The Role of Mesons in Muon g-2

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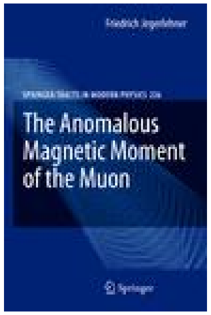
15th International Workshop on Meson Physics,
KRAKÓW, POLAND, 7th - 12th June 2018



New updated and expanded edition
Jegerlehner F., The Anomalous Magnetic Moment of the Muon.
Springer Tracts in Modern Physics, Vol 274 (2017). Springer, Cham
(693 pages on one number to 8 digits)

Muon $g-2$ is one oft the most precisely measured quantities in particle physics. A very precise measurement confronts very a precise prediction, revealing a 3 to 4 σ discrepancy of the SM prediction. It is pure loop physics, testing virtual quantum fluctuations in depth. New experiments expected to reach 140 ppb accuracy likely will enhance the significance of the deviation substantially!

Outline of Talk:

- Introduction
- To be improved: Leading Hadronic=Mesonic Effects
- Hadronic Vacuum Polarization (HVP) – Data & Status
- Hadronic Light-by-Light (HLbL): Setup and Problems
- Theory vs experiment: do we see New Physics?
- Prospects

This Talk:

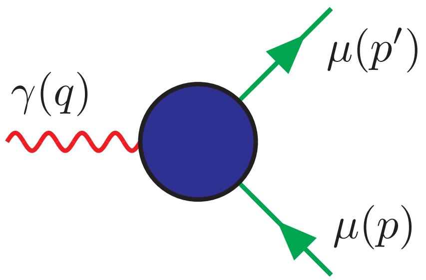
hadronic=mesonic: HVP=MVP, HLbL=MLbL to 98%

Introduction

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} \; ; \quad g_\mu = 2\,(1 + a_\mu)$$

Dirac: $g_\mu = 2 \quad , \quad a_\mu = \frac{\alpha}{2\pi} + \cdots$ muon anomaly



Electromagnetic Lepton Vertex

$$= (-\mathrm{i}e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \mathrm{i}\,\frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2)\right] u(p)$$

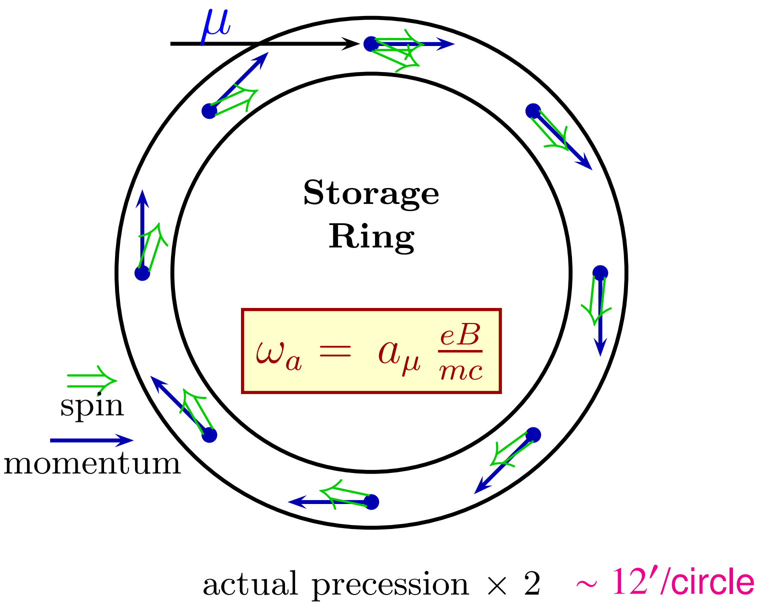
$$F_1(0) = 1 \; ; \quad F_2(0) = a_\mu$$

the simplest object
you can think of
in the static limit

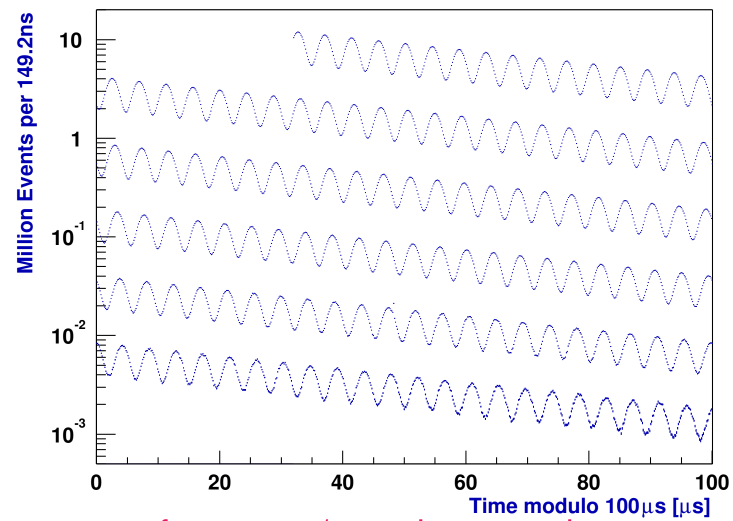
a_μ responsible for the Larmor (spin) precession $\Rightarrow$need polarized muons orbiting
Shoot protons on target producing pions which decay by P violating weak process

$$\pi^+ \to \mu^+ \nu_\mu \; ; \quad \mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$



$\sim 12'$/circle



frequency/counting experiment

Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy $\sim$ 3.1 GeV

$$\vec{\omega}_a = \frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\vec{\beta}\times\vec{E}\right]_{\text{at "magic }\gamma\text{"}}^{E\sim 3.1\,\text{GeV}} \simeq \frac{e}{m}\left[a_\mu \vec{B}\right]$$

CERN, BNL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

Crucial: 3.1 GeV muons life-time in lab frame $\gamma\tau_\mu$ 29 times longer!

$$a_\mu^{\rm exp} = (11\,659\,209.1 \pm 5.4 \pm 3.3[6.3]) \times 10^{-10} \text{ BNL updated}$$

To come – two complementary experiments: magic γ improved and $\vec{E} = 0$ novel

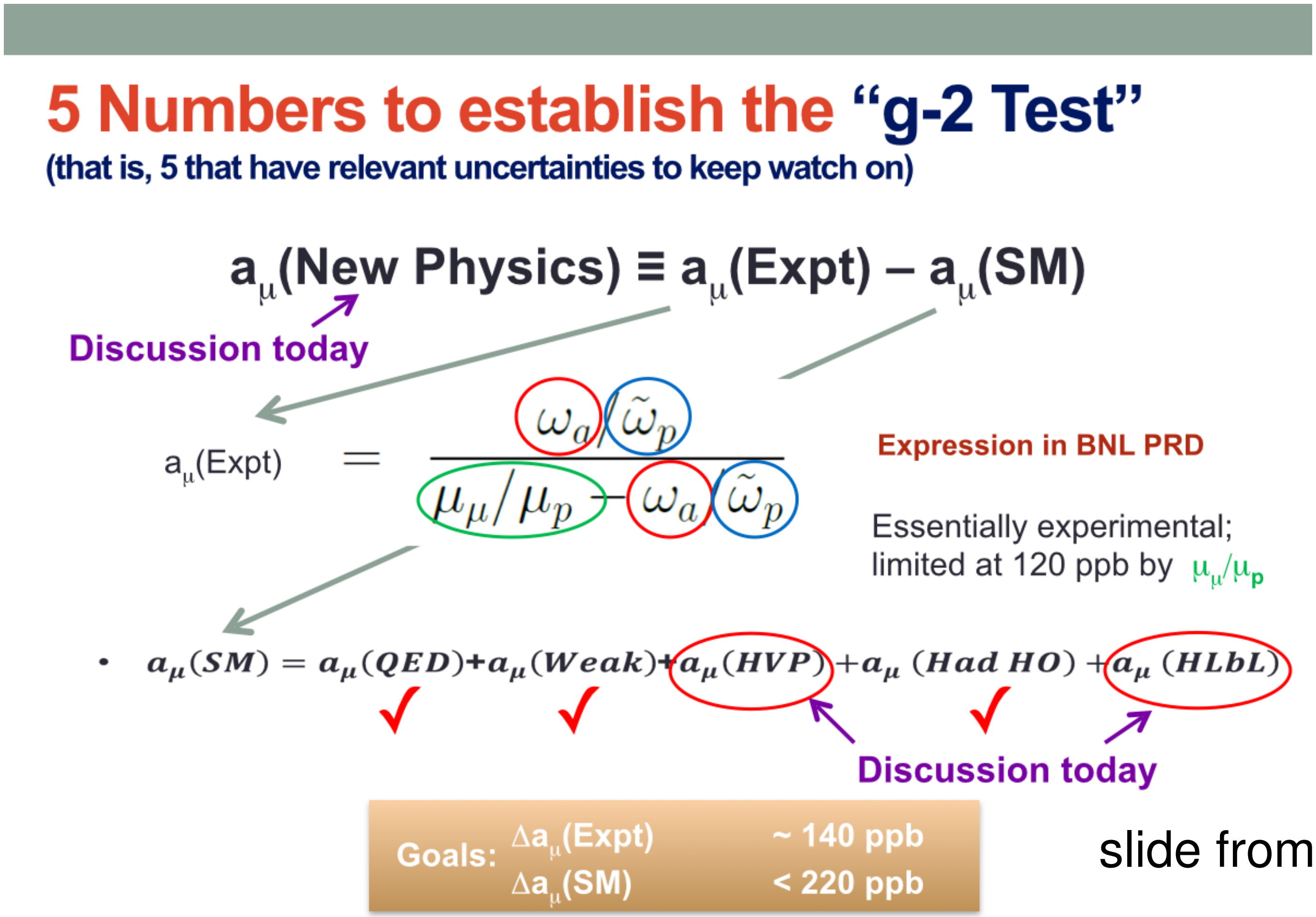
New muon $g-2$ experiments at Fermilab $\left(a_\mu - \frac{1}{\gamma^2-1}\right) = 0$ and J-PARC $\vec{E} = 0$: improve error by factor 4

- ultra relativistic (CERN, BNL, Fermilab)) vs. ultra cold (J-PARC) muons

$\Rightarrow$very different systematics

$\Rightarrow$ $\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm the} = 6.1\,\sigma$ if theory as today

