Recent results of the pion-nucleon scattering and the widths of the delta and Roper resonances in BChPT

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Talk is based on:

D. L. Yao, et al., JHEP **1605**, 038 (2016).

J. Gegelia, U.-G. Meißner, D. Siemens and D. L. Yao, Phys. Lett. B **763**, 1 (2016).

J. Gegelia, U.-G. Meißner and D. L. Yao, Phys. Lett. B **760**, 736 (2016).

Outline

- Low-energy chiral EFT;
- Pion-nucleon scattering at low energies;
- The width of the Roper resonance;
- The width of the delta resonance;
- Summary;

Low energy chiral EFT

- A very well motivated assumption:
  QCD is a correct theory of the strong interaction.
- Perturbation theory is not applicable at low energies.

  Chiral EFT provides with a solution to this problem.
- Started with:

  S. Weinberg, Physica A **96**, 327 (1979).

- Based on the symmetries of QCD, chiral EFT aims at reproducing the S-matrix of QCD in low-energy region.
- Hadronic one-particle states are represented by dynamical fields in EFT.
  Effective degrees of freedom: pions, nucleons, $\Delta(1232)$, ...
- Chiral EFT provides with a systematic expansion of physical quantities in powers of (small scale(s)/ large scale)
- Bound states require resummation of infinitely many diagrams.

- Most general EFT Lagrangian of Hadrons with symmetries of QCD gives the most general S-matrix with these symmetries.
- To obtain S-matrix of QCD one needs to fix properly the parameters of EFT $\cdots$
- $\cdots$ a finite number of them to achieve a finite accuracy!
- $\cdots$ EFT $\neq$ QCD.

  QCD calculates physical quantities in terms of fundamental parameters, EFT only relates physical quantities to each other at low-energies, like

$$\sigma_k(E) = F(E, \sigma_1(\mu_i), \sigma_2(\mu_i), \cdots, \mu).$$

What to do?

- Write down the most general effective Lagrangian.
- Consider *all* Feynman diagrams contributing to the process in question.
- Renormalize/subtract loop diagrams.
- Apply power counting - expansion in powers of small parameters due to spontaneously broken chiral symmetry.
- Sum up all *renormalized* diagrams contributing up to given order.
- Only a finite number of diagrams contribute at any given order in one-nucleon sector.

Effective Lagrangian

Effective Lagrangian of pions, nucleons, Δ and Roper resonances as an expansion in quark masses and derivatives acting on pion fields:

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} + \mathcal{L}_{\pi\Delta} + \mathcal{L}_{\pi R} + \mathcal{L}_{\pi N\Delta} + \mathcal{L}_{\pi NR} + \mathcal{L}_{\pi\Delta R}.$$

From the purely mesonic sector we need the following structures

$$\begin{aligned}
\mathcal{L}_{\pi\pi}^{(2)} &= \frac{F^2}{4}\langle\partial_\mu U \partial^\mu U^\dagger\rangle + \frac{F^2 M^2}{4}\langle U^\dagger + U\rangle, \\
\mathcal{L}_{\pi\pi}^{(4)} &= \frac{1}{8} l_4 \langle u^\mu u_\mu\rangle\langle\chi_+\rangle + \frac{1}{16}(l_3 + l_4)\langle\chi_+\rangle^2,
\end{aligned}$$

where $\langle\ \rangle$ denotes the trace in flavor space, F is the pion decay constant in the chiral limit and M is the pion mass at leading order. The pion fields are contained in the 2×2 matrix U, with $u = \sqrt{U}$ and

$$\begin{aligned}
u_\mu &= i\left[u^\dagger \partial_\mu u - u\partial_\mu u^\dagger\right], \\
\chi^+ &= u^\dagger \chi u^\dagger + u\chi^\dagger u\,, \quad \chi = \begin{bmatrix} M^2 & 0 \\ 0 & M^2 \end{bmatrix}.
\end{aligned}$$

Terms of the Lagrangian with pions and baryons:

$$
\begin{aligned}
\mathcal{L}_{\pi N}^{(1)} &= \bar{\Psi}_N \left\{ i\not{D} - m + \frac{1}{2} g \not{u} \gamma^5 \right\} \Psi_N \,, \\
\mathcal{L}_{\pi N}^{(2)} &= \bar{\Psi}_N \left\{ c_1 \langle \chi_+ \rangle - \frac{c_2}{4m^2} \langle u^\mu u^\nu \rangle (D_\mu D_\nu + h.c.) \right. \\
&\qquad \left. + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] \right\} \Psi_N \,, \\
\mathcal{L}_{\pi N}^{(3)} &= \bar{\Psi}_N \left\{ -\frac{d_1 + d_2}{4m} ([u_\mu, [D_\nu, u^\mu] + [D^\mu, u_\nu]] D^\nu + h.c.) \right. \\
&\qquad + \frac{d_3}{12m^3} ([u_\mu, [D_\nu, u_\lambda]] (D^\mu D^\nu D^\lambda + sym.) + h.c.) \\
&\qquad + i \frac{d_5}{2m} ([\chi_-, u_\mu] D^\mu + h.c.) \\
&\qquad + i \frac{d_{14} - d_{15}}{8m} (\sigma^{\mu\nu} \langle [D_\lambda, u_\mu] u_\nu - u_\mu [D_\nu, u_\lambda] \rangle D^\lambda + h.c.) \\
&\qquad \left. + \frac{d_{16}}{2} \gamma^\mu \gamma^5 \langle \chi_+ \rangle u_\mu + \frac{i d_{18}}{2} \gamma^\mu \gamma^5 [D_\mu, \chi_-] \right\} \Psi_N \,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\pi R}^{(1)} &= \bar{\Psi}_R \left\{ i\not{D} - m_R + \frac{1}{2} g_R \not{u} \gamma^5 \right\} \Psi_R \,, \\
\mathcal{L}_{\pi R}^{(2)} &= \bar{\Psi}_R \left\{ c_1^R \langle \chi^+ \rangle \right\} \Psi_R \,, \\
\mathcal{L}_{\pi NR}^{(1)} &= \bar{\Psi}_R \left\{ \frac{g_{\pi NR}}{2} \gamma^\mu \gamma_5 u_\mu \right\} \Psi_N + \text{h.c.} \,, \\
\mathcal{L}_{\pi \Delta}^{(1)} &= -\bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \left\{ \left(i\not{D}^{jk} - m_\Delta \delta^{jk} \right) g^{\mu\nu} - i \left(\gamma^\mu D^{\nu,jk} + \gamma^\nu D^{\mu,jk} \right) \right. \\
&+ i\gamma^\mu \not{D}^{jk} \gamma^\nu + m_\Delta \delta^{jk} \gamma^\mu \gamma^\nu + \frac{g_1}{2} \not{u}^{jk} \gamma_5 g^{\mu\nu} \\
&\quad \left. + \frac{g_2}{2} (\gamma^\mu u^{\nu,jk} + u^{\nu,jk} \gamma^\mu) \gamma_5 + \frac{g_3}{2} \gamma^\mu \not{u}^{jk} \gamma_5 \gamma^\nu \right\} \xi_{kl}^{\frac{3}{2}} \Psi_\nu^l \,, \\
\mathcal{L}_{\pi N \Delta}^{(1)} &= h \, \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(z_1) \, \omega_\alpha^j \Psi_N + h.c. \,, \\
\mathcal{L}_{\pi N \Delta}^{(2)} &= \bar{\Psi}_\mu^i \xi_{ij}^{\frac{3}{2}} \Theta^{\mu\alpha}(z_2) \left[i \, b_3 \omega_{\alpha\beta}^j \gamma^\beta + i \, \frac{b_8}{m} \omega_{\alpha\beta}^j i \, D^\beta \right] \Psi_N + h.c. \,,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}^{(3)}_{\pi N\Delta} \quad = \quad & \bar{\Psi}^i_\mu \xi^{\frac{3}{2}}_{ij} \Theta^{\mu\nu}(z_3) \left[\frac{f_1}{m} [D_\nu, \omega^j_{\alpha\beta}] \gamma^\alpha i D^\beta - \frac{f_2}{2m^2} [D_\nu, \omega^j_{\alpha\beta}] \{D^\alpha, D^\beta\} \right. \\
& \left. + f_4 \omega^j_\nu \langle \chi_+ \rangle + f_5 [D_\nu, i\chi^j_-] \right] \Psi_N + h.c.,
\end{aligned}
$$

