Radiative pion capture in ^2H, ^3He and ^3H

JAGIELLONIAN
UNIVERSITY

J. Golak,

R. Skibiński, K. Topolnicki, H. Witała, A. Grassi,

H. Kamada, A. Nogga, L.E. Marcucci

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Outline

- Introduction: elements of formalism
- Radiative pion capture on ^{2}H, ^{3}He and ^{3}H
- Conclusions and outlook

Introduction

A very efficient momentum space framework to deal with nucleon-nucleon scattering, nucleon-deuteron scattering and many electroweak processes has been constructed and tested:
Phys. Rept. 274, 107 (1996); Phys. Rept. 415, 89 (2005);
Eur. Phys. J. A24, 31 (2005)

Limitations: nonrelativistic character and lack of the Coulomb force in the 3N continuum

Calculations performed with semi-phenomenological 2N and 3N potentials: Bonn B, AV18, Nijmegen I and II, CD Bonn, Urbana IX, older chiral potentials from the Bonn/Bochum group and recently with the improved chiral potentials from E. Epelbaum *et al.*

Introduction

Methods developed originally for elastic and inelastic electron scattering, photodisintegration, and applied later to neutrino induced reactions and muon capture are now used to investigate the following processes

$$\pi^- + d \rightarrow \gamma + n + n$$

$$\pi^- + {}^3\mathrm{He} \rightarrow \gamma + {}^3\mathrm{H}$$

$$\pi^- + {}^3\mathrm{He} \rightarrow \gamma + d + n$$

$$\pi^- + {}^3\mathrm{He} \rightarrow \gamma + p + n + n$$

$$\pi^- + {}^3\mathrm{H} \rightarrow \gamma + n + n + n$$

These processes combine information from several areas (pion absorption ↔ pion photoproduction, weak processes, nuclear interactions) and should be ultimately studied within ChEFT

Introduction

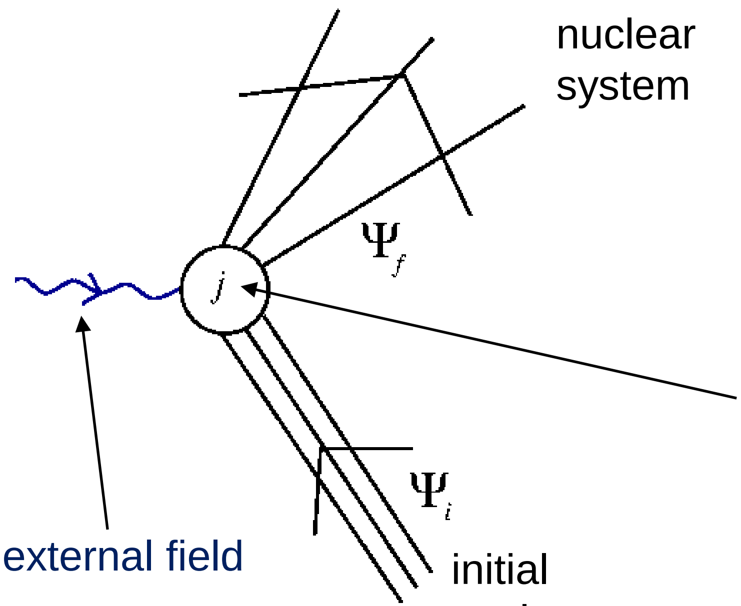
General strategy in the few-nucleon systems:

- use (**consistent)** dynamical ingredients (2N and 3N potentials, electroweak current operators)
- solve the dynamical equations (Schrödinger equation, Lippmann-Schwinger equation, Faddeev equations)
- give predictions for nuclear structure and reaction observables
- confront results of theoretical calculations with experimental data to improve your input

Presented here results have been calculated with the AV18 2N and Urbana IX 3N potentials

