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IPN INSTITUT DE PHYSIQUE NUCLÉAIRE ORSAY

Role of the $a_0(980)$, $f_0(980)$ resonances in $\eta \to 3\pi$ from the Khuri-Treiman formalism

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Introduction

Quark mass pattern and meson physics

- **if:** $m_u, \ m_d, \ m_s = O(1GeV) \longrightarrow$ Glueball is lightest meson
- **Our world:**:
  $m_u, \ m_d, \ m_s << O(1GeV) \longrightarrow \pi, K, \eta$ lightest mesons
  Glueball ?
- **isospin breaking:** $[m_u - m_d]/m_s \sim O(e^2)$ small

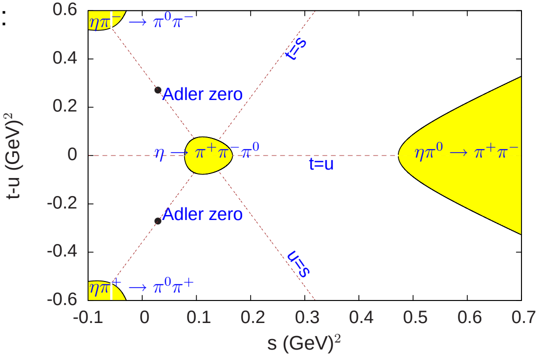
  → Determination of $[m_u - m_d]/m_s$: must disentangle QED from QCD

- $\boldsymbol{\eta \longrightarrow 3\pi}$: QED suppressed [$O(e^2)$ vanish in $m_u = m_d = 0$ limit]

- QCD in soft regime:
  - → Chiral effective theory, relates hadronic observables to m_u, m_d, m_s
  - → $\eta \to 3\pi$: [NLO: [Gasser, Leutwyler (1985)], NNLO: [Bijnens, Ghorbani (2007)]]
- Considerable experimental progress: [Crystal Ball (2001), MAMI(2009), WASA(2009,2014), KLOE(2008,2010,2016), BESIII(2015)]
- Comparison with experiment:
  Dalitz plot parameters α ($3\pi^0$ mode)

  $+0.0142$ (NLO) $\quad -0.0315 \pm 0.0015$ (PDG)
  $+0.0130$ (NNLO)
- Is chiral expansion wrong ?

- Mandelstam plane:



→ Non-linearities in Dalitz plot: NLO may not be enough

→ In physical regions: unitarity restored gradually in ChPT

→ NNLO: 6 combinations of couplings C_i: need model independent determinations

- Strategy:
  1) Effective theory in unphysical region

  2) Physical region: extrapolation based on analyticity and unitarity (nonperturbative QCD properties)

- Framework:[Khuri-Treiman (1960)] (for $K \to 3\pi$)
  $+$ Muskhelishvili-Omnès[Neveu Scherk, (1970)]
  $S+$ P-waves rescattering[Kambor,Wiesendanger, Wyler (1995), Anisovich Leutwyler (1995)]

- Our work, further extension: $\pi\pi + K\overline{K}$ inelastic channel: $a_0(980)$, $f_0(980)$ double resonance effect

Elastic Khuri-Treiman equations

Single-variable amplitudes

- In region where:

$$\mathrm{Im}\left[T_{J\geqslant 2}\right] << \mathrm{Im}\left[T_{J=0,1}\right]$$

  i.e. $|s|, |t|, |u| \lesssim 1$ GeV^2.

- Amplitudes expressed with 3 single-variable functions:

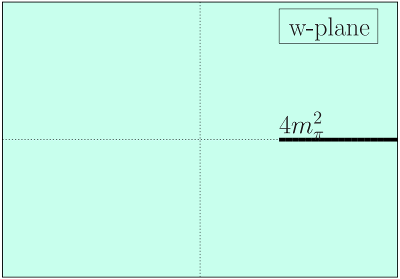
$$T^{\pi^+\pi^-\pi^0}(s,t,u) = -\epsilon_l \left[M_0(s) + (s-u)\,M_1(t) + (s-t)\,M_1(u) - \tfrac{2}{3}M_2(s) + M_2(t) + M_2(u)\right]$$

$$T^{3\pi^0}(s,t,u) = -\epsilon_l \left[M_0(s) + M_0(t) + M_0(u) + \tfrac{4}{3}\left(M_2(s) + M_2(t) + M_2(u)\right)\right]$$