where h is $\pi N\Delta$ coupling at lowest order and b_3, b_8, f_1, f_2, f_4 and f_5 are LECs of higher orders.
z_1, z_2 and z_3 are off-shell parameters.

Three chiral structures, ω^i_α, $\omega^j_{\alpha\beta}$ and χ^k_-, are given by

$$
\begin{aligned}
\omega^i_\alpha &= \frac{1}{2} \langle \tau^i u_\alpha \rangle = -\frac{1}{F} \partial_\alpha \pi^i + \frac{1}{6F^3} (\partial_\alpha \pi^i \pi^a \pi^a - \pi^i \partial_\alpha \pi^a \pi^a) + \mathcal{O}(\pi^5) , \\
\omega^j_{\alpha\beta} &= \frac{1}{2} \langle \tau^j [\partial_\alpha, u_\beta] \rangle = -\frac{1}{F} \partial_\alpha \partial_\beta \pi^j + \mathcal{O}(\pi^3) , \\
\chi^k_- &= \frac{1}{2} \langle \tau^k \chi_- \rangle = -\frac{2i}{F} M^2 \pi^k + \mathcal{O}(\pi^3) .
\end{aligned}
$$

Ψ_N and Ψ_R are the fields of the nucleon and the Roper resonance, respectively.

Rarita-Schwinger field Ψ_ν represents the Δ resonance.
$\xi^{\frac{3}{2}}$ is the isospin-3/2 projector, $\omega^i_\alpha = \frac{1}{2}\langle \tau^i u_\alpha \rangle$ and
$\Theta^{\mu\alpha}(z) = g^{\mu\alpha} + z\gamma^\mu\gamma^\nu$, where z is a so-called off-shell parameter.

We fix the off-shell structure by adopting $g_2 = -g_3 = 0$ and $z_1 = \tilde{z} = 0$.

The covariant derivatives are defined as follows:

$$
\begin{aligned}
D_\mu \Psi_{N/R} &= (\partial_\mu + \Gamma_\mu)\, \Psi_{N/R}\,, \\
(D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2\, i\, \epsilon_{ijk} \Gamma_{\mu,k} \Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i}\,, \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger\right] = \tau_k \Gamma_{\mu,k}\ .
\end{aligned}
$$

πN scattering in EFT of pions, nucleons and deltas

The amplitude of $\pi^a(q) + N(p) \to \pi^{a'}(q') + N(p')$ in the isospin limit:

$$T_{\pi N}^{a'a}(s,t,u) = \chi_{N'}^\dagger \left\{ \delta_{a'a} T^+(s,t,u) + \frac{1}{2}[\tau_{a'},\tau_a] T^-(s,t,u) \right\} \chi_N \,,$$

where a' and a are Cartesian isospin indices, τ_i - Pauli matrices and χ_N, $\chi_{N'}$ denote nucleon iso-spinors.

Lorentz decomposition of $T^\pm$:

$$T^\pm(s,t,u) = \bar{u}^{(s')}(p') \left\{ D^\pm(s,t,u) - \frac{1}{4m_N}[\not{q}', \not{q}] B^\pm(s,t,u) \right\} u^{(s)}(p) \,,$$

with (s'), (s) denoting the spins of $\bar{u}$, u, respectively.
Lorentz decomposition is not unique, a popular alternative form is

$$T^\pm(s,t,u) = \bar{u}^{(s')}(p') \left\{ A^\pm(s,t,u) + \frac{1}{2}\left(\not{q}' + \not{q}\right) B^\pm(s,t,u) \right\} u^{(s)}(p) \,.$$

Decomposition in terms of D and B is better suited for the chiral EFT due to the cancellation between contributions from A and B.

Power counting

For diagrams involving only pion and nucleon lines, we use the standard power counting considering the pion mass M and small momenta as of order $\mathcal{O}(p)$.

S. Weinberg, Nucl. Phys. B **363**, 3 (1991).
G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).

For diagrams with delta lines we apply the power counting of

T. R. Hemmert, B. R. Holstein and J. Kambor, J. Phys. G **24**, 1831 (1998),

that is we count $\Delta = m_\Delta - m_N$ as of order $\mathcal{O}(p)$.

For $s \to m_\Delta^2$ delta-propagator diverges, therefore we need to sum up self-energy insertions, i.e. consider the dressed propagator
$D^{\mu\nu}(k) \sim 1/(\not{k} - m_\Delta - \Sigma(k)) \sim 1/(-\Sigma(k)) \sim 1/p^3$.

We follow an alternative way by using the complex-mass scheme where the undressed propagator contains the width of the unstable particle and therefore the re-summation is not necessary.

