{m_{\rho'} > m_{\pi'} > m_{a_1} > m_\sigma > m_\rho > m_\pi}$



	$-\langle\bar{q}q\rangle_0^{1/3}$	f_π	m_ρ	m_σ	m_{b_1}	m_{a_1}	$m_{\pi'}$	$m_{\rho'}$	$m_{b_1'}$	$m_{a_1'}$
this work	0.228	0.095	0.83	0.86	1.17	1.23	1.34	1.43	1.54	1.68
PDG	-	0.093	0.78	0.50	1.24	1.26	1.30	1.45	-	1.64

TABLE I: The fitted spectrum and its comparison with PDG data (Full vertex, $(D\omega)^{1/3} = 0.492$ GeV, $\omega = 0.55$ GeV, $\eta = 0.35$ and $\xi = 1.30$, in the chiral limit where pion is always massless).

Summary

✦ Based on WGTIs, a **systematic** and **self-consistent method** to construct **the quark-gluon vertex** and **the scattering kernel** beyond the simplest approximation is proposed;

✦ A **demonstration** applying the method to light meson spectroscopy, including **ground** and **radially excited mesons**, is presented.

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✦ Based on WGTIs, a **systematic** and **self-consistent method** to construct **the quark-gluon vertex** and **the scattering kernel** beyond the simplest approximation is proposed;

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Outlook

◆ With the **sophisticated method** to solve the DSEs, we can push the DSE approach to a much wider range of applications in **hadron physics**, e.g., baryons.

◆ Hopefully, after more and more **successful applications** are presented, the DSE approach may provide a **new path** to understand **QCD**.

Appendices

Gluon propagator: Dynamically massive gluon

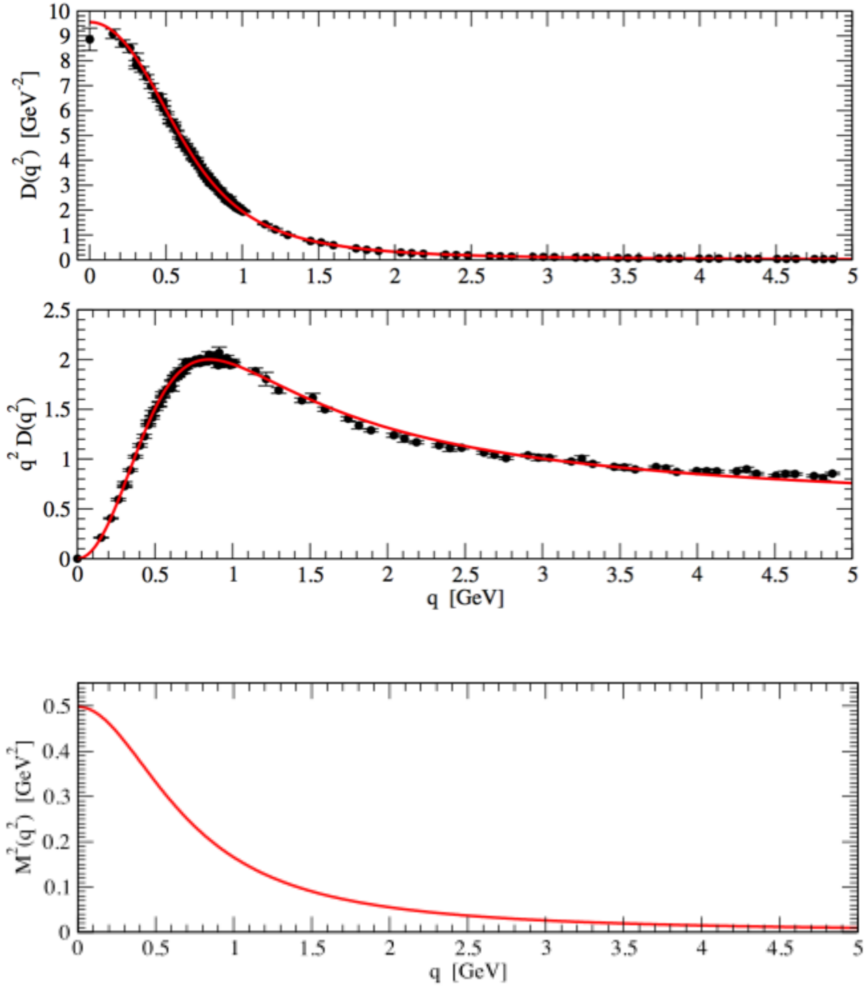
- In Landau gauge (a fixed point of the renormalization group):

$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

- Modeling the dress function: **gluon mass scale + effective running coupling constant**

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)},$$

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2},$$



O. Oliveira et. al., J.Phys. G38, 045003 (2011)

Scattering kernel: Goldstone theorem in terms of Green functions

In the chiral limit, the color-singlet axial-vector WGTI (**chiral symmetry**) is written as

$$P_\mu \Gamma_{5\mu}(k,P) = S^{-1}\left(k+\frac{P}{2}\right) i\gamma_5 + i\gamma_5 S^{-1}\left(k-\frac{P}{2}\right)$$

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Assuming **DCSB**, i.e., the mass function is generated, we have the following identity

$$\lim_{P\to 0} P_\mu \Gamma_{5\mu}(k,P) = 2i\gamma_5 B(k^2) \neq 0$$

The axial-vector vertex involves a pseudo scalar pole as

$$\Gamma_{5\mu}(k,0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \qquad f_\pi E_\pi(k^2) = B(k^2)$$

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Assuming there is a radially excited pion, its decay constant vanishes

$$\lim_{P^2\to M_{\pi_n}^2} \Gamma_{5\mu}(k,P) \sim \frac{2i\gamma_5 f_{\pi_n} E_{\pi_n}(k,P) P_\mu}{P^2 + M_{\pi_n}^2} < \infty \qquad f_{\pi_n} = 0$$

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DCSB means **much more** than massless pseudo-scalar meson.