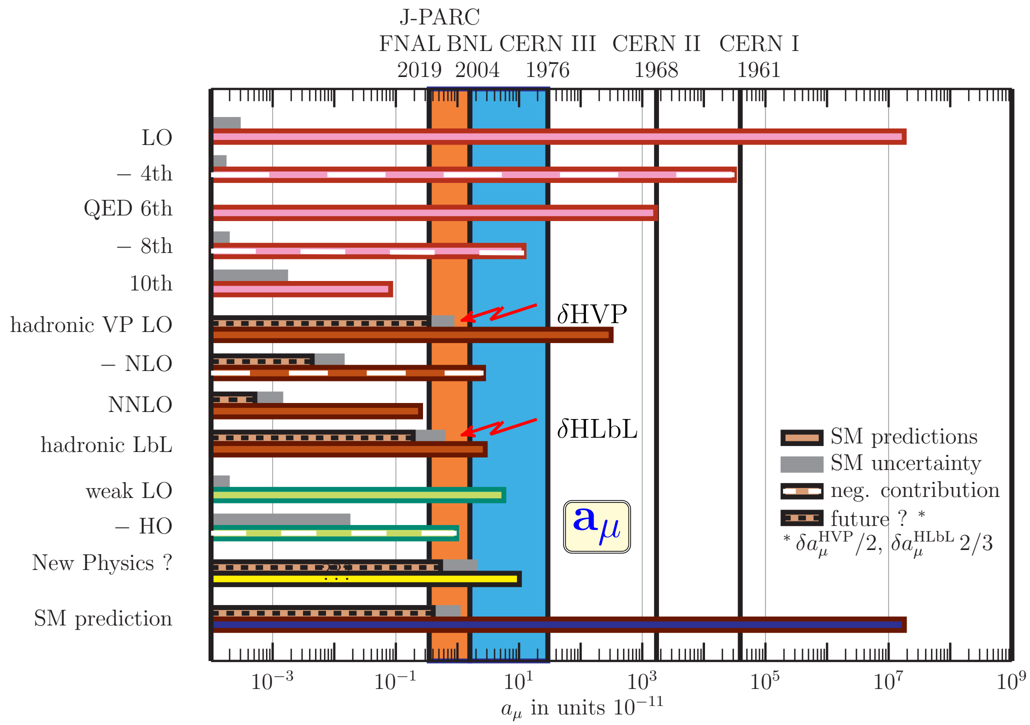
Reduction of hadronic uncertainty by factor 2 $\Rightarrow$ $\Delta a_\mu = 10.7\,\sigma$



slide from D. Hertzog

- ❐ $\tilde{\omega}_p = (e/m_\mu)\langle B\rangle$ free proton NMR frequency
- ❐ $\mathcal{R} = \omega_a/\tilde{\omega}_p$ Larmor precession from E-821
- ❐ $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

- At the present/future level of precision a_μ depends on all physics incorporated in the SM: electromagnetic, weak, and strong interaction effects and beyond that all possible new physics we are hunting for.



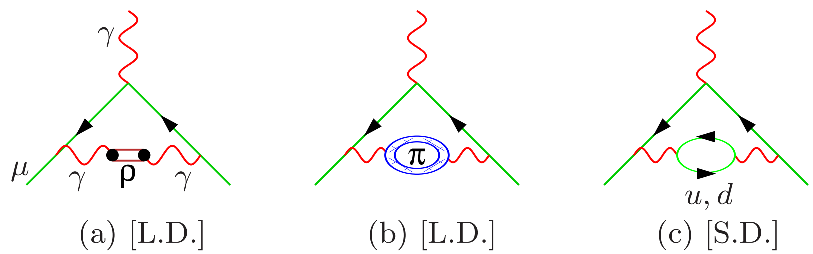
Past and future $g-2$ experiments testing various contributions.

New Physics $\stackrel{?}{=}$ deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$.

Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

To be improved: Leading Hadronic=Mesonic Effects

The problem: non-perturbative strong interaction effects, need low energy effective hadronic modeling: VMD, sQED, ENJL, HLS, etc. (Resonance Lagrangian Approach)

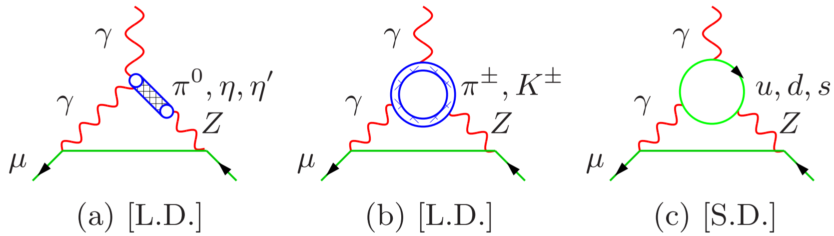


$O(\alpha^2)$

Leading is the hadronic photon vacuum polarization. Safe method: Dispersion Relation (DR) exploiting experimental data or lattice QCD (in progress).

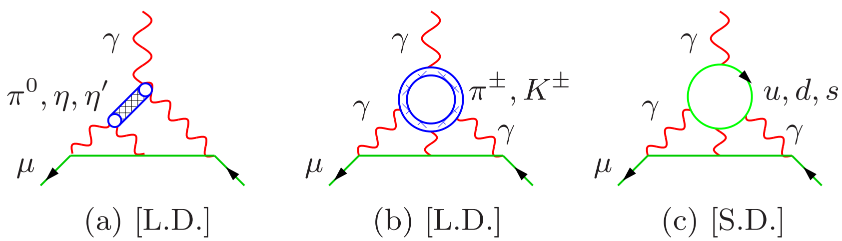
Low energy effective estimates of the leading vacuum polarization effects $a_\mu^{(4)}(\mathrm{vap})$. For comparison: 5.8420×10^{-8} for μ–loop, 5.9041×10^{-6} for e–loop

data+DR [280,810] MeV	ρ^0–exchange PDG+BW	$\pi^\pm$–loop sQED	QCD $[u,d]$ quark-loops	
			constituent quarks	current quarks
4.2666×10^{-8}	4.2099×10^{-8}	1.4154×10^{-8}	2.2511×10^{-8}	4.4925×10^{-6}



$O(\alpha\, G_\mu\, m_\mu^2 \ln M_Z^2/m_\mu^2)$

Mixed weak hadronic effects. The two CHPT diagrams (L.D.) and the QPM diagram (S.D.). Only VVA vertex [VVV≡0] anomaly cancellation at work. Potentially large effects cancel. Well under control.

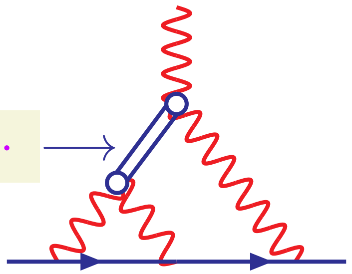


$O(\alpha^3)$

Hadronic light–by–light scattering. Diagrams (a) and (b) represent the long distance (L.D.) contributions, diagram (c) involving a quark loop which yields the short distance (S.D.) tail. Internal photon lines are dressed by $\rho - \gamma$ mixing. Most challenging part, conceptual problems.

Overview various HLbL one-particle exchange contributions: $a_\mu \times 10^{11}$

$\pi^0, \eta, \eta', a_1, f_1, f_1', a_0, f_0, f_0', a_2, f_2, f_2' \cdots$
pseudoscalar, axial, scalar, tensor, ...



PS	$a_\mu^{\text{LbL}}(\pi^0, \eta, \eta')$	64.68	14.87	15.90	95.45 ± 12.40
axials	$a_\mu^{\text{LbL}}(a_1, f_1, f_1')$	1.89	5.19	0.47	7.55 ± 2.71
scalars	$a_\mu^{\text{LbL}}(a_0, f_0, f_0')$	-0.17	-2.96	-2.85	-5.98 ± 1.20
tensors	$a_\mu^{\text{LbL}}(f_2', f_2, a_2')$	0.79	0.07	0.24	1.1 ± 0.1
	sum single meson exchange				98.12 ± 12.75
	+ $\pi^\pm, K^\pm$ loops + quark loops				103.40 ± 28.80

Relevance for new experimental result to come:
$a_\mu^{\rm exp} - a_\mu^{\rm the} = (-30.6 \pm 7.6) \times 10^{-10}$ soon $\rightarrow$ $\pm 1.6 \times 10^{-10}$

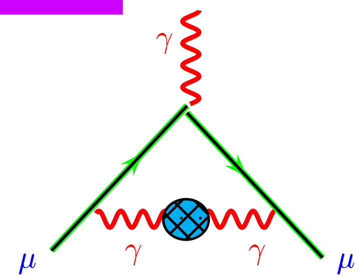
	type	contribution	SD present	SD coming
HVP	LO $O(\alpha^2)$	689.5(3.3)	90.7[0.4]	431[2.1]
	$\pi^+\pi^-$	505.7(2.7)	66.6[0.4]	316.1[1.7]
	K^+K^-	22.0(0.7)	2.9[0.1]	13.8[0.4]
	$\pi^+\pi^-2\pi^0$	20.4(0.9)	2.6[0.1]	12.8[0.6]
	1.05-2 GeV	62.2(2.5)	8.2[0.3]	38.9[1.6]
	HO $O(\alpha^n)$ $(n > 2)$	-8.7(0.1)	1.1[0.0]	5.4[0.0]
HEW	3 families	-1.5(0.0)	small by anomaly cancellation	
HLbL	all $O(\alpha^3)$	10.3(2.9)	1.4[0.4]	6.4[1.2]
	π^0	6.3(0.8)	0.9[0.1]	4.1[0.5]

in units 10^{-10} on in Standard Deviations (SD).

Hadronic Vacuum Polarization (HVP) – Data & Status

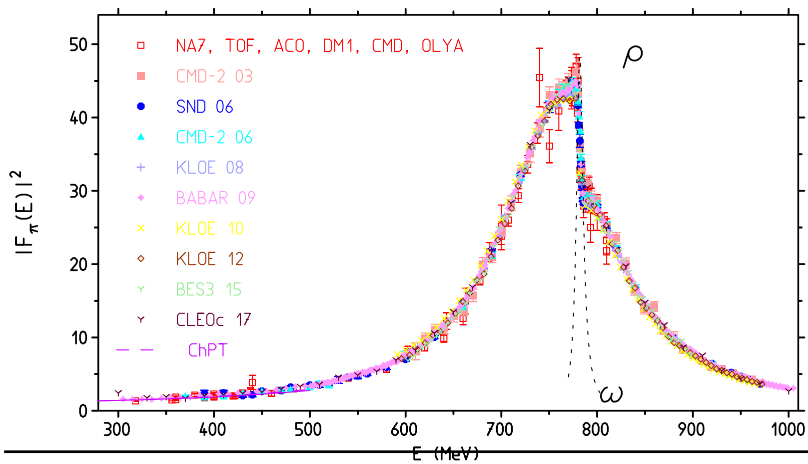
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of $R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via Dispersion Relation (DR):

$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\rm cut}^2} ds \, \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int_{E_{\rm cut}^2}^{\infty} ds \, \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$



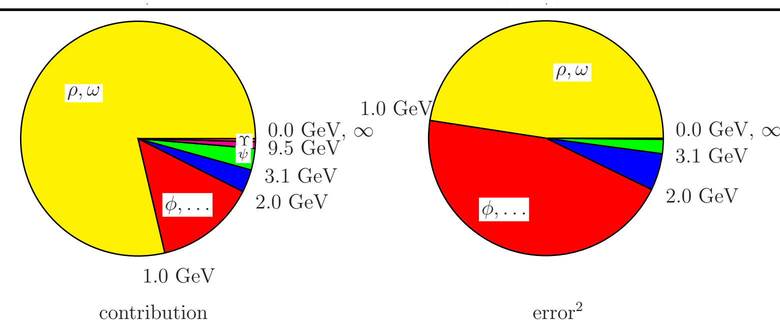
- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