Tree order contributions

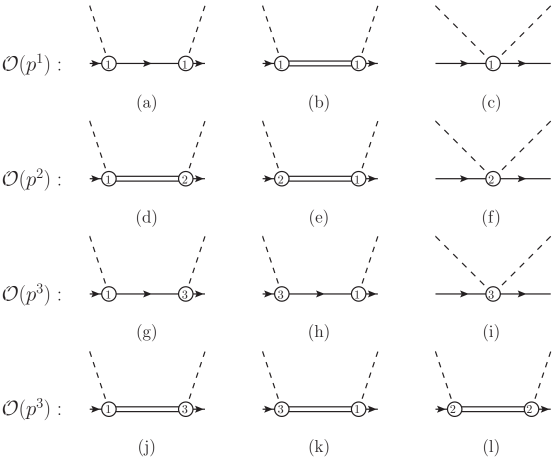


Figure: Tree diagrams contributing to πN scattering up to order $\mathcal{O}(p^3)$. Crossed diagrams are not shown.

Leading one-loop contributions

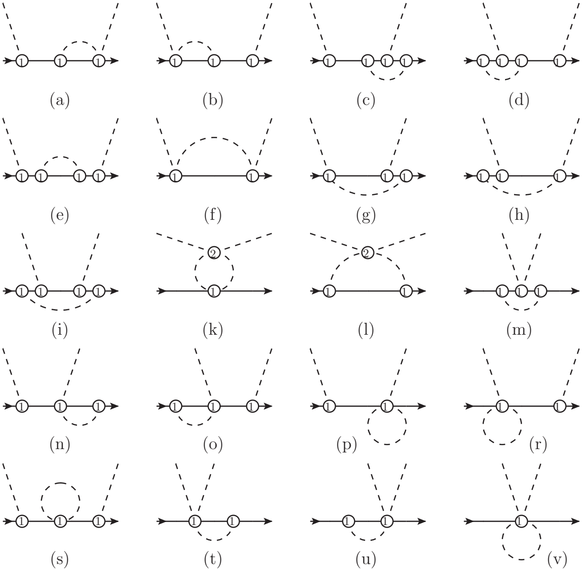


Figure: One-loop Feynman diagrams without explicit deltas to order $O(p^3)$.

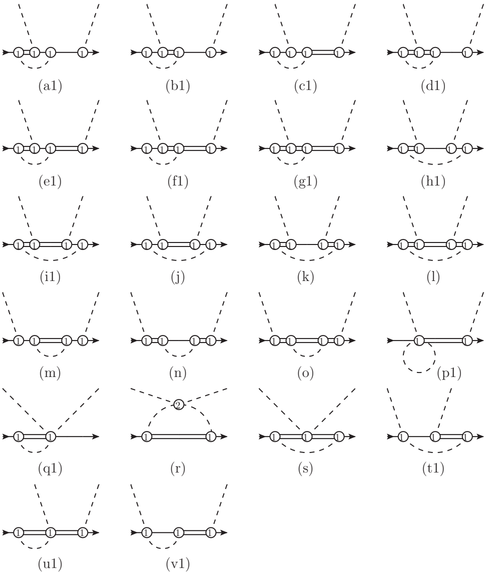


Figure: One-loop diagrams with explicit deltas to order $O(p^3)$. Crossed diagrams and diagrams with the reversed time ordering are not shown.

Renormalization

We apply EOMS renormalization scheme of

J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999),
T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).
and its generalization for delta.

To calculate loop diagrams we apply dimensional regularization.

UV divergences are removed by counter terms generated by the effective Lagrangian.

Finite pieces of counter terms are fixed such that the subtracted contributions in physical quantities satisfy the power counting.

Expressions of diagrams and subtraction terms are huge!

Final finite amplitudes respect the power counting and have the correct analytic behaviour.

We fit the unknown LECs to the phase shifts of the S- and P-waves.

Then we predict the D- and F-wave phase shifts and the threshold parameters using the determined LECs.

PW analysis of the πN amplitudes of several groups are available:

R. Koch and E. Pietarinen, Nucl. Phys. A **336**, 331 (1980).
R. Koch, Nucl. Phys. A **448**, 707 (1986).

E. Matsinos, W. S. Woolcock, G. C. Oades, G. Rasche and A. Gashi, Nucl. Phys. A **778**, 95 (2006),

R. A. Arndt, W. J. Briscoe, I. I. Strakovsky and R. L. Workman, Phys. Rev. C **74**, 045205 (2006).

Unfortunately, none of these groups provide uncertainties.

Therefore, we prefer to perform fits to the phase shifts generated by the recent RS-equation analysis of the πN scattering:

M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner, Phys. Rept. **625**, 1 (2016).

There are eleven LECs involved in the πN amplitudes in total: c_1, c_2, c_3, c_4, $d_1 + d_2$, d_3, d_5, $d_{14} - d_{15}$, $g_{\pi N}$, $g_{\pi N\Delta}$ and g_1.

We fix $g_{\pi N}$ coupling at $g_{\pi N}^2/(4\pi) = 13.69 \pm 0.20$,

V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga and D. R. Phillips, Phys. Lett. B **694**, 473 (2011).

V. Baru, C. Hanhart, M. Hoferichter, B. Kubis, A. Nogga and D. R. Phillips, Nucl. Phys. A **872**, 69 (2011).

Fit-I corresponds to the delta-less case and is performed up to $W_{\rm max}$=1.11 GeV.

Our plots for Fit-I are shown in the next page.

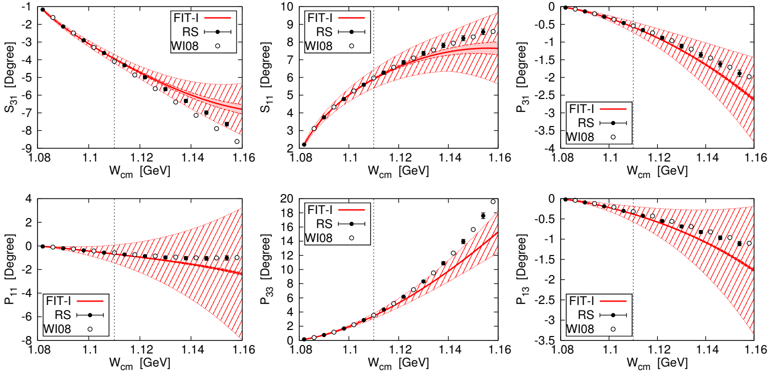


Figure: Phase shifts of fit-I.
Dots - RS phase shifts; Circles - GWU phase shifts; Red lines - our results. Red narrow and wide bands - uncertainties propagated from the errors of LECs and theoretical uncertainties, respectively.

