# Theoretical studies of $e^+ e^- \rightarrow K^+K^-$ photon reaction

Leonard Leśniak and Michał Silarski

Institute of Physics, Jagiellonian University, Kraków

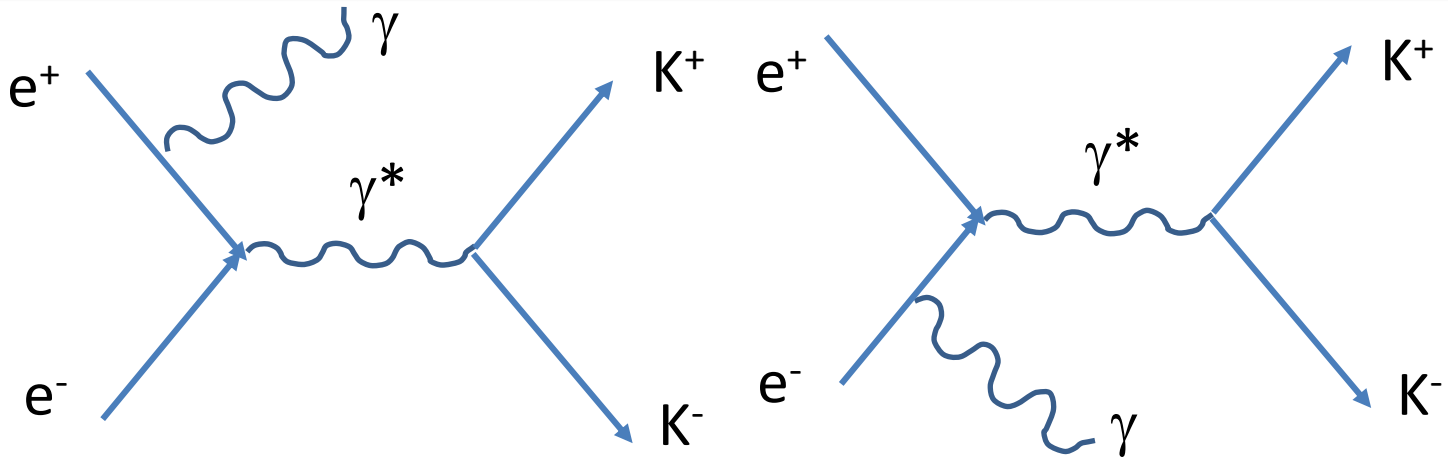
Meson2016

June 3. 2016

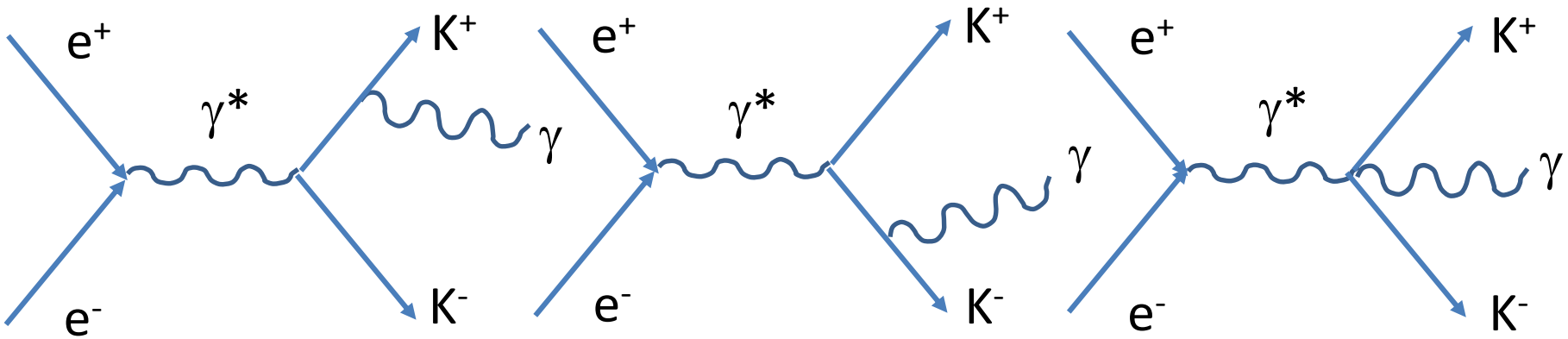
# Motivation

1. The branching fraction for the $\phi(1020)$ meson decay into the **$K^+ K^- \gamma$ channel** is yet **unknown**.
2. The **$\phi(1020) \rightarrow \pi^+ \pi^- \gamma$** branching fraction has been measured:
   $\Gamma(\pi^+ \pi^- \gamma)/ \Gamma_{total} = (4.1 \pm 1.3)\ 10^{-5}$.
3. There are also data for the decay of the $\phi(1020)$ meson into **scalar resonances plus photon**:
   $\Gamma($**$f_0(980)$** $\gamma)/ \Gamma_{total} = (3.22 \pm 0.19)\ 10^{-4}$,
   $\Gamma($**$a_0(980)$** $\gamma)/ \Gamma_{total} = (7.6 \pm 0.6)\ 10^{-5}$ .
4. Both scalar resonances decay to the $K^+ K^-$ pairs, so one should observe the reaction $e^+ e^- \rightarrow K^+ K^- \gamma$. For the decay $\phi \rightarrow K^0 K^0\text{bar}\ \gamma$ only the upper limit $1.9\ 10^{-8}$ is known (KLOE 2009).
5. Measurement of the $e^+ e^- \rightarrow \phi \rightarrow K^+ K^- \gamma$ transition could provide a new information about the **$K^+ K^-$ strong interactions near threshold**.