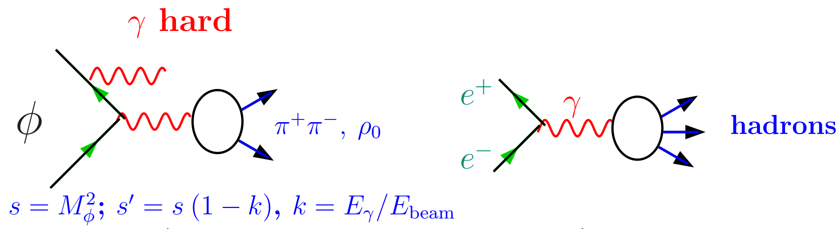
Data: NSK, KLOE, BaBar, BESIII, CLEOc



$$a_\mu^{\rm had(1)} = (686.99 \pm 4.21)[687.19 \pm 3.48]\ 10^{-10}$$

e^+e^-–data based [incl. τ]

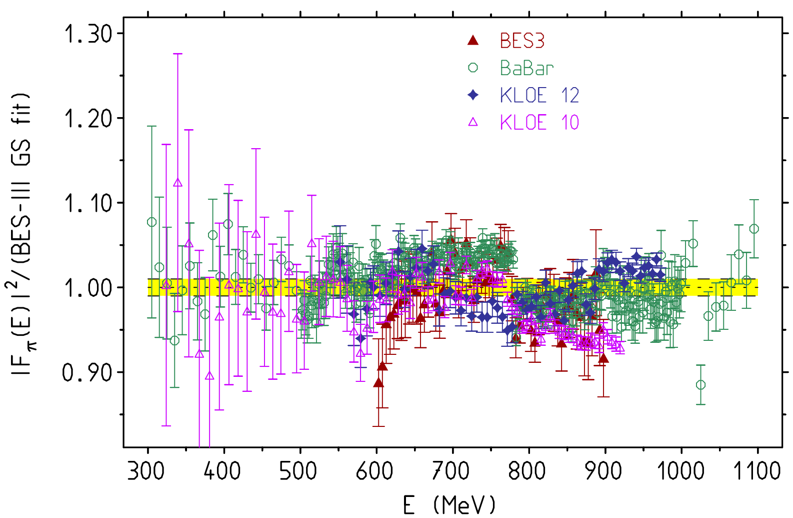


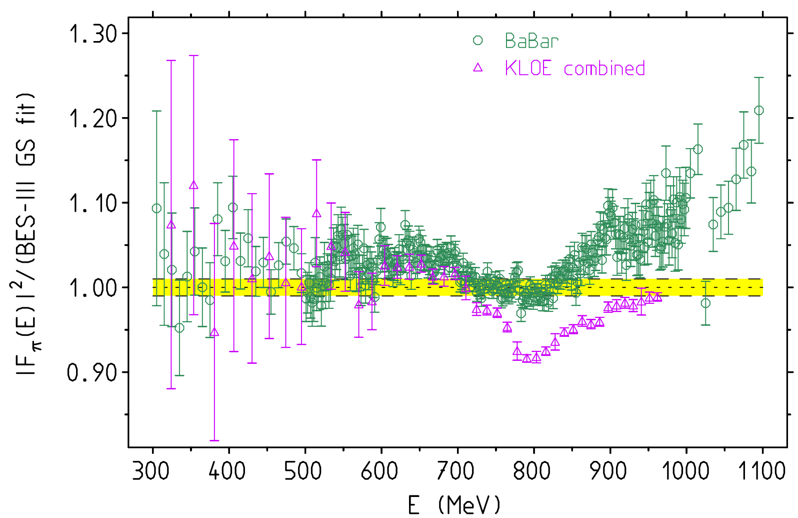


a) b)

a) Initial state radiation (ISR), b) Standard energy scan.

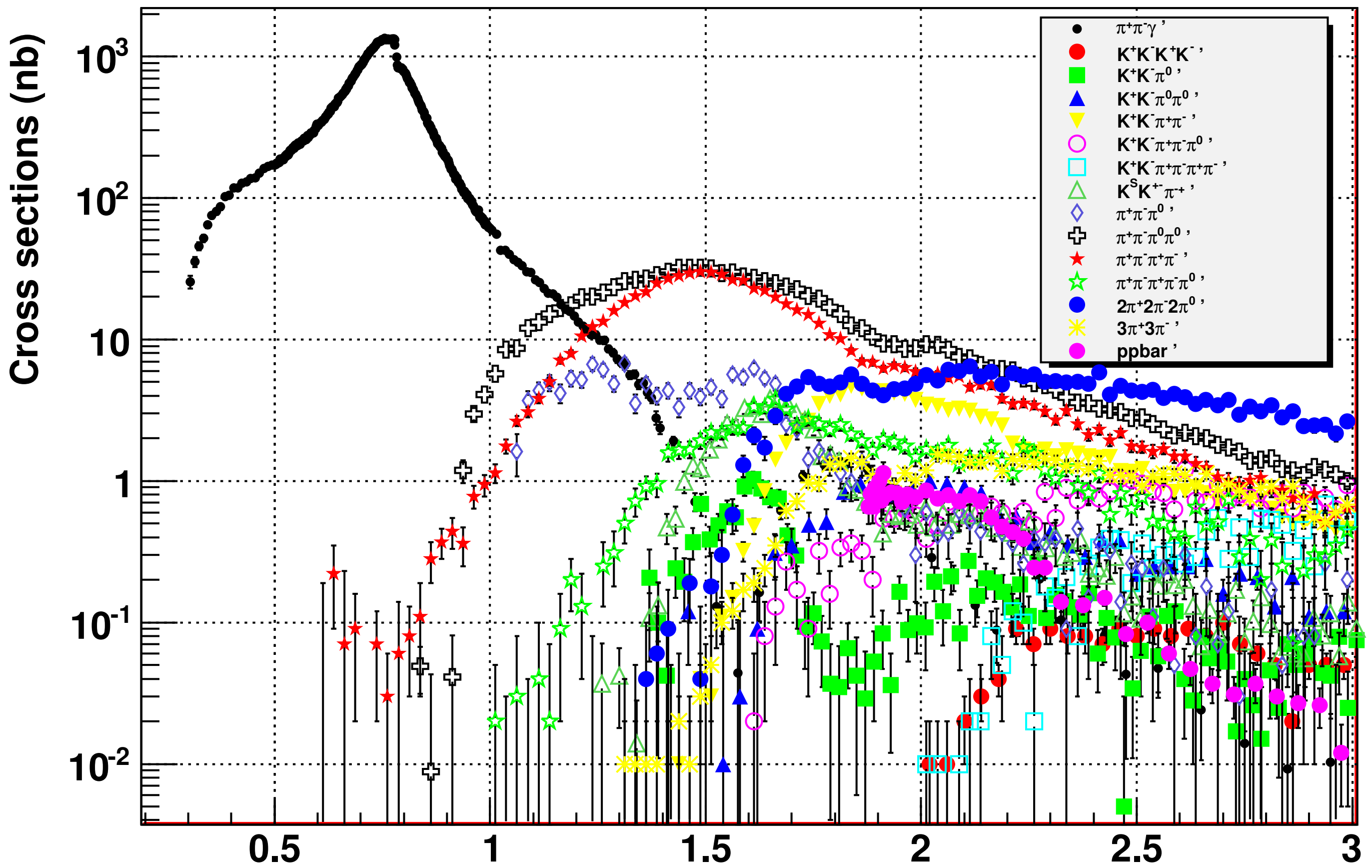
Experimental input for HVP: NSK, KLOE, BaBar, BESIII, VEPP-2000 ...





Recent BESIII vs BaBar and KLOE

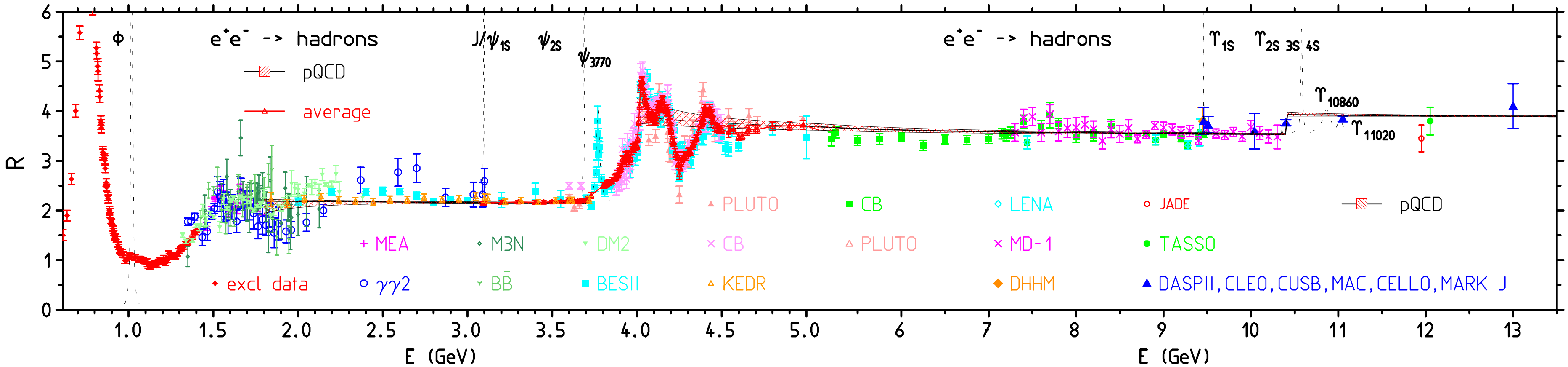
Still an issue: about 30 exclusive channels in range [1.1,2.0] GeV about 50% of HVP uncertainty



+ 15 more channels

2-body, 3π and 4π ⟵ - exclusice channels - ⟶ inclusive data

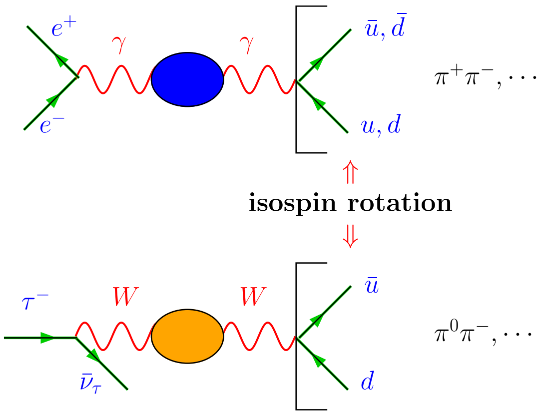
Eidelman et al 2011



The compilation of $R(s)$–data utilized.

Additional data besides e^+e^- ones providing improvements:

❶ τ–decay spectra: good idea, use isospin symmetry to include existing high quality τ–data (after isospin corrections) Alemany et al 1996



bare $\pi^+\pi^-$ spectrum after photon VP, ISR and FSR subtraction

CVC $\leftrightarrow$ isospin rotation symmetry

bare $\pi^0\pi^-$ spectrum after EW radiative and phase space correction

Why is it still a good idea to include τ CC spectra?

τ data in many respects are much simpler than e^+e^- ones, because the charged $\rho^\pm$ does not mix with other states in the dominant low energy range below about 1.05 GeV (up to above the ϕ, below ρ')!