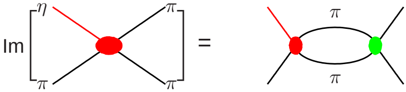
→ where: ϵ_L

$$\epsilon_L = \underbrace{\frac{m_d^2 - m_u^2}{m_s^2 - m_{ud}^2}}_{Q^{-2}\ (\text{[Leutwyler]})} \frac{(m_K^2 - m_\pi^2)\, m_K^2}{3\sqrt{3}F_\pi^2\, m_\pi^2}$$

- Analyticity properties
  - → $M_I(w)$ analytic in cut w-plane
  - → Discontinuity: unitarity relation



- Elastic unitarity



For partial-wave amplitudes $\mathcal{T}^I_J(s)$: $[\eta\pi \to (\pi\pi)^I]_J$

- Elastic Khuri-Treiman: [Anisovich, Leutwyler (1995)]

$$
\begin{aligned}
M_0(w) &= \Omega_0(w)\left[\alpha_0 + w\,\beta_0 + w^2\left(\gamma_0 + \hat{I}_0(w)\right)\right] \\
M_1(w) &= \Omega_1(w)\,w\left(\beta_1 + \hat{I}_1(w)\right) \\
M_2(w) &= \Omega_2(w)\,w^2\hat{I}_2(w)
\end{aligned}
$$

Omnès functions

$$
\Omega_I(s) = \exp\left[\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'(s'-s)}\,\delta_I(s')\right]
$$

Integrals

$$
\hat{I}_I(w) = -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} ds'\,\frac{\mathrm{Im}\left(1/\Omega_I(s')\right)}{(s')^n(s'-w)}\,\hat{M}_I(s')
$$

$\hat{M}_I(s')$: left-cut parts in $\eta \to 3\pi$ partial-waves

$$\mathcal{T}_0^0(s) = \frac{\sqrt{6}\epsilon_L}{32\pi}\left(M_0(s) + \hat{M}_0(s)\right)$$
$$\mathcal{T}_1^1(s) = \frac{\epsilon_L}{48\pi}\kappa(s)\left(M_1(s) + \hat{M}_1(s)\right)$$
$$\mathcal{T}_0^2(s) = -\frac{\epsilon_L}{16\pi}\left(M_2(s) + \hat{M}_2(s)\right)$$

- Polynomial part:
  - → Compensate for high-energy parts of integrands
  - → Four parameters: α_0, β_0, γ_0, β_1
  - → Uniquely determined by matching to chiral NLO amplitude

Chiral matching equations:

$$\boxed{\mathcal{T}_{KT}(s,t,u) - \mathcal{T}_{p^4}(s,t,u) = O(p^6)}$$

→ Satisfied for imaginary parts: expand as polynomial

→ Equations:

$$\alpha_0 = \bar{M}_0 + \frac{4}{3}\bar{M}_2 + 3(\bar{M}_2' - \bar{M}_1)\, s_0 + 9\left(\frac{1}{2}\bar{M}_2'' - \hat{I}_2\right) s_0^2$$

$$\beta_0 = \bar{M}_0' + 3\bar{M}_1 - \frac{5}{3}\bar{M}_2' - 9\left(\frac{1}{2}\bar{M}_2'' - \hat{I}_2\right) s_0 - \Omega_0'\alpha_0$$

$$\beta_1 = \bar{M}_1' + \frac{1}{2}\bar{M}_2'' - \hat{I}_1 - \hat{I}_2$$

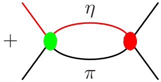
$$\gamma_0 = \frac{1}{2}\bar{M}_0'' + \frac{2}{3}\bar{M}_2'' - \hat{I}_0 - \frac{4}{3}\hat{I}_2 - \frac{1}{2}\Omega_0''\alpha_0 - \Omega_0'\beta_0$$

where $\bar{M}_I$ = chiral M_I amplitude

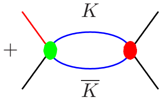
Multi-channel Khuri-Treiman

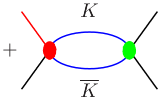
- Contributions to unitarity (two-body) beyond $\pi\pi$:

$\eta\pi$:

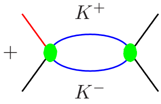


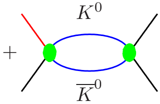
$K\overline{K}$ (I) :





$K\overline{K}$ (II) :





Part induced by $K^+ - K^0$ mass difference[Achasov, Devyanin, Shestakov (1979)]

- Isospin violating amplitudes

$I = 1 \to I = 0$ $\begin{pmatrix} \eta\pi \to [\pi\pi]^0 & \eta\pi \to [K\overline{K}]^0 \\ [K\overline{K}]^1 \to [\pi\pi]^0 & [K\overline{K}]^1 \to [K\overline{K}]^0 \end{pmatrix}$

$I = 1 \to I = 2$ $\begin{pmatrix} \eta\pi \to [\pi\pi]^2 \\ [K\overline{K}]^1 \to [\pi\pi]^2 \end{pmatrix}$

- Isospin conserving amplitudes

$$\mathbf{T}^{(0)}: \begin{pmatrix} \pi\pi \to \pi\pi & \pi\pi \to K\overline{K} \\ \pi\pi \to K\overline{K} & K\overline{K} \to K\overline{K} \end{pmatrix}_{I=0}$$

$$\mathbf{T}^{(1)}: \begin{pmatrix} \eta\pi \to \eta\pi & \eta\pi \to K\overline{K} \\ \eta\pi \to K\overline{K} & K\overline{K} \to K\overline{K} \end{pmatrix}_{I=1}$$

$$T^{(2)}: \ \pi^+\pi^0 \to \pi^+\pi^0$$

- Coupled-channel KT in matrix form: $I = 1 \to I = 0$

$$\mathbf{M}_0(w) = \Omega_0(w)\left[\mathbf{P}_0(w) + w^2\left(\hat{\mathbf{I}}_A(w) + \hat{\mathbf{I}}_B(w)\right)\right]{}^t\Omega_1(w)$$

→ Ω_I: Omnès-Muskhelishvili 2×2 matrices

→ $\mathbf{P}_0$: polynomials, 12 parameters

→ “left-cut” integrals

$$\hat{\mathbf{I}}_A(w) = -\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{(s')^2(s'-w)}\left[\mathrm{Im}\,\Omega_0^{-1}\,\hat{\mathbf{M}}_0\,{}^t\Omega_1^{-1} + \Omega_0^{-1*}\,\hat{\mathbf{M}}_0\mathrm{Im}\,{}^t\Omega_1^{-1}\right]$$

$$\hat{\mathbf{I}}_B(w) = \frac{32}{\sqrt{6}\epsilon_L}\int_{4m_\pi^2}^{\infty} \frac{ds'\,\Delta\sigma_K(s')}{(s')^2(s'-w)}\,\Omega_0^{-1*}\mathbf{T}^{(0)*}\begin{pmatrix}0 & 0\\ 0 & 1\end{pmatrix}\mathbf{T}^{(1)}\,{}^t\Omega_1^{-1}$$

- Altogether: 16 polynomial parameters
  - → Reduces exactly to uncoupled case when $\eta\pi$, $K\overline{K}$ rescattering switched off

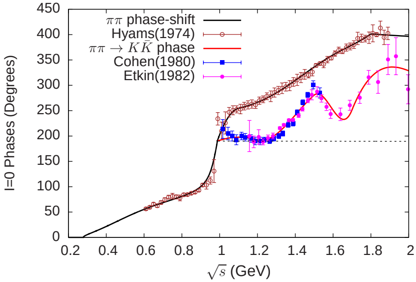
- Approximations:
  - → Left-cuts in $K\overline{K}$ amplitudes neglected
  - → Matching: $K\overline{K}$ amplitudes computed at LO only

- Coupled-channel becomes straightforward extension of single-channel

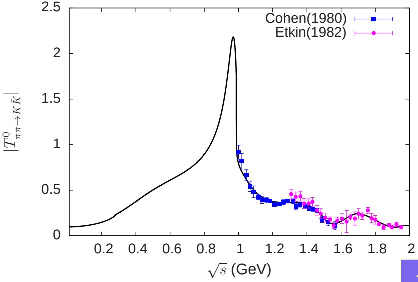
Solving the Khuri-Treiman equations

■ $I = 0$ T-matrix inputs:
Experimental data exists

$\pi\pi \to \pi\pi$ phase-shift
$\pi\pi \to K\overline{K}$ phase



$\pi\pi \to K\overline{K}$ modulus



- $I = 1$ T-matrix inputs:
  - → No phase-shift data. Model[Albaladejo, B.M. (2015)]
  - → Data on resonances + chiral symmetry
  - → T-matrix from chiral K-matrix:

$$\mathbf{T} = (1 - \mathbf{K}(s)\Phi(s))^{-1}\mathbf{K}(s)$$

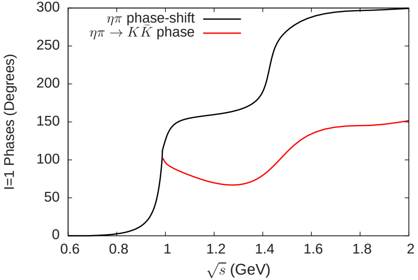
with: $\mathbf{K} = \mathbf{K}^{(2)}(s) + \mathbf{K}^{(4)}(s) + \mathbf{K}^{(6)}(s)$

When s small: $\mathbf{T} = \underbrace{\mathbf{T}^{(2)} + \mathbf{T}^{(4)}}_{\text{first two terms in Ch. exp. reproduced}} + \cdots$

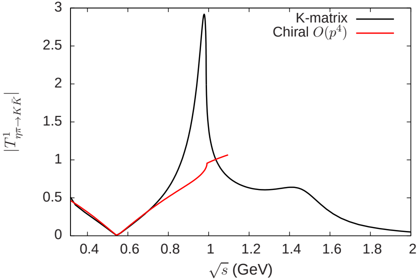
  - → Six parameters: $a_0(980)$, $a_0(1450)$ pole positions + branching fractions
    Extra constraint: $\eta - \pi$ scalar radius

- $\boxed{I=1}$ T-matrix inputs:

$\eta\pi \to \eta\pi$ phase-shift
$\eta\pi \to K\overline{K}$ phase

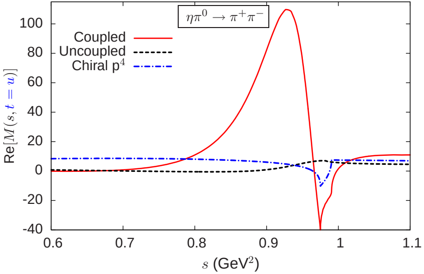


$\eta\pi \to K\overline{K}$ modulus

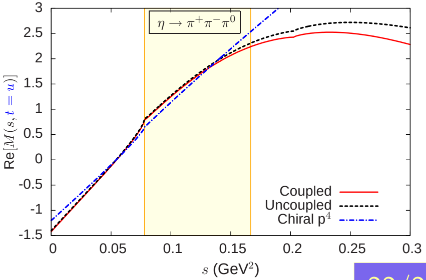


- Illustration of the solutions (I):

1) $\eta\pi^0 \to \pi^+\pi^-$: t=u line
1 GeV region



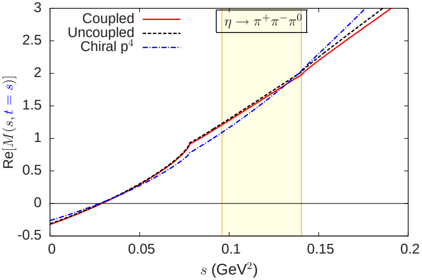
2) $\eta \to \pi^+\pi^-\pi^0$: t=u line
Decay region



- Illustration of the solutions (II):

1) $\eta \to \pi^+\pi^-\pi^0$: t=s line

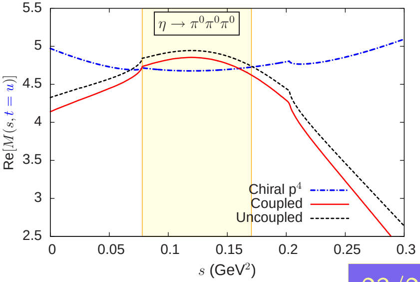
Near Adler zero: good matching KT and NLO



2) $\eta \to 3\pi^0$: t=u line

NO good matching KT and chiral NLO

Strong influence of scalar resonances



Comparison with experiment

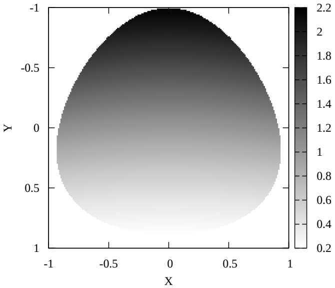
Dalitz plot parameters definitions

→ Coordinates (charged mode)

$$X = \frac{\sqrt{3}}{2m_\eta Q_c}(t-u) \ ,$$

$$Y = \frac{3}{2m_\eta Q_c}\left((m_\eta - m_{\pi^0})^2 - s\right) - 1$$

with $Q_c = 2m_\eta - 2m_{\pi^+} - m_{\pi^0}$.



→ Parametrisation: $\pi^+\pi^-\pi^0$

$$\frac{|M_c(X,Y)|^2}{|M_c(0,0)|^2} = 1 + a\,Y + b\,Y^2 + d\,X^2 + f\,Y^3 + g\,X^2 Y + \cdots$$

→ Parametrisation: $3\pi^0$

$$\frac{|M_n(X,Y)|^2}{|M_n(0,0)|^2} = 1 + 2\alpha\,(X^2 + Y^2) + \cdots$$

Dalitz plot parameters results

$\pi^+\pi^-\pi^0$	$O(p^4)$	single-ch.	coupled-ch.	KLOE	BESIII
a	-1.328	-1.154	-1.142	-1.095(4)	-1.128(15)
b	0.429	0.202	0.171	0.145(6)	0.153(17)
d	0.089	0.094	0.097	0.081(7)	0.085(16)
f	0.016	0.108	0.123	0.141(10)	0.173(28)
g	-0.081	-0.087	-0.088	-0.044(16)	–
$\pi^0\pi^0\pi^0$				PDG	
α	+0.0142	-0.0274	-0.0337	-0.0315(15)	

→ $O(p^4)$+Khuri-Treiman improves over direct chiral $O(p^4)$

→ Effect of 1 GeV scalars up to 17%

→ Some room left for sub-threshold $O(p^6)$

Quark mass ratio Q

→ Equate width $\Gamma^{KT}(Q, e^2(m_u + m_d))$ with experiment

$$\Gamma^{\pi^+\pi^-\pi^0} = (299 \pm 11)\ \text{eV} \quad \Gamma^{3\pi^0} = (427 \pm 15)\ \text{eV}$$

Central values:

	single-ch.	coupled-ch.
$\pi^+\pi^-\pi^0$:	21.8 ± 0.2	21.6 ± 0.2
$3\pi^0$:	21.9 ± 0.2	21.7 ± 0.2

[QCD scale $\mu_0 = 0.77$ GeV]

→ Error assuming 10% sub-threshold $O(p^6)$: $\Delta Q = 2.2$

→ Compatible w. lattice QCD: $Q = 22.9 \pm 0.4$[QCDSF-UKQCD (2015)], $Q = 23.4 \pm 0.6$[BMW (2016)]
but not as precise

Summary

- Khuri-Treiman formalism includes $\eta\pi$ and $K\overline{K}$ channels in rescattering/ unitarity $\longrightarrow$ account of $a_0(980)$, $f_0(980)$ resonances
- Influence of $a_0(980)$, $f_0(980)$ resonances at low energy: up to 20% in Dalitz parameters. In particular for $\eta \to 3\pi^0$
- Amplitudes from NLO-matched Khuri-Treiman are close to agreeing w. experiment in physical decay region
- Convergence of chiral exp. in unphysical region seems good, NNLO effects less than 10%
- Determination of Q: some information on sub-threshold NNLO effects needed for improved precision