14th International Workshop on Meson Production, Properties and Interaction
KRAKÓW, POLAND, 2nd - 7th June 2016

LOW-ENERGY HADRONIC CROSS SECTIONS MEASUREMENTS AT BABAR, AND IMPLICATION FOR THE G-2 OF THE MUON

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On behalf of BABAR Collaboration

OUTLINE

- Introduction
- Theory of $(g-2)_\mu$
- $e^+e^- \rightarrow$ hadrons impact to $(g-2)_\mu$
- BaBar ISR measurements
- Perspectives, conclusions

MOTIVATION: $(g-2)_\mu/2$

- Magnetic moment of muon: $\vec{\mu} = g \frac{e\hbar}{2mc} \vec{S}$
- Gyromagnetic factor g for
  - Dirac particles (point-like fermions): g = 2
  - Higher order contributions (QFT): g ≠ 2
- Muon anomaly
  - $a_\mu = (g-2)_\mu/2$

$$a_\mu^{\text{theory(SM)}} = a_\mu^{\text{QED}} + a_\mu^{\text{weak}} + a_\mu^{\text{had}}$$

E821 Experiment @ BNL

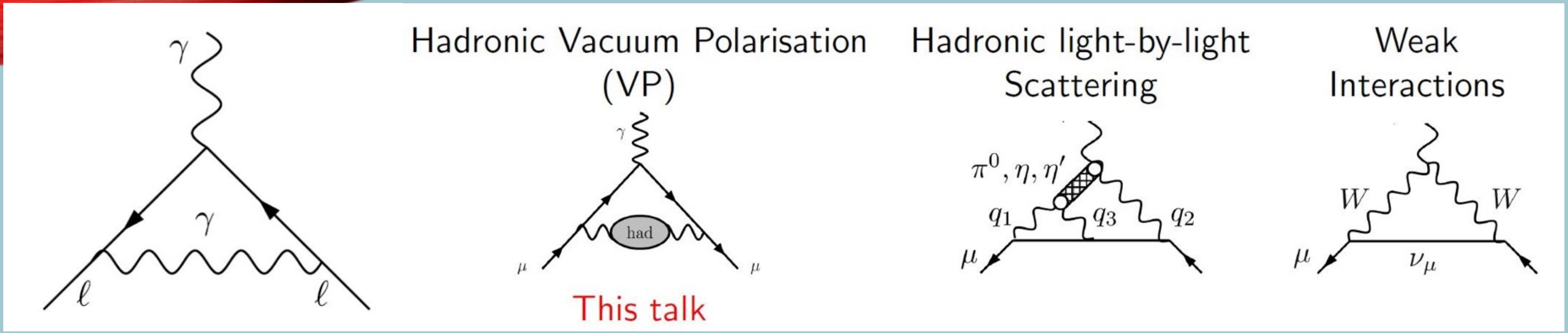
SM-to-experiment comparison [units 10^{-10}]

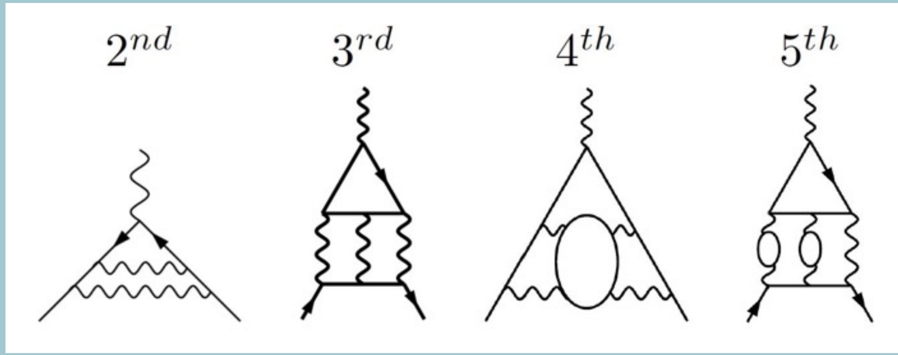
QED	11 658 471.895	± 0.008
Leading hadronic vacuum polarization (VP)	692.3	± 4.2
Sub-leading hadronic vacuum polarization	−9.8	± 0.1
Hadronic light-by-light	10.5	± 2.6
Weak (incl. 2-loops)	15.4	± 0.1
Theory	11 659 180.3	± 4.2 ± 2.6
Experiment [E821 @ BNL]	11 659 209.1	± 5.4 ± 3.3
Exp. − theory	+28.8	± 8.0

G.W. Bennett et al. (E821), PRD 73, 072003 (2006)

$$a_\mu^{EXP} - a_\mu^{SM} = 3.6\sigma$$ (M. Davier et al., EPJC71(2011)1515)

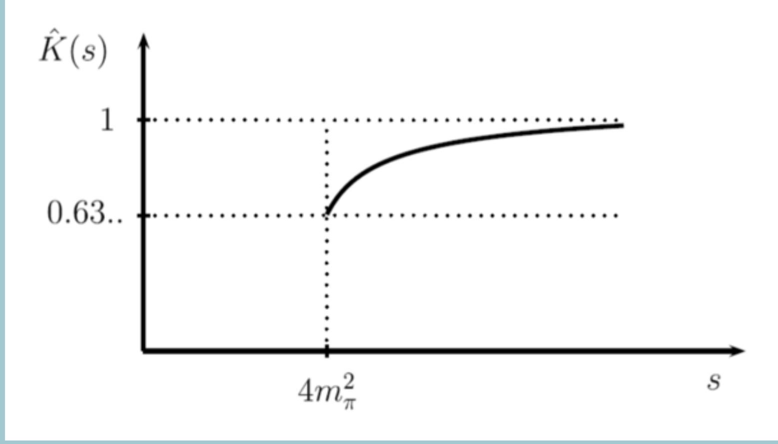
THEORY OF $(g-2)_\mu/2$





$$a_\mu^{had} = \frac{\alpha^2}{3\cdot\pi^2}\int_{4m_\pi^2}^{\infty} ds\cdot\frac{K(s)}{s}\cdot R(s)$$

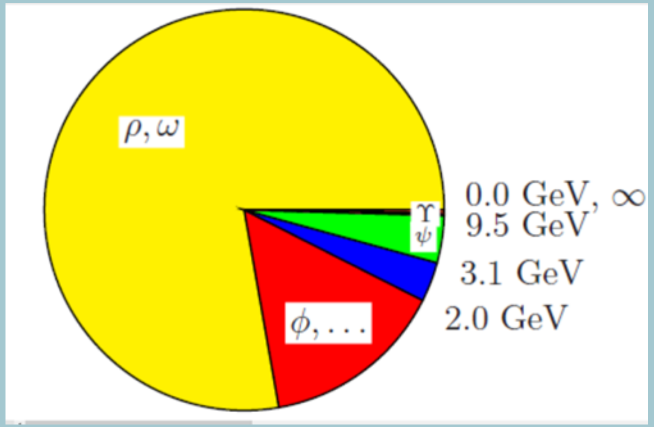
$$R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



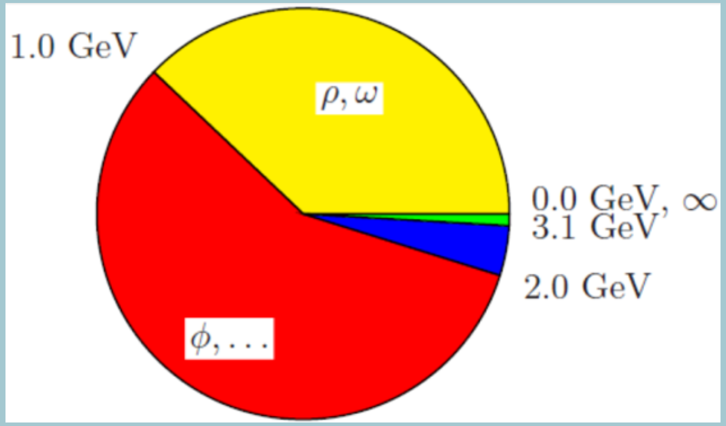
Low energy contributions are important!
Experimental input is needed!