In contrast understanding NC data requires modeling: CHPT + spin 1 resonances (VMD) $\Rightarrow$ Resonance Lagrangian Approach e.g. HLS (massive Yang-Mills) $\Rightarrow$ dynamical widths, dynamical mixing of $\gamma, \rho^0, \omega, \phi$

Effective Field Theory: Resonance Lagrangian Approach

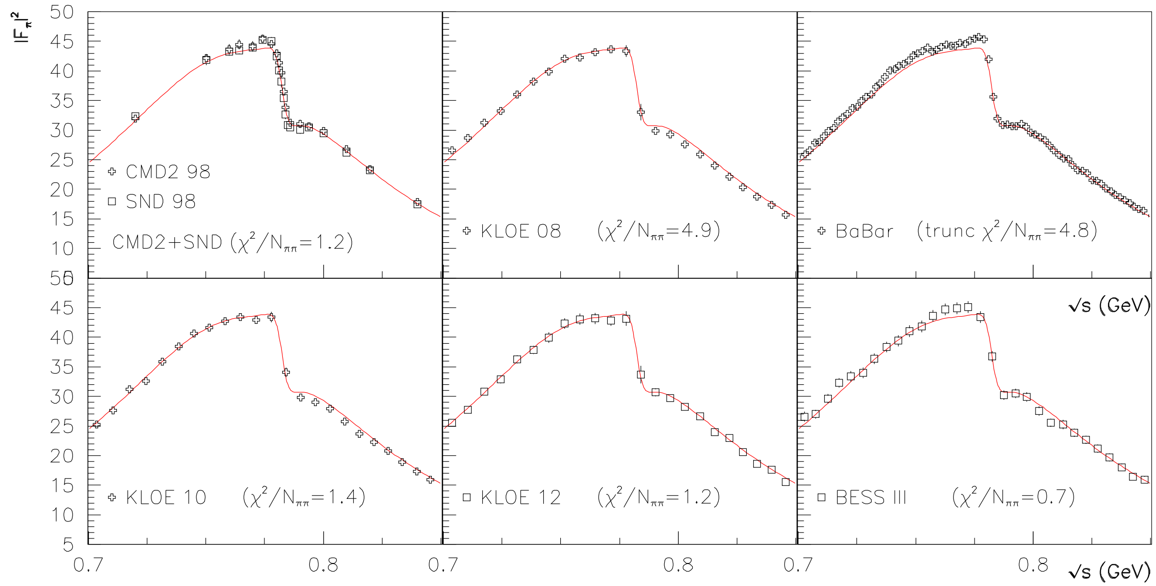
❐ Global Fit strategy based on Hidden Local Symmetry (HLS) Benayoun et al
Data below $E_0 = 1.05$ GeV (just including the ϕ) constrain effective Lagrangian couplings, using 45[47] different data sets (6[8] annihilation channels and 10 partial width decays).

❐ Effective theory predicts cross sections:
$\pi^+\pi^-,\ \pi^0\gamma,\ \eta\gamma,\ \eta'\gamma,\ \pi^0\pi^+\pi^-,\ K^+K^-,\ K^0\bar{K}^0$ (83.4%),

- Missing part: $4\pi, 5\pi, 6\pi, \eta\pi\pi, \omega\pi$ and regime $E > E_0$ evaluated using data directly and pQCD for perturbative region and tail
- Including self-energy effects is mandatory ($\gamma\rho$-mixing, $\rho\omega$-mixing ..., decays with proper phase space, energy dependent width etc)
- Method works in reducing uncertainties by using indirect constraints
- Able to reveal inconsistencies in data, e.g. KLOE vs BaBar

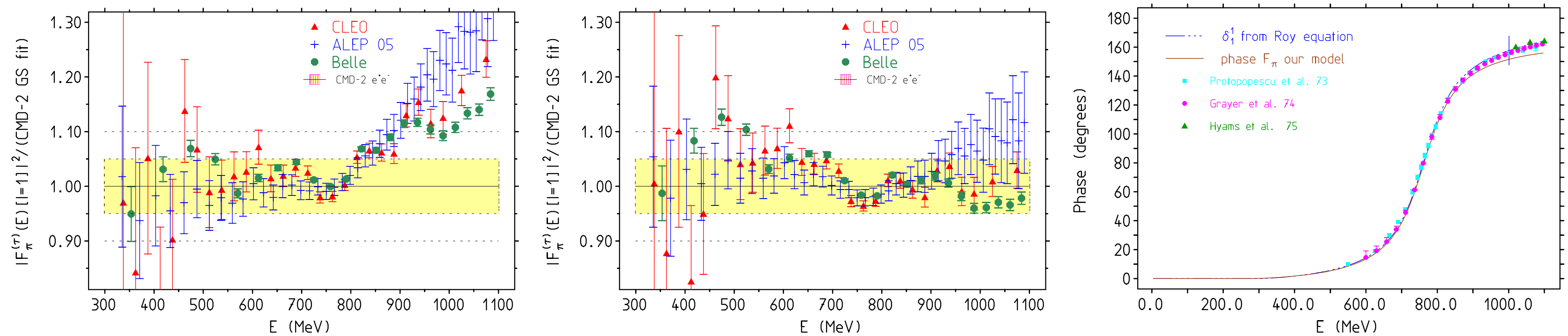
Remark: HLS supplied by symmetry breakings (isospin, $SU(3)$) is rather complicated, like EW SM with spin-1 vector bosons as gauge bosons, but with ρ, ω, ϕ (in place of one Z boson) and parity odd sector WZ type Lagrangian terms.

❑ Fit of τ +IB from PDG vs $\pi^+\pi^-$–data



Comparing the τ+PDG prediction (red curve) of the pion form factor in e^+e^- annihilation in the $\rho - \omega$ interference region. Benayoun et al

Improvements using τ decay spectra and $\pi\pi$ scattering phase shifts



$|F_\pi(E)|^2$ in units of e^+e^- $I=1$ (CMD-2 GS fit): a) τ data uncorrected for $\rho-\gamma$ mixing Szafron, F.J. 11, Benayoun et al 11, and b) after correcting for mixing. Right: $\pi\pi$ [+5% −10%] scattering phase-shift data $\delta_1(s)$ constraining $|F_\pi(E)|^2$.

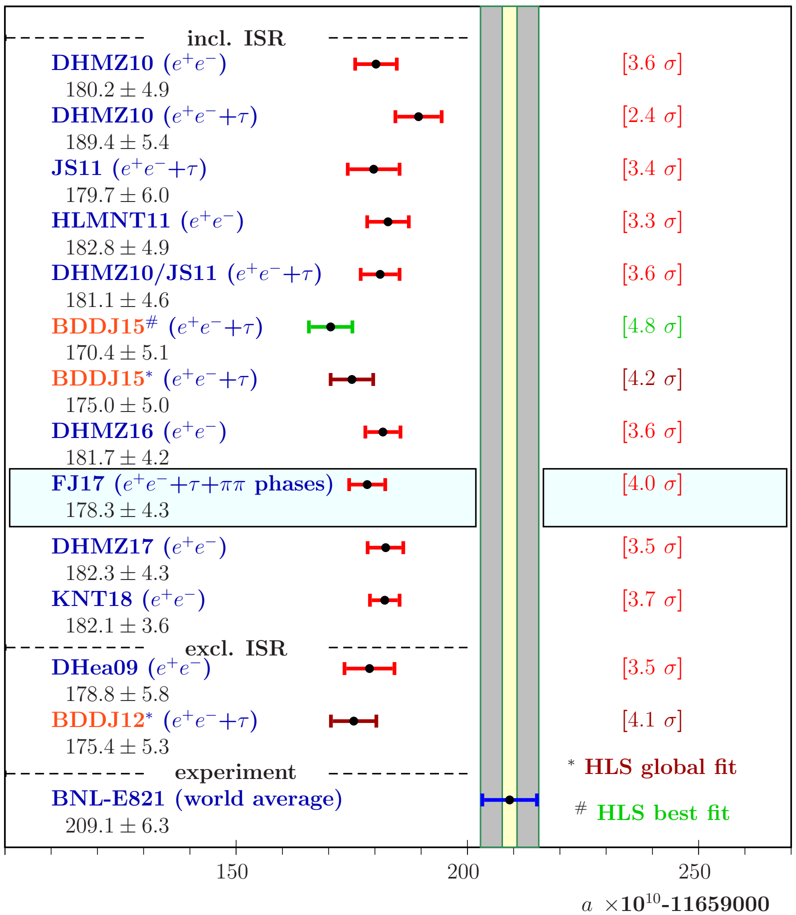
Theory: Leutwyler 02, Colangelo 03, Caprini 16
Data: Hyams 73, Grayer 74, Protopopescu 73
to constrain $|F_\pi|^2$ below 0.63 GeV, one obtains

$$F_\pi(s)=|F_\pi(s)|\,e^{i\,\delta(s)}$$
$$=P(s)\,\exp\left\{\frac{s}{\pi}\int_{4m_\pi^2}^{\infty}ds'\frac{\delta(s')}{s'\,(s'-s)}\right\}$$

$$a_\mu^{\text{had}(1)}=(689.46\pm3.25)\times10^{-10}\,,$$

best estimate combining e^+e^-, τ–decay and $\pi\pi$ scattering phase shift data.

Present leading uncertainty: hard to improve by direct $R(s)$ measurements

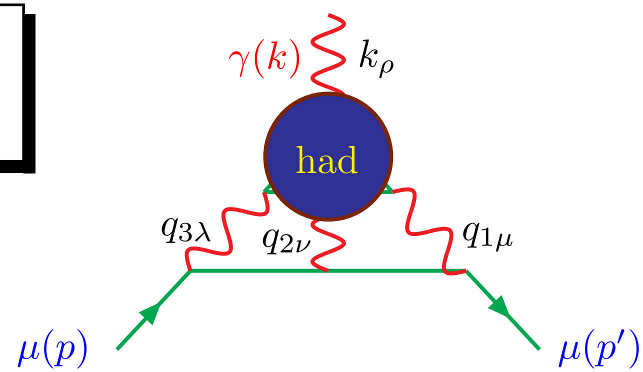


DHea	Davier, Höcker, Lopez Castro, Malaescu, Mo, Toledo Sanchez, Wang, Yuan, Zhang
HLMNT	Hagiwara, Liao, Martin, Nomura, Teubner
DHMZ	Davier, Höcker, Malaescu, Zhang
JS	Jegerlehner, Szafron
BDDJ	Benayoun, David, DelBuono, Jegerlehner
KNT	Keshavarzi, Nomura, Teubner

Comparison with other results.

