COMPASS: Meson Spectroscopy and Low-Energy Meson Dynamics

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TU München

COMPASS collaboration



Krakow June 11, 2018



Jan Friedrich (TU Munich)

Mesons at COMPASS

The COMPASS experiment commom Muon Proton Apparatus for Structure and Spectroscopy



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The COMPASS experiment

Commom Muon Proton Apparatus for Structure and Spectroscopy

CERN SPS: protons \sim 450 GeV (5 – 10 sec spills)

- tertiary muons: 4.107 / s
- secondary π, K, \overline{p} : up to 2.10⁷/s (typ. 5.10⁶/s)



Jan Friedrich (TU Munich)

The COMPASS experiment

Commom Muon Proton Apparatus for Structure and Spectroscopy

Fixed-target experiment

- two-stage magnetic spectrometer
- high-precision, high-rate tracking, PID, calorimetry

Beam

RPD + Target

SM1

RICF

E/HCAL2

SM2

E/HCAL1

The COMPASS experiment

Commom Muon Proton Apparatus for Structure and Spectroscopy

Fixed-target experiment



 high-precision, high-rate tracking, PID, calorimetry

Collaboration

SM2

E/HCAL2

E

RICH

- > 200 physicists
- currently 23 institutes
- increasing number of associated members

RPD + Target

SM

COMPASS physics Commom Muon Proton Apparatus for Structure and Spectroscopy



Hadron Spectroscopy & Chiral Dynamics



Polarised SIDIS



Polarised Drell-Yan

COMPASS-II 2012-2020

COMPASS-I 1997-2011

& 2021



DVCS (GPDs) + unp. SIDIS

The pion polarisability measurement

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- Pion polarisability: prediction $\alpha_{\pi}(ChPT) = 2.79 \cdot 10^{-4} \text{ fm}^3$
- Previous experimental determinations since 1982 were about twice as large
- COMPASS measurement confirms ChPT within the experimental uncertainties



Chiral dynamics in $\pi\gamma \rightarrow 3\pi$



Chiral anomaly in $\pi^- \gamma \rightarrow \pi^- \pi^0$



- contributions from chiral anomaly $F_{3\pi}$ and the $\rho(770)$ resonance
- can be described by a dispersive method → increased sensitivity to the chiral anomaly
- uncertainty estimate < 1%</p>



Chiral anomaly in $\pi^-\gamma \rightarrow \pi^-\pi^0$



- in-flight decay of Kaons (2.4% of beam) \rightarrow normalization
- background from $\pi^{-}\pi^{0}\pi^{0}$ subtracted
- luminosity determination ongoing (in common with $\pi^-\pi^0\pi^0$)

Radiative Coupling of $a_2(1320)$ and $\pi_2(1670)$



- \Leftrightarrow meson wave functions: $\Gamma_{i \to f} \propto |\langle \Psi_f | e^{-i \vec{q} \cdot \vec{r}} \hat{\epsilon} \cdot \vec{p} | \Psi_i \rangle|^2$
 - normalization via beam kaon decays
 - Iarge Coulomb correction

published in EPJ A50 (2014) 79

 COMPASS: World's currently largest data set for the diffractive process

$$p + \pi_{\text{beam}}^- \rightarrow p + \pi^- \pi^+ \pi^-$$

taken in 2008 ($\sim 46 \cdot 10^6$ exclusive Events)

Exclusive measurement



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- Also structure in π⁺π⁻ subsystem: Intermediate states ξ (Isobar)



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- Intermediate states appear as dynamic amplitudes Δ (m): Complex-valued functions of invariant mass m
- Simplest example: Breit-Wigner amplitude with mass m_0 and width Γ_0 :

$$\Delta_{\rm BW}\left(m\right) = \frac{m_0\Gamma_0}{m_0^2 - m^2 - im_0\Gamma_0}$$

- Analysis in bins of $m_{\chi^-} = m_{3\pi}$. Dynamic amplitude of χ^- inferred form the data
- Dynamic amplitude of ξ : Model input in conventional PWA
- True dynamic isobar amplitudes may differ from the model
- Free parameters in dynamic isobar amplitudes computationally unfeasible











Step-like isobar amplitudes details: cf. this afternoon's talk by Fabian Krinner

• Total intensity in each single $(m_{3\pi}, t')$ -bin

$$\mathcal{I}(\vec{\tau}) = \left|\sum_{i}^{\text{waves}} \mathcal{T}_{i}[\psi_{i}(\vec{\tau}) \Delta_{i}(m_{\pi^{-}\pi^{+}}) + \text{Bose Symm.}]\right|^{2}$$

as function of phase-space variables $\vec{\tau}$

Fit parameters: Production amplitudes T_i

Fixed: Angular distributions $\psi(\vec{\tau})$, dynamic isobar amplitudes $\Delta_i(m_{\pi^-\pi^+})$

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$$\Delta_{i}(m_{\pi^{-}\pi^{+}}) \rightarrow \sum_{\text{bins}} \mathscr{T}_{i}^{\text{bin}} \underbrace{\Delta_{i}^{\text{bin}}(m_{\pi^{-}\pi^{+}})}_{1 \text{ in the bin,}} \equiv [\pi\pi]_{J^{PC}}$$

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as function of phase-space variables $ec{ au}$

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$$1 \text{ in the bin,}$$

$$0 \text{ otherwise}$$

• Each bin introduces an independent Partial Wave $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathcal{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_{i}^{\text{waves}} \sum_{\text{bins}} \mathcal{T}_{i}^{\text{bin}} \left[\psi_{i}(\vec{\tau}) \Delta_{i}^{\text{bin}} \left(m_{\pi^{-}\pi^{+}} \right) + \text{Bose S.} \right] \right|^{2}$$

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Partial-Wave Analysis

$$\mathcal{I}(\vec{\tau}) = \left| \sum \mathcal{T}_{i}\psi_{i}(\vec{\tau}) \Delta_{i}(m_{\pi^{-}\pi^{+}}) \right|^{2}$$
Waves specified by:

$$\int PCM^{\mathcal{E}}\xi\pi L$$

$$p_{\text{target}}$$

$$p_{\text{recoil}}$$

$$P$$

$$p_{\text{recoil}}$$

- J^{PC}: spin and eigenvalues under parity and charge conjugation of X⁻ (or its multiplet)
- M^e: spin projection and naturality of the exchange particle
- π : the bachelor π^- (always the same)
- ξ : the fixed or freed isobar, e.g. ρ (770) or $[\pi\pi]_{1--}$
- L: orbital angular momentum between isobar and bachelor pion

88 waves needed to describe the data ("hand-selected") interference terms \rightarrow get (relative) phases

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Mesons at COMPASS

Selected Waves (1 of 88) in two of the 11 independent t' bins



Selected Waves (2 of 88) in two of the 11 independent t' bins



Selected Waves (3 of 88) in two of the 11 independent t' bins



ngn *t*

Selected Waves (4 of 88) in two of the 11 independent t' bins



Step 2: Resonance model fit



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Towards an exotic signal: $J^{PC} = 1^{++}$ sector



 $a_1(1260)$

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- coupling strength does (form factors) and non-resonant parts may vary with t'



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- $a_1(1260)$ reproduced: $m^{fit} = 1298^{+13}_{-22} \text{ MeV}/c^2$ $m^{PDG} = 1230 \pm 40 \text{ MeV}/c^2$ $\Gamma^{fit} = 403^{+0}_{-100} \text{ MeV}/c^2$ $\Gamma^{PDG} = 250 - 600 \text{ MeV}/c^2$



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- weak signal for a₁(1640)





11 June 2018 18 / 27

 $a_1(1420)$ a new - quite exotic - signal



Mesons at COMPASS

- new signal: *a*₁(1420)
- decay into $f_0(980)\pi$







- new signal: a₁(1420)
- decay into *f*₀(980)π
- possible explanations:
 - triangle diagram Mikhasenko, Ketzer, Sarantsev PRD91 (2015) 094015
 - two-channel unitarized Deck amplitude Basdevant, Berger PRL114 (2015) 192001

 $m_{3\pi}$ [GeV/c²]

2.2

1.8

1.6





- new signal: a₁(1420)
- decay into $f_0(980)\pi$
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 - triangle diagram Mikhasenko, Ketzer, Sarantsev PRD91 (2015) 094015
 - two-channel unitarized Deck amplitude Basdevant, Berger PRL114 (2015) 192001
- Mass:

 $m_{a_1(1420)} = 1411.8^{+1.0}_{-4.4} \,\mathrm{MeV}/c^2$ Width:

$$\Gamma_{a_1(1420)} = 158^{+8}_{-8}\,{
m MeV}/c^2$$

2.2

Resonance parameters



- resonance parameters with unprecedented precision and systematic investigations of 6 *a*-like and 5 π-like states
- 75-pages PRD recently accepted: Light isovector resonances in $\pi^- p \rightarrow \pi^- \pi^- \pi^+ p$ at 190 GeV/c
- recently published: PLB779(2018)464 New analysis of $\eta\pi$ tensor resonances measured at the COMPASS experiment (together with JPAC): better constraints on $a'_2(1700)$

Some math: zero mode in the spin-exotic wave What is a "zero mode"?