# Reaction mechanisms

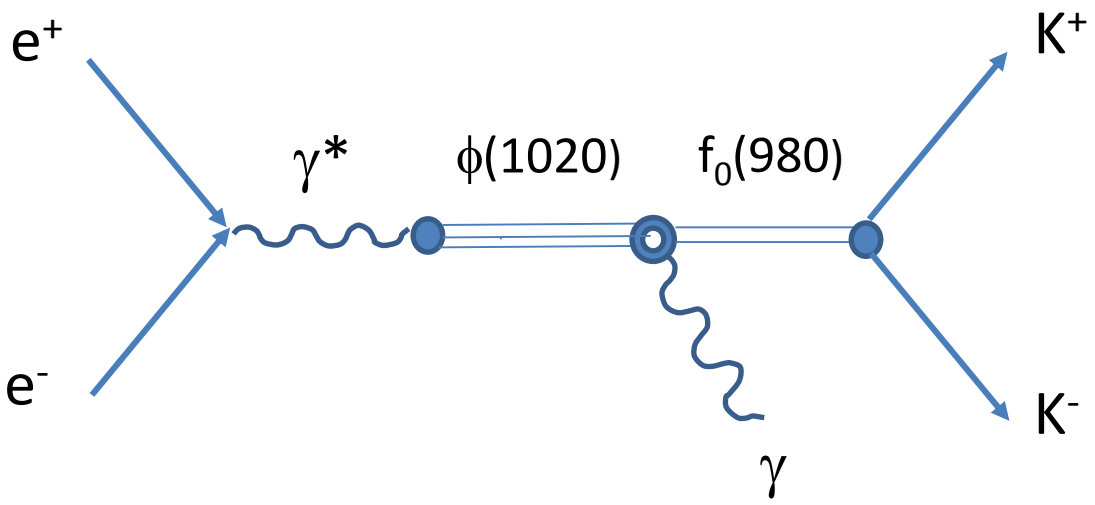



**ISR** – initial state radiation




**FSR** – final state radiation

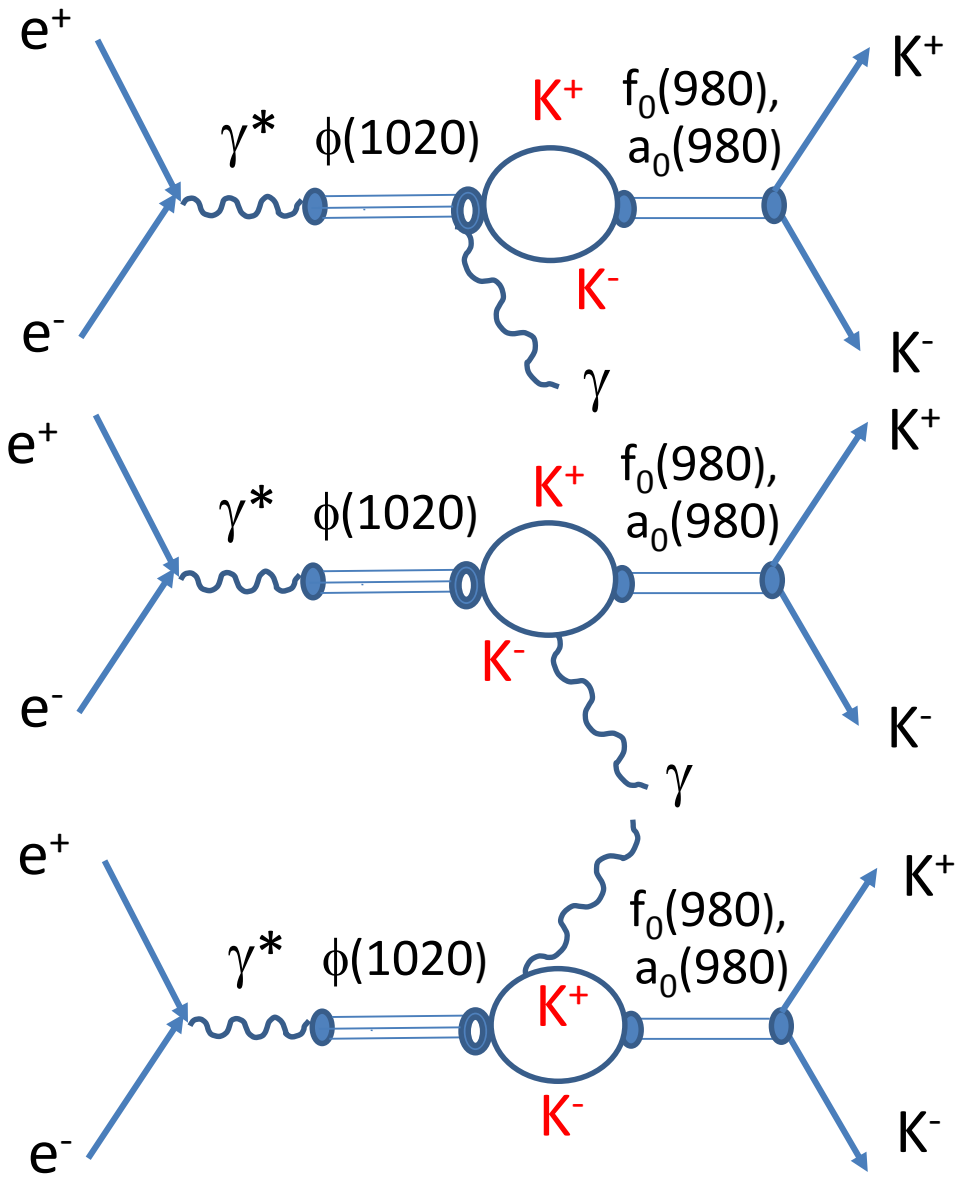
# Reaction mechanisms




**NS** – no-structure model

Ref.: G. Isidori, L. Maiani, M. Nicolaci, S. Pacetti, JHEP 0605 (2006)049.

# Reaction mechanisms




kaon-loop model

Ref.: N. N. Achasov, V. V. Gubin and V. I. Shevchenko, PRD 56(1997)203.

# Differential cross-section

Reaction: $e^+(p_{e^+})\, e^-(p_{e^-}) \rightarrow K^+(p_{K^+})\, K^-(p_{K^-})\, \gamma\,(q)$

$$d\sigma = \frac{(2\pi)^4}{2\sqrt{s(s-4m_e^2)}} |\mathrm{M}|^2 d\Phi_3$$

M - matrix element

$\Phi_3$ – phase space

5 invariants: $\boldsymbol{s} = (p_{e^+} + p_{e^-})^2$

2 momentum transfers: $\mathbf{t} = (p_{e^-} - q)^2$, $\boldsymbol{t_1} = (p_{e^-} - p_{K^-})^2$