Hadronic Light-by-Light (HLbL): Setup and Problems

Hadrons in $\langle 0|T\{A^\mu(x_1)A^\nu(x_2)A^\rho(x_3)A^\sigma(x_4)\}|0\rangle$



Key object full rank-four hadronic vacuum polarization tensor

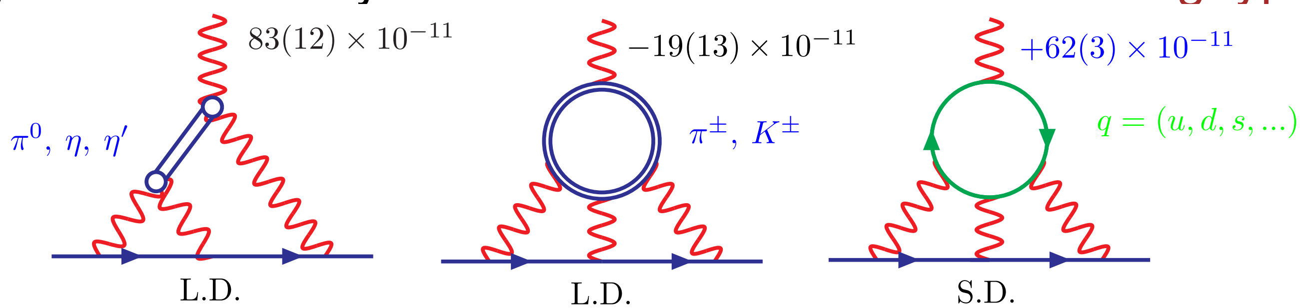
$$\Pi_{\mu\nu\lambda\rho}(q_1,q_2,q_3) = \int d^4x_1\, d^4x_2\, d^4x_3\, e^{i\,(q_1x_1+q_2x_2+q_3x_3)} \times \langle\, 0\,|\, T\{j_\mu(x_1)j_\nu(x_2)j_\lambda(x_3)j_\rho(0)\}\,|\,0\,\rangle\ .$$

- non-perturbative physics
- covariant decomposition involves 138 Lorentz structures (43 gauge invariant)
- 28 can contribute to $g-2$; by permutation symmetry 19 independent
- fortunately, dominated by the pseudoscalar exchanges $\pi^0, \eta, \eta', \ldots$ described by the effective Wess-Zumino Lagrangian

- generally, pQCD useful to evaluate the short distance (S.D.) tail

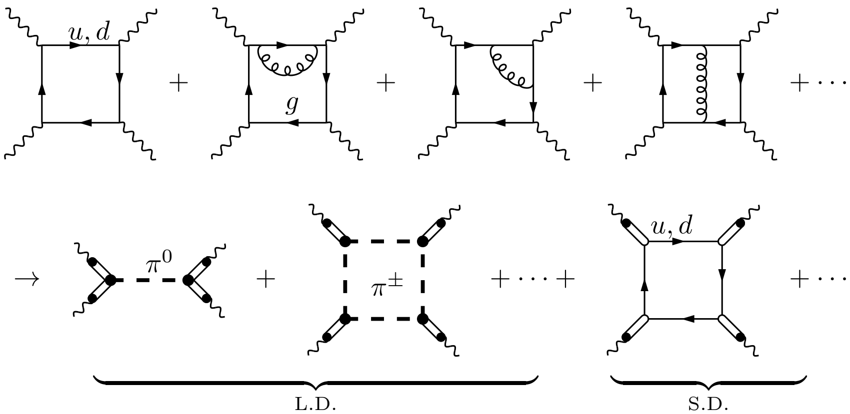
- the dominant long distance (L.D.) part must be evaluated using some low energy effective model which includes the pseudoscalar pions as well as the vector mesons ($\rho, \cdots$) which play a dominant role (vector meson dominance mechanism); HLS, ENJL, general RLA, large N_c inspired ansätze, and others

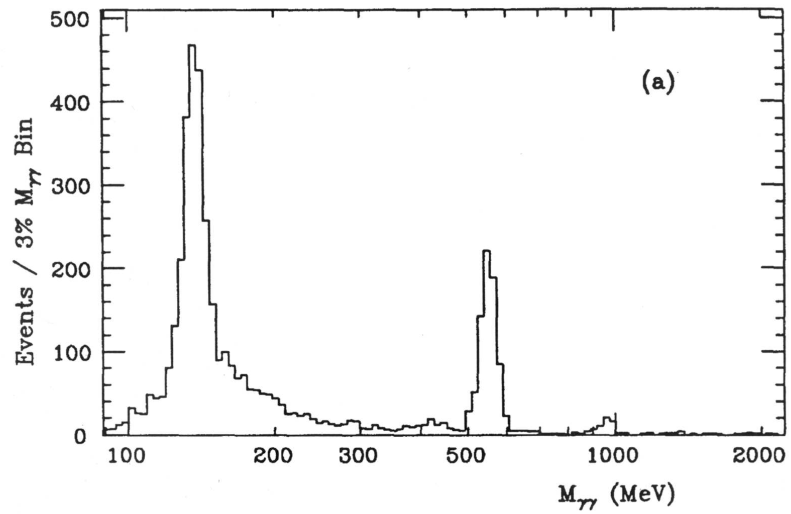
In low energy effective theory ⇒amount to calculate the following type diagrams



simplest case and leading one: π^0 exchange, needs $\pi^0\gamma\gamma$ form factor

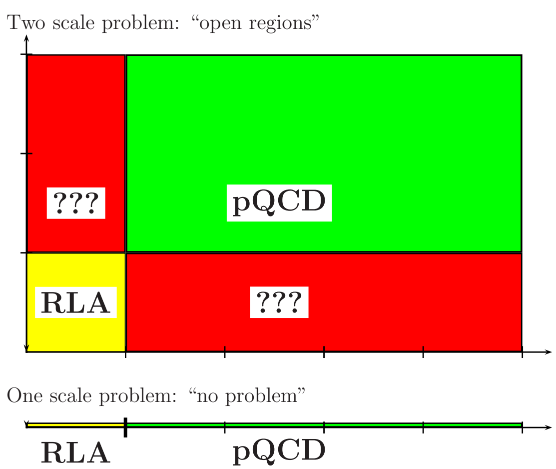
Crystal Ball 1988





Data show almost background free spikes of the PS mesons!

Basic problem: (s, s_1, s_2)–domain of $\mathcal{F}_{\pi^{0*}\gamma^*\gamma^*}(s, s_1, s_2)$; here $(0, s_1, s_2)$–plane



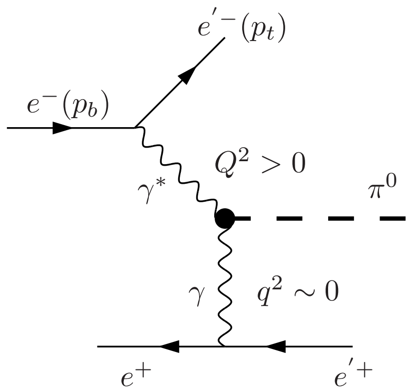
???

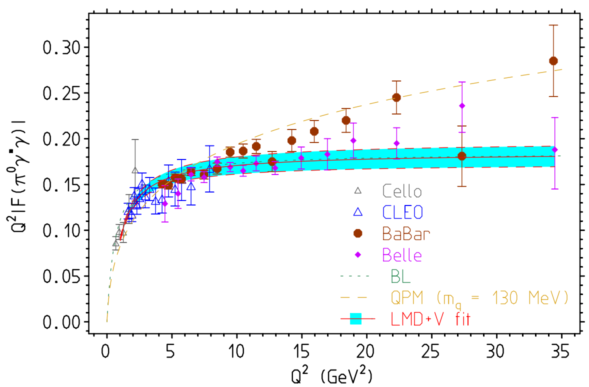
– Data + Dispersion Relation, OPE,
– QCD factorization,
– Brodsky-Lepage approach
– Models constrained by data

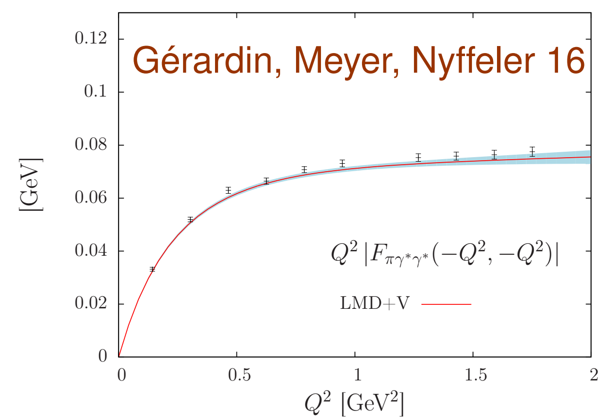
❐ Constraint I: $\Gamma(\pi^0\gamma\gamma) \leftrightarrow$ effective WZ-Lagrangian

❖ The constant $e^2\,\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0) = \frac{e^2 N_c}{12\pi^2 f_\pi} = \frac{\alpha}{\pi f_\pi} \approx 0.025$ GeV^{-1} well determined by $\pi^0 \to \gamma\gamma$ decay rate.

❖ Information on $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ from $e^+e^- \to e^+e^-\pi^0$ experiments







CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0)$ at high space–like Q^2. Outdated by BaBar? Belle conforms with theory expectations!.
First measurements of $\mathcal{F}_{\pi^{0*}\gamma\gamma}(m_\pi^2;-Q^2,-Q^2)$ in lattice QCD

❒ Constraint II: VMD mechanism ↔ Brodsky-Lepage behavior

$$\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2,-Q^2,0) \simeq \frac{1}{4\pi^2 f_\pi}\frac{1}{1+(Q^2/8\pi^2 f_\pi^2)} \sim \frac{2f_\pi}{Q^2}$$

then cannot miss to get reasonable result!

The large–N_c QCD inspired (OPE constrained) LMD+V ansatz Knecht, Nyffeler

$$\mathcal{F}^{\mathrm{LMD+V}}_{\pi^{0*}\gamma^*\gamma^*}(p_\pi^2,q_1^2,q_2^2) = \frac{F_\pi}{3}\frac{\mathcal{P}(q_1^2,q_2^2,p_\pi^2)}{\mathcal{Q}(q_1^2,q_2^2)},$$

←Polynomial fixed by OPE

←VMD denominator

$$\mathcal{P}(q_1^2,q_2^2,p_\pi^2) = h_0\, q_1^2\, q_2^2\, (q_1^2+q_2^2+p_\pi^2) + h_1\,(q_1^2+q_2^2)^2 + h_2\, q_1^2\, q_2^2 + h_3\,(q_1^2+q_2^2)\, p_\pi^2 + h_4\, p_\pi^4 + h_5\,(q_1^2+q_2^2) + h_6\, p_\pi^2 + h_7,$$

$$\mathcal{Q}(q_1^2,q_2^2) = (M_{V_1}^2-q_1^2)\,(M_{V_2}^2-q_1^2)\,(M_{V_1}^2-q_2^2)\,(M_{V_2}^2-q_2^2),$$

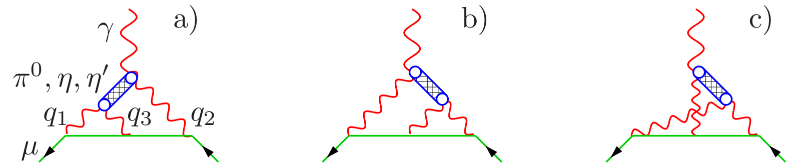
$p_\pi^2=m_\pi^2$, **well constrained now, i.e. parameters** $h_i\ (i=0,\cdots,7)$ **rather well under control (QCD asymptotics, experimental and lattice data)**

QCD: $h_0=-1$, $h_1=0$ **+ constraints by data ;** $h_3,\ h_4,\ h_6$ **absent in chiral limit**

remain $h_2,\ h_5$ **and** h_7 **as essential parameters**

VMD masses: $M_{V_1}^2,\ M_{V_2}^2$ **identified with** $\rho,\ \rho'$ **masses. So far mostly pole approximation!**