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
- introduces continuous mathematical ambiguities in the model

arxiv.org/abs/1710.09849 F. Krinner *et al*, Resolving ambiguities in model-independent partial-wave analysis of three-body decay *talk this afternoon by Fabian Krinner*

Some math: zero mode in the spin-exotic wave What is a "zero mode"?

- Freed-isobar analysis: much more freedom than fixed-isobar analysis
- introduces continuous mathematical ambiguities in the model
- "Zero mode": dynamic isobar amplitudes $\Omega(m_{\pi^-\pi^+})$ that do not contribute to the **total** 3π -amplitude
- Spin-exotic wave:

$$\psi(\vec{\tau})\Omega(m_{\pi^-\pi^+}) + \text{Bose S.} = \mathbf{0}$$

at every point $\vec{\tau}$ in phase space

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- Example: One bin in $(m_{3\pi}, t')$
 - $1.58 < m_{3\pi} < 1.62 \, {\rm GeV}/c^2$
 - $0.326 < t' < 1.000 (GeV/c)^2$



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- Zero-mode ambiguity resolved with ρ (770) used as constraint
- Dynamic isobar amplitude dominated by ρ (770)



Conclusions and Outlook

COMPASS on chiral dynamics:

- Measurement of the pion polarisability at COMPASS
 - via the Primakoff reaction, COMPASS has determined

$$\alpha_{\pi} =$$
 (2.0 \pm 0.6_{stat} \pm 0.7_{syst}) \times 10⁻⁴ fm³

- most direct access to the $\pi\gamma \to \pi\gamma$ process
- most precise experimental determination
- control of systematics: $\mu\gamma \to \mu\gamma$, $K^- \to \pi^-\pi^0$
- more data (×4) on tape
- Related topics at COMPASS: radiative widths and chiral dynamics in $\pi^-\gamma \rightarrow \pi^-\pi^0$ and $\pi\gamma \rightarrow \pi\pi\pi$
 - chiral anomaly on the way



some of the new COMPASS entries in the RPP2016 edition

Citation: C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016)

COMPASS on exotic mesons:

- partial-wave decomposition of $\pi^-\pi^+\pi^-$ with 88 waves
- o conclusions on two exotic signals:
 - ► a₁(1420) supernumerous
 - * matches a Breit-Wigner description with $\Gamma = 158 \text{ MeV}/c^2$
 - * position at $K^*\bar{K}$ threshold \rightarrow rescattering interpretation
 - * and/or Deck interference
 - $\pi_1(1600)$ spin-exotic
 - * at small t' dominant background
 - * slow phase motion, much broader than previous analyses
- ongoing developments
 - refine non-resonant (Deck) background description
 - include unitary constraints / dispersion relations

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- more channels to come, including π^0, η, K
- Iow statistics for incoming K[−] beams → future option: dedicated RF-separated beam, part of upcoming Letter of Intent Mini-Workshop (half-day) on June 20, 2pm, CERN / vidyo

Thank you for your attention!



Mathematical origin

- Process: $X^- \to \xi \pi_3^- \to \pi_1^- \pi_2^+ \pi_3^-$.
- Partial-wave amplitude

$$\psi\left(\vec{\tau}
ight) \Omega\left(m_{12}
ight) + \mathrm{Bose~S.} = \mathbf{0}$$

Tensor formalism (X⁻ rest frame) for 1⁻⁺

ų

 $\psi\left(ec{ au}
ight)\proptoec{p}_{1} imesec{p}_{3}$

(1)

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• Bose symmetrization $(\pi_1^- \leftrightarrow \pi_3^-)$:

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• Fulfill (1) at every point in phase space $\Rightarrow \Omega(m_{\xi}) = \text{const.}$

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- then intensity is not altered:

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for any complex-valued zero-mode coefficient $\ensuremath{\mathcal{C}}$

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for any complex-valued zero-mode coefficient \mathcal{C}

C: complex-valued ambiguity in the model

The $1^{-+}1^+\rho(770)\pi P$ wave



at low t' very weak resonant component

The 1⁻⁺1⁺ho(770) πP wave



at higher t' resonant component dominant

The 1⁻⁺1⁺ho(770) πP wave



at higher t' resonant component dominant

The 1⁻⁺1⁺ ρ (770) πP wave Phase motion



resonance with mass \sim 1600 MeV/ c^2 very broad $\Gamma \sim$ 600 MeV/ c^2