$\boldsymbol{m^2} = (p_{K^+} + p_{K^-})^2$

$\boldsymbol{m^2_{K^-\gamma}} = (p_{K^-} + q)^2$

$$\frac{d\sigma}{dm^2\, dm^2_{K^-\gamma}\, dt\, dt_1} = \frac{1}{(2\pi)^4} \frac{|\mathrm{M}|^2}{16\, s\,(s-4m_e^2)(s-m^2)r}$$

$$r = \sqrt{-(t_1 - t_{1min})(t_1 - t_{1max})}$$

m – $K^+K^-$ effective mass, $m_{K^-\gamma}$ – $K^-$ photon effective mass

# Kinematical relations (1)

$\theta_1$ , $\theta_\gamma$ - $K^-$ and photon **polar angles** in the **$e^+e^-$ center-of-mass frame**, z-axis along the $e^-$ momentum

Relations:

$$t_1 \approx m_K^2 - \sqrt{s}\, E_1^l (1 - v_1 \cos \Theta_1)$$

$$t \approx -\sqrt{s}\, \omega^l (1 - \cos \Theta_\gamma)$$

$E_1^l$ , $\omega^l$ are $K^-$ and photon energies, $v_1$ is $K^-$ velocity.

$$E_1^l = \frac{m^2 + m_{K^-\gamma}^2 - m_K^2}{2\sqrt{s}} \qquad \omega^l = \frac{s - m^2}{2\sqrt{s}}$$