❑ Results π^0, η, η'-exchanges

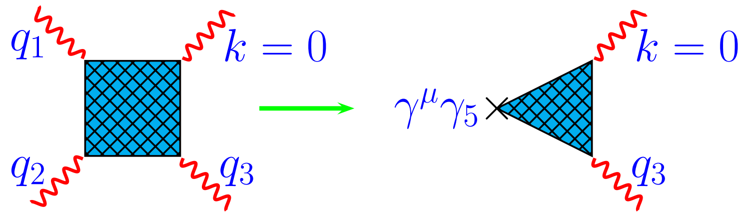


Various model predictions of the π^0 and PS exchange contribution. In units 10^{-10}

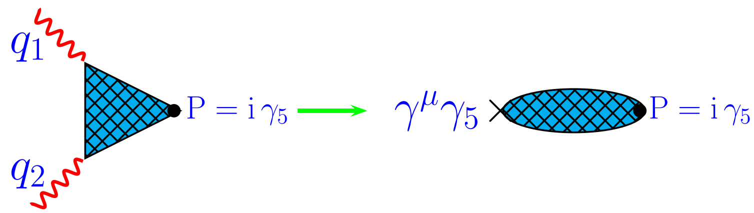
model/method	$a_\mu(\pi^0)$	$a_\mu(\pi^0, \eta, \eta')$	Ref.
HLS	5.7 ± 0.4	8.3 ± 0.6	Hayakawa, Kinoshita, Sanda (2002)
ENJL	5.9 ± 0.9	8.5 ± 1.3	Bijnens, Pallante, Prades (2002)
LMD+V (on-shell, $h2 = -10$ GeV^2)	6.3 ± 1.0	8.8 ± 1.2	Knecht, Nyffeler (2002)
LMD+V (on-shell, WZ at external vertex)	7.7 ± 0.7	11.4 ± 1.0	Melnikov, Vainshtein (2004)
LMD+V (off-shell, $\chi = -(3.3 \pm 1.1)$ GeV^{-2})	7.2 ± 1.2	9.9 ± 1.6	Jegerlehner, Nyffeler (2009)
linearized ENJL	8.2 ± 1.6	9.5 ± 1.7	Bartos, Dubnickova et al (2001)
RLA – VV'P model	6.7 ± 0.2	10.5 ± 0.5	Roig, Guevara, López Castro (2014)
RLA resonance saturation	6.6 ± 1.2		Kampf, Novotny (2011)
Non–local quark model	5.0 ± 0.4	5.9 ± 0.9	Dorokhov, Broniowski (2008)
Chiral Quark Model	6.8 ± 0.3		Greynat, de Rafael (2012)
AdS/QCD inspired FF	6.9 ± ?	10.7 ± ?	Hong, Kim (2009)
AdS/QCD inspired FF	6.5 ± 0.3		Cappiello, Cata, D'Ambrosio (2010)
DSE (truncated Dyson-Schwinger equations)	5.8 ± 1.0	8.1 ± 1.2	Goecke, Fischer, Williams (2012)
Padéized data		9.4 ± 0.5	Masjuan, Sánchez-Puertas (2017)
LMD+V (on-shell, lattice QCD constraint)	6.5 ± 0.8		Gérardin, Meyer, Nyffeler (2016)
Data driven resonance saturation		8.3 ± 0.3	Czyż, Kisza, Tracz (2018)
RLA	5.8 ± 0.1	8.5 ± 0.2	Guevara, Roig, Sanz-Cillero (2018)
Dispersive model	6.3 ± 0.3		Hoferichter, Hoid, Kubis et al (2018)
LMD+V (my estimate)	6.5 ± 1.2	9.5 ± 1.3	JN updated

A crucial difference in use of OPE:
The OPE for the photon–photon scattering Green function, for $q_1 \approx -q_2 >> q_3 = -(q_1 + q_2)$ the triangle "blob" is dominated by pseudoscalar exchanges but also accounts for whatever QCD permits. In the sum of all hadronic states it is given by the perturbatively calculable triangle anomaly.

Melnikov-Vainshtein considering the $\gamma\gamma \to \gamma\gamma$ amplitude. The external vertex $k = 0$ is then the constant WZ form factor (no VMD damping). Hard-hard asymptotics.



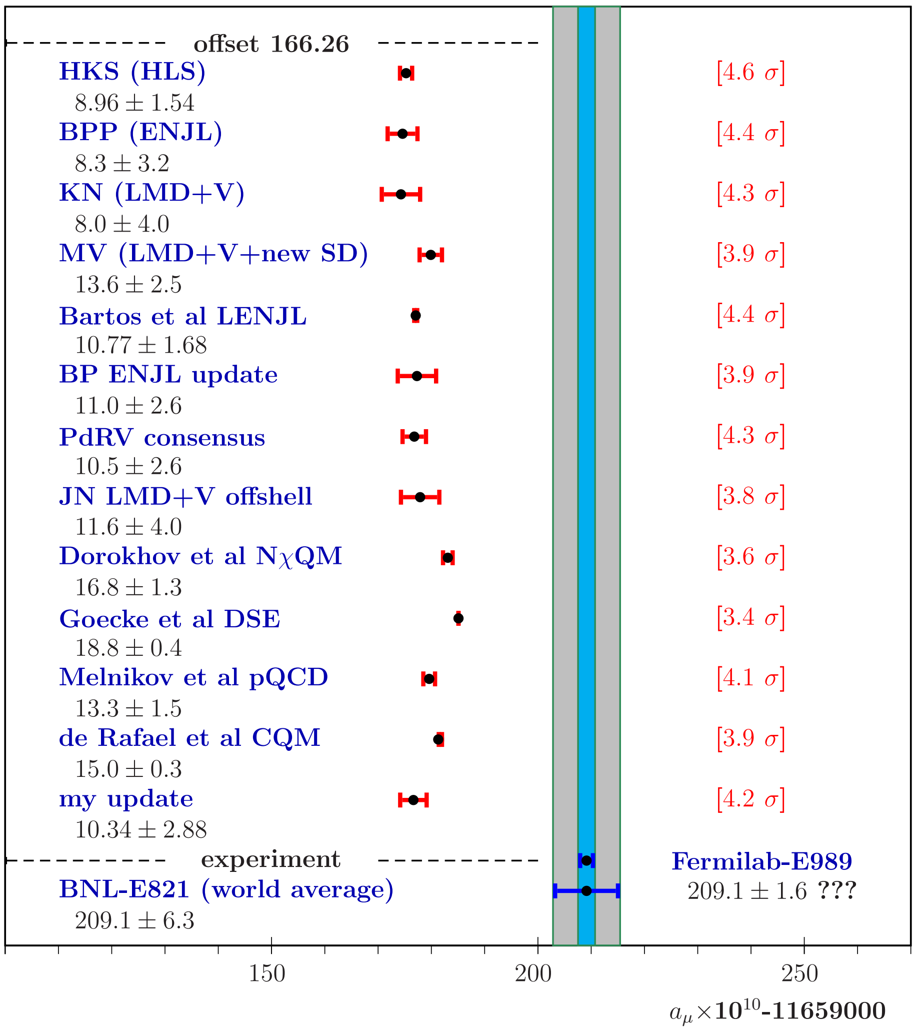
Knecht-Nyffeler considering the $\gamma\gamma \to \pi^0$ form factor. The subsequent $\pi^0 \to \gamma\gamma$ transition is taken into account as an extra factor. External vertex VMD suppressed. Soft-hard asymptotics.



Different splitting/bookkeeping of various effects. Individual contributions may differ, only total HLbL has to agree!

$a_\mu^{\rm HLbL} = [95.45(12.40)+7.55(2.71)-5.98(1.20)+20(5)-20(4)+2.3(0.2)+1.1(0.1)+3(2)] \times 10^{-11}$
$= 103.4(28.8) \times 10^{-11}$

my estimate



Results from various HLbL calculations. The plot also illustrates the history of HLbL evaluations.

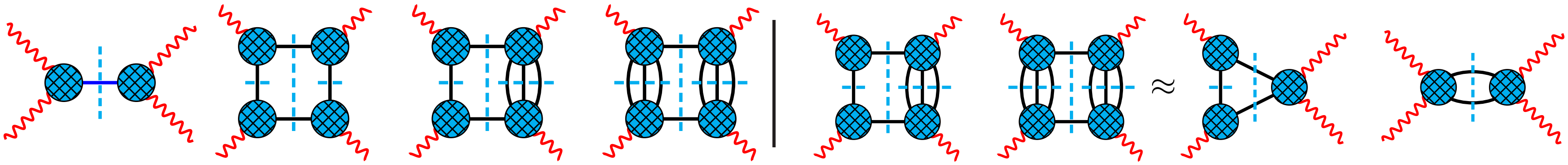
Issues:
- pole approximation,
- damping at external vertex
- possible missing effects
- model dependence

My best estimate the sum of PS, axials, scalars, pion-loops, quark-loops, c-quark, tensor and LbLNLO

I added errors quadratically and multiplied the resulting error by a factor 2 to account for possible so far unaccounted model uncertainties.

Agreement between different estimates not yet satisfactory!

A way out: lattice QCD and/or Dispersive Approach (Colangelo et al, Pauk, Vanderhaeghen: fix HLbL amplitudes via data and DR's.

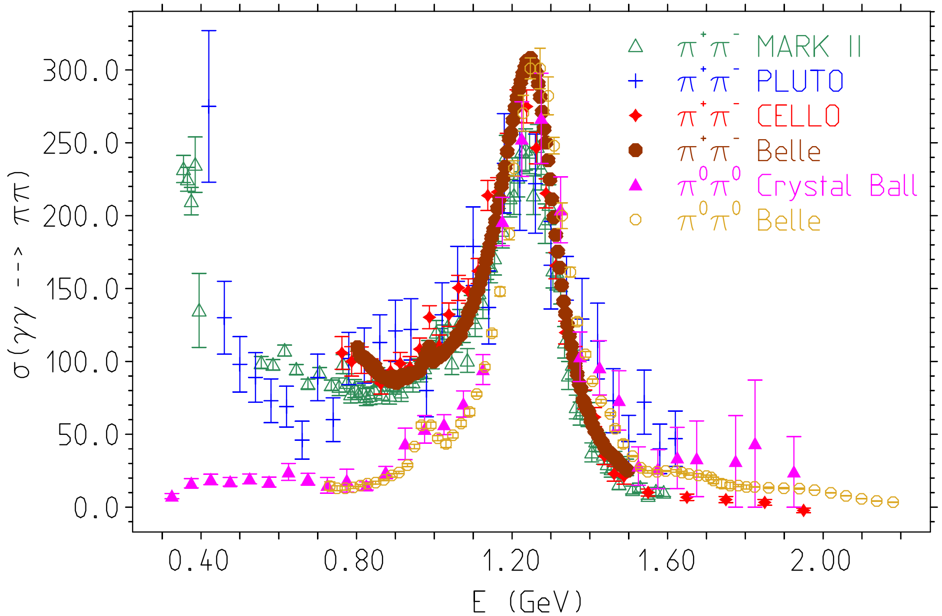


Left: Unitarity diagrams according to the Mandelstam representation. Cuts are represented by the dashed lines, where cut lines represent on-sell particles. Crossed diagrams have to be included as well. Right: Partial–wave approximation of multi–particle intermediate states by leading two-particle cut contributions

Dispersive approach to $\gamma^* \gamma^* \to \gamma^* \gamma^*$: Colangelo, Hoferichter, Procura, Stoffer