Formalism

interaction of nuclear system with external probe



$$N^{\alpha} \equiv \left\langle \Psi_f \left| j^{\alpha} \right| \Psi_i \right\rangle$$

crucial dynamical quantity

Formalism

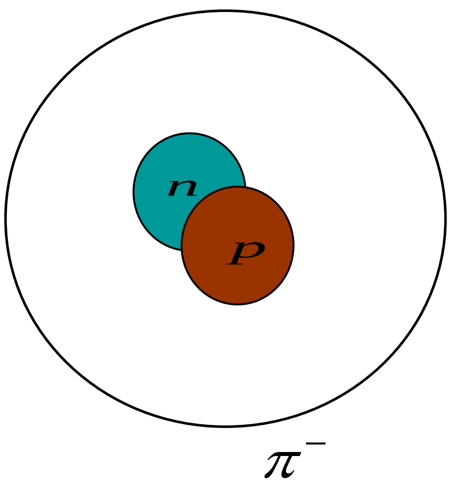
Pion capture from the lowest K-shell of the pionic atom followed by photon emission studied with impulse approximation:

$$T_{fi} \propto \vec{\varepsilon} \cdot \left\langle \Psi_f \left| \vec{j}_A \right| \Psi_i \right\rangle$$

final photon polarization vector

$$\vec{j}_A \approx \sum_{i=1}^{A} \tau_-(i)\, \vec{\sigma}(i)$$

nuclear axial current



$$\psi_K(r) \equiv \psi_{100}(r) = \sqrt{\frac{(Z\, m'\alpha)^3}{\pi}}\, e^{-Z\, m'\alpha\, r}$$

$$m' \equiv \frac{m_\pi m_Z}{m_\pi + m_Z}$$

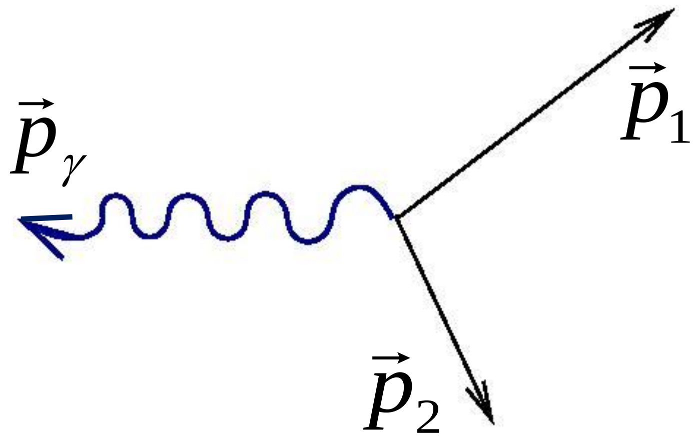
reduced mass

$$E_1 = -\frac{Z^2 \alpha^2 m'}{2}$$

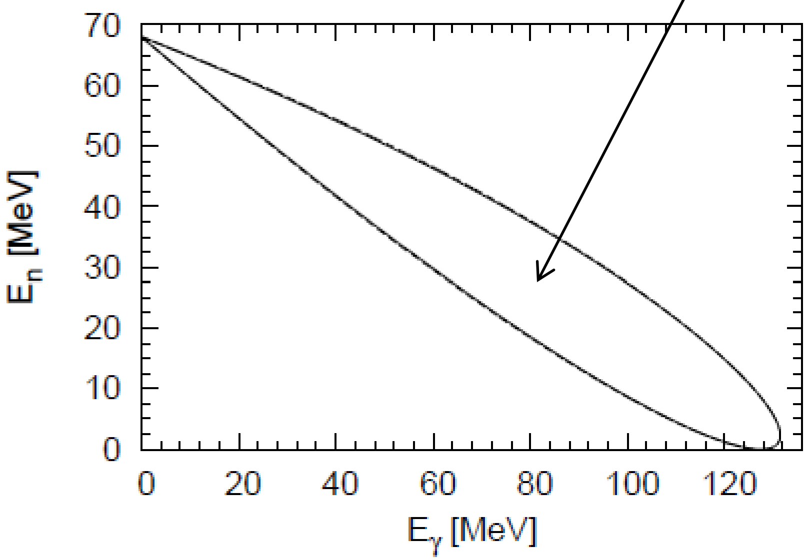
negligible for *Z=1,2* when compared to the pion or nucleon mass

Radiative pion capture on ^{2}H

$$\pi^- + d \rightarrow \gamma + n + n$$

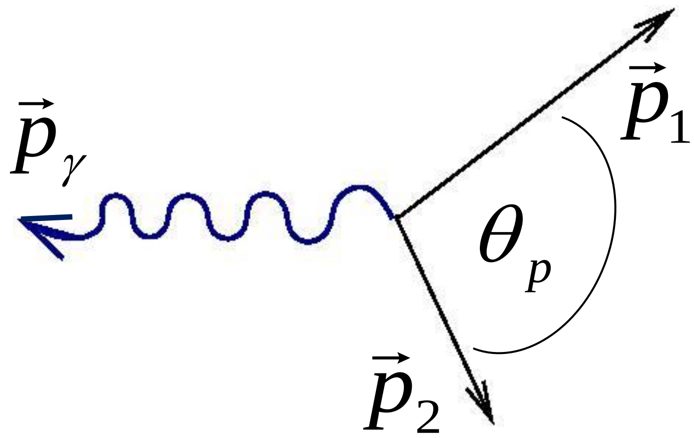


kinematically allowed region



Radiative pion capture on ^{2}H

$$\pi^- + d \rightarrow \gamma + n + n$$



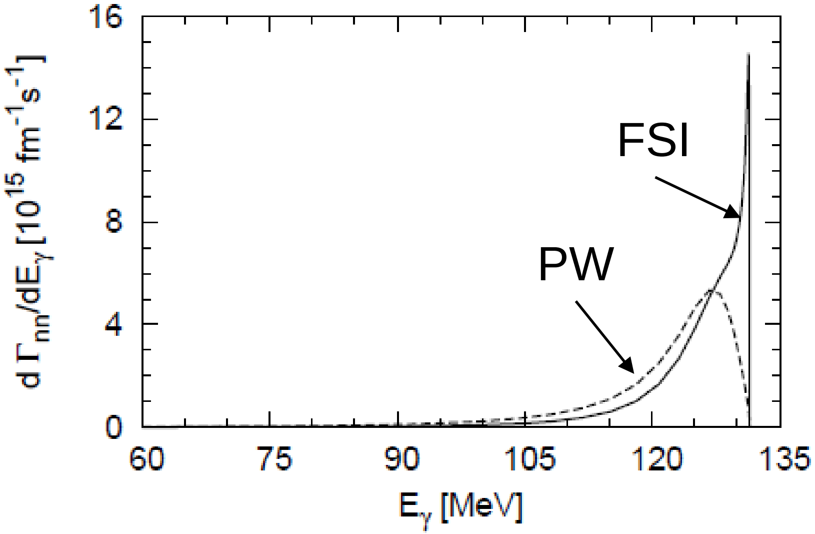
$$\Gamma_{nn} = \frac{1}{2}\frac{1}{(2\pi)^2}\frac{2\pi\alpha}{f_\pi^2 M_\pi}\frac{(M_d'\alpha)^3}{\pi}\int_0^\pi d\theta_{p_\gamma}\sin\theta_{p_\gamma}\int_0^{2\pi} d\phi_{p_\gamma}\int_0^{E_\gamma^{max,nn}} dE_\gamma E_\gamma \frac{1}{2}M_n p$$

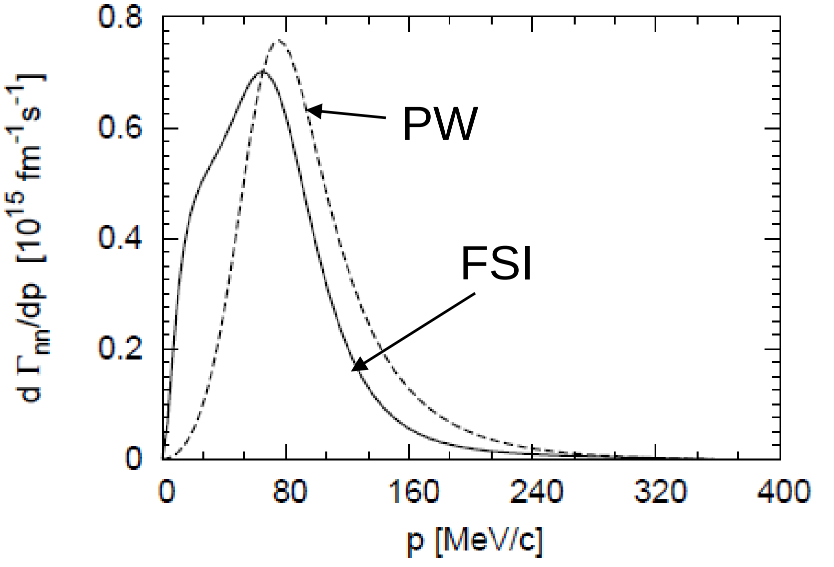
$$\int_0^\pi d\theta_p \sin\theta_p \int_0^{2\pi} d\phi_p \frac{1}{3}\sum_{m_d}\sum_{m_1,m_2}\left(|N_{+1}(m_1,m_2,m_d)|^2 + |N_{-1}(m_1,m_2,m_d)|^2 \right)$$

best way to get Γ_{nn} from $d\Gamma_{nn}/dE_\gamma$

Radiative pion capture on ^2H

$$\pi^- + d \rightarrow \gamma + n + n$$





$\Gamma_{nn} = 0.318 \times 10^{15}$ 1/s (PW) similar results

$\Gamma_{nn} = 0.328 \times 10^{15}$ 1/s (FSI) but completely different physics !

Radiative pion capture on ^{2}H

$$\pi^- + d \to \gamma + n + n$$

Earlier theoretical predictions for Γ_{nn}:

A. Reitan, Nucl. Phys. 87, 232 (1966):
3.32×10^{14} 1/s $\rightarrow 4 \times 10^{14}$ 1/s (corrected by ST)

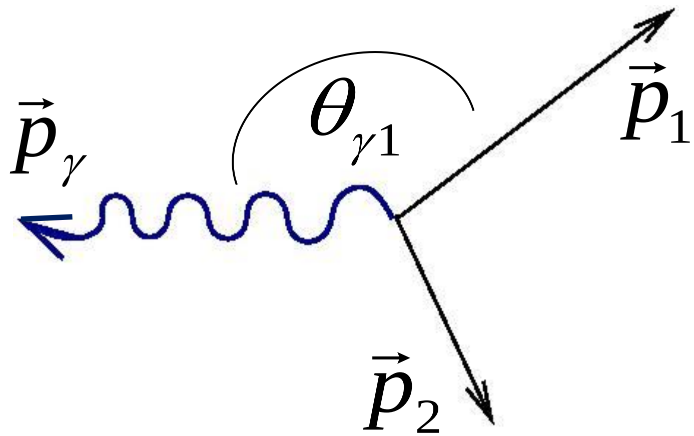
M. Sotona and E. Truhlik, Nucl. Phys. A262, 400 (1976):
3.75×10^{14} 1/s (based on pion photoproduction data)
3.83×10^{14} 1/s (based on soft-pion limit+ corrections)

W. R. Gibbs, B. F. Gibson, and Q. J. Stephenson, Jr.,
Phys. Rev. C16, 327 (1977); 17, 856 (1978) (E)
$(4.2 \pm 0.5) \times 10^{14}$ 1/s

this contribution: 3.28×10^{14} 1/s

Radiative pion capture on ^{2}H

$$\pi^- + d \rightarrow \gamma + n + n$$



another way to get Γ_{nn}
from neutron spectrum

$$\Gamma_{nn} = \frac{1}{2}\frac{1}{(2\pi)^2}\,\frac{2\pi\alpha}{f_\pi^2 M_\pi}\,\frac{(M_d'\alpha)^3}{\pi}\int_0^\pi d\theta_{p_\gamma}\sin\theta_{p_\gamma}\int_0^{2\pi} d\phi_{p_\gamma}$$

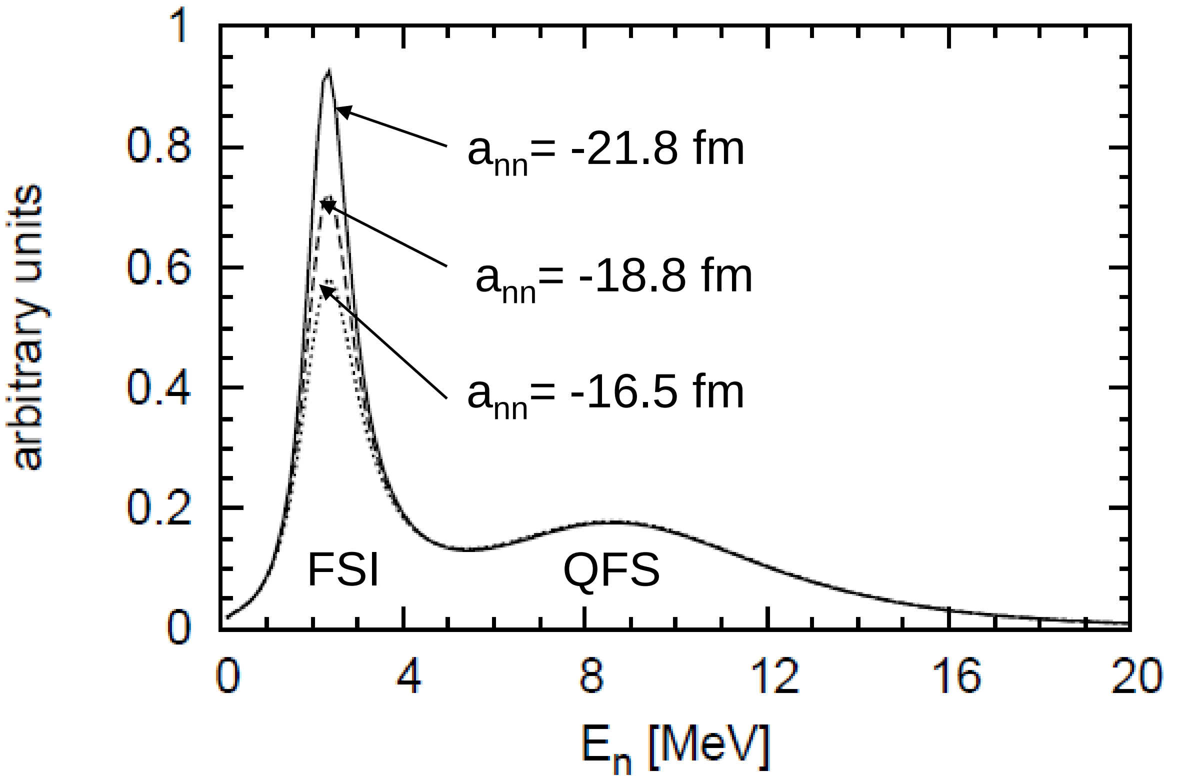
$$\int_0^\pi d\theta_{p_1}\sin\theta_{p_1}\int_0^{2\pi} d\phi_{p_1}\int_0^{E_1^{max}} dE_1\,\frac{M_n^2 p_1 E_\gamma}{E_\gamma + M_n + p_1\cos\theta_{\gamma 1}}$$

$$\frac{1}{3}\sum_{m_d}\sum_{m_1,m_2}\left(\,|N_{+1}(m_1,m_2,m_d)\,|^2 + |N_{-1}(m_1,m_2,m_d)\,|^2\,\right)$$

Radiative pion capture on ^{2}H

$$\pi^{-} + d \rightarrow \gamma + n + n$$

$$d\Gamma^{5}_{nn} / (d\Omega_{\gamma}\, d\Omega_{1}\, dE_{1})$$



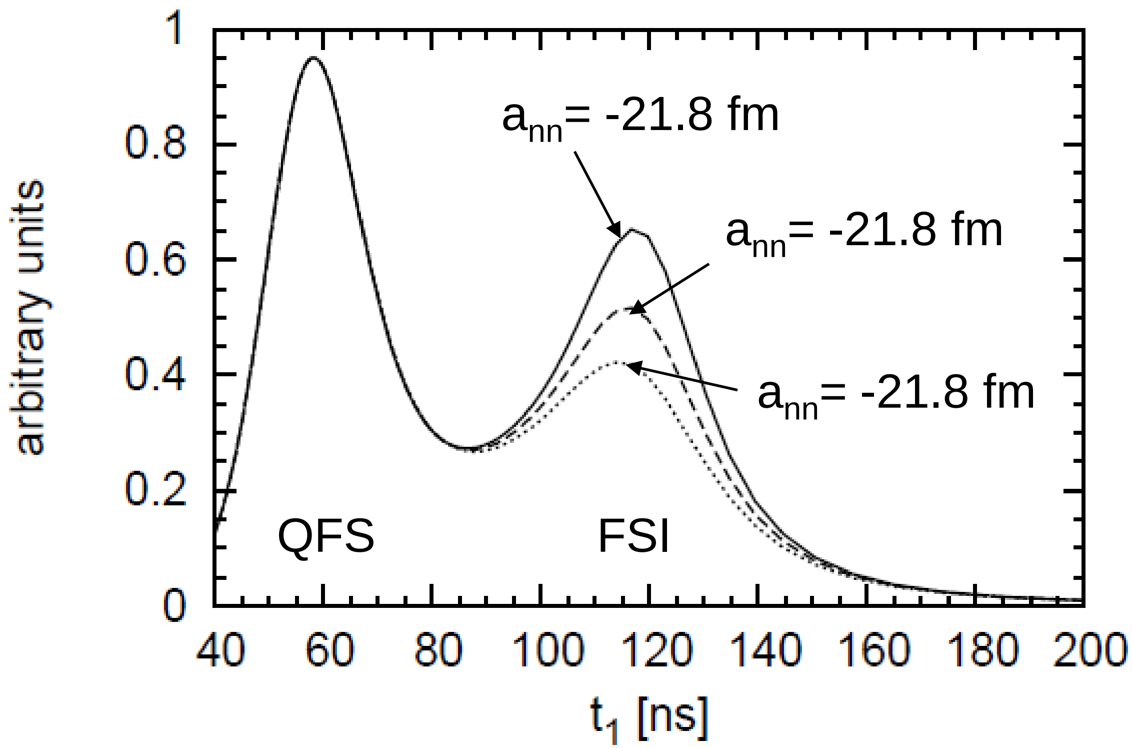
neutron-neutron potential is changed by 1 % only in the $^{1}S_{0}$ channel