Error bands in P_{33} and S_{31} PWs do not cover the RS and GWU data beyond the fitting range → theoretical errors underestimated.

Adding the delta degree of freedom should mostly improve the description of the P_{33} wave in the Δ-resonance region.

We performed two fits (Fit-II and Fit-III) using 1.2 GeV as $W_{\rm max}$ for the P_{33} partial wave and 1.11 GeV for the other five PWs.

Fit II is done by adding the LO tree contribution of the delta-exchange diagrams to the delta-less case and serves only the purpose of estimating the effect of the loop diagrams with delta.

Unlike previous works,

J. M. Alarcon, J. Martin Camalich and J. A. Oller, Annals Phys. **336**, 413 (2013),
Y. H. Chen, D. L. Yao and H. Q. Zheng, Phys. Rev. D **87**, 054019 (2013),

we incorporated the complex pole position and obtained better results with smaller uncertainties, for instance, the large errors in $d_{14} - d_{15}$ are substantially reduced.

Our plots for Fit-II are shown in next page.

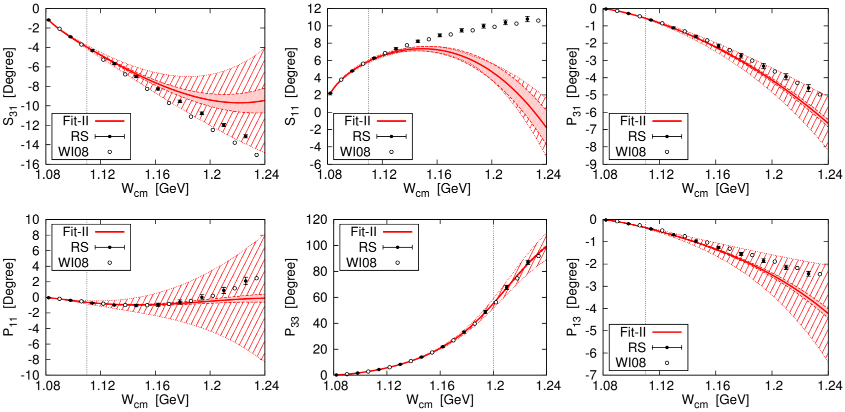


Figure: Phase shifts of Fit-II.

Dots with error bars - RS phase shifts; Circles - GWU phase shifts; The red line - our results.

The red narrow error bands - the uncertainties propagated from the errors of LECs.The wide dashed error bands - the theoretical uncertainties.

Fit-III is done with the full contributions of pions, nucleons and deltas up to NNLO.

The obtained LECs of Fit-III are different from those of Fit-II due to the inclusion of contributions of loop diagrams involving delta lines.

All the c_i and most of the higher order LECs are of natural size.

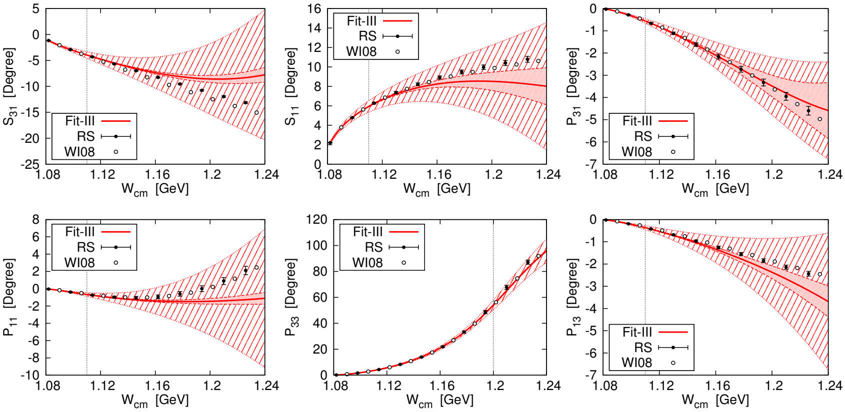


Figure: Phase shifts from BChPT with explicit delta - Fit-III.
Dots with error bars - RS phase shifts; Circles - GWU phase shifts. The red line - the result of Fit III.
The red narrow error bands - uncertainties propagated from the errors of LECs. The wide dashed error bands - theoretical uncertainties.

Compared to the plots in Fit-II, Fit-III improves the predictions beyond fitting ranges in most of the partial waves, especially for the S_{11} wave.

The larger theoretical error in Fit-III compared to Fit-II is due to the large contributions of delta-loop diagrams, which are not taken into account in estimation of the theoretical error of Fit-II.

Imaginary part of h_A from Fit-III is small compared to the corresponding real part $\mathrm{Re}[h_A]$ and our determination for $\mathrm{Re}[h_A]$ is close to the large-N_c prediction.

The obtained g_1 for Fit-III is nearly consistent (within the error bars) with the corresponding large-N_C result, $|g_1| = 9g_A/5 \simeq 2.28$. g_1 appears only in the loop contribution, hence a precise determination of its value is not to be expected.

Table: LECs for various fits. c_i and d_j are in GeV^{-1} and GeV^{-2}, resp. Stat. and syst. uncert. shown in the first and the second brackets, respectively.

LEC	Fit-I N (i.e. $\not\Delta$)	Fit-II N+LO Δ	Fit-III $N+\Delta$
c_1	$-1.22(2)(2)$	$-0.99(2)(1)$	$-1.31(2)(1)$
c_2	$3.58(3)(6)$	$1.38(3)(1)$	$0.78(4)(2)$
c_3	$-6.04(2)(9)$	$-2.33(3)(1)$	$-2.55(10)(7)$
c_4	$3.48(1)(3)$	$1.71(2)(1)$	$1.20(4)(2)$
d_{1+2}	$3.25(4)(9)$	$0.14(4)(3)$	$4.85(68)(64)$
d_3	$-2.88(8)(14)$	$-0.97(8)(15)$	$-0.62(10)(15)$
d_5	$-0.15(6)(14)$	$0.39(6)(11)$	$-0.93(11)(15)$
d_{14-15}	$-6.19(7)(12)$	$-1.08(8)(3)$	$5.54(2.79)(2.01)$
$g_{\pi N}$	13.12^*	13.12^*	13.12^*
h_A	–	$1.28(1)(1)$	$1.42(1)(1) - i\,0.16(1)(1)$
g_1	–	–	$-1.21(46)(39)$
χ^2/dof	$\frac{272.0(23.7)}{216-8} = 1.31(11)$	$\frac{339.8(27.4)}{328-9} = 1.07(9)$	$\frac{373.8(29.9)}{328-10} = 1.18(9)$

Using the fitted LECs we predict the phase shifts of higher PWs.

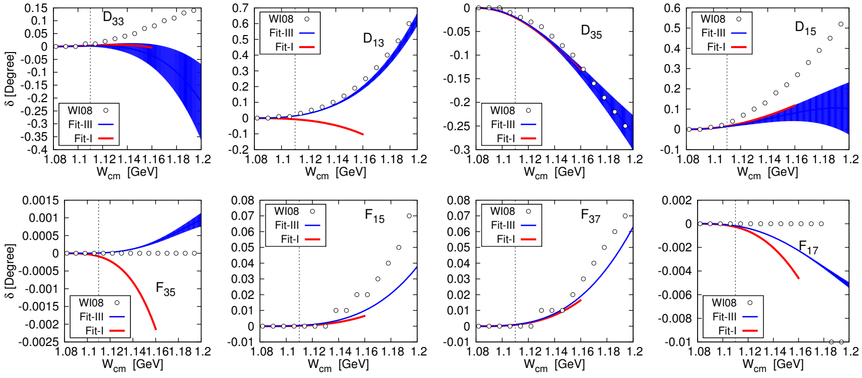


Figure: Phase shifts of the *D* and *F* PWs from the delta-less and delta-full BChPT using the parameters of Fit-I (red) and Fit-III (blue), respectively. The circles correspond to phase shifts by the GWU group.

Except for D_{33} channel, our predictions agree qualitatively with the GWU results and the predictions of the delta-full theory are somewhat better than those of the delta-less theory.

Scattering lengths and volumes

General form of the effective range expansion is given by

$$|\mathbf{p}|^{2\ell+1}\cot[\delta^{I}_{\ell\pm}] = \frac{1}{a^{I}_{\ell\pm}} + \frac{1}{2} r^{I}_{\ell\pm}|\mathbf{p}|^2 + \sum_{n=2}^{\infty} v^{I}_{n,\ell\pm}|\mathbf{p}|^{2n}\,,$$

where $\mathbf{p}$ is the three-momentum of the nucleon in center-of-mass frame, a is the threshold parameter, r - the effective range, and v_n - the shape parameters.

Results of the threshold parameters corresponding to the three different fits are presented below, together with the determinations from the Roy-Steiner equation analysis.

Table: Scattering lengths and volumes. The numbers in brackets correspond to the errors propagated from the uncertainties of LECs and the theoretical errors, respectively.

Th. Par.	Fit-I	Fit-II	Fit-III	RS
a^{+}_{0+} $[10^{-3}M_\pi^{-1}]$	$-0.6(7)(3.4)$	$-1.1(7)(3.0)$	$-0.5(7)(7.1)$	$-0.9(1.4)$
a^{-}_{0+} $[10^{-3}M_\pi^{-1}]$	85.7(5)(3.3)	85.8(4)(1.1)	85.8(3)(1.0)	85.4(9)
a^{+}_{1-} $[10^{-3}M_\pi^{-3}]$	$-49.8(1)(16)$	$-52.5(4)(4.7)$	$-51.0(5)(6.7)$	$-50.9(1.9)$
a^{-}_{1-} $[10^{-3}M_\pi^{-3}]$	$-9.7(3)(9.5)$	$-11.3(3)(3.2)$	$-9.5(2)(1.7)$	$-9.9(1.2)$
a^{+}_{1+} $[10^{-3}M_\pi^{-3}]$	139.9(1.8)(12)	131.0(4)(4.)	131.5(5)(8.8)	131.2(1.7)
a^{-}_{1+} $[10^{-3}M_\pi^{-3}]$	$-84.0(6)(4.)$	$-80.3(1)(1.4)$	$-80.4(2)(2.3)$	$-80.3(1.1)$

Nucleon sigma term

There are many studies of the πN σ-term, e.g., see
M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner, Phys. Rev. Lett. **115**, 092301 (2015),
V. Bernard, N. Kaiser and U.-G. Meißner, Z. Phys. C **60**, 111 (1993),
J. M. Alarcon, J. Martin Camalich and J. A. Oller, Phys. Rev. D **85**, 051503 (2012).

A high-precision determination of the $\sigma_{\pi N}$ was done from RS-equation analysis based on the improved Cheng-Dashen low-energy theorem and $\sigma_{\pi N} = (59.1 \pm 3.5)$ MeV was reported in
M. Hoferichter, J. Ruiz de Elvira, B. Kubis and U.-G. Meißner, Phys. Rev. Lett. **115**, 092301 (2015).

The πN sigma term $\sigma_{\pi N}$ can be obtained from the nucleon mass by applying the Hellmann-Feynman theorem,

$$\sigma_{\pi N} = \hat{m}\frac{\partial m_N}{\partial \hat{m}}, \quad \hat{m} = \frac{(m_u + m_d)}{2} ,$$

where m_N is the nucleon pole mass.

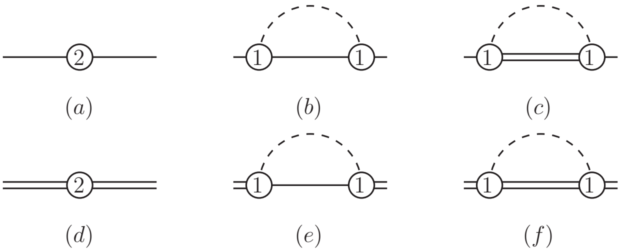


Figure: Tree and one-loop diagrams contributing to the self-energies of the nucleon and the delta resonance up to the order $O(p^3)$.

The results for $\sigma_{\pi N}$ based on the different sets of fitted parameters are shown in next page.

Table: The πN sigma term in units of MeV. The numbers in brackets correspond to the errors propagated from the uncertainties of LECs and the theoretical errors, respectively.

	Fit-I	Fit-II	Fit-III	RS
LO	94.3	76.5	101.2	–
NLO	−19.5	−19.5	−32.7	–
Sum	74.8(2.2)(11.4)	57.1(1.9)(7.0)	68.5(1.9)(7.6)	59.1(3.5)

Our prediction for Fit-I is marginally consistent with the RS determination when the large uncertainties are taken into account.

For Fit-II we obtained $\sigma_{\pi N} = 57.1(1.9)(7.0)$ MeV, which agrees with the RS determination very well.

Result of Fit-III $\sigma_{\pi N} = 68.5(1.9)(7.6)$ MeV improves the delta-less result and within the error it overlaps the value of the RS analysis.