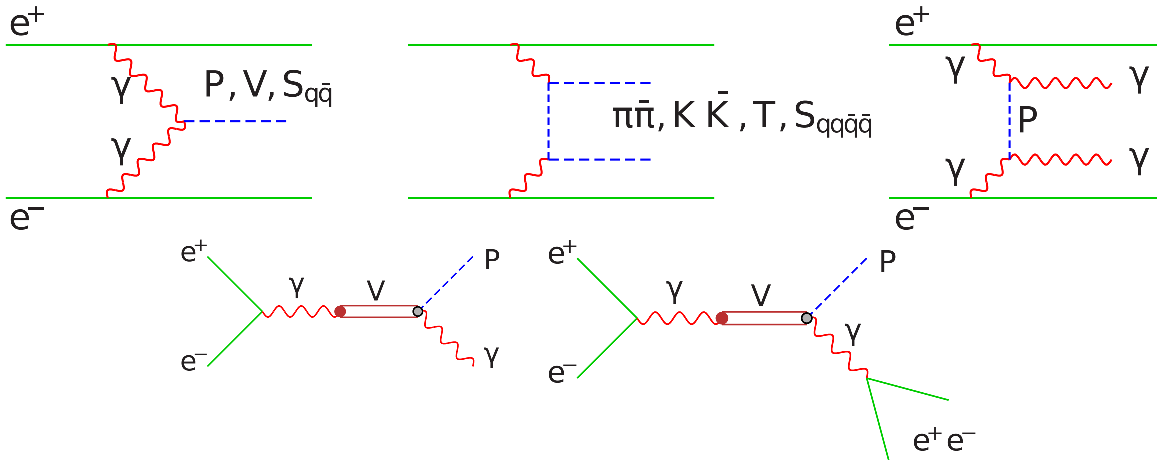
- Very ambitious long term project, requires all kind of data not yet available

Example: pion–loop contribution may be evaluated in terms of $\sigma(\gamma\gamma \to \pi\pi)$ evaluated for the first time 2017 by the Bern group



Di-pion production in $\gamma\gamma$ fusion. At low energy we have direct $\pi^+\pi^-$ production and by strong rescattering $\pi^+\pi^- \to \pi^0\pi^0$, however with very much suppressed rate. With increasing energy, above about 1 GeV, the strong $q\bar{q}$ resonance $f_2(1270$ appears produced equally at expected isospin ratio $\sigma(\pi^0\pi^0)/\sigma(\pi^+\pi^-) = \frac{1}{2}$. This demonstrates convincingly that we may safely work with point-like pions below 1 GeV

● For future improvements one desperately needs more information from $\gamma\gamma \to$ hadrons in order to have better constraints on modeling of the relevant hadronic amplitudes. The goal is to exploit possible new experimental constraints from $\gamma\gamma \to$ hadrons and crossed channels if possible



mostly single-tag events: KLOE, KEDR (taggers), BaBar, Belle, BESIII;
Dalitz-decays: $\rho, \omega, \phi \to \pi^0(\eta)e^+e^-$ Novosibirsk, NA60,JLab, Mainz, Bonn, Jülich, BES would be interesting, but some buried in the background.

Theory vs experiment: do we see New Physics?

Standard model theory and experiment comparison

Contribution	Value$\times 10^{10}$	Error$\times 10^{10}$	Reference
QED incl. 4-loops + 5-loops	11 658 471.886	0.003	Aoyama et al 12,Laporta 17
Hadronic LO vacuum polarization	689.46	3.25	
Hadronic light–by–light	10.34	2.88	
Hadronic HO vacuum polarization	-8.70	0.06	
Weak to 2-loops	15.36	0.11	Gnendiger et al 13
Theory	11 659 178.3	4.3	–
Experiment	11 659 209.1	6.3	BNL 04
The. - Exp. 4.0 standard deviations	-30.6	7.6	–

Standard model theory and experiment comparison [in units 10^{-10}]. What represents the 4 σ deviation: ❒ new physics? ❒ a statistical fluctuation?
❒ underestimating uncertainties (experimental, theoretical)?

❖do experiments measure what theoreticians calculate?

☼ could it be unaccounted for real photon radiation effects?
(is the BMT equation [Dirac particle in external e.m. field] sufficiently accurate?)

Prospects

- The muon $g-2$ prediction is limited by hadronic uncertainties, which are dominated by meson form factors
- Substantial progress would be possible if one could reach agreement on what QCD predicts for the various meson form factors
- Dispersive Methods require primarily improved data
- Lattice QCD is making big progress and begins to help to settle hadronic issues
- a “New Physics” interpretation of the persisting 3 to 4 σ requires relatively strongly coupled states in the range below about 250 GeV.

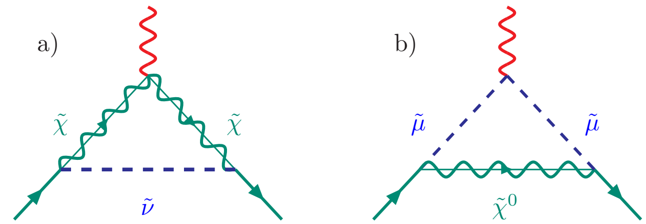
Problem: LEP, Tevatron and LHC direct bounds on masses of possible new states

[typically $M_X > 800$ GeV]

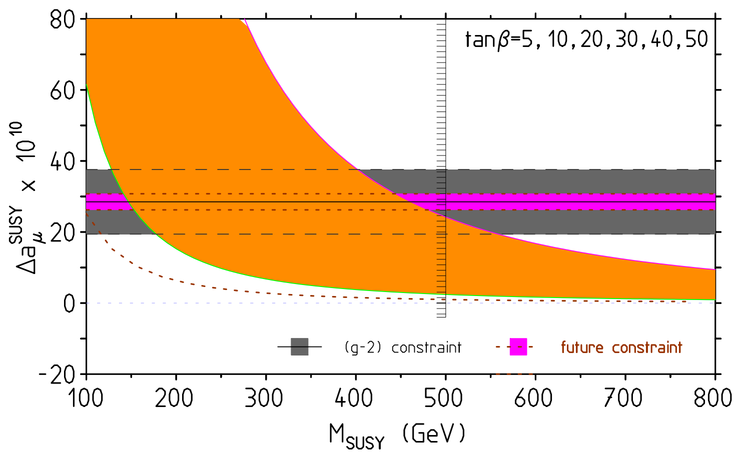
Need enhanced couplings! as in SUSY extensions of SM

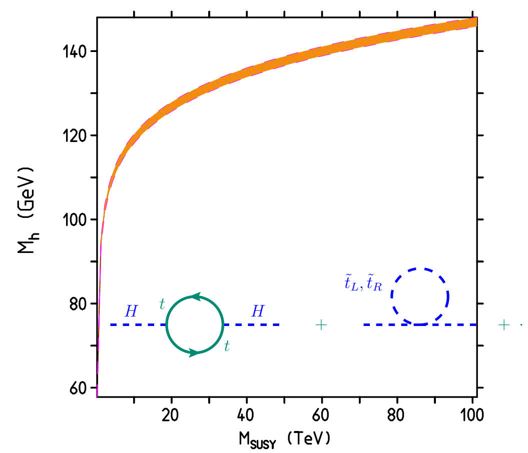
$$a_\mu^{\rm SUSY} \simeq \frac{{\rm sign}(\mu M_2)\,\alpha(M_Z)}{8\pi \sin^2\Theta_W}\,\frac{\left(5+\tan^2\Theta_W\right)}{6}\,\frac{m_\mu^2}{M_{\rm SUSY}^2}\,\tan\beta\left(1-\frac{4\alpha}{\pi}\ln\frac{M_{\rm SUSY}}{m_\mu}\right)$$

- $\tan\beta = \frac{v_1}{v_2}$, $v_i = < H_i >$; $i = 1, 2$;
  $\tan\beta \sim m_t/m_b \sim 40 \quad [4-40]$



Before LHC: MSSM just would fit great





Physics beyond the SM: leading SUSY contributions to $g-2$ in a supersymmetric extension of the SM. The Higgs mass is constrained, here $\tan\beta = 5$ and $m_A = 60$ GeV.

In any case a_μ constrains BSM scenarios distinctively and at the same time challenges a better understanding of the SM prediction.

The big challenge: two complementary experiments: Fermilab with ultra hot muons and J-PARC with ultra cold muons (very different radiation) to come

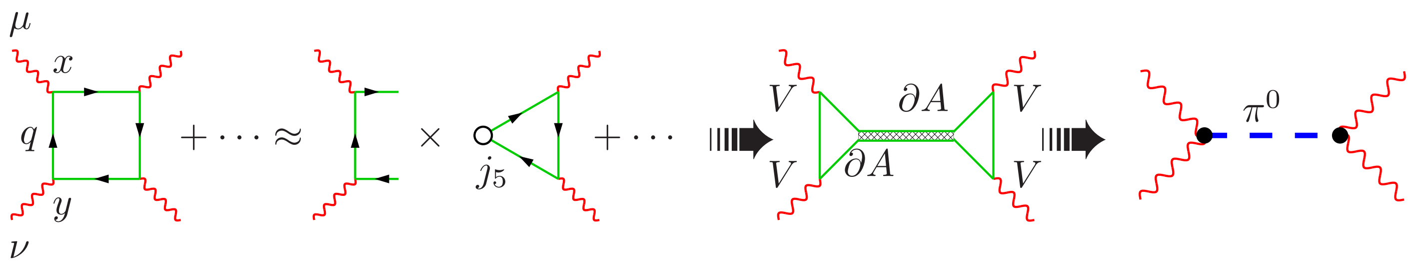
Provided deviation is real and theory and needed cross section data improves the same as the muon $g-2$ experiments $3\sigma \to 9\sigma$ possible?!

Key: more/better data and/or progress in non-perturbative QCD

A lot remains to be done! while a new a_μ^{exp} is approaching!

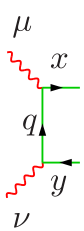
Thanks!