neutron energy spectra for $\theta_{\gamma 1} = 179^{\circ}$

Radiative pion capture on ^{2}H

$$\pi^- + d \rightarrow \gamma + n + n$$

$$d\Gamma^5_{nn} / (d\Omega_\gamma \, d\Omega_1 dt_1)$$

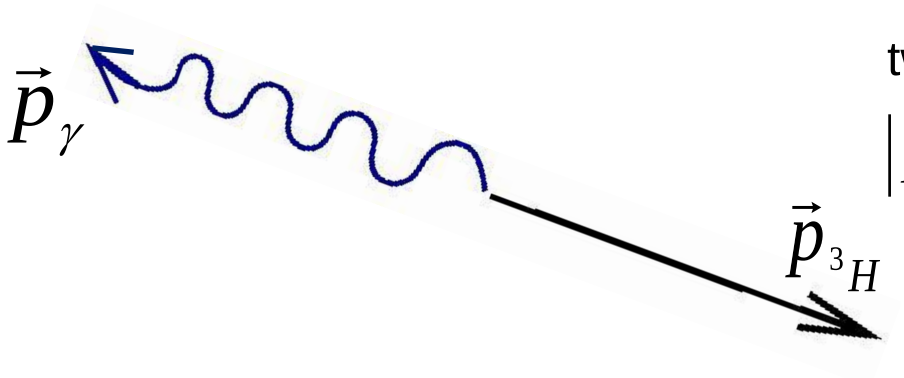


neutron-neutron potential is changed by 1 % only in the $^1S_0$ channel

neutron TOF spectra for $\theta_{\gamma 1} = 179^\circ$ normalized at QFS peak for s= 2.55 m

Radiative pion capture on ^{3}He: triton channel

$$\pi^- + {}^3He \rightarrow \gamma + {}^3H$$



two-body kinematics

$|\vec{p}_\gamma| = |\vec{p}_{^3H}| =$ 135.7 MeV/c

Γ_{3H} = 2.059 × 10^{15} 1/s (2NF)
Γ_{3H} = 2.132 × 10^{15} 1/s (2NF+3NF)

Radiative pion capture on ^{3}He: triton channel

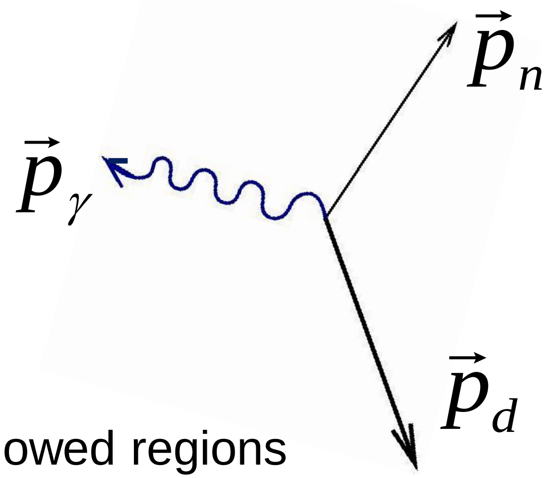
Earlier theoretical predictions for Γ_{3H}

