The chiral anomaly and the heterochiral and homochiral classification for mesons

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Motivation

Chiral (or axial) anomaly: a classical symmetry of QCD broken by quantum fluctuations

Chiral anomaly important for η and η'. What about other mesons?

Classification of mesons in heterochiral and homochiral multiplets.

Other effects of the chiral anomaly: in baryonic sector, N(1535) -> Nη, and for the pseudoscalar glueball

Summary

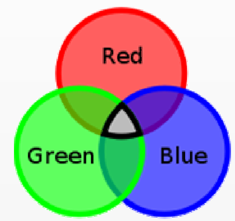
QCD Lagrangian: symmetries and anomalies

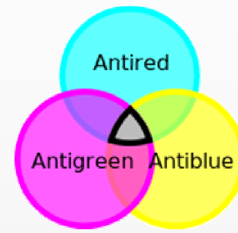
Born	Giuseppe Lodovico Lagrangia 25 January 1736 Turin
Died	10 April 1813 (aged 77) Paris

Francesco Giacosa

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B



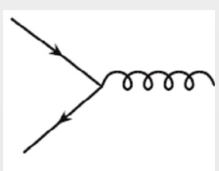


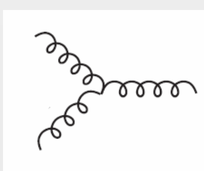
$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix};\; i = u,d,s,\ldots$$

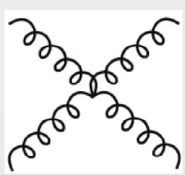
8 type of gluons ($R\bar{G}$, $B\bar{G}$,…)

$$A_\mu^a \;;\; a = 1,\ldots,8$$

$$\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \bar{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$

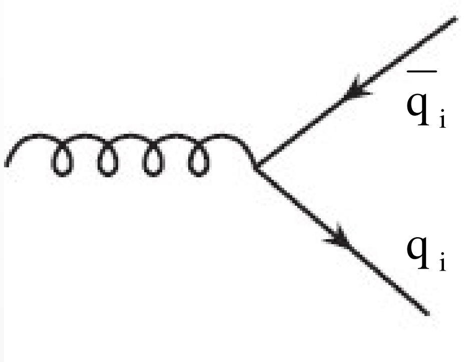






Btw: **where are glueballs?**

Flavor symmetry



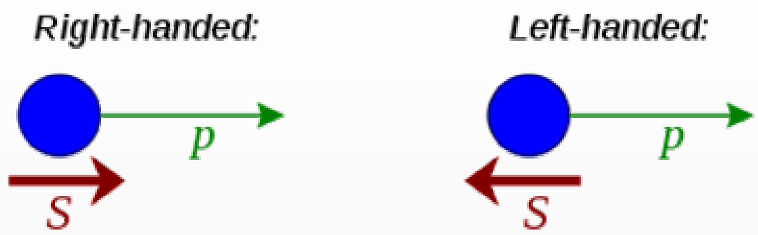
Gluon-quark-antiquark vertex.

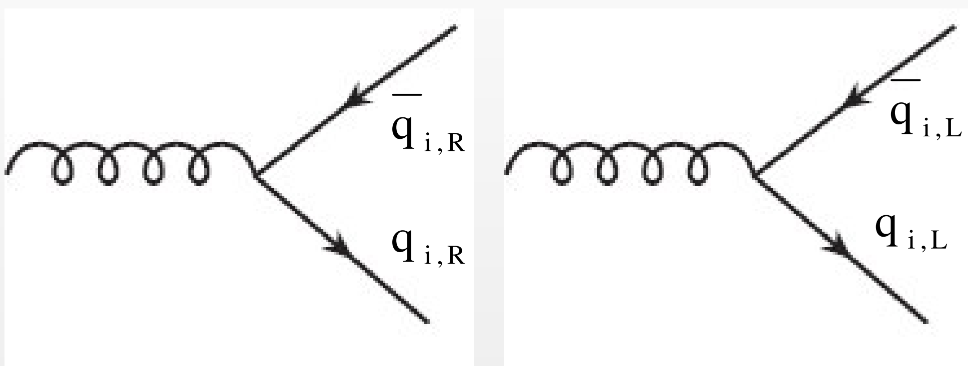
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$U \in U(3)_V \rightarrow U^+ U = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$q_i = q_{i,R} + q_{i,L} \rightarrow U^R_{ij} q_{j,R} + U^L_{ij} q_{j,L}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (m_i=0) chiral symmetry is exact

Chiral transformations and axial anomaly

$$G_{\text{fl}} \times U(1)_{\text{A}} = SU(3)_{\text{L}} \times SU(3)_{\text{R}} \times U(1)_{\text{A}}$$

$$q_{\text{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\text{L,R}}\, q_{\text{L,R}}$$

U(1)A chiral

Axial anomaly:

$$\partial^{\mu}(\bar{q}^{i}\gamma_{\mu}\gamma_{5}q^{i}) = \frac{3g^{2}}{16\pi^{2}}\,\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})$$

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A quark-antiquark state is a conventional meson.

