News from the Lattice

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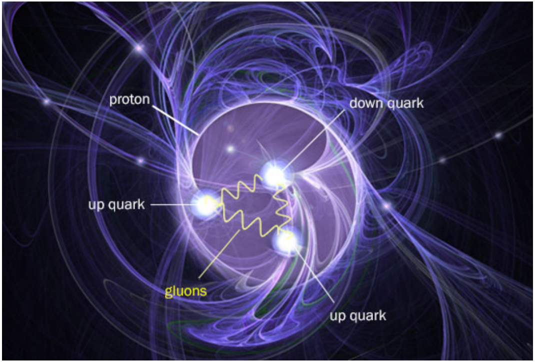
Hadrons' internal structure



Credit: *Brookhaven National Lab website*

Experiment: HERA, LHC, Electron-Ion Collider study hadron structure functions and try to discover the origin of mass,
Theory: Quantum Chromodynamics (QCD) is the theory describing the interations of quarks and gluons.

Lattice Quantum Chromodynamics

Theoretical 'experiments'

- ✓ 'experiments' with objects present in a quantum field theory
- ✓ they follow the rules of quantum field theory
- ✓ no model assumptions
- ✓ once you devise an 'experiment', you can make 'measurements'
- × you have to deal with systematic effects
- × several limitations reduce the final precision

How do we do this?

$\Rightarrow$ numerical simulations on a computer

Lattice Quantum Chromodynamics

Lattice Quantum Chromodynamics: what is it?

- space-time is discretized $\Rightarrow$ finite dimensional problem fits into a computer,
- equations of QCD are solved numerically,
- the only practically available *ab initio* approach.

Lattice Quantum Chromodynamic: how does it work?

- *physical observable* = very high dimensional integral,
- Monte Carlo integration with Boltzmann probability distribution,
- Markov chains to generate samples = configurations,
- many different observables can be estimated using one ensemble of configurations.

Do it yourself!

$\Rightarrow$ design your 'experiment', take your favorite gauge configurations and do the measurements!

Limitations and solutions

- × physical pion mass
  - ✓ developement of multigrid algorithms reduced the cost of simulations
  - ✓ the cost became nearly constant as a function of the quark mass, it still grows as a function of the volume

- × growing autocorrelations
  - ✓ due to topological charge trapping autocorrelations grow substantially towards the continuum limit
  - ✓ open boundary conditions allow to reach lattice spacings below 0.05 fm with managable autocorrelation times

Lattice Quantum Chromodynamics

× signal-to-noise ratio problem

- ✓ momentum smearing creates hadron's wavefunction with a substantially better overlap with the wavefunction of a moving particle

× structure functions as light-cone correlations

- ✓ on an Euclidean lattice only space-like correlations can be constructed, hence up to recently only moments of structure functions were available
- ✓ Ji's proposal allows to extract the full x-dependent function from space-like correlations from Monte Carlo simulations
- ✓ several practical implementations of Ji's idea have been published

Example: Pion distribution amplitude

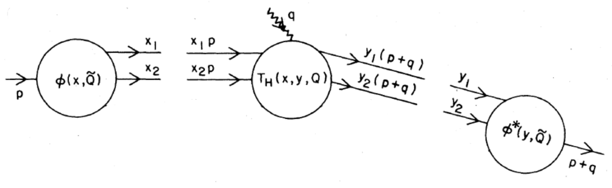
Definition

Pion DA is the quantum amplitude that the pion moving with momentum P is built of a pair of quark and antiquark moving with momentum xP and $(1-x)P$ respectively. Introduced by Radyushkin in '77 and Brodsky in '80. It is process independent, scheme and scale dependent.

Relevance: pion photoproduction

Two off-shell photons provide the hard scale. Transition form factor measured most recently experimetaly by BaBar '09 and Belle '12.