Roper resonance in chiral EFT

Roper resonance is the first nucleon resonance that decays into $N\pi\pi$, besides decaying into $N\pi$.

Despite the fact that the Roper resonance was found a long time ago

L. D. Roper, Phys. Rev. Lett. **12**, 340 (1964).

a satisfactory theory of this state is still missing.

First steps in this direction within chiral EFT have been made in

B. Borasoy, P. C. Bruns, U.-G. Meißner and R. Lewis, Phys. Lett. B **641**, 294 (2006).
D. Djukanovic, J.Gegelia, S. Scherer, Phys. Lett. B **690**, 123 (2010).
B. Long and U. van Kolck, Nucl. Phys. A **870-871**, 72 (2011).
T. Bauer, J.Gegelia, S. Scherer, Phys. Lett. B **715**, 234 (2012).
E. Epelbaum, J.Gegelia, U.-G. Meißner and D. L. Yao, Eur. Phys. J. C **75**, no. 10, 499 (2015).

We present the calculation of the width of the Roper resonance at leading two-loop order in BChPT of pions, nucleons, the delta and Roper resonances.

J.Gegelia, U.-G. Meißner and D. L. Yao, “The width of the Roper resonance in baryon chiral perturbation theory,” Phys. Lett. B **760**, 736 (2016).

The width of the Roper resonance

The dressed propagator of the Roper resonance can be written as

$$i\,S_R(p) = \frac{i}{\not{p} - m_{R0} - \Sigma_R(\not{p}\,)}\,,$$

where $-i\,\Sigma_R(\not{p}\,)$ is the self-energy.

The pole of the dressed propagator S_R is obtained by solving

$$S_R^{-1}(z) \equiv z - m_{R0} - \Sigma_R(z) = 0\,.$$

We define the physical mass and the width of the Roper resonance by parameterizing the pole as

$$z = m_R - i\,\frac{\Gamma_R}{2}\,.$$

Topologies of the one- and two-loop Roper self-energy diagrams

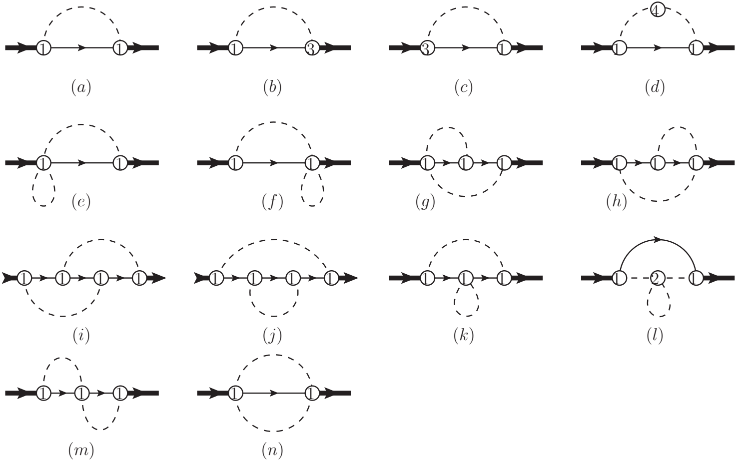


Figure: One and two-loop self-energy diagrams of the Roper resonance. The dashed and thick solid lines represent the pions and the Roper resonances, respectively. The thin solid lines in the loops stand for either nucleons, Roper or delta resonances.

We parameterize the pole as

$$z = m_2 + \hbar\delta z_1 + \hbar^2 \delta z_2 + \mathcal{O}(\hbar^3),$$

where $m_2 = m_R^0 + 4c_1^R M^2$, with m_R^0 the Roper mass in the chiral limit and write the self-energy as an expansion:

$$\Sigma_R = \hbar\Sigma_1 + \hbar^2\Sigma_2 + \mathcal{O}(\hbar^3)\,.$$

By expanding the equation for z in powers of $\hbar$, we get

$$\hbar\delta z_1 + \hbar^2\delta z_2 - \hbar\Sigma_1(m_2) - \hbar^2\delta z_1\Sigma_1'(m_2) - \hbar^2\Sigma_2(m_2) + \mathcal{O}(\hbar^3) = 0\,.$$

Solving order by order we obtain

$$\begin{aligned}\delta z_1 &= \Sigma_1(m_2),\\ \delta z_2 &= \Sigma_1(m_2)\,\Sigma_1'(m_2) + \Sigma_2(m_2).\end{aligned}$$

The width takes the form

$$
\begin{aligned}
\Gamma_R &= \hbar\, 2i\, \mathrm{Im}\left[\Sigma_1(m_2)\right] \\
&+ \hbar^2\, 2i \left\{ \mathrm{Im}\left[\Sigma_1(m_2)\right] \mathrm{Re}\left[\Sigma_1'(m_2)\right] + \mathrm{Re}\left[\Sigma_1(m_2)\right] \mathrm{Im}\left[\Sigma_1'(m_2)\right] \right\} \\
&+ \hbar^2\, 2i\, \mathrm{Im}\left[\Sigma_2(m_2)\right] + \mathcal{O}(\hbar^3).
\end{aligned}
$$

It turns out that the contribution of the second term is $\mathcal{O}(\delta^6)$.

To calculate the contributions of the first and third terms we use the Cutkosky cutting rules.

Only contributions obtained by cutting the lines, corresponding to stable particles, are needed.

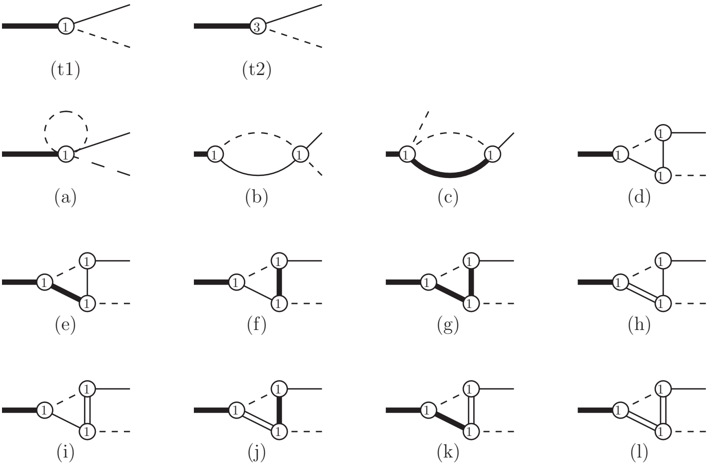


Figure: Diagrams contribution to the decay $R \to N\pi$ up to leading one-loop order. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

The $R(p) \to N(p')\pi^a(q)$ decay width reads

$$\Gamma_{R\to\pi N} = \frac{\lambda^{1/2}(m_R^2, m_N^2, M^2)}{16\pi\, m_R^3}\, |\mathcal{M}_1|^2\,,$$

with $\lambda(x,y,z) = (x-y-z)^2 - 4yz$ and $\mathcal{M}_1$ the corresponding decay amplitude.