Mesons: review of quark-antiquark from PDG

$n\ ^{2s+1}\ell_J$	J^{PC}	I = 1 $u\bar{d}$, $\bar{u}d$, $\frac{1}{\sqrt{2}}(d\bar{d}-u\bar{u})$	I = $\frac{1}{2}$ $u\bar{s}$, $d\bar{s}$; $\bar{d}s$, $-\bar{u}s$	I = 0 f'	I = 0 f
$1\ ^1S_0$	0^{-+}	π	K	η	$\eta'(958)$
$1\ ^3S_1$	1^{--}	$\rho(770)$	$K^*(892)$	$\phi(1020)$	$\omega(782)$
$1\ ^1P_1$	1^{+-}	$b_1(1235)$	$K_{1B}{}^{\dagger}$	$h_1(1380)$	$h_1(1170)$
$1\ ^3P_0$	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1710)$	$f_0(1370)$
$1\ ^3P_1$	1^{++}	$a_1(1260)$	$K_{1A}{}^{\dagger}$	$f_1(1420)$	$f_1(1285)$
$1\ ^3P_2$	2^{++}	$a_2(1320)$	$K_2^*(1430)$	$f_2'(1525)$	$f_2(1270)$
$1\ ^1D_2$	2^{-+}	$\pi_2(1670)$	$K_2(1770)^{\dagger}$	$\eta_2(1870)$	$\eta_2(1645)$
$1\ ^3D_1$	1^{--}	$\rho(1700)$	$K^*(1680)$		$\omega(1650)$
$1\ ^3D_2$	2^{--}		$K_2(1820)$		

Strange-nonstrange mixing in the isoscalar sector: recall and the strange case of pseudotensor mesons

based on

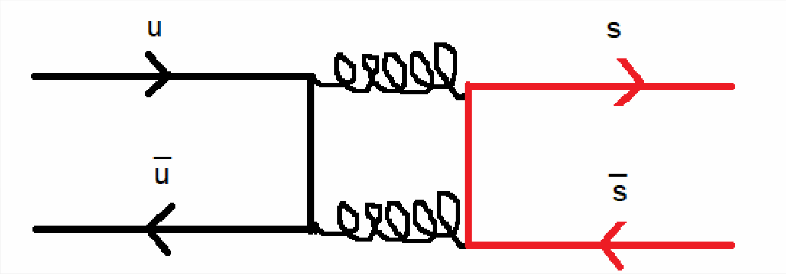
A . Koenigstein and F.G.

Eur. Phys.J. A52 (2016) no.12, 356, arXiv: 1608.8777

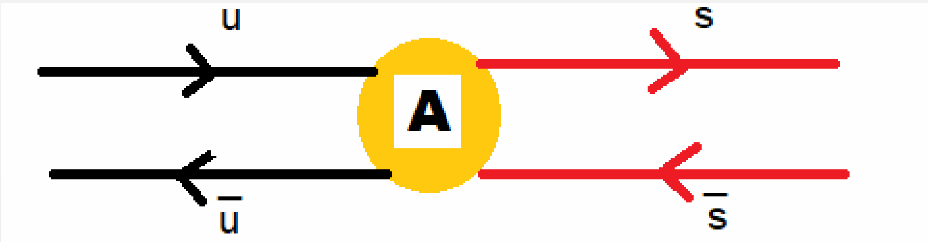
What physical processes we look at:
mixing in the isoscalar sector in a certain multiplet

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{mix} & \sin\theta_{mix} \\ -\sin\theta_{mix} & \cos\theta_{mix} \end{pmatrix} \begin{pmatrix} M_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ M_S = \bar{s}s \end{pmatrix}$$

Such a mixing is suppressed...



But this can be large



Known mixing angles

$$\begin{pmatrix} M_1 \\ M_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{mix} & \sin\theta_{mix} \\ -\sin\theta_{mix} & \cos\theta_{mix} \end{pmatrix} \begin{pmatrix} M_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ M_S = \bar{s}s \end{pmatrix}$$

- For pseudoscalar mesons: $M_1 = \eta(547)$ and $M_2 = \eta'(958)$.
  $\Theta_{mix} = -42°$ Large mixing caused by the axal anomaly.

- For vector mesons: $M_1 = \omega(782)$ and $M_2 = \varphi(1020)$.
  $\Theta_{mix} = -3°$ Very small mixing. Why?

- For tensor mesons: $M_1 = f_2(1270)$ and $M_2 = f'_2(1525)$
  $\Theta_{mix} = 3°$ Also very small mixing. Why?

Pseudotensor meson: suprising large mixing?

A. Koenigstein, F.G., Eur.Phys.J. A**52** (2016) no.12, 356, arXiv: 1608.8777

Phenomenology of pseudotensor mesons and the pseudotensor glueball

Pseudotensor mesons: {$\pi_2(1670)$, $K_2(1770)$, $\eta_2(1645)$, $\eta_2(1870)$}

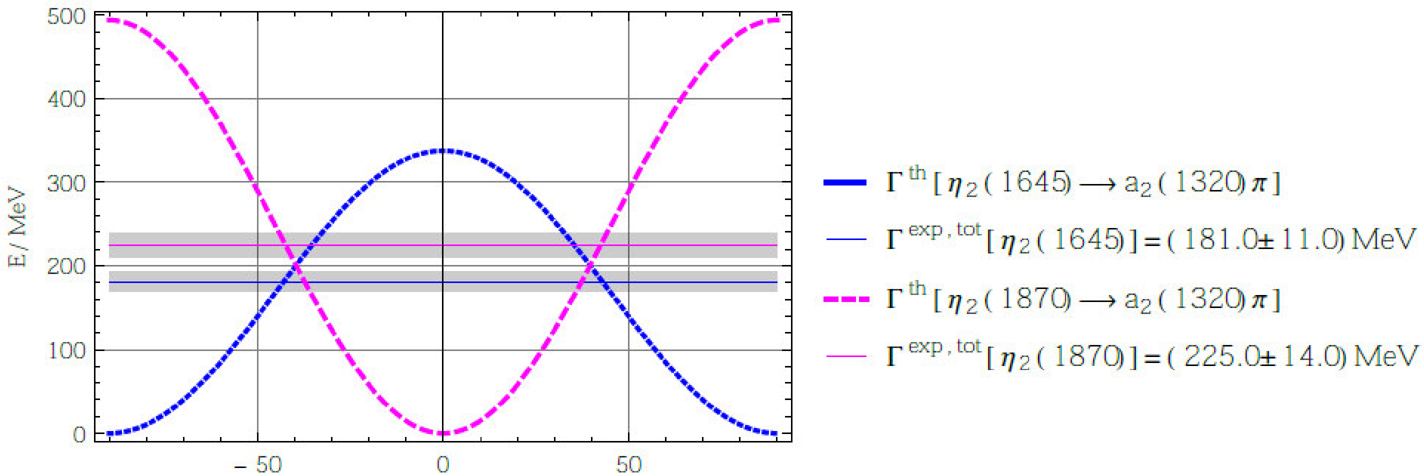
For pseudotensor mesons: $M_1 = \eta_2(1645)$ and $M_2 = \eta_2(1870)$
Only a large mixing angle $\Theta_{mix} = -40°$ is compatible with present experimental data.

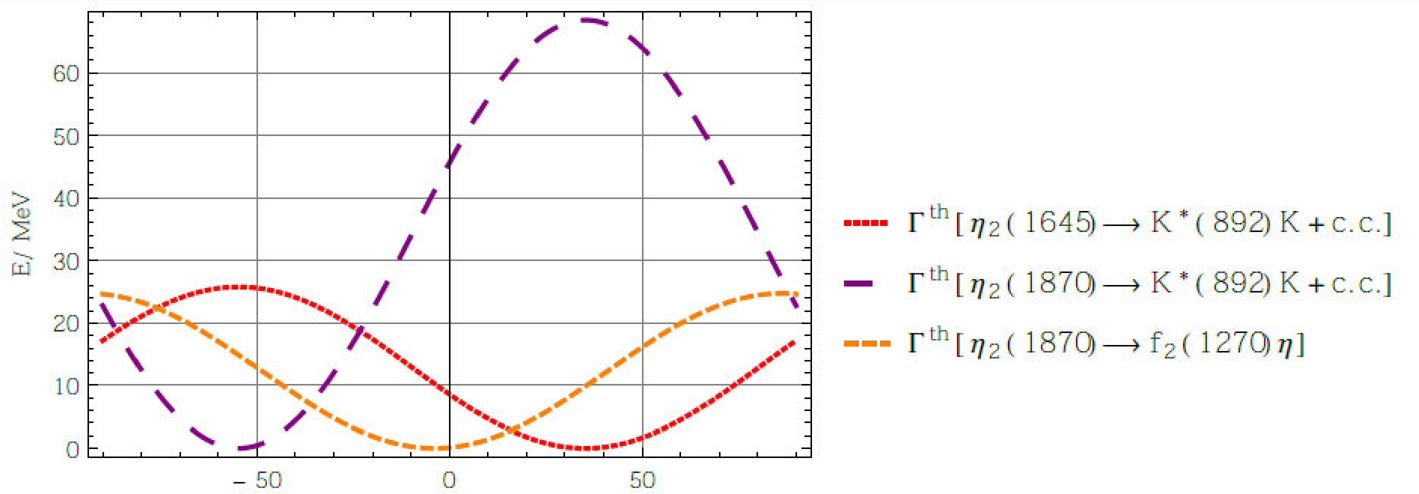
$\pi_2(1670)$, $K_2(1770)$ used to fix coupling constant. Good description of these states.
A small mixing angle generates a too large η2(1645) (exp 181 MeV).

Decay process	Theory (MeV) ($\beta_{pt} = 14.8°$)	Theory (MeV) ($\beta_{pt} = 0.0°$)
$\eta_2(1645) \to \bar{K}^*(892)\, K + c.c.$	3.2 ± 0.4	8.6 ± 1.1
$\eta_2(1645) \to a_2(1320)\, \pi$	315.6 ± 21.2	337.8 ± 22.6

$\eta_2(1645)$ and $\eta_2(1870)$

Only a large mixing angle Θmix = -40° is compatible with present experimental data.





Θmix

Axial anomaly and strange-nonstrange mixing

based on

F.G., A . Koenigstein, R.D. Pisarski

How the axial anomaly controls flavor mixing among mesons

Phys. Rev. D97 (2018) no.9, 091901 arXiv: 1709.07454

(Pseudo)scalar mesons: heterochiral scalars

Pseudoscalar mesons: {π, K, η(547), η'(958)}
Scalar mesons: {a_0(1450), K_0*(1430), f_0(1370), f_0(1500)}

f0(1710) mostly glueball
See 1408.4921

$$\Phi = S + iP = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\ a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S \end{pmatrix} + \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_S \end{pmatrix}$$

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow e^{\mp i\alpha/2} U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

$$\Phi \longrightarrow e^{-2i\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^{\dagger}$$

We call the transformation of the matrix Φ **heterochiral**!
We thus have heterochiral scalars.

$\mathrm{tr}(\Phi^\dagger\Phi),\ \mathrm{tr}(\Phi^\dagger\Phi)^2$ are clearly invariant; typical terms for a chiral model.

$\det(\Phi)$ is interesting, since it breaks only U(1)A axial anomaly $\det\Phi \to e^{-i6\alpha}\det\Phi$

How to describe the mixing: Anomaly Lagrangian for heterochiral scalars

$$\det(\Phi) = \frac{1}{6}\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi^{jj'}\Phi^{kk'} \to e^{-3i\alpha}\det(\Phi)$$

$$\mathscr{L}_{\Phi}^{\text{anomaly}} = -a_{\text{A}}^{(3)}[\det(\Phi) - \det(\Phi^{\dagger})]^2 + \dots$$

- invariant under $SU(3)_R \times SU(3)_L$, but breaks $U(1)_A$

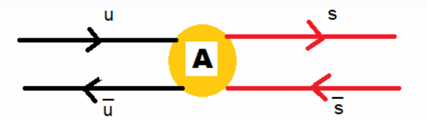
- third term: affects **only** η and η'

- other terms which affect the also scalar mixing and generate decays are possible, see paper.

Recall the condensation:
$\langle\Phi\rangle \sim \sqrt{3/2}\, f_{\pi}\, \mathbb{1}$

Pseudoscalar mixing

$$\mathcal{L}_{\Phi}^{\text{anomaly}} = -\alpha_A \eta_0^2 + ... = -\alpha_A \left(\sqrt{2}\eta_N + \eta_S\right)^2 + ...$$



$$\theta_P \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\alpha_A}{m_K^2 - m_\pi^2 - \alpha_A}\right]$$

$$\begin{pmatrix} \eta \equiv \eta(547) \\ \eta' \equiv \eta(958) \end{pmatrix} = \begin{pmatrix} \cos\theta_P & \sin\theta_P \\ -\sin\theta_P & \cos\theta_P \end{pmatrix} \begin{pmatrix} \eta_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_P \simeq -42°$$

The numerical value can be correctly described, see e.g.

S. D. Bass and A. W. Thomas, Phys. Lett. B **634** (2006) 368 doi:10.1016/j.physletb.2006.01.071 [hep-ph/0507024].

(Axial-)vector mesons: homochiral vectors

Vector mesons: {ρ(770), K*(892), ω(782), φ(1020)}
Axial-vector mesons: {a1(1230), K1A,, f1(1285), f1(1420)}

$$V^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{\star +} \\ \rho^{\mu -} & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{\star 0} \\ K^{\star -} & \bar{K}^{\star 0} & \omega_S \end{pmatrix}^{\mu}$$

$$A^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{f_{1N} + a_1^0}{\sqrt{2}} & a_1^+ & K_{1,A}^+ \\ a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_{1,A}^0 \\ K_{1,A}^- & \bar{K}_{1,A}^0 & f_{1S} \end{pmatrix}^{\mu}$$

$$R^{\mu} = V^{\mu} - A^{\mu} \text{ and } L^{\mu} = V^{\mu} + A^{\mu}$$

Chiral transformations

$$q_{\mathrm{L,R}} \longrightarrow \mathrm{e}^{\mp \mathrm{i}\alpha/2}\, U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

➡

$$L_{\mu} \longrightarrow U_{\mathrm{L}} L_{\mu} U_{\mathrm{L}}^{\dagger}$$

$$R_{\mu} \longrightarrow U_{\mathrm{R}} R_{\mu} U_{\mathrm{R}}^{\dagger}$$

We have here a **homochiral** multiplet. We call these states as homochiral vectors.

Mixing among vector mesons

$$\begin{pmatrix} \omega(782) \\ \phi(1020) \end{pmatrix} = \begin{pmatrix} \cos\theta_V & \sin\theta_V \\ -\sin\theta_V & \cos\theta_V \end{pmatrix} \begin{pmatrix} \omega_N = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

$$\theta_V \simeq -3.2^\circ$$

The mixing is very small.
This is understandable: there is no term analogous to the determinant.
Namely, anomlay-driven terms are more complicated, involve derivatives and do not affect isoscalar mixing, e.g. Wess-Zumino like terms:

$$\varepsilon^{\mu\nu\alpha\beta}\mathrm{tr}[L_\mu\Phi(\partial_\nu\Phi^\dagger)\Phi(\partial_\alpha\Phi^\dagger)\Phi(\partial_\beta\Phi^\dagger) + R_\mu\Phi^\dagger(\partial_\nu\Phi)\Phi^\dagger(\partial_\alpha\Phi)\Phi^\dagger(\partial_\beta\Phi)]$$

Mixing among axial-vector mesons

$$\begin{pmatrix} f_1(1285) \\ f_1(1420) \end{pmatrix} = \begin{pmatrix} \cos\theta_{AV} & \sin\theta_{AV} \\ -\sin\theta_{AV} & \cos\theta_{AV} \end{pmatrix} \begin{pmatrix} f_{1,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ f_{1,S} = \bar{s}s \end{pmatrix}$$

θ_{AV} should be small!

Small mixing angle found in the following phenomenological studies:
L. Olbrich, F. Divotgey, F.G., Eur.Phys.J. A**49** (2013) 135 arXiv:1306.1193
Parganlija et al, Phys. Rev. D**87** (2013) no.1, 014011

Ground-state tensors (and their chiral partners): Homochiral tensors

Tensor mesons: {a2(1320), K_2*(1430), f2(1270), f2(1535)}
Axial-vector mesons: {ρ_2(???), K_2(1820), ω_2(???), φ2(???)}

Chiral transformations

$q_{L,R} \longrightarrow e^{\mp i\alpha/2} U_{L,R}\, q_{L,R}$

$$L_{\mu\nu} \longrightarrow U_L L_{\mu\nu} U_L^\dagger$$

$$R_{\mu\nu} \longrightarrow U_R R_{\mu\nu} U_R^\dagger$$

Thus, we have **homochiral** tensors. We do not expect large mixing.

Tensor mixing

$$\begin{pmatrix} f_2(1270) \\ f_2'(1525) \end{pmatrix} = \begin{pmatrix} \cos\theta_T & \sin\theta_T \\ -\sin\theta_T & \cos\theta_T \end{pmatrix} \begin{pmatrix} f_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ f_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_T \simeq 3.2°$$

As expected, the mixing is very small.

A small mixing is also expected for the (yet unknown) chiral partners of tensor mesons.

Pseudovectors and orbitally excited vectors: Heterochiral vectors

Pseudovextor mesons: {b1(1230), K1B, h1(1170), h1(1380)}
Excited vector mesons: {ρ(1700), K*(1680) , ω(1650), φ(???)}

$$B^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{h_{1,N}+b_1^0}{\sqrt{2}} & b_1^+ & K_{1,B}^{\star+} \\ b_1^- & \frac{h_{1,N}+b_1^0}{\sqrt{2}} & K_{1,B}^{\star 0} \\ K_{1,B}^{\star -} & \bar{K}_{1,B}^{\star 0} & h_{1,S} \end{pmatrix}^\mu$$

$$E_{\text{ang}}^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_{\text{ang},N}+\rho_{\text{ang}}^0}{\sqrt{2}} & \rho_{\text{ang}}^+ & K_{\text{ang}}^{\star+} \\ \rho_{\text{ang}}^- & \frac{\omega_{\text{ang},N}-\rho_{\text{ang}}^0}{\sqrt{2}} & K_{\text{ang}}^{\star 0} \\ K_{\text{ang}}^{\star -} & \bar{K}_{\text{ang}}^{\star 0} & \omega_{\text{ang},S} \end{pmatrix}^\mu$$

$$\tilde{\Phi}^\mu = E_{\text{ang}}^\mu - iB^\mu$$

Chiral transformations

$$q_{\text{L,R}} \longrightarrow e^{\mp i\alpha/2}\, U_{\text{L,R}}\, q_{\text{L,R}}$$

➡

$$\Phi_\mu \longrightarrow e^{-i\alpha}\, U_{\text{L}}\, \Phi_\mu\, U_{\text{R}}^\dagger$$

The pseudovector mesons and the excited vector mesons form a **heterochiral** multiplet. We thus call them heterochiral vectors.

Excited vector mesons: φ(1930) predicted to be the missing state, see M. Piotrowska, C. Reisinger and FG., arXiv:1708.02593 [hep-ph]

Anomalous Lagrangian for heterochiral vectors

$$\mathscr{L}_{\Phi_\mu}^{\text{anomaly}} = -b_{\text{A}}^{(3)} (\varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi_\mu^{kk'} - h.c.)^2 + \ldots,$$

It is $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

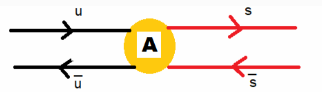
Other terms are possible, see paper.

Recall that for (pseudo)scalar states it is::

$$\det(\Phi) = \frac{1}{6} \varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'} \to e^{-3i\alpha} \det(\Phi) \qquad \mathscr{L}_{\Phi}^{\text{anomaly}} = -a_{\text{A}}^{(3)} [\det(\Phi) - \det(\Phi^\dagger)]^2 + \ldots$$

Pseudovector mixing

$$\mathcal{L}_{\Phi_\mu}^{\text{anomaly}} = -\beta_A \left(\sqrt{2} h_{1,N} + h_{1,SS}\right)^2 + \ldots$$



$$\theta_{PV} \simeq -\frac{1}{2}\arctan\left[\frac{2\sqrt{2}\beta_A}{m_{K_{1,B}}^2 - m_{b_1(1235)}^2 - \beta_A}\right]$$

$$\begin{pmatrix} h_1(1170) \\ h_1(1380) \end{pmatrix} = \begin{pmatrix} \cos\theta_{PV} & \sin\theta_{PV} \\ -\sin\theta_{PV} & \cos\theta_{PV} \end{pmatrix} \begin{pmatrix} h_{1,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ h_{1,S} = \bar{s}s \end{pmatrix}$$

θ_{PV} can be large! (and negative...)

This is a prediction.
Experimental knowledge poor; it does not allow for a phenonemonological study yet.

Pseudotensor mesons (and their chiral partners): heterochiral tensors

Pseudotensor mesons: {π2(1670), K2(1770),η2(1645),η2(1870)}
Chiral partners: {a_2(???), K_2^*(???) , f_2(???), f_2(???)}

Chiral transformations

$$q_{L,R} \longrightarrow e^{\mp i\alpha/2} U_{L,R}\, q_{L,R}$$

$$\Phi_{\mu\nu} \longrightarrow e^{-i\alpha} U_L\, \Phi_{\mu\nu}\, U_R^\dagger$$

Thus, we have **heterochiral** tensor states.
Transformation just as heterochiral scalars.
Mixing between strange-nonstrange possible.

Anomalous Lagrangian for heterochiral tensors

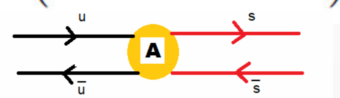
$$\mathscr{L}^{\text{anomaly}}_{\Phi_{\mu\nu}} = c_A^{(3)} (\varepsilon^{ijk} \varepsilon^{i'j'k'} \Phi^{ii'} \Phi^{jj'} \Phi^{kk'}_{\mu\nu} - h.c.)^2 + \ldots,$$

Again, the various terms are $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

First term generates mixing for pseudotensors and also for their chiral partners.
Second term generates decays of pseudotensor (and partners) into (pseudo)scalars.
Third term generates mixing for pseudotensors only.

Pseudotensor mixing

$$\mathcal{L}_{\Phi_{\mu\nu}}^{\text{anomaly}} = -\beta_A \left(\sqrt{2} h_{1,N} + h_{1,SS}\right)^2 + \ldots$$



$$\begin{pmatrix} \eta_2(1645) \\ \eta_2(1870) \end{pmatrix} = \begin{pmatrix} \cos\theta_{PT} & \sin\theta_{PT} \\ -\sin\theta_{PT} & \cos\theta_{PT} \end{pmatrix} \begin{pmatrix} \eta_{2,N} = \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_{2,S} = \bar{s}s \end{pmatrix}$$

$$\theta_{PV} \simeq -\frac{1}{2} \arctan\left[\frac{2\sqrt{2}\beta_A}{m_{K_{1,B}}^2 - m_{b_1(1235)}^2 - \beta_A}\right]$$

According to the phenomenological study in
A. Koenigstein, F.G., Eur.Phys.J. A**52** (2016) no.12, 356, arXiv: 1608.8777:

$$\theta_{PT} \approx -40^0$$

Other effects of the axial anomaly

based on

L. Olbrich, M. Zetenyi, F.G., D.H. Rischke

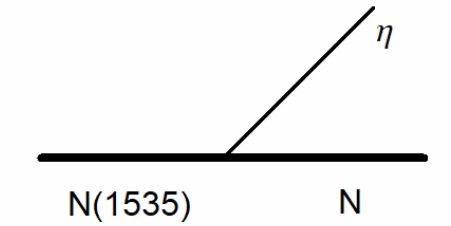
Influence of the axial anomaly on the decay $N(1535) \to N\eta$

Phys.Rev. D97 (2018) no.1, 014007 ArXiv: 1708.01061

Violation of flavour symmetry in N(1535) decays?

Flavour symmetry predicts:

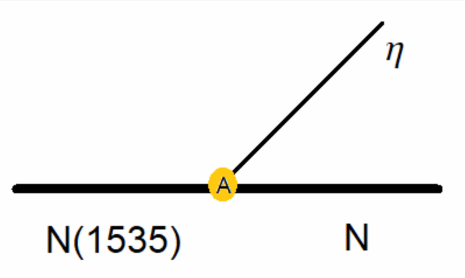
$$\frac{\Gamma_{N(1535)\to N\eta}}{\Gamma_{N(1535)\to N\pi}} \approx \frac{1}{3}\cos^2\theta_P \approx 0.17$$



This is in evident conflict with the experiment (see below).

A simple idea: axial anomaly and N(1535)

There is a simple explanation for an enhanced copling of N(1535) to Nη: the anomaly. Namely, one can write (in the mirror assignment) an anomalous term which couples the nucleon and its chiral partner to the η.



$$\mathcal{L}_A^{N_f=2} = \lambda_A^{N_f=2}(\det\Phi - \det\Phi^\dagger)(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2)$$

Consequences

N(1535) is the chiral partner of the nucleon

Extension to Nf =3 straightforward (see paper).

One can understand the enhanced decay $N(1535) \to N\eta$

Further predictions possible, e.g.: $\Gamma_{\Lambda(1670)\to\Lambda\eta} = (5.1 \pm {}^{2.7}_{2.1})$ MeV
(Experimentally between 2.5 and 12.5 MeV)

Enhanced coupling to η' follows: $g_{\eta' N N_*} \simeq 4.1$

Study of $pn \to pn\eta'$
delivered $g_{\eta' N N_*} \simeq 3.7$

X. Cao and X. G. Lee, Phys. Rev. C **78** (2008) 035207
[arXiv:0804.0656 [nucl-th]].

Details in L. Olbrich, M. Zetenyi, F.G., D. Rischke,
Phys.Rev. D97 (2018) no.1, 014007 [arXiv:1708.01061 [hep-ph]].

The pseudoscalar glueball and the anomaly

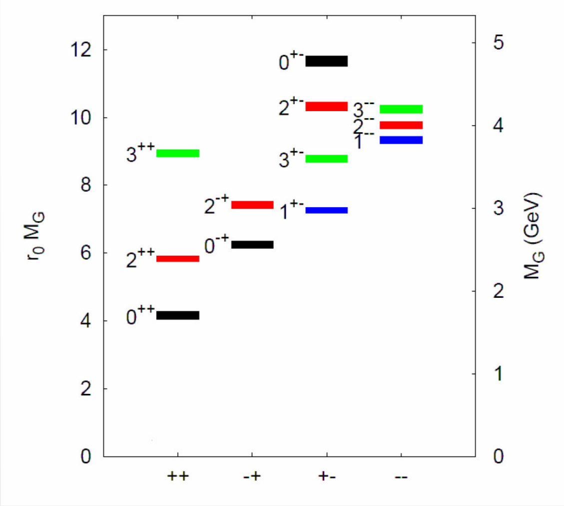
Uniwersytet Jana Kochanowskiego w Kielcach

$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = ic_{\tilde{G}\Phi}\tilde{G}\left(\det\Phi - \det\Phi^{\dagger}\right)$$

M_G = 2.6 GeV from lattice as been used as an input.

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to\eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to\eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to\eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to\eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to\eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

$$\Gamma_{\tilde{G}\to\pi\pi\pi} = 0$$

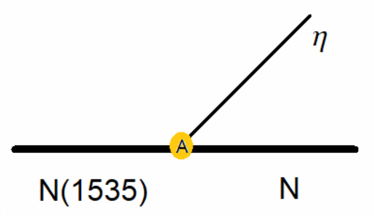


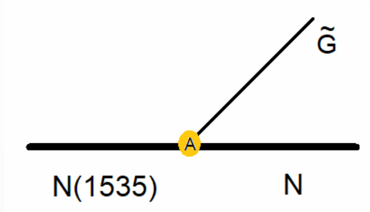
X(2370) found at BESIII is a possible candidate.

Future experimental search, e.g. at BESIII, GlueX, CLAS12, and PANDA.

W. Eshraim, S. Janowski, F.G., D.H. Rischke, Phys.Rev. D**87** (2013) no.5, 054036, ArXiv: 1208.6474

Coupling of the pseudoscalar glueball to baryons





$$\mathcal{L}_A^{N_f=2} = \lambda_A^{N_f=2}(\det\Phi - \det\Phi^\dagger)(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2) \;\Rightarrow\; \mathcal{L}_{\tilde{G}\Phi}^{N_f=2} = ig_{\tilde{G}\Phi}^{N_f=2}\tilde{G}(\bar{\Psi}_2\Psi_1 - \bar{\Psi}_1\Psi_2)$$

Strong coupling of the pseudoscalar glueball to NN and also N(1535)N

$$\frac{\Gamma_{\tilde{G}\to\bar{N}N}}{\Gamma_{\tilde{G}\to\bar{N}_*N+\text{ h.c.}}} \simeq 1.96$$

A promising reaction to search the pseudoscalar glueball is:

$$p + \bar{p} \to p + \bar{p}(1535) + \text{ h.c.}$$

See also:

L Olbrich, M. Zetenyi, F.G., D. H. Rischke, Phys.Rev. D97 (2018) no.1, 014007 arXiv: 1708.01061
W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., Acta Phys. Pol. B, Prc. Suppl. 5/4, arXiv: 1209.3976

Concluding remarks

- Concept of homochirality and heterochirality.
- For heterochiral multiplets an axial-anomalous strange-nonstrange mixing is possible.
  (η-η', but also η_2(1645)-η_2(1870) and evt h1 states)
- For homochiral multiplets no anomalous mixing.
  (ω-phi(1020), f_2(1270)-f_2'(1525),..., are nonstrange and strange, resp.)
- Baryons: anomalous coupling of N(1535) to Nη
  Consequences: decay Λ(1670) into Λη and N(1535)Nη' coupling
- Pseudoscalar glueball: anomalous coupling to mesons and baryons.
- Outlook: anomalous decays, detailed study of mixing, anomaly and nucleon-nucleon interation,...

  (Looking forward for exp. results BESIII, Compass, GlueX, CLAS12, PANDA, ...)

Thanks

(Pseudo)scalar mesons: heterochiral scalars

$$q_{\mathrm{L,R}} \longrightarrow e^{\mp i\alpha/2} U_{\mathrm{L,R}}\, q_{\mathrm{L,R}}$$

J^{PC}, $^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
0^{-+}, $^1S_0$	$\begin{cases} \pi \\ K \\ \eta, \eta'(958) \end{cases}$	$P^{ij} = \frac{1}{2}\bar{q}^j i\gamma^5 q^i$	$\Phi = S + iP$ $(\Phi^{ij} = \bar{q}^j_{\mathrm{R}} q^i_{\mathrm{L}})$	$\Phi \longrightarrow e^{-2i\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$
0^{++}, $^3P_0$	$\begin{cases} a_0(1450) \\ K_0^*(1430) \\ f_0(1370), f_0(1710)^* \end{cases}$	$S^{ij} = \frac{1}{2}\bar{q}^j q^i$		

$$\Phi \longrightarrow e^{-2i\alpha} U_{\mathrm{L}} \Phi U_{\mathrm{R}}^\dagger$$

We call the transformation of the matrix Φ **heterochiral**!
We thus have heterochiral scalars.

$\mathrm{tr}(\Phi^\dagger\Phi)$, $\mathrm{tr}(\Phi^\dagger\Phi)^2$ are clearly invariant; typical terms for a chiral model.

$\det(\Phi)$ is interesting, since it breaks only U(1)A axial anomaly $\det\Phi \rightarrow e^{-i6\alpha} \det\Phi$

(Axial-)vector mesons: homochiral vectors

J^{PC}, $^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
1^{--}, $^1S_1$	$\begin{cases} \rho(770) \\ K^*(892) \\ \omega(782), \phi(1020) \end{cases}$	$V_\mu^{ij} = \frac{1}{2}\bar{q}^j\gamma_\mu q^i$	$L_\mu = V_\mu + A_\mu$ $(L_\mu^{ij} = \bar{q}_{\mathrm{L}}^j\gamma_\mu q_{\mathrm{L}}^i)$	$L_\mu \longrightarrow U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^\dagger$
1^{++}, $^3P_1$	$\begin{cases} a_1(1260) \\ K_{1,A} \\ f_1(1285), f_1(1420) \end{cases}$	$A_\mu^{ij} = \frac{1}{2}\bar{q}^j\gamma^5\gamma_\mu q^i$	$R_\mu = V_\mu - A_\mu$ $(R_\mu^{ij} = \bar{q}_{\mathrm{R}}^j\gamma_\mu q_{\mathrm{R}}^i)$	$R_\mu \longrightarrow U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$

$$L_\mu \longrightarrow U_{\mathrm{L}} L_\mu U_{\mathrm{L}}^\dagger$$

$$R_\mu \longrightarrow U_{\mathrm{R}} R_\mu U_{\mathrm{R}}^\dagger$$

We have here a **homochiral** multiplet.
We call these states as homochiral vectors.

Ground-state tensors (and their chiral partners): Homochiral tensors

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\rm L} \times SU(3)_{\rm R} \times$ $\times U(1)_{\rm A}$
2^{++}, ${}^3P_2$	$\begin{cases} a_2(1320) \\ K_2^*(1430) \\ f_2(1270), f_2'(1525) \end{cases}$	$V_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q^i$	$L_{\mu\nu} = V_{\mu\nu} + A_{\mu\nu}$ $(L_{\mu\nu}^{ij} = \bar{q}_{\rm L}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q_{\rm L}^i)$	$L_{\mu\nu} \longrightarrow U_{\rm L} L_{\mu\nu} U_{\rm L}^\dagger$
2^{--}, ${}^3D_2$	$\begin{cases} \rho_2(?) \\ K_2(1820) \\ \omega_2(?), \phi_2(?) \end{cases}$	$A_{\mu\nu}^{ij} = \frac{1}{2}\bar{q}^j(\gamma^5\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q^i$	$R_{\mu\nu} = V_{\mu\nu} - A_{\mu\nu}$ $(R_{\mu\nu}^{ij} = \bar{q}_{\rm R}^j(\gamma_\mu i\overleftrightarrow{D}_\nu + \ldots)q_{\rm R}^i)$	$R_{\mu\nu} \longrightarrow U_{\rm R} R_{\mu\nu} U_{\rm R}^\dagger$

$$L_{\mu\nu} \longrightarrow U_{\rm L} L_{\mu\nu} U_{\rm L}^\dagger$$

$$R_{\mu\nu} \longrightarrow U_{\rm R} R_{\mu\nu} U_{\rm R}^\dagger$$

Thus, we have **homochiral** tensors. We do not expect large mixing.

Pseudovectors and orbitally excited vectors: Heterochiral vectors

J^{PC}, $^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{**} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_\mathrm{L} \times SU(3)_\mathrm{R} \times$ $\times U(1)_\mathrm{A}$
1^{+-}, $^1P_1$	$\begin{cases} b_1(1235) \\ K_{1,B} \\ h_1(1170), h_1(1380) \end{cases}$	$P_\mu^{ij} = -\frac{1}{2}\bar{q}^j\gamma^5\overleftrightarrow{D}_\mu q^i$	$\Phi_\mu = S_\mu + \mathrm{i}P_\mu$ $(\Phi_\mu^{ij} = \bar{q}_\mathrm{R}^j \mathrm{i}\overleftrightarrow{D}_\mu q_\mathrm{L}^i)$	$\Phi_\mu \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha}U_\mathrm{L}\Phi_\mu U_\mathrm{R}^\dagger$
1^{--}, $^3D_1$	$\begin{cases} \rho(1700) \\ K^*(1680) \\ \omega(1650), \phi(?) \end{cases}$	$S_\mu^{ij} = \frac{1}{2}\bar{q}^j\mathrm{i}\overleftrightarrow{D}_\mu q^i$		

$$\Phi_\mu \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_\mathrm{L}\, \Phi_\mu\, U_\mathrm{R}^\dagger$$

The pseudovector mesons and the excited vector mesons form a **heterochiral** multiplet. We thus call them heterochiral vectors.
The chiral transformation is just as the (pseudo)scalar mesons (which is also hetero).
Hence, an anomalous Lagrangian is possible for heterochiral vectors.

Excited vector mesons: phi(1930) predicted to be the missing state, see
M. Piotrowska, C. Reisinger and FG.,
``Strong and radiative decays of excited vector mesons and predictions for a new phi(1930)$ resonance,'' arXiv:1708.02593 [hep-ph], to appear in PRD.

Anomalous Lagrangian for heterochiral vectors

$$\mathcal{L}_{\Phi_\mu}^{\text{anomaly}} = - b_{\text{A}}^{(1)}[\text{tr}(\Phi \times \Phi_\mu \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(2)}[\text{tr}(\Phi \times \partial_\mu \Phi \cdot \Phi^\mu) + \text{c.c.}] - b_{\text{A}}^{(3)}[\text{tr}(\Phi \times \Phi \cdot \Phi_\mu) - \text{c.c.}]^2 + \ldots$$

$$(A \times B)^{ii'} = \frac{1}{3!}\epsilon^{ijk}\epsilon^{i'j'k'} A^{jj'} B^{kk'}$$

The first term contains objects as: $\varepsilon^{ijk}\varepsilon^{i'j'k'}\Phi^{ii'}\Phi_\mu^{jj'}\Phi_\mu^{kk'}$

So for the other terms. Such objects are $SU(3)_R \times SU(3)_L$ invariant but break $U(1)_A$.

The first term generates mixing among both nonets (pseudovector and excited vector).

The second term generates decay into (pseudo)scalar states (interesting for future works).

The third terms generates mixing for pseudovectors only.

Pseudotensor mesons (and their chiral partners): heterochiral tensors

J^{PC}, ${}^{2S+1}L_J$	$\begin{cases} I=1 & (\bar{u}d, \bar{d}u, \frac{\bar{d}d-\bar{u}u}{\sqrt{2}}) \\ I=1 & (-\bar{u}s, \bar{s}u, \bar{d}s, \bar{s}d) \\ I=0 & (\frac{\bar{u}u+\bar{d}d}{\sqrt{2}}, \bar{s}s)^{\star\star} \end{cases}$	microscopic currents	chiral multiplet	transformation under $SU(3)_{\mathrm{L}} \times SU(3)_{\mathrm{R}} \times$ $\times U(1)_{\mathrm{A}}$
2^{-+}, ${}^1D_2$	$\begin{cases} \pi_2(1670) \\ K_2(1770) \\ \eta_2(1645), \eta_2(1870) \end{cases}$	$P^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\mathrm{i}\gamma^5\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu + \ldots)q^i$	$\Phi_{\mu\nu} = S_{\mu\nu} + \mathrm{i}P_{\mu\nu}$ $(\Phi^{ij}_{\mu\nu} = \bar{q}^j_{\mathrm{R}}(\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu + \ldots)q^i_{\mathrm{L}})$	$\Phi_{\mu\nu} \longrightarrow \mathrm{e}^{-2\mathrm{i}\alpha}U_{\mathrm{L}}\Phi_{\mu\nu}U^\dagger_{\mathrm{R}}$
2^{++}, ${}^3F_2$	$\begin{cases} a_2(?) \\ K_2^*(?) \\ f_2(?), f_2'(?) \end{cases}$	$S^{ij}_{\mu\nu} = -\frac{1}{2}\bar{q}^j(\overleftrightarrow{D}_\mu\overleftrightarrow{D}_\nu + \ldots)q^i$		

$$\Phi_{\mu\nu} \longrightarrow \mathrm{e}^{-\mathrm{i}\alpha}\, U_{\mathrm{L}}\, \Phi_{\mu\nu}\, U^\dagger_{\mathrm{R}}$$

Thus, we have heterochiral tensor states.
Transformation just as heterochiral scalars.
Mixing between strange-nonstrange possible.

Proposed explanations for N(1535)

Dynamical generation through KΛ and KΣ channels

N. Kaiser, P. B. Siegel and W. Weise, Phys. Lett. B **362** (1995) 23 [nucl-th/9507036].
T. Inoue, E. Oset and M. J. Vicente Vacas, Phys. Rev. C **65** (2002) 035204 [hep-ph/0110333].
P. C. Bruns, M. Mai and U. G. Meissner, Phys. Lett. B **697** (2011) 254 [arXiv:1012.2233 [nucl-th]].
E. J. Garzon and E. Oset, Phys. Rev. C **91** (2015) no.2, 025201 [arXiv:1411.3547 [hep-ph]].
C. S. An and B. S. Zou, Sci. Sin. G **52** (2009) 1452 [arXiv:0910.4452 [nucl-th]].
B. C. Liu and B. S. Zou, Phys. Rev. Lett. **96**, 042002 (2006) [nucl-th/0503069].
X. Cao, J. J. Xie, B. S. Zou and H. S. Xu, Phys. Rev. C **80** (2009) 025203 [arXiv:0905.0260 [nucl-th]].

Basically, an sbar-s component is present in N(1535) and explains the coupling to η