Factorization in QCD



Example: Pion distribution amplitude

Definition: formulae

$$\langle 0|\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n)|\pi(p)\rangle =$$

$$= i f_\pi (p\cdot n)\int_0^1 dx e^{-i(z_1 x + z_2(1-x))p\cdot n}\phi_\pi(x,\mu^2)$$

Neglecting isospin breaking effects $\phi_\pi(x)$ is symmetric under the interchange of momentum fraction $x \to (1-x)$

$$\phi_\pi(x,\mu^2) = \phi_\pi(1-x,\mu^2)$$

Moments of the momentum fraction difference $\xi = x-(1-x)$ are interesting

$$\langle \xi^n \rangle = \int_0^1 dx (2x-1)^n \phi_\pi(x,\mu^2)$$

$$\phi_\pi(x,\mu^2) = 6u(1-u)\left[1+\sum_n a_{2n}^\pi(\mu) C_{2n}^{3/2}(2u-1)\right]$$

Moments and local operators

The nonlocal operator can be Taylor expanded and expressed in terms of local operators with derivatives

$$\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n) = \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!l!} n^\rho n^{\mu_1} \ldots n^{\mu_{k+l}} \mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{i+1}}$$

where

$$\mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{k+l}} = \bar{d}(0)\overleftarrow{D}_{(\mu_1} \ldots \overleftarrow{D}_{\mu_k} \overrightarrow{D}_{\mu_{k+1}} \ldots \overrightarrow{D}_{\mu_{k+l}} \gamma_{\rho)}\gamma_5 u(0)$$

Consequently,

$$i^{k+l}\langle 0|\mathcal{M}^{(k,l)}_{\rho,\mu_1,\ldots,\mu_{k+l}}|\pi(p)\rangle = if_\pi p_{(\rho} p_{\mu_1} \ldots p_{\mu_{k+l})}\langle x^l(1-x)^k\rangle$$

2nd moment of the pion distribution amplitude

Lattice operators for the 2nd moment

Two operators local are relevant

$$\mathcal{O}^{-}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} - 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

and

$$\mathcal{O}^{+}_{\rho\mu\nu}(x) = \bar{d}(x)\left[\overleftarrow{D}_{(\mu}\overleftarrow{D}_{\nu} + 2\overleftarrow{D}_{(\mu}\overrightarrow{D}_{\nu} + \overrightarrow{D}_{(\mu}\overrightarrow{D}_{\nu}\right]\gamma_{\rho)}\gamma_5 u(x)$$

We estimate the following correlation functions

$$C_\rho(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}_\rho(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

$$C^{\pm}_{\rho\mu\nu}(t,\mathbf{p}) = a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\mathbf{x}} \langle \mathcal{O}^{\pm}_{\rho\mu\nu}(\mathbf{x},t) J_{\gamma_5}(0)\rangle$$

2^{nd} moment of the pion distribution amplitude

Lattice operators for the 2^{nd} moment

From the correlation functions we construct ratios

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$

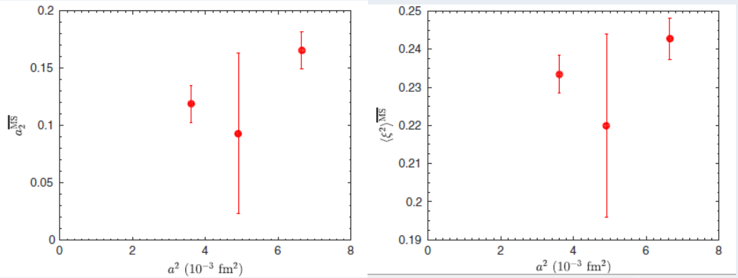
which exhibit plateaux and which we fit to extract the values $R^{\pm}_{\rho\mu\nu,\sigma}$. Finally,

$$\begin{aligned}
\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} &= \zeta_{11} R^{-} + \zeta_{12} R^{+}, \\
a_2^{\overline{\mathrm{MS}}} &= \frac{7}{12}\left[5\zeta_{11} R^{-} + (5\zeta_{12} - \zeta_{22}) R^{+}\right]
\end{aligned}$$

where ζ_{ij} are renormalization constants estimated nonperturbatively.