Non-perturbative mechanism easy understood



Single particle exchange ⇒OPE $x \to y$ i.e. large momentum transfer $\hat{q} = (q_1 - q_2)/2$ ⇒Wilson coefficient $\times$ quark triangle

Furry: VVV≡0 ⇒only axial vector contributes i.e. AVV ⇒leading are pseudoscalar exchanges! Parity odd states in pure vector-like environment (QED). Wilson coefficient (only a matter of Dirac algebra):



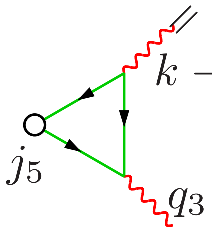
$$\sim \frac{\gamma_\mu \hat{q}\!\!\!/ \gamma_\nu}{\hat{q}^2} \propto \frac{1}{\hat{q}^2} \left\{ \left(g_{\mu\alpha} g_{\nu\beta} + g_{\mu\alpha} g_{\nu\beta} - g_{\mu\alpha} g_{\nu\beta} \right) \hat{q}^\alpha \, \underline{\gamma^\beta} + \mathrm{i} \varepsilon_{\mu\nu\alpha\beta} \, \hat{q}^\alpha \, \underline{\gamma^\beta \gamma_5} \right\}$$

↳ VVV ↳ AVV

$q_3 = q_1 + q_2, \; 2 q_3 \hat{q} = q_1^2 - q_2^2 \approx 0$

Triangle AVV anomaly graph:

$$\mathcal{T}_{\lambda\beta} = -\mathrm{i}\, e\, \epsilon^{\rho}(k)\mathcal{T}_{\rho\lambda\beta} = \mathrm{i} \int \mathrm{d}^4 x_3\, \mathrm{e}^{\mathrm{i} q_3 x_3} \langle 0|T\, j_\lambda(x_3)\, j_{5\,\beta}(0)\, |\gamma(k)\rangle \text{ for } k \to 0$$



$$\mathcal{T}_{\lambda\beta} = \frac{\mathrm{i}}{4\pi^2}\left\{ w_L\left(q_3^2\right) q_{3\beta} q_3^\sigma \tilde{f}_{\sigma\lambda} + w_T\left(q_3^2\right)\left[-q_3^2 \tilde{f}_{\lambda\beta} + q_{3\lambda} q_3^\sigma \tilde{f}_{\sigma\beta} - q_{3\beta} q_3^\sigma \tilde{f}_{\sigma\lambda}\right]\right\}$$

$$q_3^\beta \mathcal{T}_{\lambda\beta} = \frac{\mathrm{i}\, e}{2\pi^2} q_3^\sigma \tilde{f}_{\sigma\lambda}$$

axial anomaly

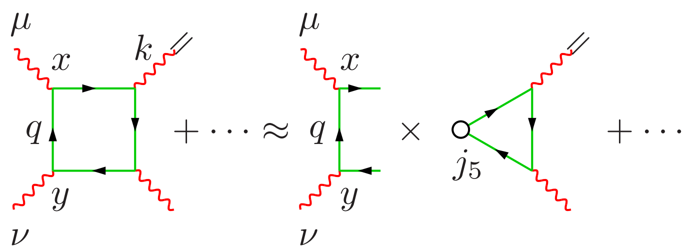
Here $\tilde{f}_{\alpha\beta} = \varepsilon_{\alpha\beta\rho\sigma} f^{\rho\sigma}$ is the dual of $f_{\alpha\beta} = k_\alpha \varepsilon_\beta - k_\beta \varepsilon_\alpha$ with $\varepsilon_\alpha = \varepsilon_\alpha(k)$ the (external) photon polarization vector.

Longitudinal part entirely fixed by the anomaly and $w_T = \frac{1}{2} w_L$ Vainshtein 2003

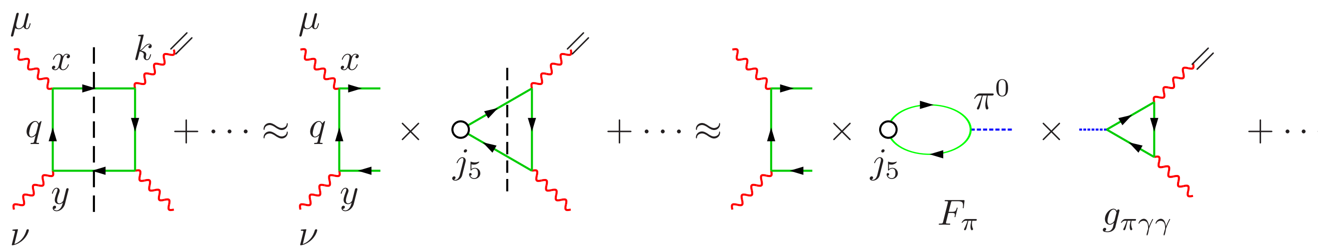
$$w_L\left(Q^2\right) = \frac{2N_c}{Q^2}, \quad w_L = 2 w_T = 2N_c \int_0^1 \frac{\mathrm{d}\alpha\, \alpha(1-\alpha)}{\alpha(1-\alpha)Q^2 + m^2} = 2N_c \left[\frac{1}{Q^2} - \frac{2m^2}{Q^4} \ln \frac{Q^2}{m^2} + \cdots\right]$$

which in chiral limit ($m = 0$) is exact to all orders of perturbation theory
(Adler-Bardeen non-renormalization theorem) and
mediates non-perturbative pseudoscalar exchanges Adler, Bell, Jackiw

This is perturbative non-perturbative physics!



$$\langle\, 0\,|\, T\{j_\mu(x) j_\nu(y) j_\lambda(0)\}\,|\,\gamma(k)\,\rangle \quad\approx\quad \frac{2i}{q^2}\varepsilon_{\mu\nu\alpha\beta}q^\alpha\,\langle 0|T\, j_5^\beta(z)\; j_\lambda(0)|\gamma(k)\rangle + \cdots$$



$$\frac{2\mathrm{i}}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha \langle 0|j_5^\beta(z)\, j_\lambda(0)|\gamma(k)\rangle \approx \frac{2\mathrm{i}}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha \int \frac{\mathrm{d}^4 p_n}{(2\pi)^3} \sum\!\!\!\!\!\!\int_n \langle 0|j_5^\beta(z)|n\rangle\langle n|j_\lambda(0)|\gamma(k)\rangle$$

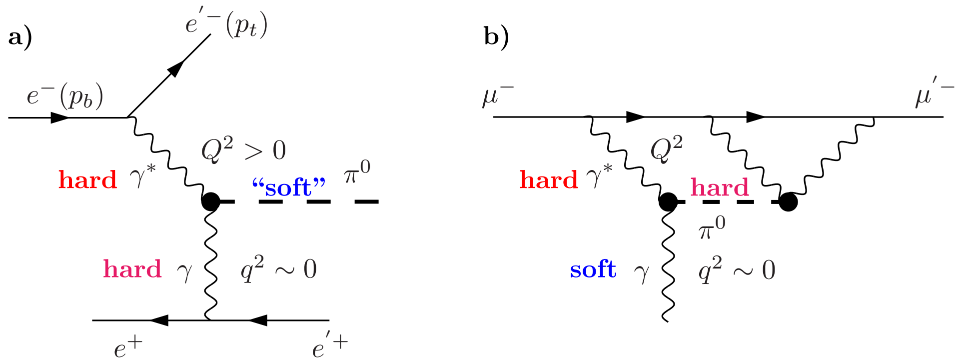
$$\approx \frac{2\mathrm{i}}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha \langle 0|j_5^\beta(z)|\pi^0(p)\rangle \times \langle \pi^0(p)|j_\lambda(0)|\gamma(k)\rangle + \cdots$$

$$= \frac{2\mathrm{i}}{\hat{q}^2}\varepsilon_{\mu\nu\alpha\beta}\hat{q}^\alpha\, 2\mathrm{i}\, F_\pi\, p_\pi^\beta\, \mathrm{e}^{\mathrm{i}p_\pi z} \times -4\, e\, g_{\pi^0\gamma\gamma}\, p_\pi^\sigma\, \tilde{f}_{\sigma\lambda} + \cdots$$

The first ME defines F_π, the second fixes $g_{\pi^0\gamma\gamma} = 1/(16\pi^2 F_\pi)$ by the anomaly in the chiral limit $m_\pi^2 \to 0$ (product independent of F_π).

Another issue:

the need of analytic continuation

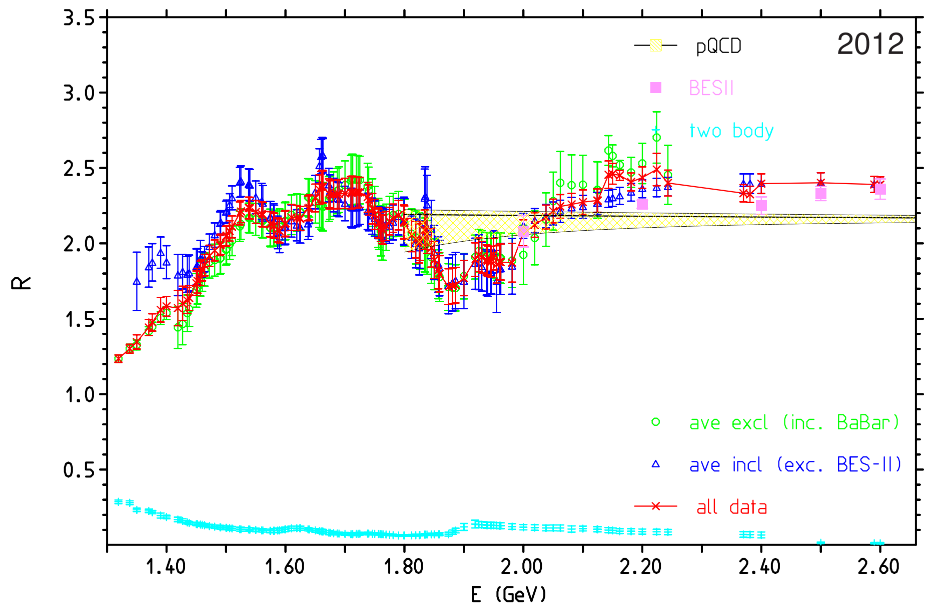


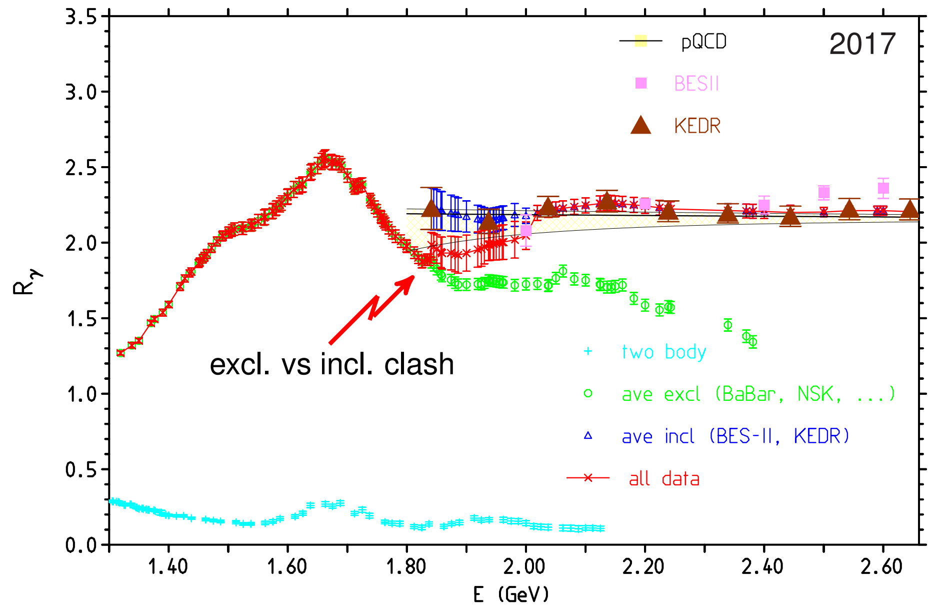
Measured is $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space–like Q^2, needed at external vertex is $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(-Q^2, -Q^2, 0)$ or $\mathcal{F}_{\pi^{0*}\gamma^*\gamma}(q^2, q^2, 0)$ if integral to be evaluated in Minkowski space.

Can we check such questions experimentally or in lattice QCD?

Still an issue in HVP

- region 1.2 to 2 GeV data; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty





- illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR