PWA with full rank density matrix of the $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ systems at VES setup

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Preface

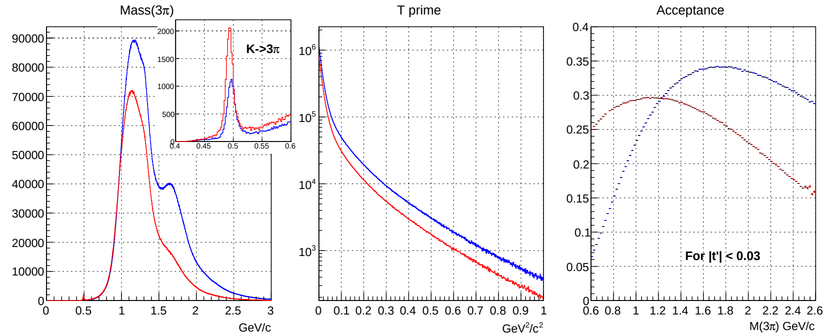
Comparison of two final states permit us to know better:

- isospin relation between these states
- what is resonant and what is not
- bugs in hardware, methods, programs...
- PWA is model dependent. It is useful to compare one model for two systems and two model (full rank and rank 1) for the same system.

We have:

- full featured magnetic spectrometer with 29 GeV/c $\pi-$ beam, Be target, $|t'| = 0 \ldots 1\ GeV^2/c^2$
- $20 \cdot 10^6$ events in $\pi^-\pi^0\pi^0$ (leading statistics in the world)
- $30 \cdot 10^6$ events in $\pi^+\pi^-\pi^-$ (next to leading)
- Analysis is done for $|t'| =$ 0–0.03–0.15–0.30–0.80 GeV^2/c^2

Raw data

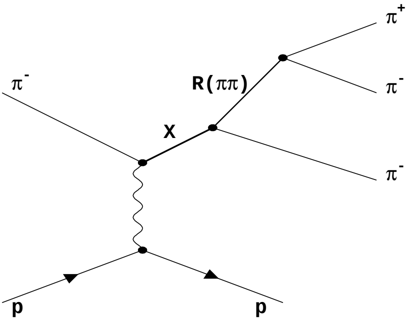


Here and below:

Blue line - $\pi^+\pi^-\pi^-$

Red line - $\pi^-\pi^0\pi^0$

PWA method. Partial waves



PWA amplitudes are constructed using isobar model, sequential decay via $\pi\pi$ subsystem. Wave has quantum numbers $J^P L M^\eta R$ where J^P is spin-parity for 3π system, M^η is its projection of spin and naturality, R is the known resonance in $\pi\pi$ system, L is orbital momentum in $R\pi$ decay. $I^G = 1^-$ is implicit for 3π charged states.

Method details

- Amplitudes use d-functions (Hansen, Illinois PWA)
- Amplitudes are relativistic (Chung, Filippini)
- resonanses are relativistic Breit-Wigners
  $R = f_0(980)$, $\varepsilon(1300)$, $f_0(1500)$, $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$
  To describe $\pi\pi$ S-wave we use modified Au, Morgan, Pennnington M-solution with $f_0(980)$ withdrawn. We name it $\varepsilon(1300)$
- Notation for some known 3π resonances:
  $a_1(1260)$ — $1^+S0^+\rho(770)$, $a_2(1320)$ — $2^+D1^+\rho(770)$,
  $\pi_2(1670)$ — $2^-S0^+f_2(1270)$, $\pi(1800)$ — $0^-S0^+f_0(980)$
- fit parameters are elements of positive definite density matrix. No rank constraints. For small number of waves (where $C \neq 1$, see below) we have 100% coherence.

Extended likelihood function

$$\ln\mathcal{L} = \sum_{e=1}^{N_{ev}} \ln \sum_{i,j=1}^{N_w} C_{k(i)} R_{m(i)m(j)} C^*_{k(j)} \mathcal{M}_i(\tau_e) \mathcal{M}^*_j(\tau_e)$$

$$- \quad N_{ev} \sum_{i,j=1}^{N_w} C_{k(i)} R_{m(i)m(j)} C^*_{k(j)} \int \varepsilon(\tau) \mathcal{M}_i(\tau) \mathcal{M}^*_j(\tau)\, d\tau$$

- N_{ev} — number of events, N_w — number of waves
- $\mathcal{M}(\tau_e)$ — amplitudes for e-th event (data)
- R — positive definite density matrix (parameters)
- C — coupling coeff (most of $C = 1$, some are parameters)
- $m(i)$, $k(i)$ — describes wave to C and R correspondence
- $\tau = s, t, m(3\pi), \ldots$ — phase space variables
- $\varepsilon(\tau)$ — acceptance of the setup

Coherent part of density matrix

Coherent part of the density matrix R is the largest part of the matrix which has rank 1 and behaves like vector of amplitudes. Let

$$R = \sum_{k=1}^{d} e_k * V_k * V_k^+ \quad \text{where} \quad \begin{cases} e_k \text{ is k-th eigenvalue} \\ V_k \text{ is k-th eigenvector} \end{cases}$$

Let $e_1 \gg e_2 > \ldots > e_d > 0$. Leading term R_L is coherent part of density matrix and R_S is the rest (incoherent part). This decomposition is stable w.r.t. variations of R matrix elements.

$$R = R_L + R_S, \quad R_L = e_1 * V_1 * V_1^+, \quad R_S = \sum_{k=2}^{d} e_k * V_k * V_k^+$$

Experience shows that resonances tend to concentrate in R_L.

Isospin relations

If we neglect phase space factors

$$R = \frac{\sigma(\pi^-\pi^0\pi^0)}{\sigma(\pi^+\pi^-\pi^-)} = \begin{cases} 1 & \text{for waves with } \rho(770),\ \rho_3(1690) \\ 1/2 & \text{for waves with } f_0(...),\ f_2(1270) \end{cases}$$

All waves coupled to $\pi^0\pi^0$ have factor 1/2

To simplify comparison, they are scaled 2x.

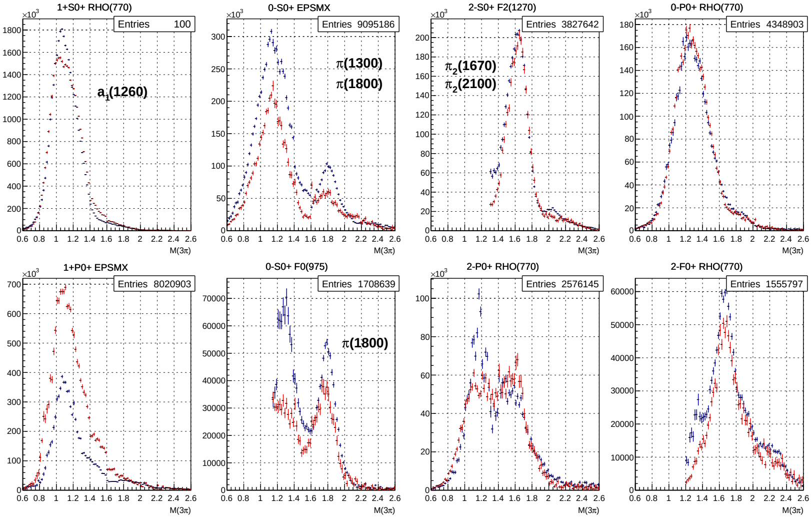
To compensate for some losses, all $\pi^-\pi^0\pi^0$ are scaled by 1.25

This factor is obtained comparing signals of $a_2(1320)$

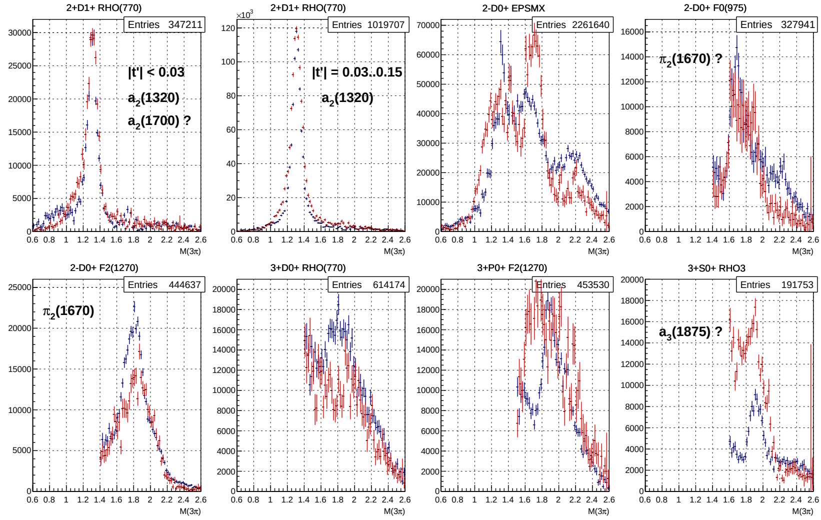
Blue line - $\pi^+\pi^-\pi^-$

Red line - $\pi^-\pi^0\pi^0$

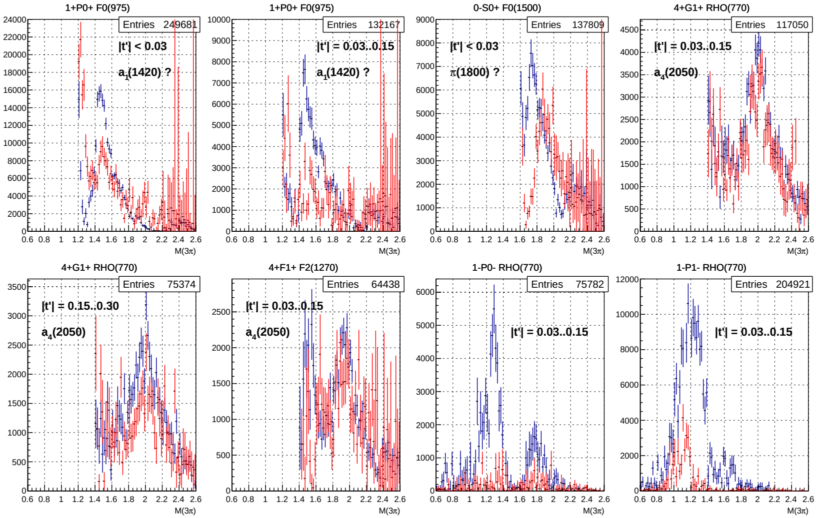
Largest waves, $|t'| < 0.03\ GeV^2/c^2$



Minor waves — 1, $|t'| < 0.03\ GeV^2/c^2$



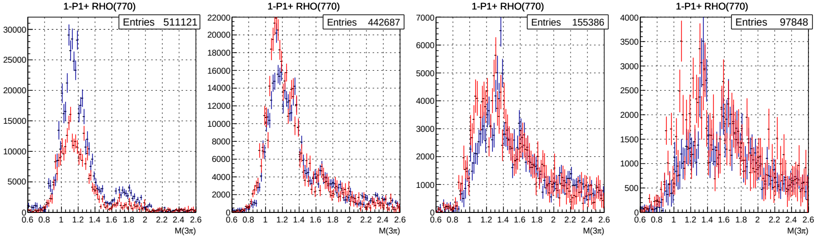
Minor waves — 2



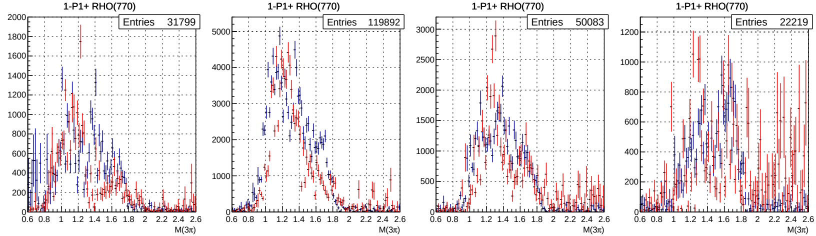
Exotic (non $q\bar{q}$) wave $J^{PC} = 1^{-+}$

Results for $|t'| = 0\text{–}0.03\text{–}0.15\text{–}0.30\text{–}0.80\ \mathrm{GeV}^2/c^2$

Full density matrix



Largest Eigenvalue



Conclusions

- Mass-independent PWA is done for $\pi^+\pi^-\pi^-$ and $\pi^-\pi^0\pi^0$ data. A lot of waves look alike in both reactions, some are not. Naive isospin relations can be violated by the difference in phase space and in interference on Dalitz plot for $I = 0$ isobars.
- Unestablished decay modes $\pi(1800) \to f_0(1500)\pi$, $a_3(1875) \to \rho_3(1690)\pi$ are much better seen in $\pi^+\pi^-\pi^-$. The same is true for 1^+S $f_0(980)\pi$ bump at $M = 1.4$ GeV/c^2, $a_1(1420)$. State $a_2(1700)$ is not seen in both reactions.
- State $a_2(1320)$ is in good agreement in both systems for all t'. For $a_4(2050)$ this is true for $|t'| > 0.03$ GeV^2/c^2. We have not enough statictics for low t'.
- Waves with $J^{PC} = 1^{-+}$ are small, in largest eigenvalue are even smaller, albeit in agreement for $|t'| > 0.03$ GeV^2/c^2. In negative naturality in $\pi^-\pi^0\pi^0$ they are severely suppressed.

Backup slides

Wave set used in the analysis

$\mathbf{J^{PC}}$	Waves	
FLAT		
$\mathbf{0^{-+}}$	$0^-S0^+\,\varepsilon$	
	$0^-S0^+f_0(975)$	$0^-S0^+f_0(1500)$
	$0^-P0^+\,\rho$	
$\mathbf{1^{++}}$	$1^+S0^+\,\rho$	
	$1^+P0^+\,\varepsilon$	
	$1^+D0^+\,\rho$	$1^+P0^+f_0(975)$
	$1^+P0^+f_2$	
	$1^+S1^+\,\rho$	
	$1^+P1^+\,\varepsilon$	
	$1^+S1^-\,\rho$	
$\mathbf{1^{-+}}$	$1^-P1^+\,\rho$	
	$1^-P0^-\,\rho$	
	$1^-P1^-\,\rho$	
$\mathbf{2^{-+}}$	$2^-S0^+f_2$	
	$2^-P0^+\,\rho$	
	$2^-D0^+\,\varepsilon$	
	$2^-F0^+\,\rho$	
	$2^-D0^+f_0(975)$	
	$2^-D0^+f_2$	
	$2^-S1^+f_2$	
	$2^-P1^+\,\rho$	$2^-F1^+\,\rho$
	$2^-D1^+\,\varepsilon$	$2^-D1^+f_2$
$\mathbf{2^{++}}$	$2^+D1^+\,\rho$	$2^+P1^+f_2$
	$2^+D0^-\,\rho$	
	$2^+D1^-\,\rho$	
$\mathbf{3^{++}}$	$3^+S0^+\,\rho_3$	$3^+P0^+f_2$
	$3^+D0^+\,\rho$	
$\mathbf{4^{-+}}$	$4^-F0^+\,\rho$	
$\mathbf{4^{++}}$	$4^+F0^+f_2$	$4^+G0^+\,\rho$

Largest waves, $|t'| < 0.03\ \mathrm{GeV}^2/c^2$, Largest EigenValue