2^{nd} moment of the pion distribution amplitude

State-of-the-art calculation (Braun et al., PRD 2015)

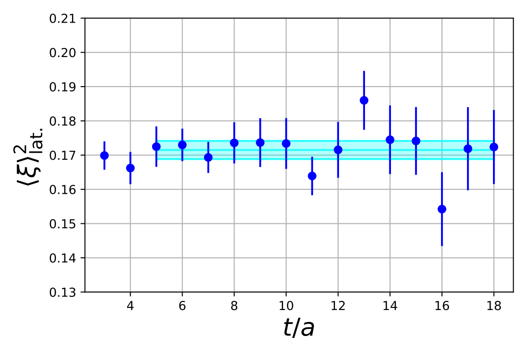


Lattice spacing dependence of $a_2^{\overline{\text{MS}}}$ and $\langle \zeta^2 \rangle^{\overline{\text{MS}}}$ for $m_\pi \approx 280$ MeV. Statistical accuracy does not allow for a reliable continuum extrapolation.

2^{nd} moment of the pion distribution amplitude

Plateau fit example

$$R^{\pm}_{\rho\mu\nu,\sigma}(t,\mathbf{p}) = \frac{C^{\pm}_{\rho\mu\nu}(t,\mathbf{p})}{C_{\sigma}(t,\mathbf{p})}$$



We use momentum smearing (Bali *et al.* '16) to reduce signal-to-noise problem (Braun *et al.* '17).

2^{nd} moment of the pion distribution amplitude

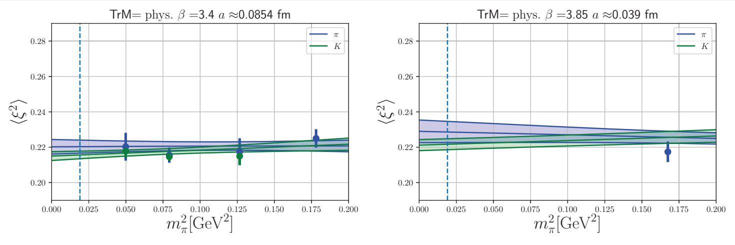
Continuum extrapolation: combined fit

$$\langle \xi^2 \rangle_\alpha = \left(1 + c_0 a + c_1 a \overline{M}^2 + c_2^\alpha a \delta M^2\right) \langle \xi^2 \rangle_0 + \overline{A} \overline{M}^2 - 2 \delta A \delta M^2$$

with

$$\overline{M}^2 = \frac{2 m_K^2 + m_\pi^2}{3}, \qquad \delta M^2 = m_K^2 - m_\pi^2$$

$\overline{A}$, δA combinations of low energy constants $\Rightarrow$ 7 fit parameters.



Full x-dependence of the pion distribution amplitude

Ji's proposal:

- X. Ji (Phys. Rev. Lett. 110, 262002 (2013)) proposed to recover the light-cone definition of SF from purely space-like correlations with hadron moving at large momentum calculable in Lattice QCD,
- In the infinite momentum limit one recovers the light-cone distributions,
- In the framework of Large Momentum Effective Theory one can systematically calculate corrections.

