



# Resonances in forward $\pi^+\pi^-$ photoproduction on hydrogen

15th International Workshop on Meson Physics  
KRAKÓW, POLAND  
7th - 12th June 2018

JPAC Collaboration

**Łukasz Bibrzycki\*** (Pedagogical University, Kraków)

**Adam Szczepaniak** (Indiana University/JLab)

**Petr Bydžovský** (Nuclear Physics Institute, Czech Academy of Sciences, Řež)

**Robert Kamiński** (Institute of Nuclear Physics PAS, Kraków)

**et al.**

\*-speaker

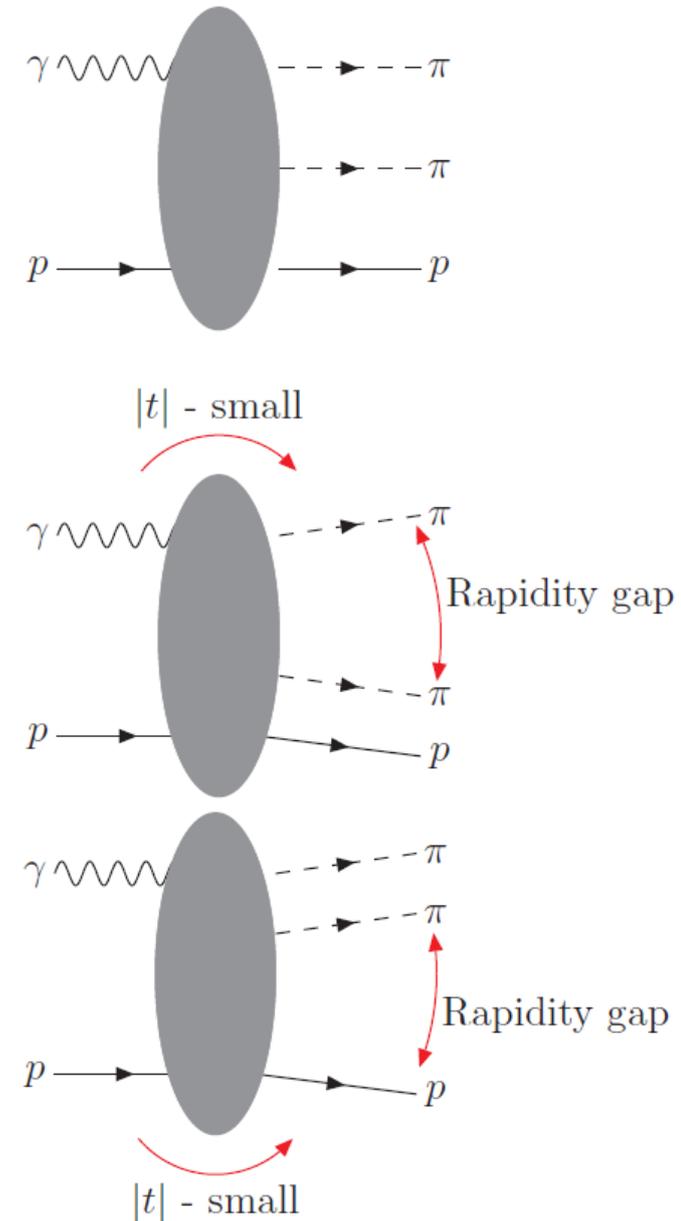
# Motivation

## Studying the $\pi\pi$ photoproduction:

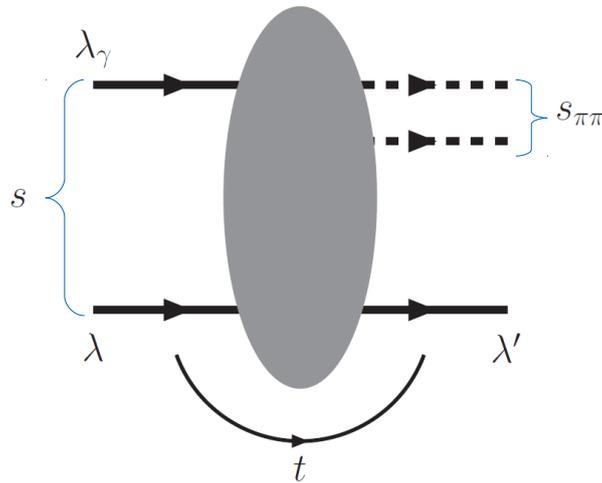
- Is a means to produce a reach spectrum of baryon ( $N^*$ ,  $\Delta$ ) and meson ( $f_0$ ,  $\rho$ ,  $f_2, \dots$ ) resonances
- It is a laboratory of methods which can be applied in more complex systems, ie. containing a few mesons or spinning mesons
- For many resonances observed in this system production mechanisms (exchanges involved) are still poorly known
- Polarized photon beams at CLAS12 and GlueX experiments allow for detailed study of production mechanisms
- We want to utilize as much as possible the known information on  $\pi\pi$  scattering (SAID, MAID, others)

# Kinematics of interest

- Out of all 3-body final states
- ... we are interested in those with small momentum transfer to spectator particle
- Such kinematics favors the production of resonances in  $\pi\pi$  and  $\pi p$  systems



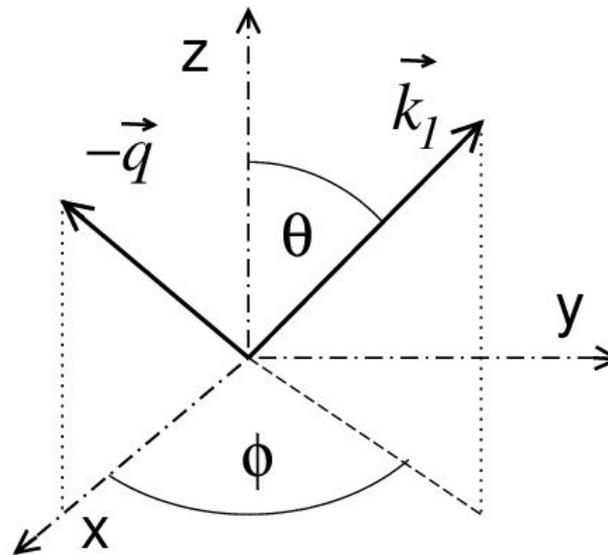
# General description of the 3-particle production



The system is described in terms of 5 kinematic variables:

- 3 Lorentz invariants –  $s$ ,  $s_{\pi\pi'}$ ,  $t$
- $\varphi$ ,  $\theta$  – angles, which describe the outgoing pions direction (in their CM system), with respect to z-axis directed opposite to the recoil proton momentum (helicity system)
- and 3 spins

## Definition of the frame of reference



$\hat{z}$  is opposite to recoil proton momentum

$\hat{y}$  is perpendicular to production plane

$$\hat{x} = \hat{y} \times \hat{z}$$

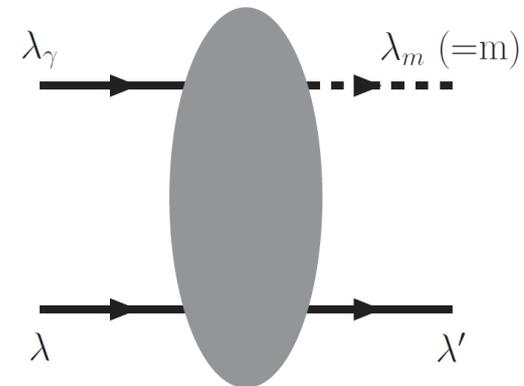
# Assume that we analyze the $\pi\pi$ system with the following properties:

- Total CM energy  $\sqrt{s}$  is “large” ( $\sim 10$  GeV)
- Effective mass  $\sqrt{s_{\pi\pi}}$  is low – so that partial wave expansion of the amplitude is valid

$$A(s, s_{\pi\pi}, t, \theta, \varphi) = \sum_{l=0}^{l_{\max}} \sum_{m=-l}^l a_m^l(s, s_{\pi\pi}, t) Y_m^l(\theta, \varphi)$$

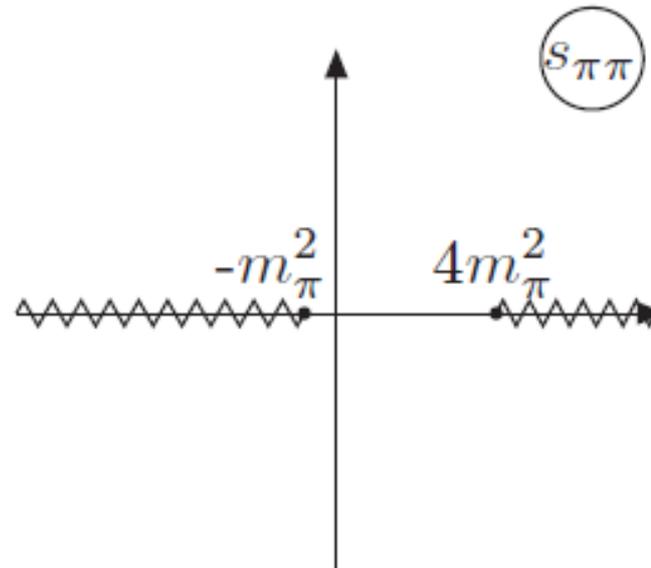
- For any given partial wave, we can think about the reaction as of the quasi  $2 \rightarrow 2$  scattering
- For fixed  $s, t, \lambda, \lambda', \lambda_m$  we can treat the partial wave amplitude as a function of only  $s_{\pi\pi}$ , ie.

$$a_{lm}(s, s_{\pi\pi}, t) = a(s_{\pi\pi})$$



**We can proceed similarly to describe resonances in the  $\pi p$  system**

Now assume that (approximate)  
analytical structure of  $a(s_{\pi\pi})$   
is the following:

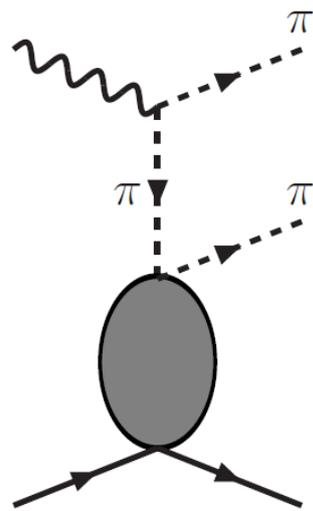


1. Right hand cuts of  $a(s_{\pi\pi})$  are determined by unitarity
2. Left hand cut is close to physical region (to be explicitly calculated) but there may be other dynamical singularities far from the physical region, which can be parametrized (eg. by polynomials) – famous Chew-Low method

- For the  $\gamma p \rightarrow \pi^+ \pi^- p$  reaction the left hand cut nearest to the physical region is due to 1-pion exchange

- So schematically we write:

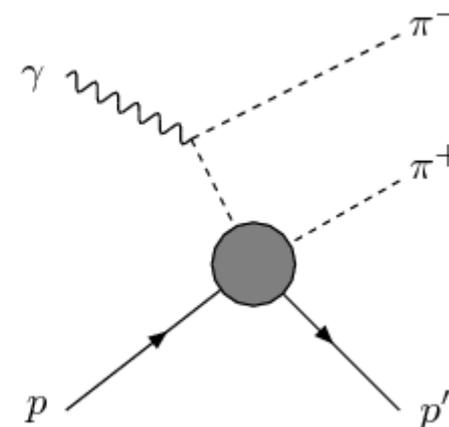
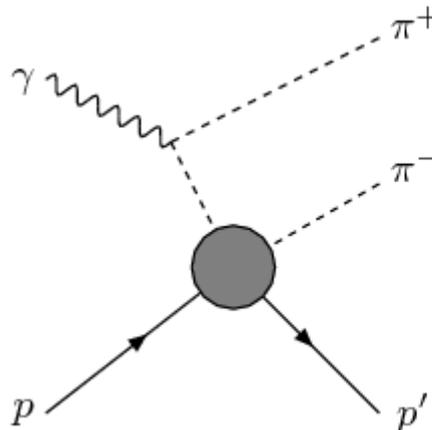
$$a_L(s_{\pi\pi}) \approx$$



Part of the amplitude dominated by the nearest left hand cut singularity – pion exchange

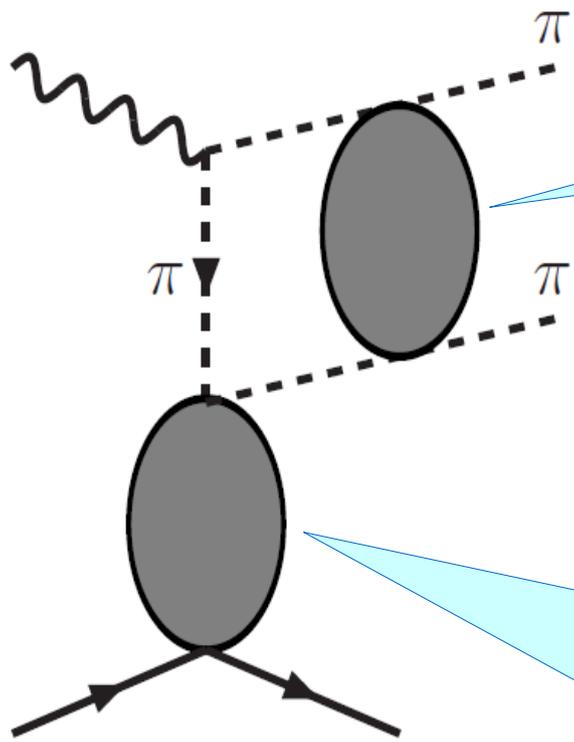
$\pi p \rightarrow \pi p$  elastic amplitude we need to parametrize (we use the SAID parametrization)

- For charged pions we have 2 diagrams of this type
- We call this approach the **Deck model (1964)** (sometimes called Drell model)





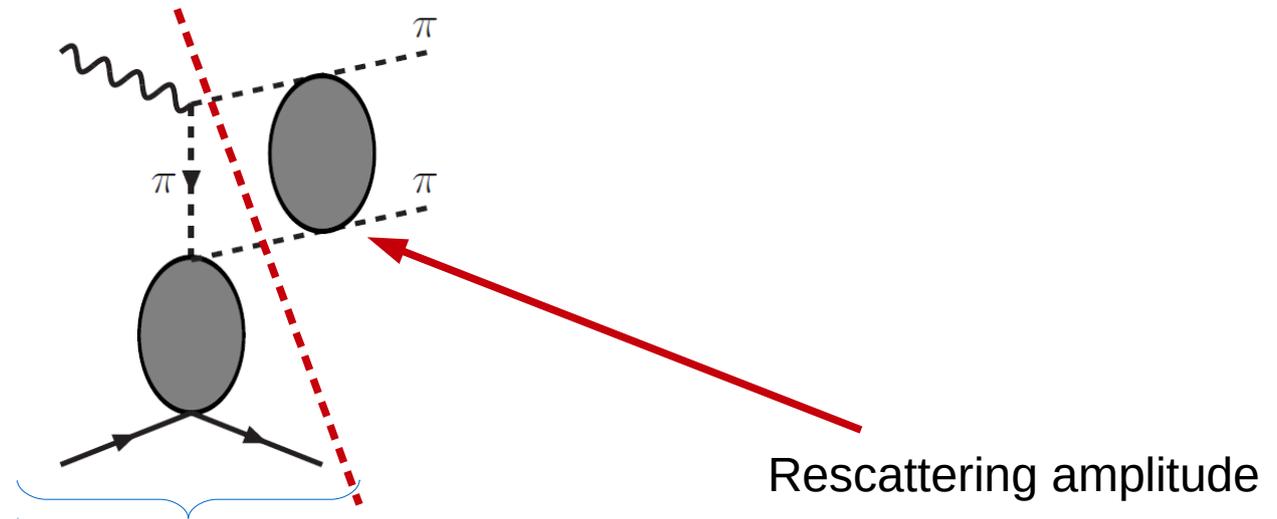
- In our model meson resonances arise due to final state interaction
- $\pi\pi$  FSI parametrised using dispersive amplitudes by Bydzovsky et al. ( Phys.Rev. D94 (2016) 11601)



- $I=1/2$  and  $I=3/2$  baryon resonances  $N^*$  and  $\Delta$  are encoded in SAID amplitudes
- In what follows we focus on  $\Delta^{++}(1232)$
- To reconstruct the  $\Delta^{++}(1232)$  we integrate  $\pi p$  mass distribution

# Translating diagrams into amplitude structure

- Structure of the production amplitude



Initial state amplitude (Deck type amplitude)

$$A_{\pi\pi} = V_{\pi\pi} + \langle \pi\pi | \hat{t}_{FSI} | m'n' \rangle G_{m'n'}(\kappa') V_{m'n'}$$

or in integral form

$$A_{\pi\pi} = V_{\pi\pi} + 4\pi \sum_{m'n'} \int_0^\infty \frac{\kappa'^2 d\kappa'}{(2\pi)^3} \langle \pi\pi | \hat{t}_{FSI} | m'n' \rangle G_{m'n'}(\kappa') V_{m'n'}$$

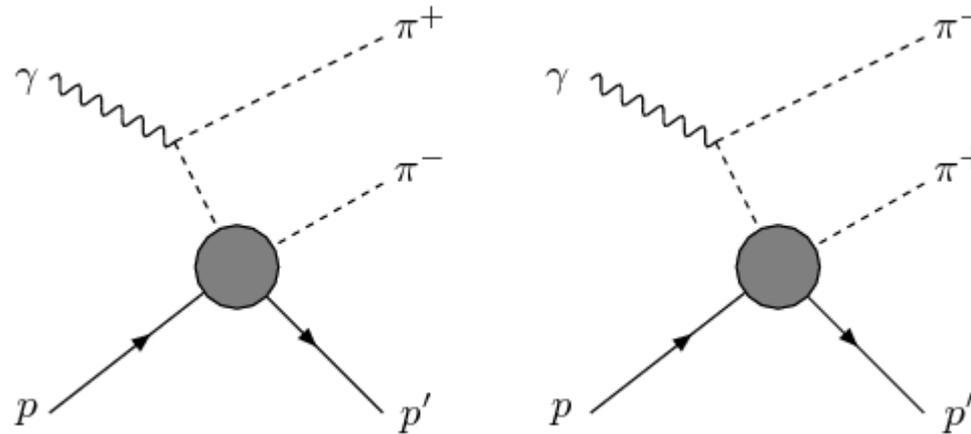
where:

$A_{\pi\pi}$  – photoproduction amplitude of the meson pair  $\pi\pi$ ,

$V_{\pi\pi}$  - initial state amplitude,

$t_{FSI}$  -rescattering amplitude

# Deck amplitude



- **General form of the amplitude** [Pumplin 1970]

$$\mathcal{M}_{\lambda_2\lambda_1} = \frac{-1}{\sqrt{4\pi}} \left\{ e\varepsilon \cdot \left[ \frac{\hat{\kappa}}{|\mathbf{q}|} \frac{1}{x + \hat{\mathbf{q}} \cdot \hat{\kappa}} + \frac{\mathbf{p}_1 + \mathbf{p}_2}{\mathbf{q} \cdot (\mathbf{p}_1 + \mathbf{p}_2)} \right] T^+_{\lambda_2\lambda_1} + e\varepsilon \cdot \left[ \frac{\hat{\kappa}}{|\mathbf{q}|} \frac{1}{x - \hat{\mathbf{q}} \cdot \hat{\kappa}} - \frac{\mathbf{p}_1 + \mathbf{p}_2}{\mathbf{q} \cdot (\mathbf{p}_1 + \mathbf{p}_2)} \right] T^-_{\lambda_2\lambda_1} \right\}$$

- **The amplitude is gauge invariant**

# How do we know what particle is exchanged ?

- It was shown by Stichel 1964 and Ader et al. 1968 that to leading order in  $t/s$ :
- $\frac{d\sigma_{\parallel}}{dt}$  receives contributions only from unnatural exchange
- $\frac{d\sigma_{\perp}}{dt}$  receives contributions only from natural exchange
- Where  $\frac{d\sigma_{\parallel}}{dt}$  and  $\frac{d\sigma_{\perp}}{dt}$  are photoproduction cross sections for photons polarized parallel and perpendicular to production plane, respectively
- Naturalness is defined as:  
$$P(-1)^J=1 \text{ natural} \qquad P(-1)^J=-1 \text{ unnatural}$$
- So, eg.:  
 $\pi$  and  $b$  - unnatural exchanges  
 $\rho$  and  $a_2$  – natural exchanges

# $\pi p \rightarrow \pi p$ amplitude – partial wave expansion

- General form of the  $\pi p$  scattering amplitude (Chew, Goldberger, Low, Nambu (1957))

$$T_{\alpha\beta} = \bar{u}(p_2)(A_{\alpha\beta} + \gamma \cdot Q B_{\alpha\beta})u(p_1)$$

Where:

$$Q = \frac{1}{2}(q - k_1 + k_2) \quad \text{and} \quad \frac{A}{4\pi} = \frac{W + m}{E + m} f_1 - \frac{W - m}{E - m} f_2,$$
$$\frac{B}{4\pi} = \frac{f_1}{E + m} + \frac{f_2}{E - m}.$$

Then the  $f_1$  and  $f_2$  functions are partial wave expanded (separately for  $l=1/2$  and  $l=3/2$ ):

$$f_1 = \sum_{l=0}^{\infty} f_{l+} P'_{l+1}(\cos \theta^*) - \sum_{l=2}^{\infty} f_{l-} P'_{l-1}(\cos \theta^*),$$
$$f_2 = \sum_{l=1}^{\infty} (f_{l-} - f_{l+}) P'_l(\cos \theta^*),$$

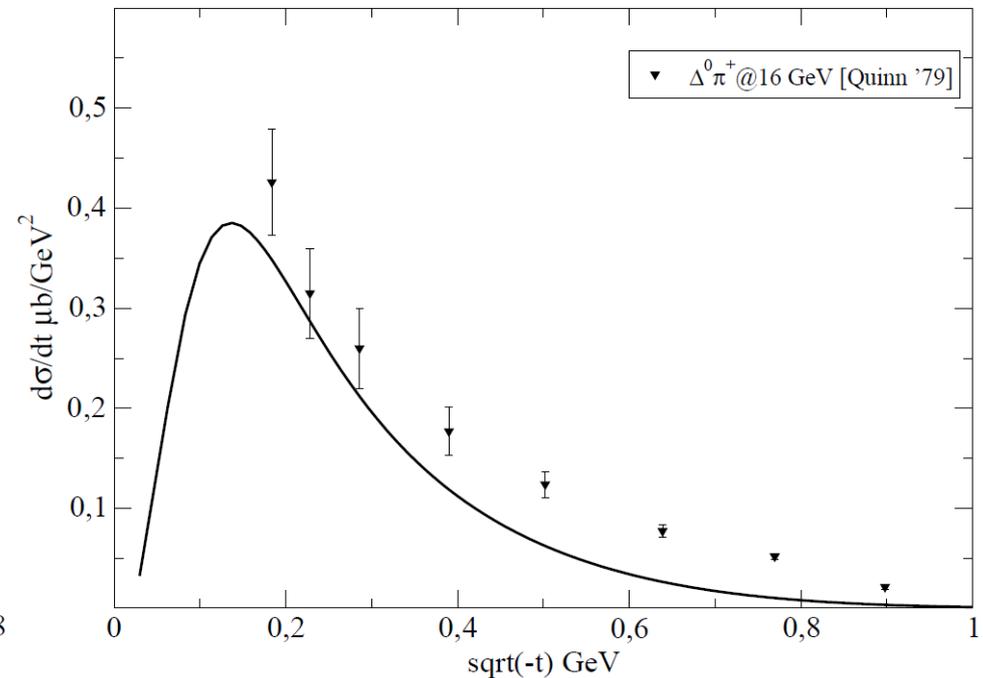
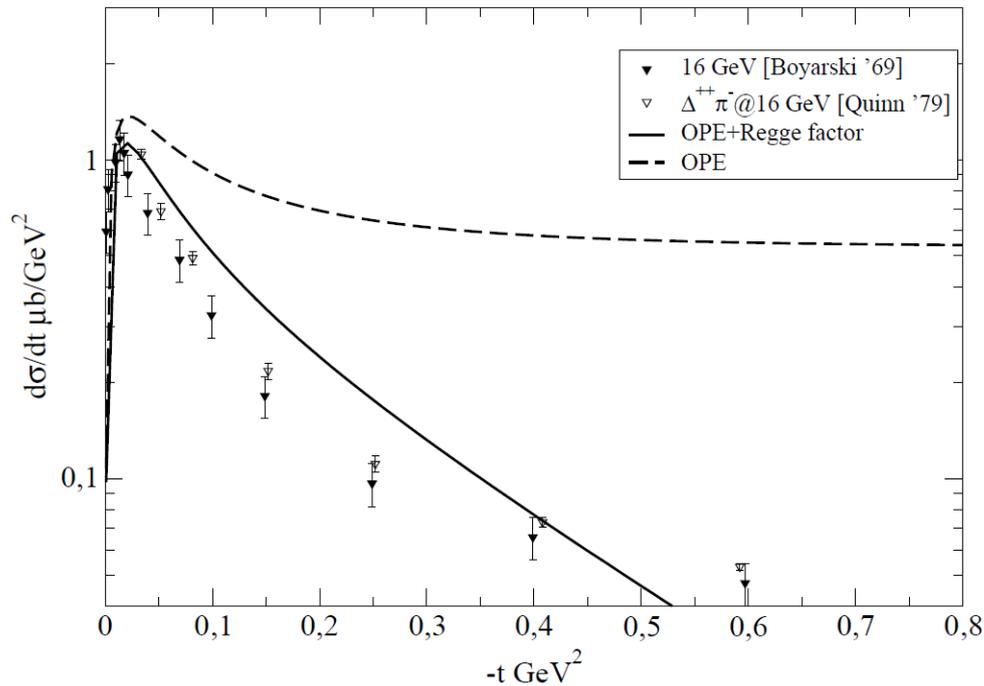
There are a few experimental/phenomenological analyses in order to fit data to this expansion: Bonn-Gatchina, MAID, SAID.



# Resonances in the $p\pi$ subsystem

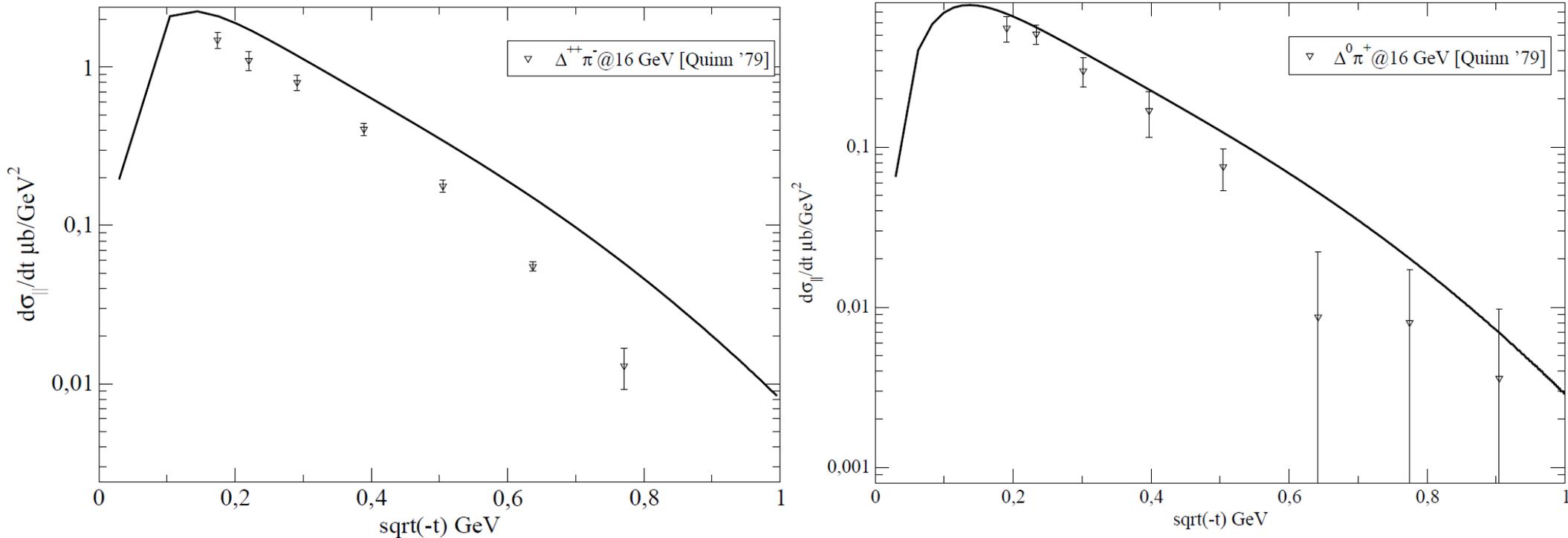
## Cross section results

# Unpolarised cross sections for $\gamma p \rightarrow \Delta^{++} \pi^-$ and $\gamma p \rightarrow \Delta^0 \pi^+$



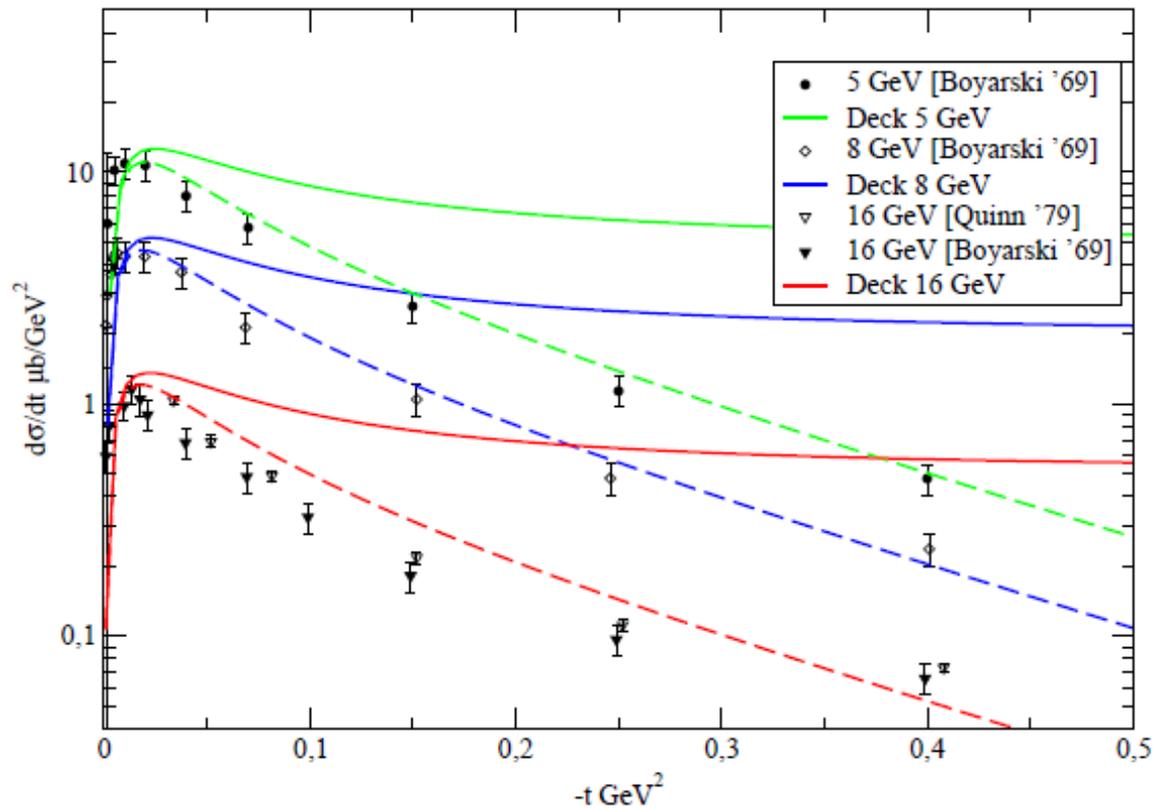
- Pure gauge invariant one pion exchange model overshoots the data
- One can make the  $t$ -dependence correct by using collimation factor of  $\exp(3t)$  in the amplitude or by applying Regge energy factor  $(s/s_0)^{\alpha_\pi(t)}$
- We use the Regge factor to make our analysis comparable with Regge theory predictions

# Paralell cross sections for $\gamma p \rightarrow \Delta^{++} \pi^-$ and $\gamma p \rightarrow \Delta^0 \pi^+$



- Pure gauge invariant one pion exchange model predicts the  $\frac{d\sigma_{\perp}}{dt} \approx 0$
- To correct for this, one needs to take into account also the s-channel diagrams apart from t-channel ones.

# Energy dependence of $\gamma p \rightarrow \Delta^{++} \pi^-$ cross section



- We reproduce the cross section behavior throughout the wide interval of photon energies



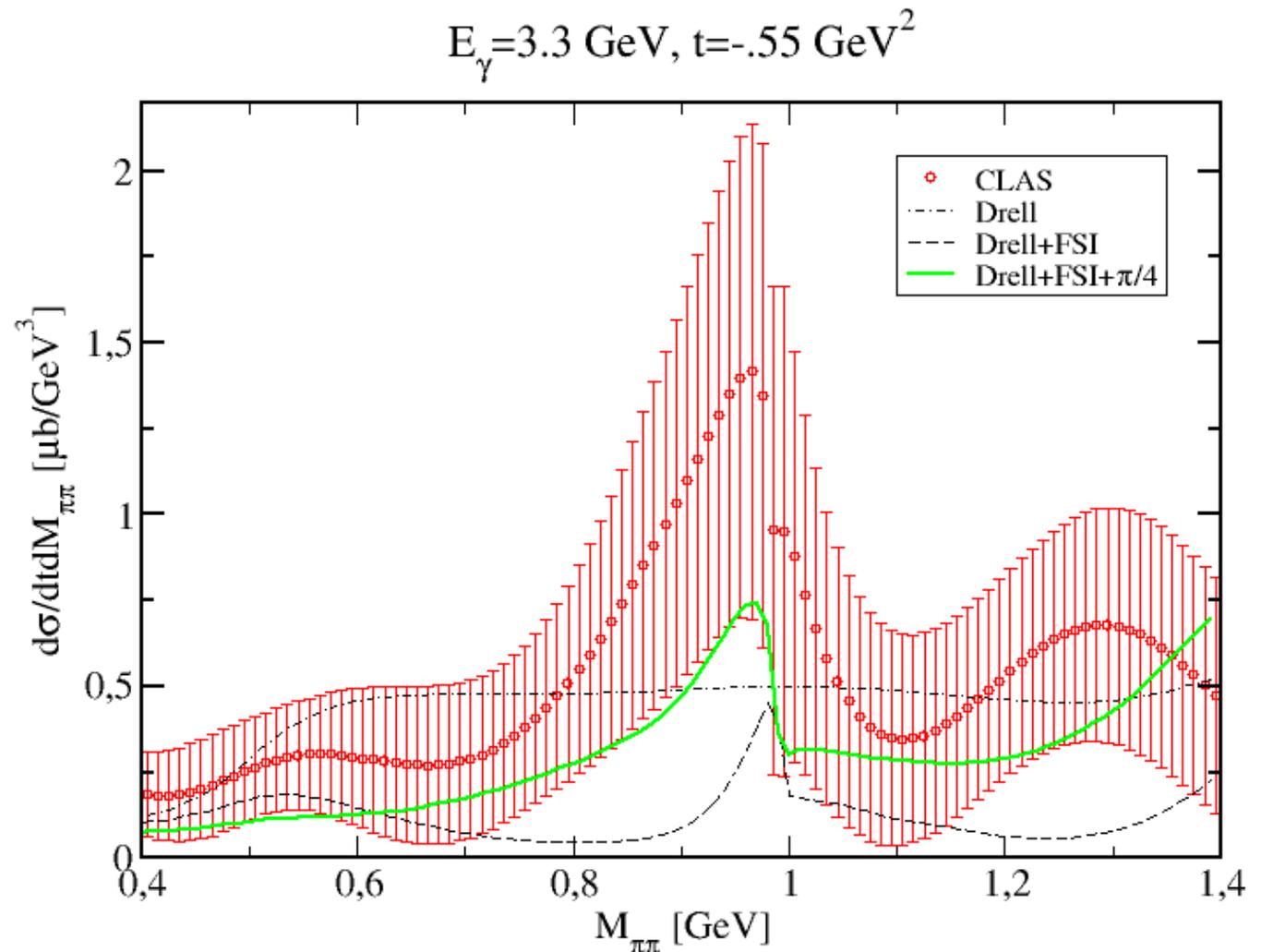
# Resonances in the $\pi\pi$ subsystem

## Mass distributions

# S-wave

## Notice:

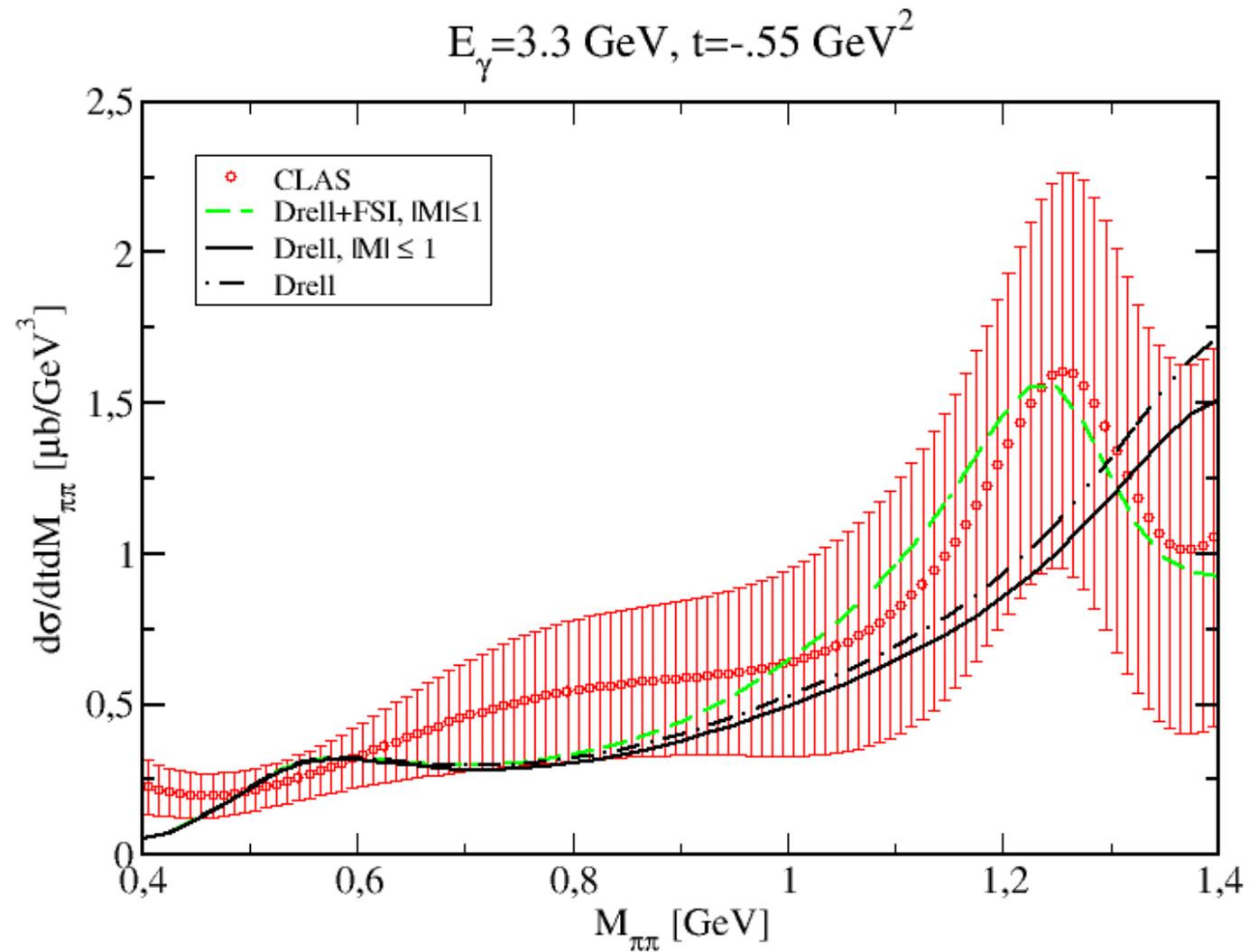
- Very good distribution description already at the level of Drell amplitudes
- Clear  $f_0(980)$  resonance contribution
- Correction relative phase of  $\pi/4$  needed to get the proper distribution behavior
- Otherwise the Drell-FSI interference is destructive and the theoretical distribution is too small
- Indication of the influence of the coupled  $K\bar{K}$  channel above 1 GeV



# D-wave

## Notice:

- Very good distribution description already at the level of Drell amplitudes
- Clear  $f_2(1270)$  resonance contribution
- Correction relative phase of  $\pi/2$  needed to get the proper distribution behavior
- Small contribution of helicities  $M=\pm 2$
- No indication of the influence of the coupled  $K\bar{K}$  channel – quite understandable,  $f_2(1270)$  decays to  $K\bar{K}$  only in  $<5\%$  (84% to  $\pi\pi$ )
- No additional background needed to describe the data



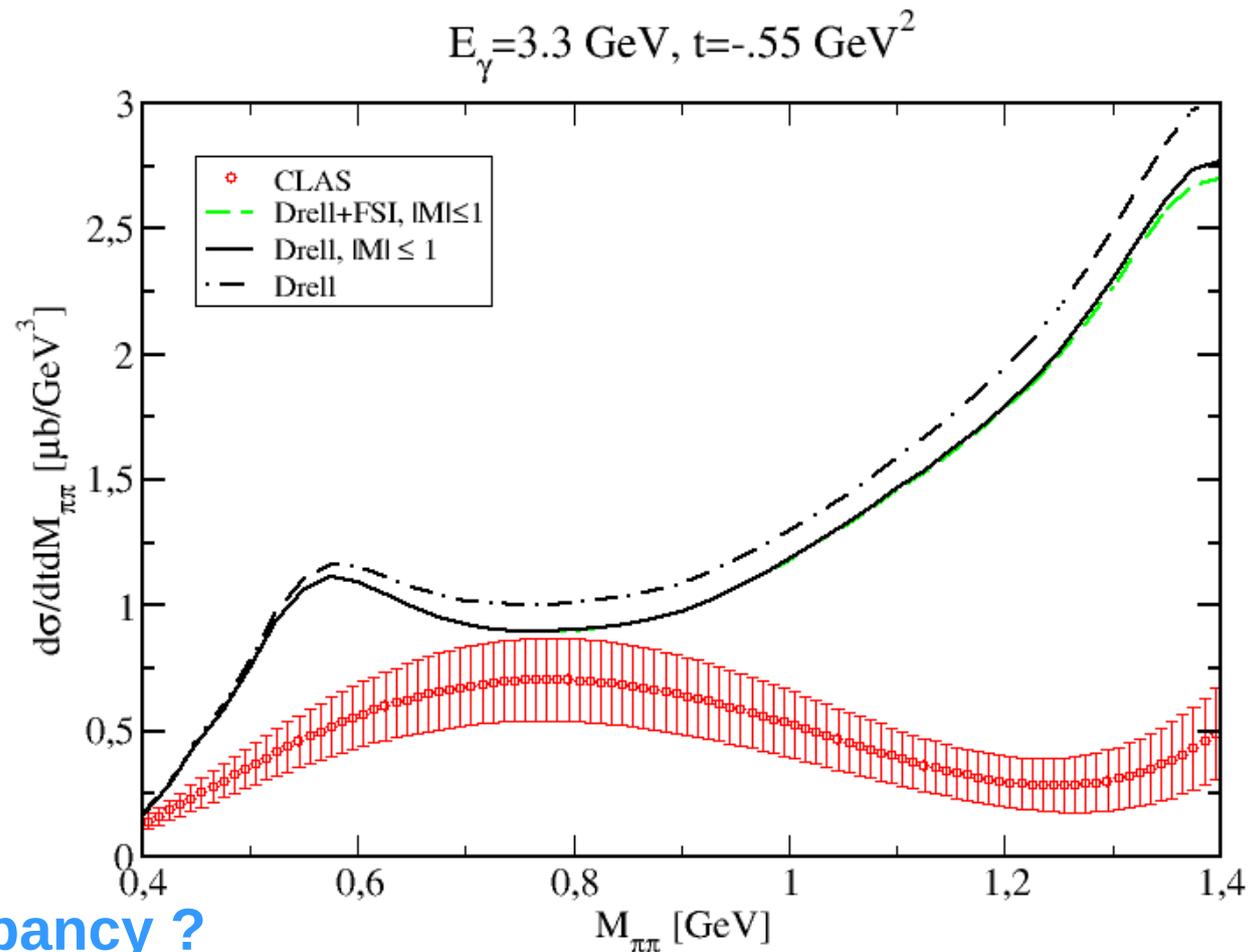
# F-wave

## Notice:

- Model prediction is to large in almost whole mass interval
- Helicities  $|M|>1$  bring small contribution to the distribution
- No indication of the FSI

## Can we explain the discrepancy ?

- The F-wave FSI model we use, is almost  $\pi\pi$  elastic up to  $M_{\pi\pi}=1.5$  GeV. This may be disputed given that there are many isovector channels already opened below this energy, eg.  $\pi\pi\pi\pi$ ,  $\pi\omega$ ,  $K\bar{K}$ .
- The dominant F-wave resonance  $\rho_3(1690)$  decays to  $\pi\pi$  only in  $<24\%$  while it mostly (71%) decays to  $\pi\pi\pi\pi/\pi\omega$  channel – we may expect large interchannel coupling and destructive interference
- Higher partial waves may be affected by vector meson exchange amplitudes which may destructively interfere with one-pion-exchange amplitudes



# Summary:

- The OPE Deck model gives good predictions of unpolarised and fairly good predictions of parallel cross sections (dominated by unnatural exchanges)
- In order to describe perpendicular cross sections one needs to include the s-channel diagrams and/or absorption effects
- Deck amplitudes with the final state  $\pi\pi$  interaction very well describe the S- and D-wave photoproduction from the  $\pi\pi$  threshold up to the resonance region ( $f_0(980)$  and  $f_2(1270)$  respectively)