# Kinematical relations (2)

Definitions: $\theta_1^*$- polar angle of $K^-$ with respect to the photon axis in the **$K^+$ $K^-$ center- of- mass frame**,

$z = \cos\theta_1^*$

Relation to the $K^- \gamma$ effective mass squared $m^2_{K^-\gamma}$:

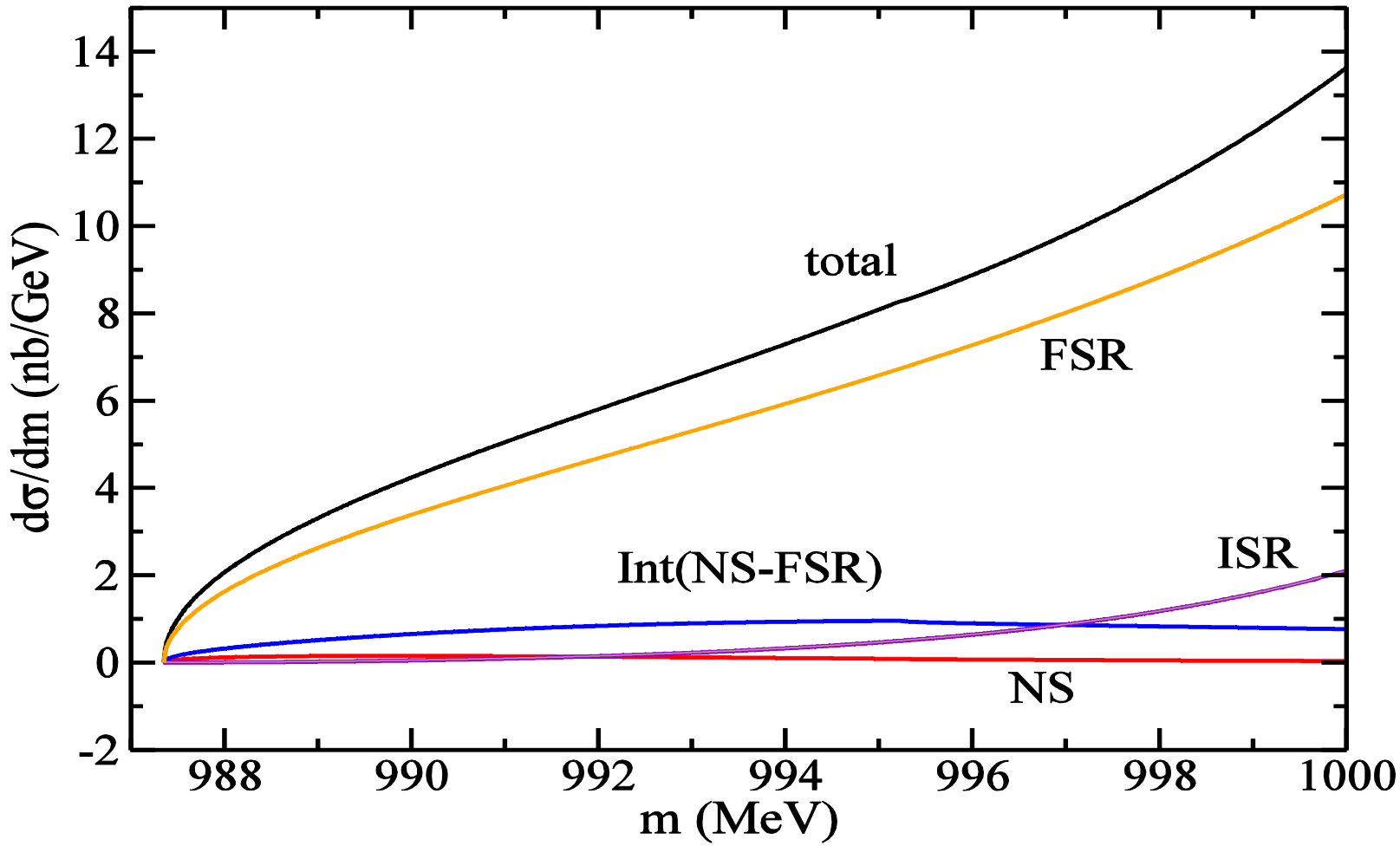
$$m^2_{K^-\gamma} = m_K^2 + \frac{1}{2}(s - m^2)(1 - \boldsymbol{v}\, z)$$

$\boldsymbol{v}$ = $K^-$ velocity in the $K^+$ $K^-$ center of mass frame, $\boldsymbol{v} = \sqrt{1 - \frac{4m_K^2}{m^2}}$

z=+1 corresponds to minimum of $m^2_{K^-\gamma}$, **z= -1** to **maximum** of $m^2_{K^-\gamma}$.

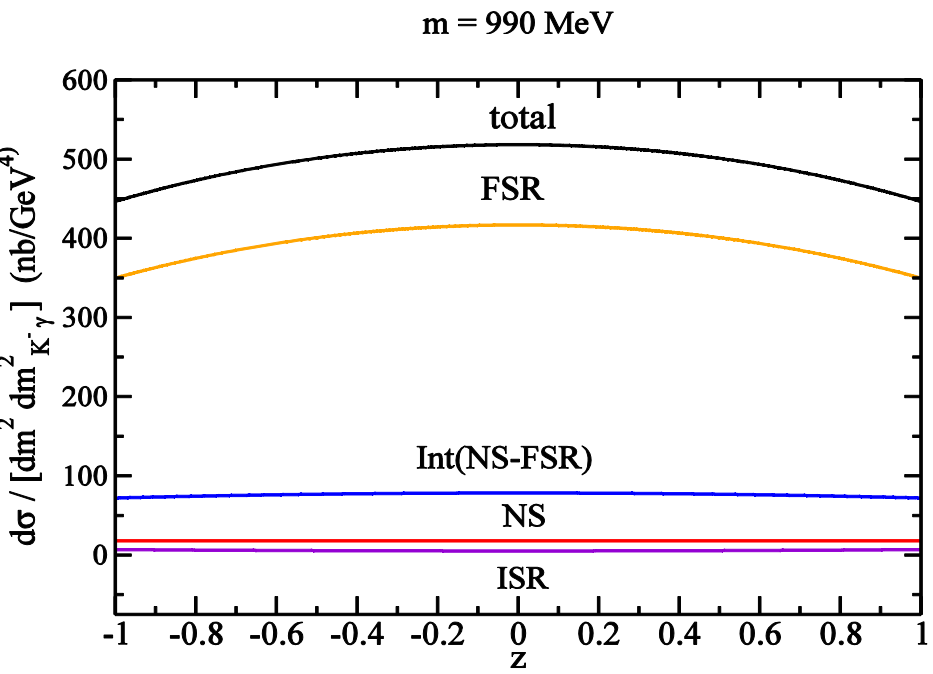
# $K^+K^-$ effective mass distributions

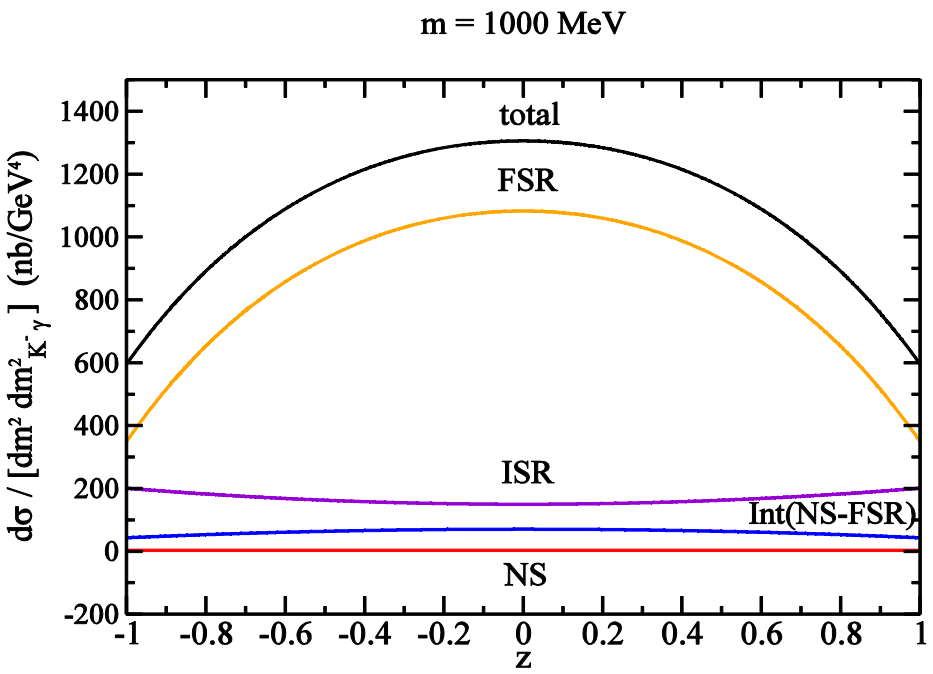
## for $45^0 < \theta_\gamma < 135^0$



# Distributions of the polar angle of $K^-$ with respect to the photon axis in the $K^+$ $K^-$ center of mass frame

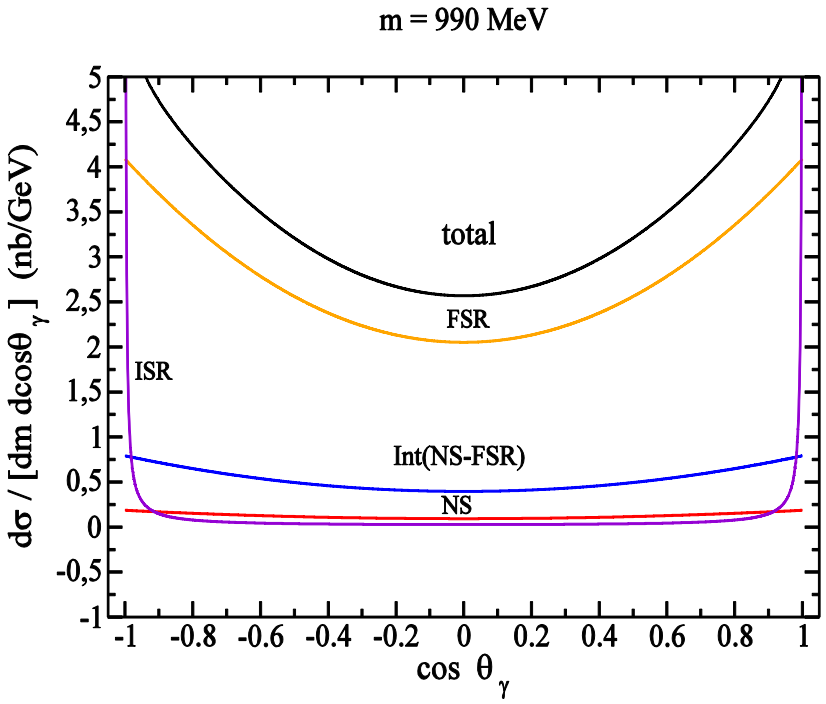
for $45^0 < \theta_\gamma < 135^0$

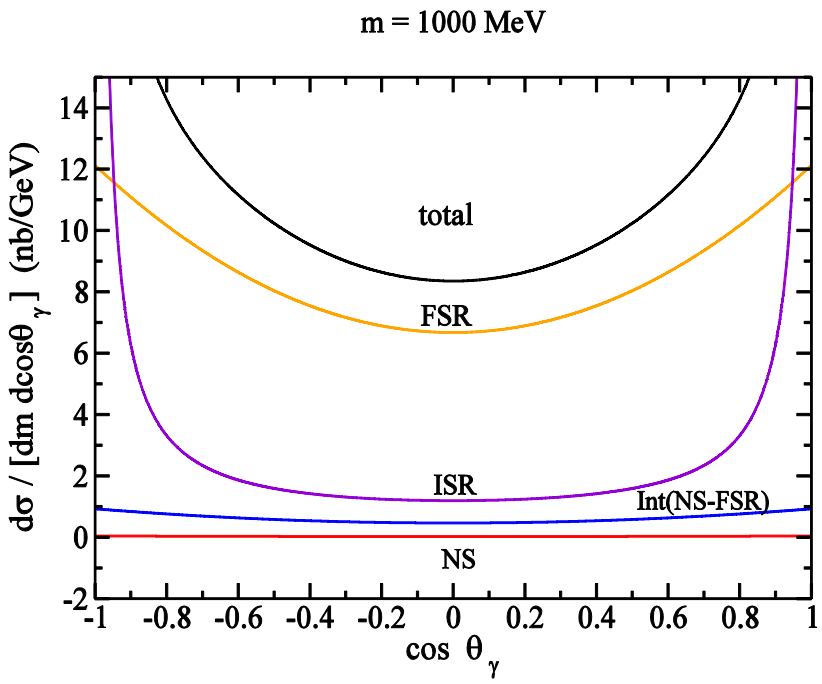



$z = \cos \theta_1^*$