Applications

- unpolarized nucleon parton distribution functions
- polarized nucleon parton distribution functions
- pion distribution amplitude

Full x-dependence of the pion distribution amplitude

Pion quasi-distribution amplitude

Start with a bare matrix element

$$\tilde{\phi}(x,P_z) = \frac{i}{f_\pi}\int \frac{dz}{2\pi} e^{-i(x-1)P_z z}\langle \pi(P)|\bar{\psi}(z)\Gamma W(z)\psi(0)|0\rangle$$

with $\langle \pi(P)|$ describes a pion with momentum P in the direction of the Wilson line $W(z)$, typically

$$P = (P_0,0,0,P_3) \qquad\qquad z = (0,0,0,z)$$

Pion distribution amplitude

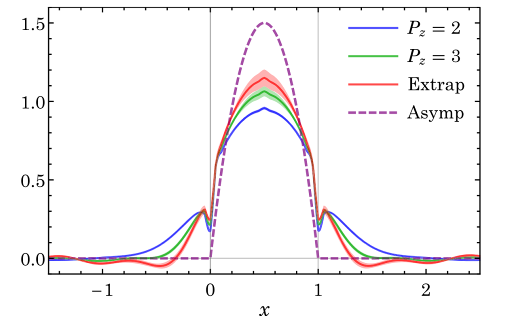
We match the effective theory to QCD perturbatively

$$\tilde{\phi}(x,\Lambda,P_z) = \int_0^1 dy Z_\phi(x,y,\Lambda,\mu,P_z)\phi(y,\mu) + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{m_\pi^2}{P_z^2})$$

with $Z_\phi(x,y,\Lambda,\mu,P_z)$ the matching factor between the LaMET and QCD, and $\Lambda = \frac{\pi}{a}$ being the ultraviolet lattice cut-off.

Full x-dependence of the pion distribution amplitude

Results (J.-H. Zhang *et al*, PRD 17)



Pion distribution amplitude at $\mu = 2$ GeV for $P_z = 4\frac{\pi}{L}$ and $P_z = 6\frac{\pi}{L}$ and extrapolation to infinite-momentum limit along with the asymptotic form $6x(1-x)$.

Full x-dependence of the pion distribution amplitude

Problematic Wilson line (Braun *et al*, '18)

One can replace the Wilson line by a fermionic line. The approach was tested for the pion DA on a single ensemble.

$$T(p\cdot z, z^2) = \langle 0|[\bar{u}q](z/2)[\bar{q}\gamma_5 u](-z/2)|\pi(p)\rangle$$

where the brackets [] denote operator renormalization in $\overline{\mathrm{MS}}$ scheme and the renormalization scale is fixed to $\mu = 2/\sqrt{-z^2}$.
Using continuum perturbation theory and standard QCD factorization techniques one gets

$$T(p\cdot z, z^2) = F_\pi \frac{p\cdot z}{2\pi^2 z^4}\Phi(p\cdot z)$$

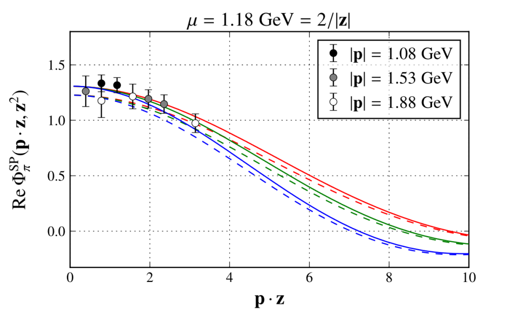
and

$$\Phi(p\cdot z) = \int_0^1 du e^{i(u-\frac{1}{2})(p\cdot z)}\phi_\pi(u)$$

Results

For illustration we consider three models for the pion DA

$$\phi_\pi^1(u) = 6u(1-u)$$
$$\phi_\pi^2(u) = \frac{8}{\pi}\sqrt{u(1-u)}$$
$$\phi_\pi^3(u) = 1$$



Conclusions

- ✓ Lattice QCD provides non-perturbative, *ab initio* results for hadron structre functions
- ✓ The tools is our toolbox are becoming more taylored to our needs - the estimates are more and more precise
- ✓ New tools appear - new observables can be estimated

- systematic effects for the moments of pion distribution amplitude under control
- new method to extract the full x-dependence of structure functions gives good qualitative results
- further studies needed to push that to a quantitative level

Thank you for your attention!