The decay width corresponding to the $\pi\pi N$ final state is given by:

$$\Gamma_{R\to\pi\pi N} = \frac{1}{32 m_R^3 (2\pi)^3} \int_{4M_\pi^2}^{(m_R-m_N)^2} \mathrm{d}s_1 \int_{s_{2-}}^{s_{2+}} \mathrm{d}s_2 \, |\mathcal{M}|^2 \,,$$

where $\mathcal{M}$ is the $R(p) \to N(p')\pi^a(q_1)\pi^b(q_2)$ decay amplitude and

$$s_{2\pm} = \frac{m_R^2 + m_N^2 + 2M_\pi^2 - s_1}{2} \pm \frac{1}{2s_1}\lambda^{1/2}(s_1, m_R^2, m_N^2)\lambda^{1/2}(s_1, M_\pi^2, M_\pi^2) \,.$$

We consider $m_R - m_N \sim 400$ MeV as a small parameter of the order δ^1 and count $M_\pi \sim \delta^2$.

As s_1 varies from $4M_\pi^2$ to $(m_R - m_N)^2$, we assign the order δ^2 to it. We also count $m_R - m_\Delta \sim \delta^2$.

LO tree diagrams contributing to the $R \to \pi\pi N$ decay:

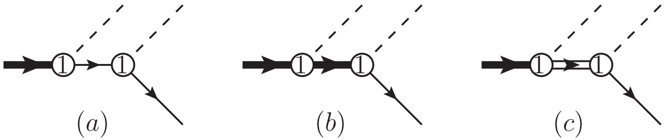


Figure: Tree diagrams contributing to the $R \to \pi\pi N$ decay. Crossed diagrams are not shown. Dashed, solid, double and thick solid lines correspond to pions, nucleons, deltas and Roper resonances, respectively. The numbers in the circles give the chiral orders of the vertices.

The delta propagators in these diagrams are dressed. The non-pole parts are of higher orders and therefore can be dropped.

The contributions of the loop diagrams are suppressed by additional powers of δ so that they do not contribute at order δ^5.

Thus, the contributions of the one- and two-loop self-energy diagrams in the width of the Roper resonance at order δ^5 sum up to

$$\Gamma_R = \Gamma_{R\to\pi N} + \Gamma_{R\to\pi\pi N}.$$

Numerical results

We use the following standard values of the parameters from PDG

$$M_\pi = 139\ \mathrm{MeV}, \quad m_N = 939\ \mathrm{MeV}, \quad m_\Delta = 1210 \pm 1\ \mathrm{MeV},$$
$$\Gamma_\Delta = 100 \pm 2\ \mathrm{MeV}, \quad m_R = 1365 \pm 15\ \mathrm{MeV}, F_\pi = 92.2\ \mathrm{MeV},$$

and obtain

$$\begin{aligned}
\Gamma_{R\to\pi N} &= 550(57.7)\, g_{\pi NR}^2\ \mathrm{MeV}, \\
\Gamma_{R\to\pi\pi N} &= \Big[1.49(0.58)\, g_A^2\, g_{\pi NR}^2 - 2.76(1.07)\, g_A\, g_{\pi NR}^2\, g_R \\
&+ 1.48(0.59)\, g_{\pi NR}^2\, g_R^2 + 2.96(0.94)\, g_A\, g_{\pi NR}\, h h_R \\
&- 3.79(1.37)\, g_{\pi NR}\, g_R\, h h_R + 9.93(5.45)\, h^2 h_R^2\Big]\ \mathrm{MeV}.
\end{aligned}$$

Further, we substitute $g_A = 1.27$ and $h = 1.42 \pm 0.02$.
The latter value is taken from

D. L. Yao, et.al. JHEP **1605**, 038 (2016).

We pin down $g_{\pi NR}$ by reproducing

$\Gamma_{R\to\pi N} = (123.5 \pm 19.0)$ MeV

from PDG, which yields

$g_{\pi NR} = \pm(0.47 \pm 0.11)$.

Following
S. R. Beane and U. van Kolck, J. Phys. G **31**, 921 (2005)
we assume $g_R = g_A$ and $h_R = h$ and obtain:

$$\begin{aligned}\Gamma_{R\to\pi\pi N} &= \Big[0.53(32) - 0.98(60) + 0.53(32) \pm 3.57(1.41) \\ &\mp 4.57(1.97) + 40.4(22.2)\Big] \,\mathrm{MeV} = 40.5(22.3) \,\mathrm{MeV}.\end{aligned}$$

The largest contribution comes from the decay diagram with intermediate Δ state.

Further, using the approach of

E. Epelbaum, H. Krebs and U.-G. Meißner, Eur. Phys. J. A **51**, no. 5, 53 (2015)

we estimate the theoretical error due to the omitting the higher order contributions and obtain

$$\Gamma_{R\to\pi\pi N} = (40.5 \pm 22.3 \pm 16.8)\ \mathrm{MeV},$$

which is consistent with

$$\Gamma_{\pi\pi N} = (66.5 \pm 9.5)\ \mathrm{MeV}$$

quoted by PDG.

Width of the Δ-resonance up to order p^5

J.Gegelia, U.-G.Meißner, D.Siemens, D.L.Yao, Phys. Lett. B **763**, 1 (2016).