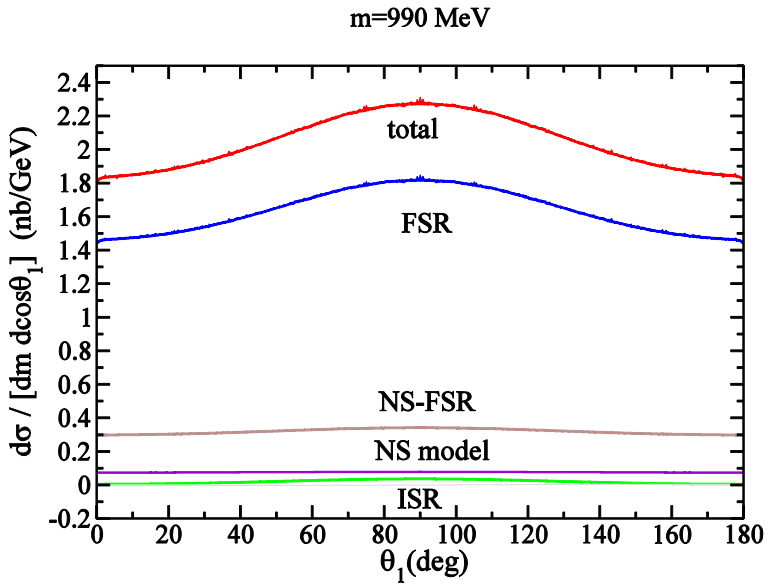
# Photon angular distributions

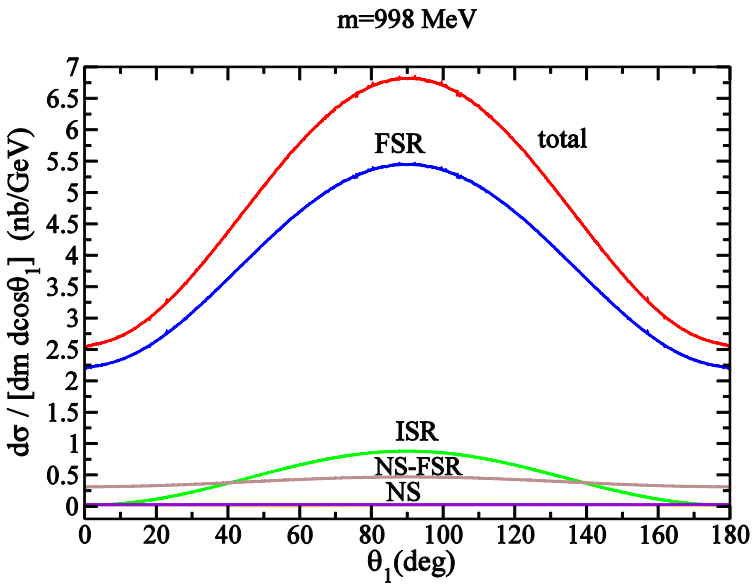


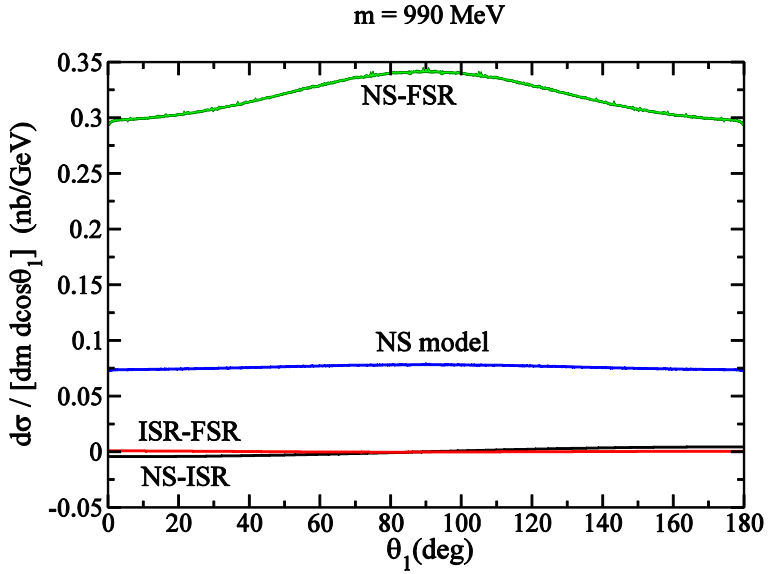



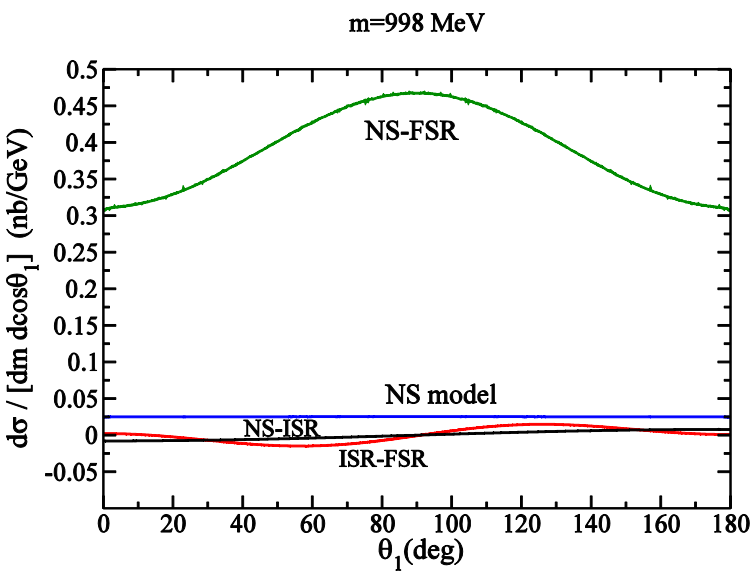
# $K^-$ angular distributions at fixed m

## for $45^0 < \theta_\gamma < 135^0$



# Integrated cross sections

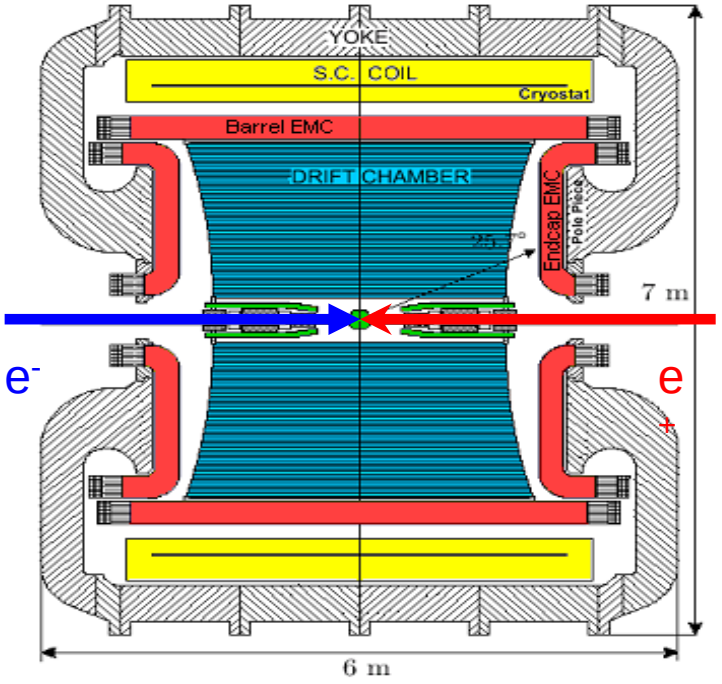
The **cross sections integrated** over the $K^+K^-$ effective mass up to 1009 MeV for two photon angle ranges (units are nanobarns):

| reaction mechanism | $24^0 < \theta_\gamma < 156^0$ | $45^0 < \theta_\gamma < 135^0$ |
|---|---|---|
| **FSR** | 0.330 nb | 0.238 nb |
| **NS** | 0.0020 nb | 0.0014 nb |
| **Int(NS-FSR)** | 0.021 nb | 0.015 nb |
| **ISR** | 0.183 nb | 0.104 nb |
| **total** | 0.536 nb | 0.358 nb |

# Experimental search for $e^+ e^- \rightarrow K^+K^- \gamma$ reaction

- This can be done with the **KLOE** **data** (analysis has already started)

- **Advantages**:
  - very good kaon momentum determination
  - high statistics
- **Problems**:
  - low energy photons (< 32 MeV) $\Rightarrow$ lower efficiency and energy resolution
  - slow transverse momentum tracks for low $K^+K^-$ effective masses




- Challenging measurement

# Experimental implications

**Expected number of events** integrated over the $K^+K^-$ effective mass up to 1009 MeV for two photon angle ranges and for the integrated luminosity of 1.7 $fb^{-1}$:

| reaction mechanism | $24^0 < \theta_\gamma < 156^0$ | $45^0 < \theta_\gamma < 135^0$ |
|---|---|---|
| **FSR** | $5.6\ 10^5$ | $4.0\ 10^5$ |
| **NS** | $3.4\ 10^3$ | $2.4\ 10^3$ |
| **Int(NS-FSR)** | $3.6\ 10^4$ | $2.5\ 10^4$ |
| **ISR** | $3.1\ 10^5$ | $1.8\ 10^5$ |
| **total** | $9.1\ 10^5$ | $6.1\ 10^5$ |

# Summary

The results of the theoretical calculations presented here can be used in experimental analyses of the $e^+ e^- \rightarrow K^+ K^- \gamma$ reaction.

They can also serve in determination of:

the $K^+ K^-$ **threshold parameters** of the strong interaction amplitudes

and in a better specification of the **properties** of the **scalar meson resonances** $f_0(980)$ and $a_0(980)$.