$e^+e^- \to$ hadrons impact to $(g-2)_\mu/2$

Contributions
of the different energy regions to:

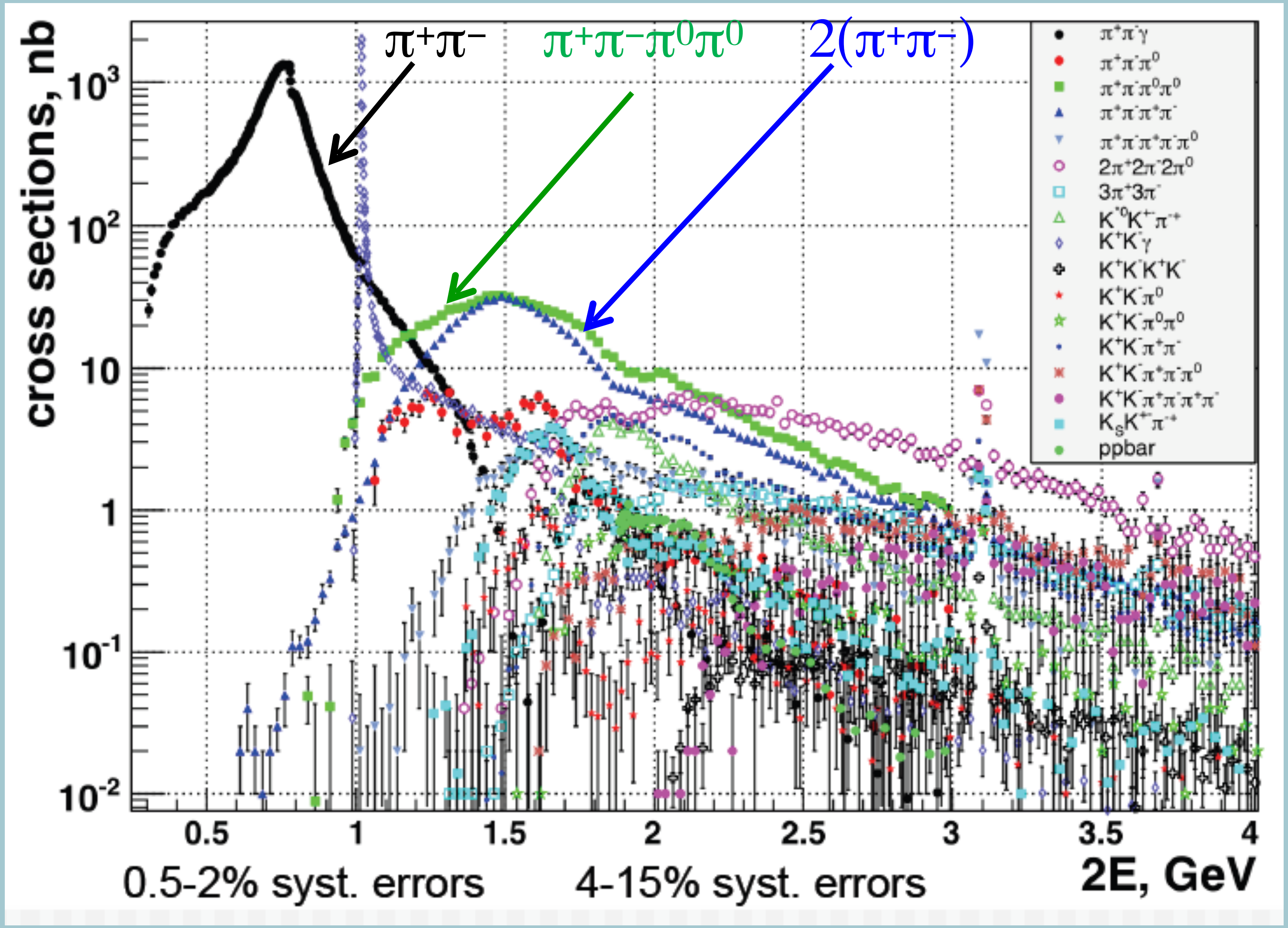


a_μ integral



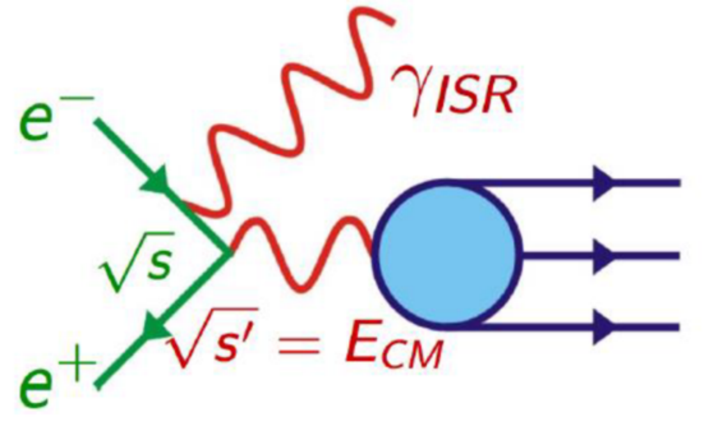
a_μ uncertainty

BaBar ISR measurements



Precise measurements of σ_{had} in $1 < E_{cm} < 2$ GeV are needed

ISR – Initial State Radiation



- Optimal use of available luminosity
- Covers whole energy range with the same detector conditions and analysis
- Asymmetric e^+e^- ⇒ good efficiency down to threshold
- If the whole final state (γ + hadrons) is observed ⇒ overconstrained kinematic fit ⇒ powerful background noise rejection

$$\frac{d\sigma_{[e^+e^-\to f\gamma]}}{ds'}(s') = \frac{2m}{s} W(s,x)\sigma_{[e^+e^-\to f]}(s') \ , \qquad x = \frac{E_\gamma}{\sqrt{s}} = 1 - \frac{s'}{s}$$

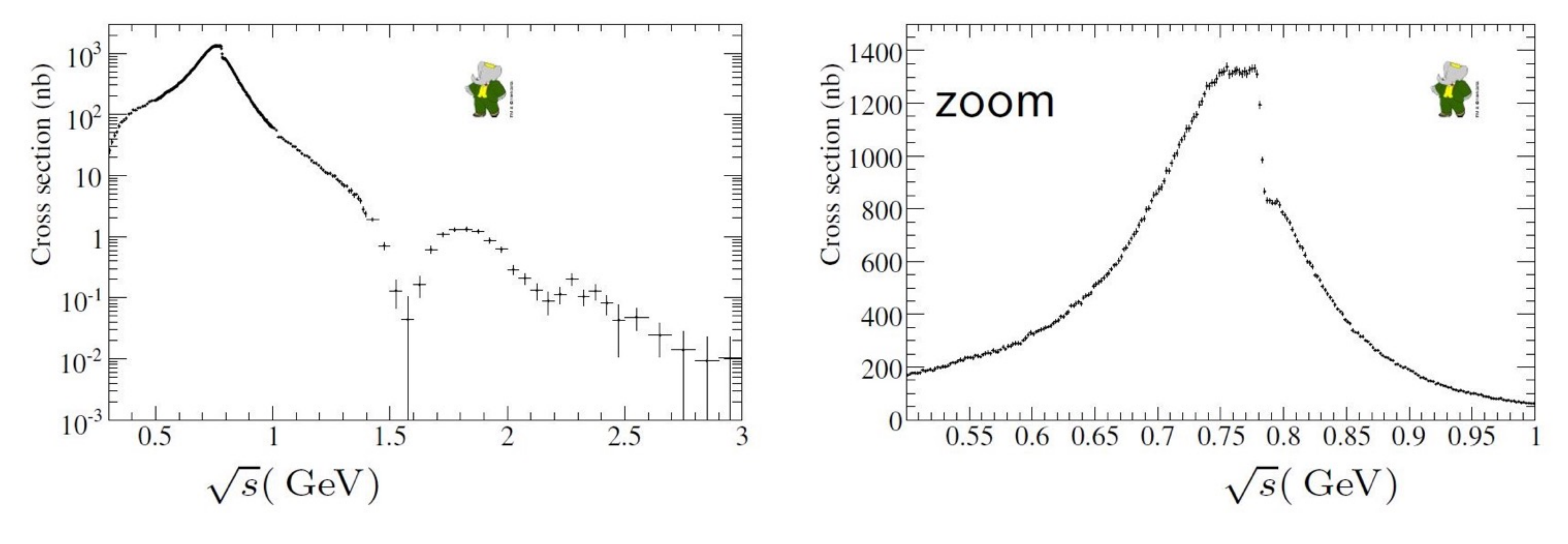
W(s,x) – "Radiator function", density of probability to radiate a photon with energy $E_\gamma = x\cdot\sqrt{s}$

$e^+e^- \to \pi^+\pi^-(\gamma)\gamma$

- Systematics mastered at the 10^{-3} level for first time in BaBar
- ISR γ in EMC (thus: at large angle)
- Good quality tracks, particle identification (PID)
- Kinematic Fit (using only direction of ISR γ)
  - Possibly including 1 additional γ: NLO!
- All efficiencies (trigger, filter, tracking, PID, fit) from data. $\pi^+\pi^-/\ \mu^+\mu^-$ ratio:
  - Cancelation of ee luminosity, additional ISR, VP, ISR γ efficiency
- Correct for lowest order FSR in $\mu^+\mu^-$ and for ISR + additional FSR, both calc. in QED and checked in data

$$R_{\text{exp}}(s') = \frac{\sigma_{[\pi\pi\gamma(\gamma)]}(s')}{\sigma_{[\mu\mu\gamma(\gamma)]}(s')} = \frac{\sigma^0_{[\pi\pi(\gamma)]}(s')}{(1+\delta^{\mu\mu}_{\text{FSR}})\sigma^0_{[\mu\mu(\gamma)]}(s')} = \frac{R(s')}{(1+\delta^{\mu\mu}_{\text{FSR}})(1+\delta^{\mu\mu}_{add,FSR})}$$

$e^+e^- \to \pi^+\pi^-(\gamma)$ Cross Section



Bare (incl. additional FSR, VP removed) unfolded $\sigma(e^+e^- \to \pi^+\pi^-)$ 232 fb^{-1} @ $\sqrt{s}$ = 10.6 GeV

- **Excellent precision down to threshold:**

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\text{GeV}] = (514.1 \pm 2.2 \pm 3.1) \times 10^{-10}$$

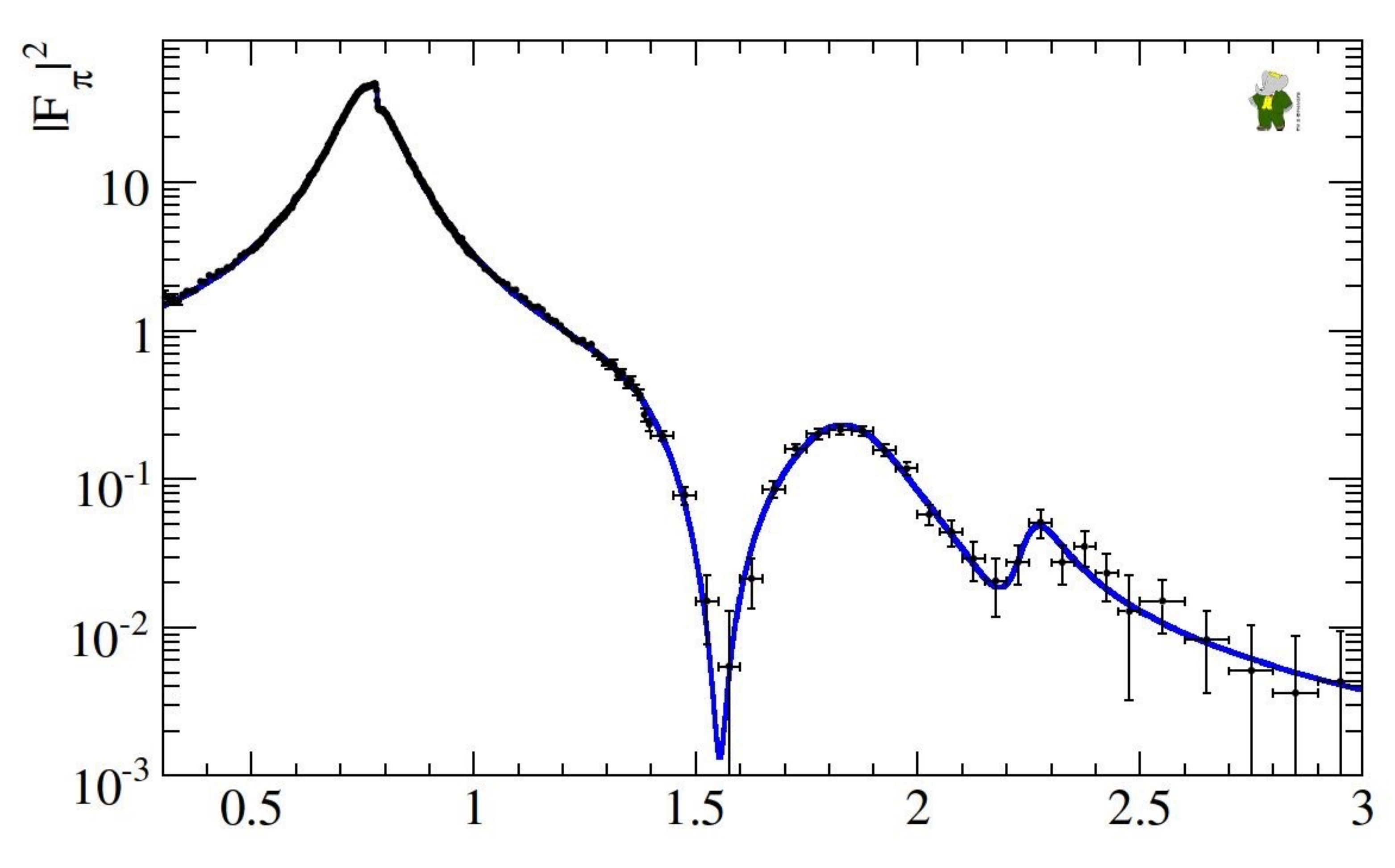
- **Similar precision as combination of previous e^+e^- results:**

$$a_\mu^{\pi^+\pi^-}[2m_\pi, 1.8\,\text{GeV}] = (503.5 \pm 4.5) \times 10^{-10}$$

1.7 σ **larger than previous e^+e^- average:** $\Delta = +(10.6 \pm 5.9) \times 10^{-10}$

Phys.Rev.Lett. 103 (2009) 231801
Phys.Rev.D86 (2012) 032013

$e^+e^- \to \pi^+\pi^-(\gamma)$: VDM Fit of $|F_\pi(s)|^2$



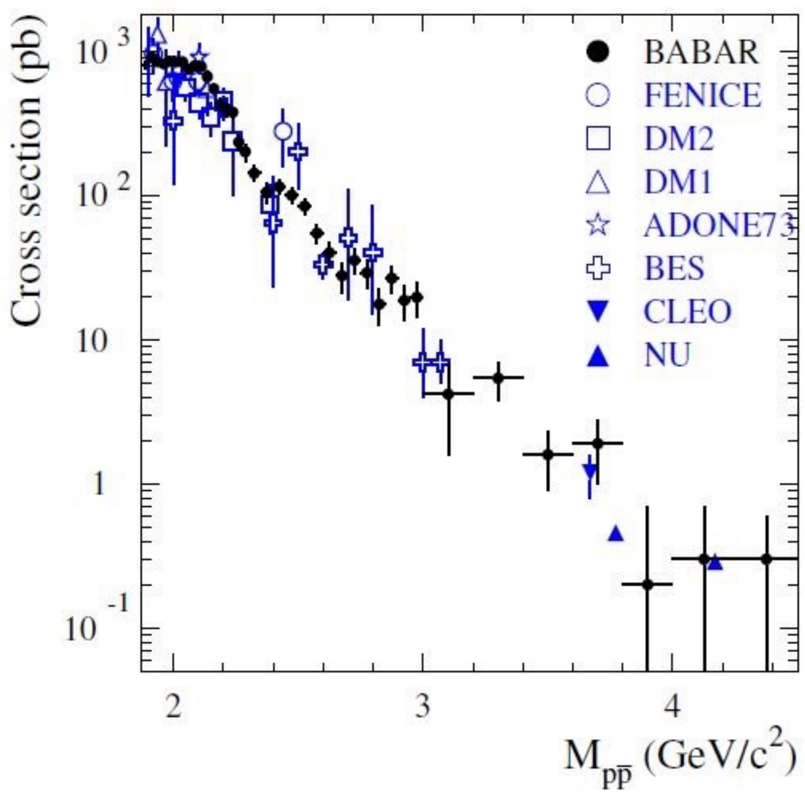
- $|$Form Factor$|^2$ fitted with a vector dominance model ρ, ρ', ρ'', ω'
- ρ' s described by the Gounaris-Sakurai model χ^2/n.d.f = 334/323

Phys.Rev.Lett. 103 (2009) 231801

Phys.Rev.D86 (2012) 032013

pp measurements @ BaBar

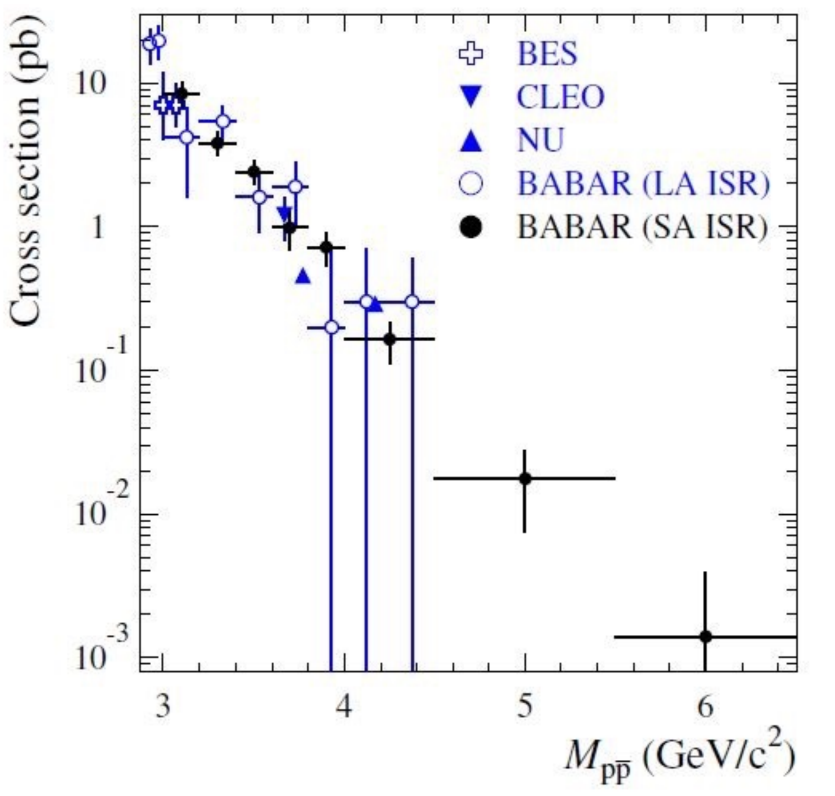
γ detected



Phys. Rev. D87 (2013) 092005

Supersedes our Phys. Rev. D73 (2006) 012005

γ undetected



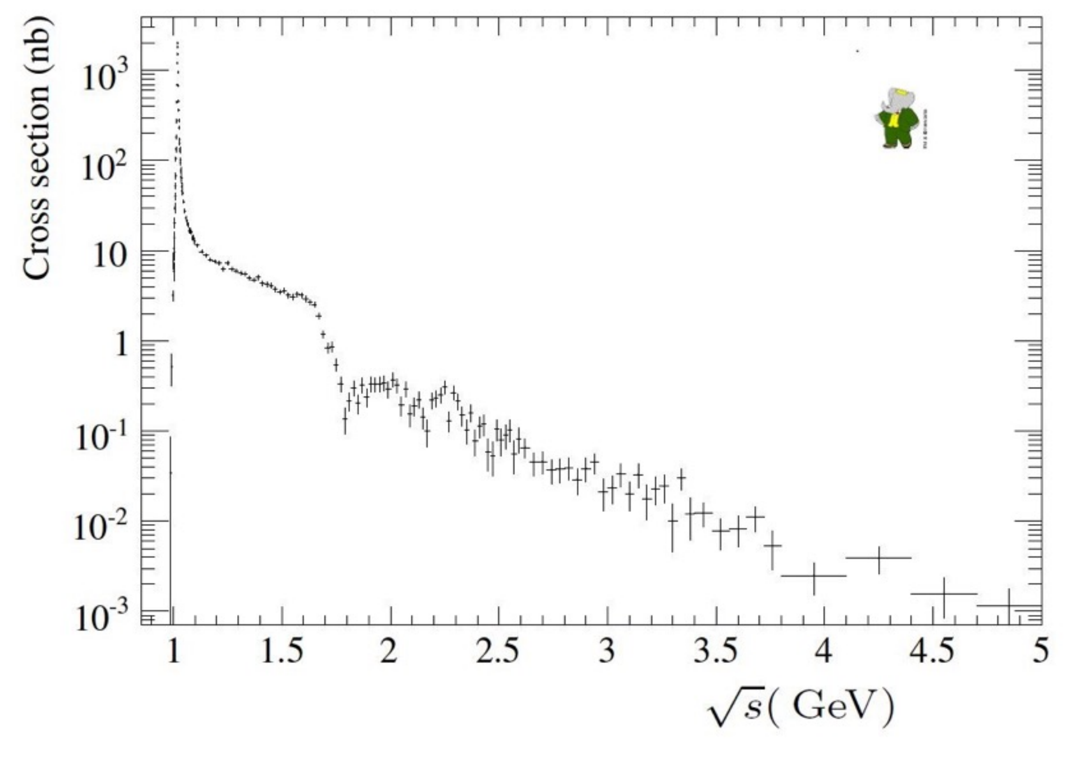
Phys. Rev. D. 88 (2013) 072009

These pp data are not used for a_μ

$e^+e^- \to K^+K^-(\gamma)$ Cross Section

Same method, as for $\pi^+\pi^-$:

- **NLO ISR, data sample, evt generators …**
- **Data driven, step-by-step evaluation of $\varepsilon^{data}/\varepsilon^{MC}$**
- **Resolution unfolding (almost no FSR for kaons) Malaescu arXiv:0907.3791**

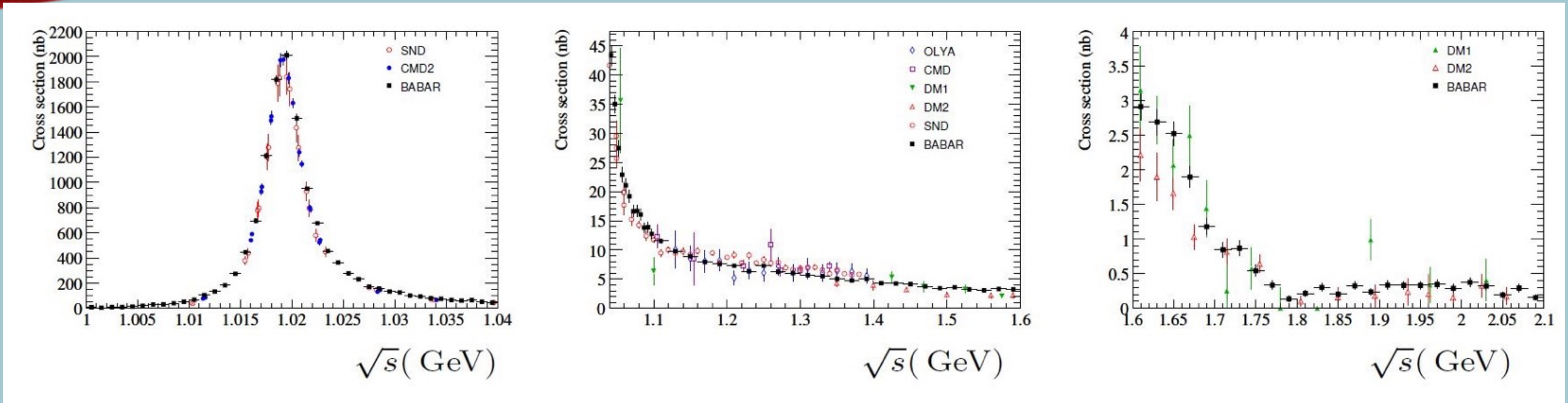


$e^+e^- \to K^+K^-(\gamma)$ bare cross section (including FSR), J/ψ and ψ(2S) removed.

Phys.Rev.D88 (2013) 032013

$e^+e^- \to K^+K^-(\gamma)\gamma$

Zooms and comparisions with previous (direct measurements) results



- **Unprecedented precision in the ϕ region [1.01 - 1.03] GeV, of 7.3×10^{-3}**
- **Dispersion integral:**

$$a_\mu^{KK}\ (\sqrt{s} < 1.8\,\mathrm{GeV}) \qquad \boxed{(22.93 \pm 0.18_{\mathrm{stat}} \pm 0.22_{\mathrm{syst}} \pm 0.03_{\mathrm{VP}})\ 10^{-10}}$$

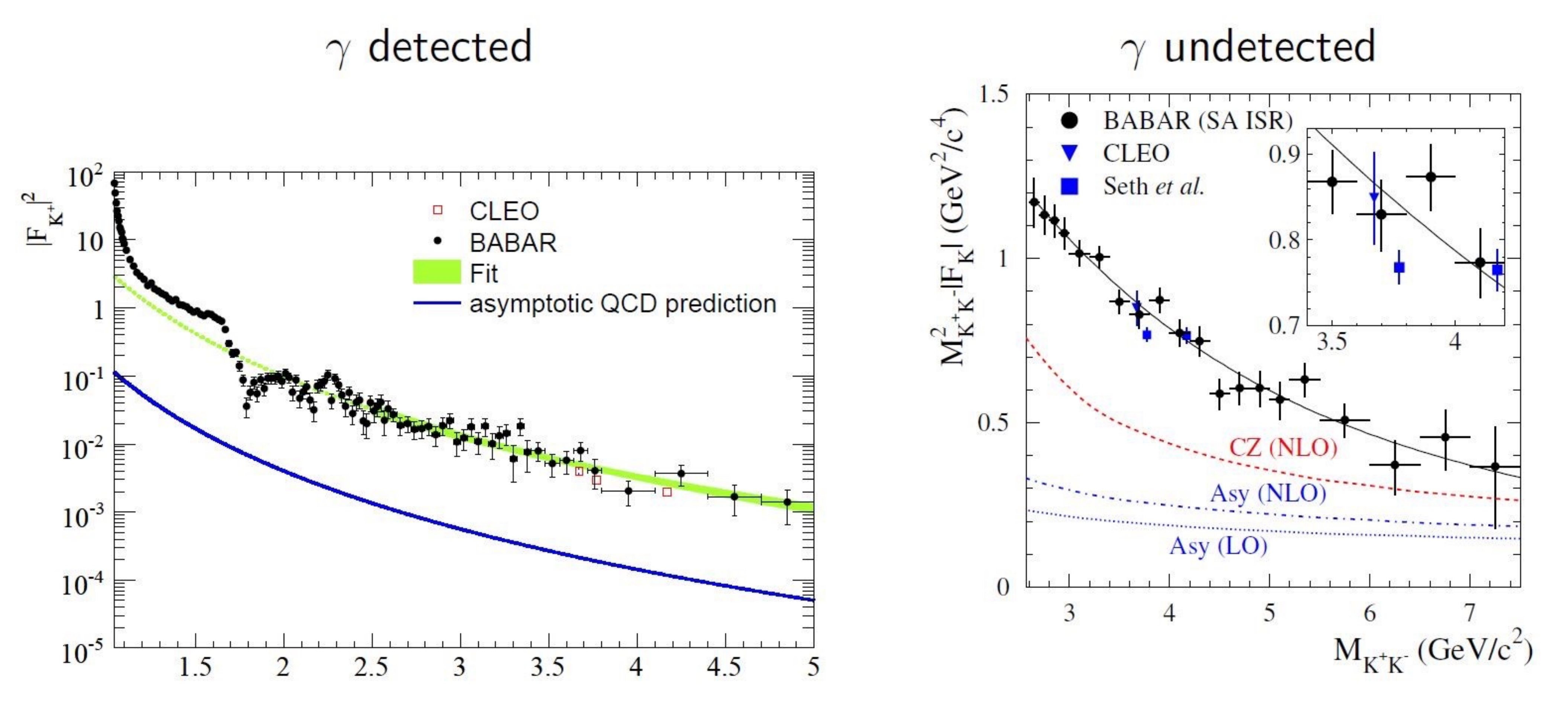
Combination of previous results: **$(21.63 \pm 0.27\ \mathrm{stat} \pm 0.68\ \mathrm{syst})\ 10^{-10}$**

Difference: $+(1.30 \pm 0.79)10^{-10}$ **a 1.6 σ difference**

Precision better than combination of previous results by a factor 2.7

Phys.Rev.D88 (2013) 032013

$e^+e^- \to K^+K^-$ comparison with QCD



Blue: Chernyak-Zhitnitsky NLO pQCD

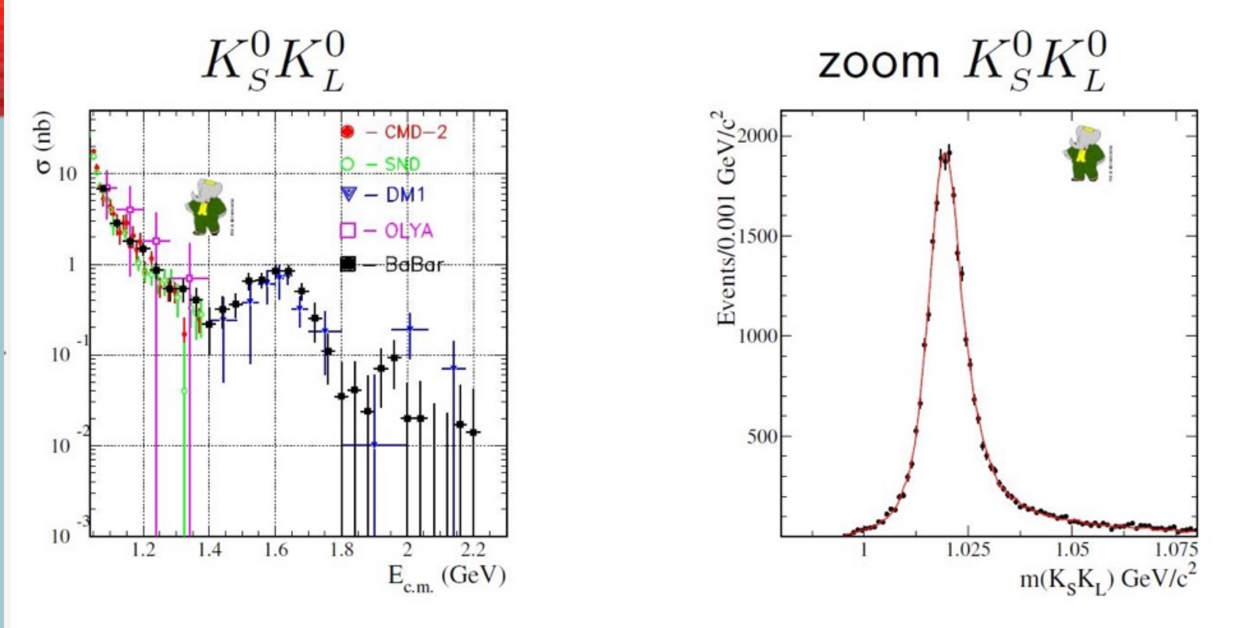
pQCD fails to describe the data.

Black: fit to BaBar data
Red: Chernyak-Zhitnitsky NLO pQCD
pQCD with asymptotic kaon distribution amplitude.

Phys. Rev. D87 (2013) 092005

Phys. Rev. D 92, 072008 (2015)

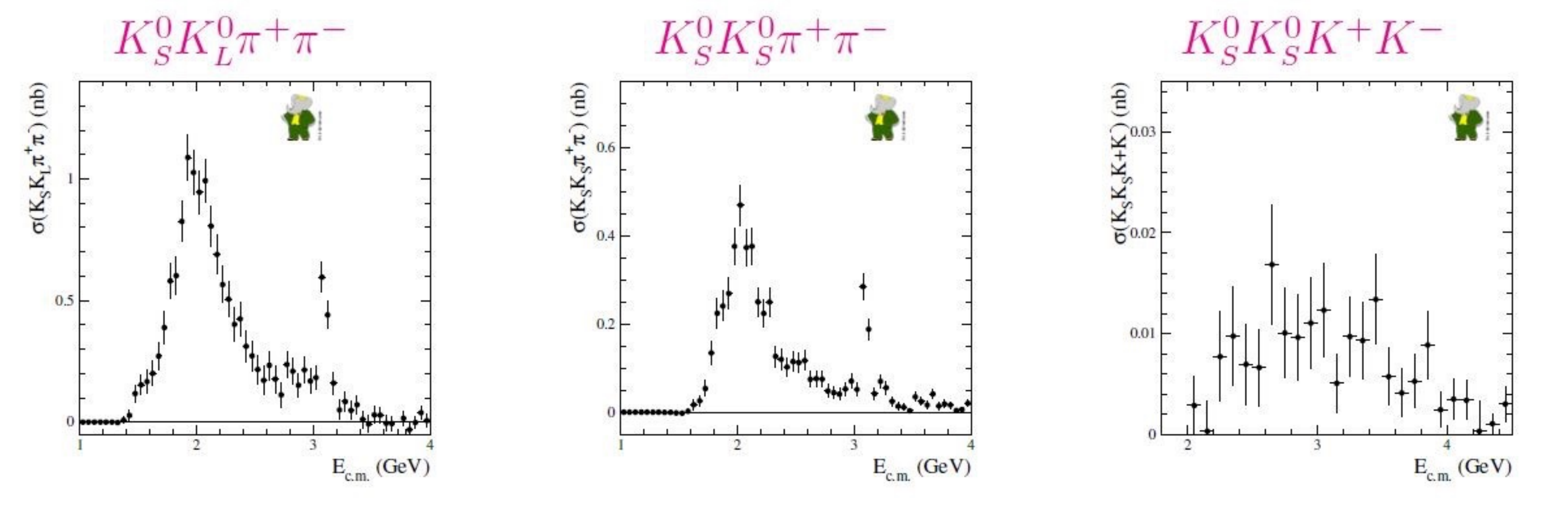
Processes with 2 neutral kaons



BaBar efforts are to remove iso-spin relations from (g-2) calculations.
The way - just to measure all missing channels

Contributions to muon (g-2)/2 (units 10^{-10}):

	$\sqrt{s}$ < 1.8 GeV	$\sqrt{s}$ < 2.0 GeV
Isospin relations	1.35±0.39	3.31±0.58
Sum of BaBar measurements	0.85±0.05	2.41±0.11

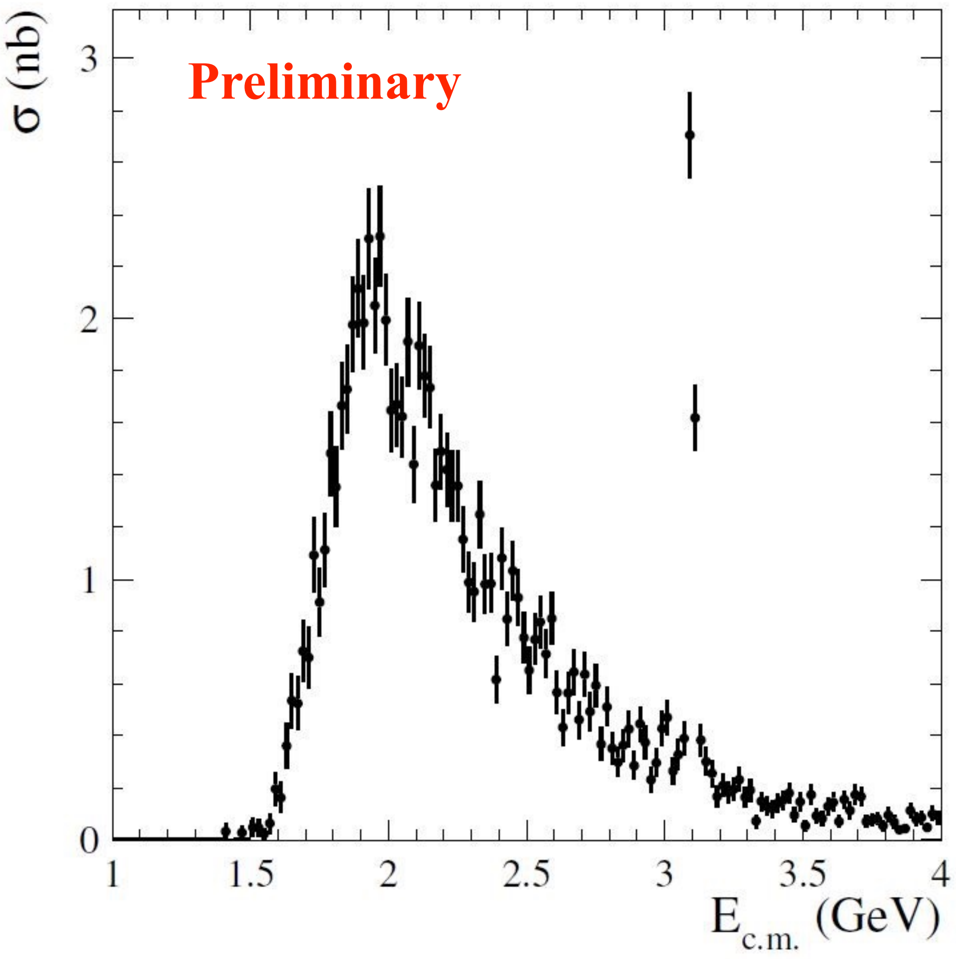


Phys. Rev. D 89, 092002 (2014)

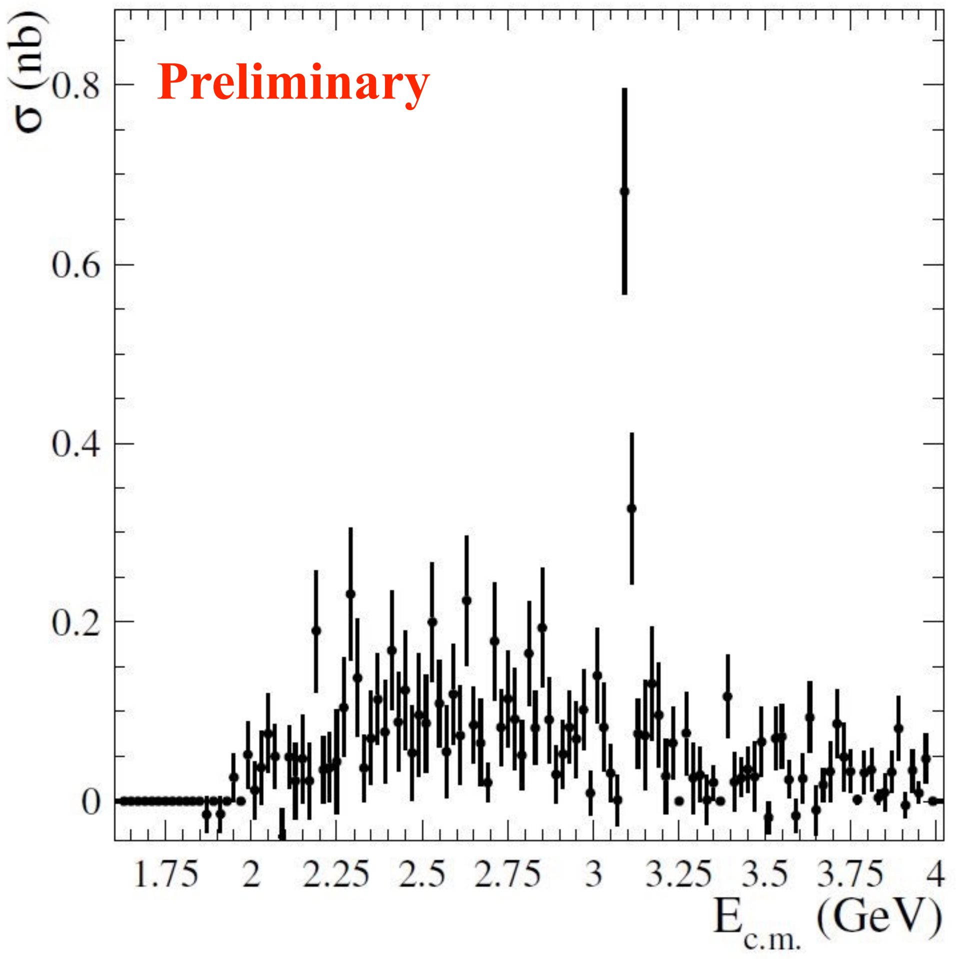
Magenta: first measurements

$e^+e^- \to K_S K^+\pi^-\pi^0/\eta$

$K_S K^+\pi^-\pi^0$



$K_S K^+\pi^-\eta$



$e^+e^- \to K_S K_L \pi^0\pi^0, (K_S K_S \pi^0\pi^0)$
$e^+e^- \to K_S K_L \pi^0(\eta), (K_S K_S \pi^0)$

… are coming soon

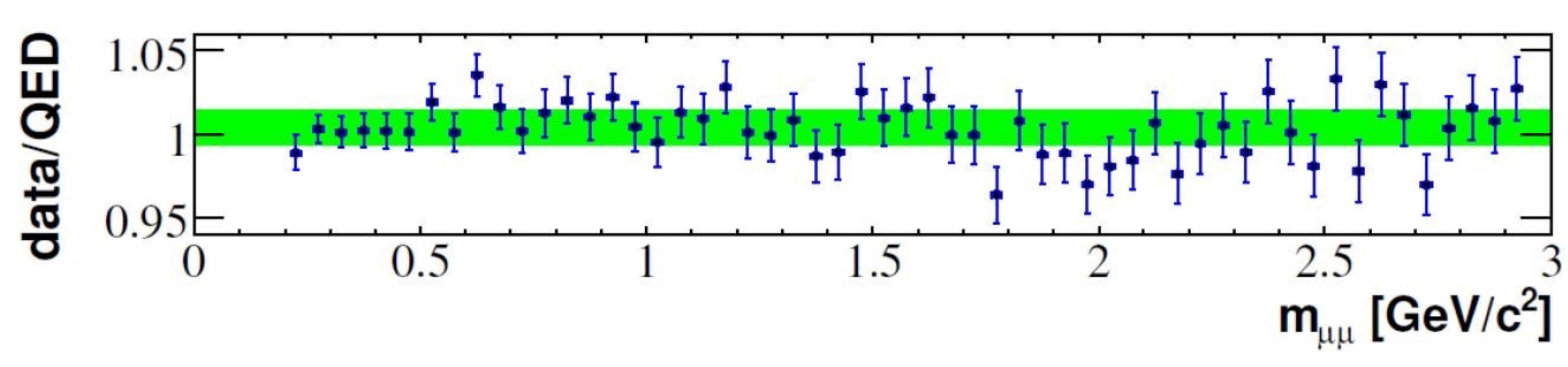
Preliminary, publication in preparation

Conclusion

- BaBar ISR programme well advanced.
  Most precise $\sigma(e^+e^- \to \text{had})$, from threshold to $E_{\text{cut}} = 1.8\,\text{GeV}$ (and above)
- a_μ: SM prediction uncertainty $\approx 5.2 \times 10^{-10}$, $a_\mu^{\text{had,LO}}$- and $a_\mu^{\text{l-by-l}}$- dominated
  BNL measurement uncertainty $\approx 6.3 \times 10^{-10}$
- Study of $\rho - \gamma$ mixing solves former discrepancy between e^+e^- and τ-based $a_\mu^{\pi^+\pi^-,LO}$
  Jegerlehner Eur.Phys.J. C71 (2011) 1632
- Experiment-to-SM discrepancy still at 3 – 4 σ
- New measurements of a_μ are eagerly awaited:
  - Fermilab Nucl.Phys.Proc.Suppl. 225-227 (2012) 277-281 and
  - J-PARC Nucl.Phys.Proc.Suppl. 218 (2011) 242-246

BACKUP

BaBar sanity check: comparision $\mu\mu$ spectra with QED

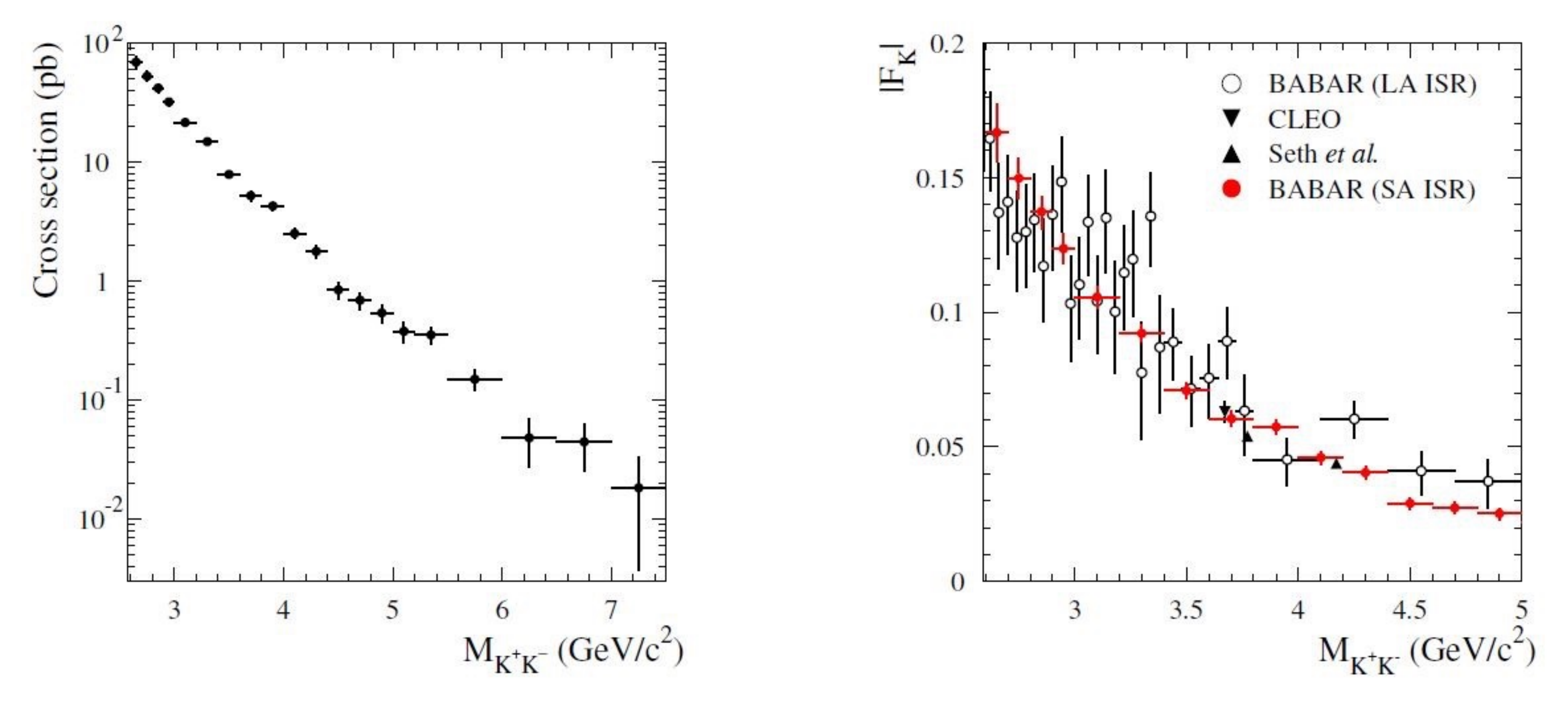


- Here the radiator function and the collider integrated luminosity are needed
- MC simulation corrected for all known MC/data dierences.
  - ISR γ efficiency measured in data, from $\mu\mu$-only reco'ed evts.
  - MC corrected for known NLO deciencies by comparing to PHOKHARA

Good agreement within (0.4 ± 1.1)% ; Dominated by $\mathcal{L}_{ee}$ (± 0.9%)

Phys.Rev.Lett. 103 (2009) 231801
Phys.Rev.D86 (2012) 032013

$e^+e^- \to K^+K^-$, γ undetected



Phys. Rev. D 92, 072008 (2015)

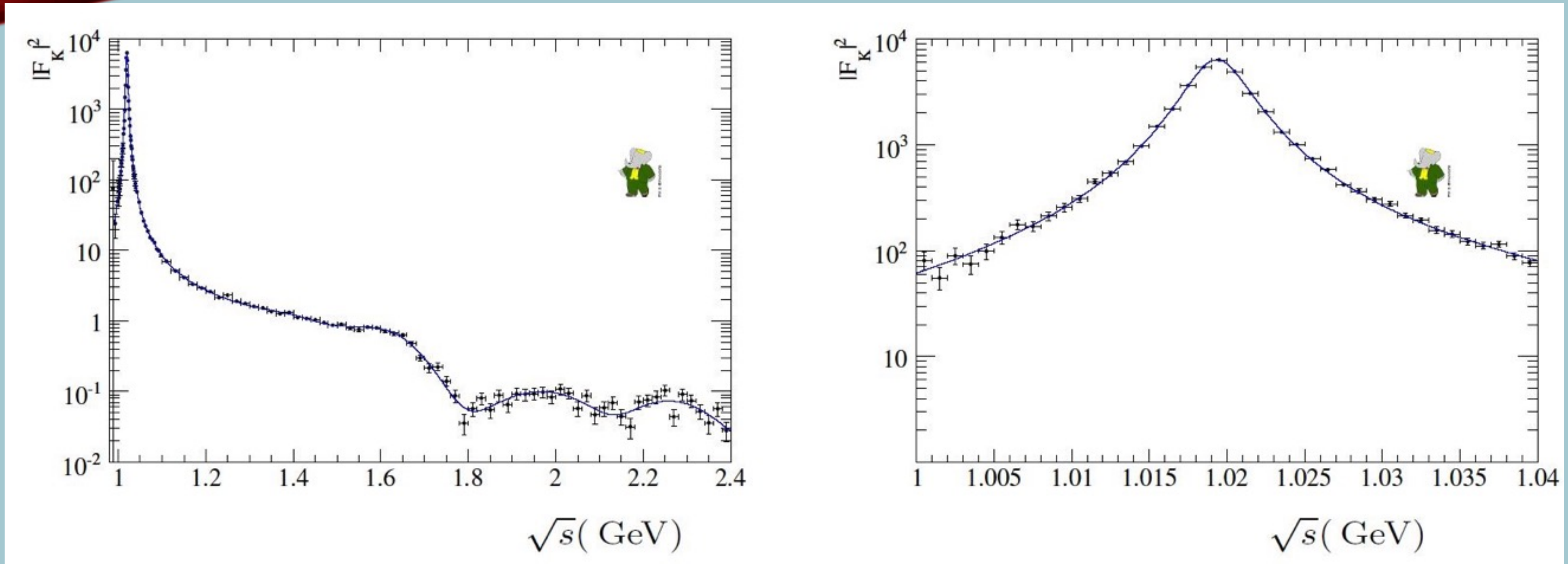
ISR: Recent LO measurements @ BaBar

- LO: hadronic final state ISR γ either detected (E^*_γ > 3 GeV) or undetected
- ISR luminosity computed from MC

	$\mathcal{L}$ fb^{-1}	$\sqrt{s}_{\mathrm{min}}$ GeV	$\sqrt{s}_{\mathrm{max}}$ GeV	
$K^0_S K^0_L$	469		2.3	Phys. Rev. D 89, 092002 (2014)
$K^0_S K^0_L \pi^+\pi^-$			4.0	
$K^0_S K^0_S \pi^+\pi^-$			4.0	
$K^0_S K^0_S K^+K^-$			4.5	
$\bar{p}p$	454		4.5	Phys. Rev. D87 (2013) 092005
$\bar{p}p$	469	3.0	6.5	Phys. Rev. D. 88 (2013) 072009
K^+K^-	469	2.6	8.0	Phys. Rev. D 92, 072008 (2015)
$K^0_S K^+\pi^-\pi^0$	454		4.0	Preliminary
$K^0_S K^+\pi^-\eta$				

Blue: ISR γ undetected **Magenta: first measurements**

$e^+e^- \rightarrow K^+K^-(\gamma)$: VDM Fit of $|F_K(s)|^2$



$F_K = \hat{\phi}/3 + \hat{\rho}/2 + \hat{\omega}/6$, with:

$$\hat{\phi} = \sum_{i=\phi}^{\phi''} a_i BW_i, \quad \hat{\rho} = \sum_{i=\rho}^{\rho'''} a_i BW_i, \quad \hat{\omega} = \sum_{i=\omega}^{\omega'''} a_i BW_i$$

$$\sum_{i=\phi}^{\phi''} a_i = 1, \quad \sum_{i=\rho}^{\rho'''} a_i = 1, \quad \sum_{i=\omega}^{\omega'''} a_i = 1.$$

		This work	PDG	
m_ϕ	=	$1019.51 \pm 0.02 \pm 0.05$	1019.455 ± 0.020	MeV/c^2
Γ_ϕ	=	$4.29 \pm 0.04 \pm 0.07$	4.26 ± 0.04	MeV
$\Gamma_\phi^{ee} \times \mathcal{B}(\phi \to K^+K^-)$	=	$0.6340 \pm 0.0070_{\rm exp} \pm 0.0037_{\rm fit} \pm 0.0013_{\rm cal}$		keV

Phys.Rev.D88 (2013) 032013

BaBar ISR measurements: present situation

- A vigorous campaign that is almost completed @ 10.6 GeV
- radiation: LO: $e^+e^- \to X\gamma$ or NLO: $e^+e^- \to X\gamma(\gamma)$
- γ detected (lower X mass), or γ undetected (higher X mass)
- For most channels, precision improvement by a factor of ≈ 3 wrt previous averages

$K_S^0K^+\pi^-\pi^0$, $K_S^0K^+\pi^-\eta$		Preliminary
K^+K^-	γ undet.	Phys. Rev. D 92, 072008 (2015)
$K_S^0K_L^0$, $K_S^0K_L^0\pi^+\pi^-$, $K_S^0K_S^0\pi^+\pi^-$, $K_S^0K_S^0K^+K^-$		Phys. Rev. D 89, 092002 (2014)
$\overline{p}p$	γ undet.	Phys.Rev. D88 (2013) 7, 072009
$\overline{p}p$		Phys. Rev. D87 (2013) 092005
K^+K^-	NLO	Phys. Rev. D88 (2013) 032013
$\pi^+\pi^-$	NLO	Phys. Rev. Lett. 103 (2009) 231801 Phys. Rev. D86 (2012) 032013
$2(\pi^+\pi^-)$		Phys. Rev. D85 (2012) 112009
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$		Phys. Rev. D86 (2012) 012008
$K^+K^-\eta$, $K^+K^-\pi^0$, $K^0K^\pm\pi^\mp$		Phys. Rev.D77 (2008) 092002
$2(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-)\eta$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-\pi^+\pi^-\eta$		Phys. Rev.D76 (2007) 092005
$K^+K^-\pi^+\pi^-$, $K^+K^-\pi^0\pi^0$, $K^+K^-K^+K^-$		Phys. Rev.D76 (2007) 012008
$\Lambda\overline{\Lambda}$, $\Lambda\Sigma^0$, $\Sigma^0\Sigma^0$		Phys. Rev. D76 (2007) 092006
$3(\pi^+\pi^-)$, $2(\pi^+\pi^-\pi^0)$, $K^+K^-2(\pi^+\pi^-)$		Phys. Rev.D73 (2006) 052003
$\overline{p}p$		Phys. Rev.D73 (2006) 012005
$2(\pi^+\pi^-)$, $K^+K^-\pi^+\pi^-$, $K^+K^-K^+K^-$		Phys. Rev.D71 (2005) 052001
$\pi^+\pi^-\pi^0$		Phys. Rev.D70 (2004) 072004

Magenta: First measurements, Orange: superseded, Green: 454 - 469 fb^{-1}, Cyan: 232 fb^{-1}, Blue: 89 fb^{-1}