Dressed Δ-propagator in d space-time dimensions:

$$
\begin{aligned}
-iD^{\mu\nu}(p) &= \left[g^{\mu\nu} - \frac{\gamma^\mu\gamma^\nu}{d-1} - \frac{p^\mu\gamma^\nu - \gamma^\mu p^\nu}{(d-1)m_\Delta^0} - \frac{d-2}{(d-1)(m_\Delta^0)^2}\, p^\mu p^\nu \right] \\
&\times \frac{-i}{\not p - m_\Delta^0 - \Sigma_1 - \not p\,\Sigma_6} + \text{pole free terms}\,,
\end{aligned}
$$

where the self-energy of the Δ-resonance is parameterised as:

$$
\begin{aligned}
\Sigma^{\mu\nu} &= \Sigma_1(p^2)\, g^{\mu\nu} + \Sigma_2(p^2)\,\gamma^\mu\gamma^\nu + \Sigma_3(p^2)\, p^\mu\gamma^\nu \\
&+ \Sigma_4(p^2)\,\gamma^\mu p^\nu + \Sigma_5(p^2)\, p^\mu p^\nu + \Sigma_6(p^2)\,\not p\, g^{\mu\nu} + \Sigma_7(p^2)\,\not p\,\gamma^\mu\gamma^\nu \\
&+ \Sigma_8(p^2)\,\not p\, p^\mu\gamma^\nu + \Sigma_9(p^2)\,\not p\,\gamma^\mu p^\nu + \Sigma_{10}(p^2)\,\not p\, p^\mu p^\nu\,.
\end{aligned}
$$

Pole position z of the Δ-propagator can be found by solving

$$z - m_\Delta^0 - \Sigma_1(z^2) - z\,\Sigma_6(z^2) \equiv z - m_\Delta^0 - \Sigma(z) = 0\,.$$

Pole mass and the width are defined by parameterizing z as

$$z = m_\Delta - i\,\frac{\Gamma_\Delta}{2}\,.$$

We calculate z perturbatively order by order in $\hbar$ (loop expansion). For that purpose we write:

$$\Sigma = \hbar\,\Sigma_{(1)} + \hbar^2\Sigma_{(2)} + \mathcal{O}(\hbar^3)\,,$$

and obtain the following expression for the width

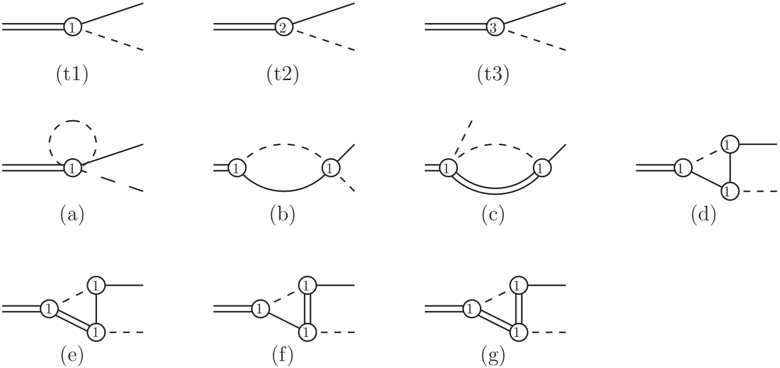
$$\begin{aligned}\Gamma_\Delta &= \hbar\,2i\,\mathrm{Im}\left[\Sigma_{(1)}(m_\Delta)\right] + \hbar^2\,2i\left\{\mathrm{Im}\left[\Sigma_{(1)}(m_\Delta)\right]\mathrm{Re}\left[\Sigma'_{(1)}(m_\Delta)\right]\right.\\ &+ \left.\mathrm{Re}\left[\Sigma_{(1)}(m_\Delta)\right]\mathrm{Im}\left[\Sigma'_{(1)}(m_\Delta)\right]\right\} + \hbar^2 2i\mathrm{Im}\left[\Sigma_{(2)}(m_\Delta)\right] + \mathcal{O}(\hbar^3).\end{aligned}$$

For $\Sigma_{(1)}$ we used the corresponding explicit expressions.

For the two-loop contribution we use the Cutkosky cutting rules, that is we relate it to $A_{\Delta\to\pi N}$ amplitude via

$$\begin{aligned}\Gamma_\Delta &= \frac{\left[(m_\Delta+m_N)^2-M_\pi^2\right]\left[\left(m_\Delta^2-m_N^2-M_\pi^2\right)^2-4M_\pi^2m_N^2\right]^{3/2}}{192\pi m_\Delta^5}\\ &\times \left|A_{\Delta\to\pi N}\right|^2.\end{aligned}$$

The tree and one-loop diagrams contributing to the $\Delta \to \pi N$ decay up to order p^3:



Calculating one- and two-loop contributions in the delta width we extracted h_A from the experimental value of the delta width for a given value of the leading $\pi\Delta$ coupling constant g_1:

$$\Gamma_\Delta = 53.91\, h_A^2 + 0.87 g_1^2 h_A^2 - 3.31 g_1 h_A^2 - 0.99\, h_A^4\,.$$

Substituting $\Gamma_\Delta = 100 \pm 2\ \mathrm{MeV}$ from the PDG we extract h_A as a function of g_1.

The obtained result is plotted in next page.

Such a correlation between $\pi N\Delta$ and $\pi\Delta$ couplings exists in the large N_C limit but, as far as we know, we obtained it for the first time for the real world with $N_C = 3$.

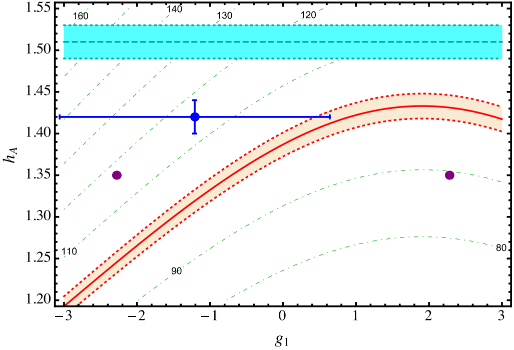


Figure: h_A as a function of g_1. The central line corresponds to $\Gamma_\Delta = 100$ MeV, while the band is obtained by varying $\Gamma_\Delta \sim 98 - 102$ MeV.

The dot-dashed lines correspond to delta widths indicated by their values.

The blue dot with error bars represents the real part of the coupling fitted to πN scattering, the purple dots stand for the leading order $\pi N\Delta$ coupling in the large-N_c limit and the horizontal dashed line with cyan band corresponds to the value (with error represented by the band) from V. Bernard, E. Epelbaum, H. Krebs and U.-G. Meißner, Phys. Rev. D **87**, no. 5, 054032 (2013).

Summary

- Pion-nucleon scattering in BChPT with delta:
  - One-loop full order p^3 calculation in a chiral EFT with pions, nucleons and delta resonances.
  - Fit of unknown parameters to S and P waves.
  - Reasonable description of phase shifts and threshold parameters.
- Width of the Roper resonance calculated up to NLO of BChPT:
  - One of the three unknown couplings we fix by reproducing the PDG value for $\Gamma_{R\to\pi N}$.
  - Assuming that the remaining two couplings of the Roper interaction take values equal to those of the nucleon, we obtain the result for $\Gamma_{R\to\pi\pi N}$ consistent with the PDG value.
  - To improve the accuracy of our calculation, three-loop self-energy diagrams need to be calculated.
- Width of the delta resonance.