- Fujii and D. Hall, Nucl. Phys. 32, 102 (1962).
  $(8.32 \rightarrow 4.28) \times 10^{15}$ 1/s (corrected by Truöl 1974)
- P. Divakaran, Phys. Rev. 139, 3887 (1965).
  $(0.97 \rightarrow 3.88) \times 10^{15}$ 1/s (corrected by Truöl 1974)
- D. Griffiths and C. Kim, Phys. Rev. 173, 1584(1968)
  2.32×10^{15} 1/s
- P. Pascual and A. Fujii, Nuovo Cimento 65, 411 (1970)
  $(3.37 \rightarrow 2.25) \times 10^{15}$ 1/s (corrected by Truöl 1974)
- P. Truöl et al., Phys. Rev. Lett. 32, 1268 (1974)
  3.60×10^{15} 1/s
- A. C. Phillips and F. Roig, Nucl. Phys. A234, 378 (1974)
  $(3.1 - 3.7) \times 10^{15}$ 1/s
- W. R. Gibbs et al., Phys. Rev. C18, 1761 (1978)
  3.30×10^{15} 1/s

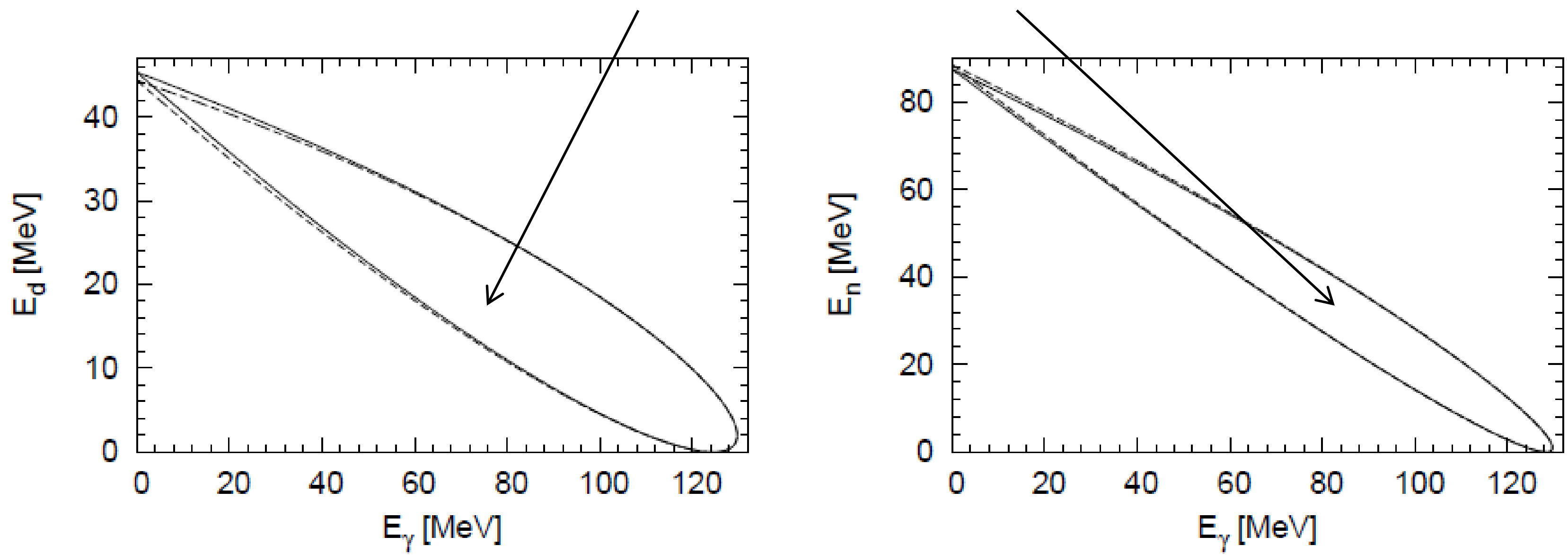
this contribution: 2.132×10^{15} 1/s

Radiative pion capture on ^{3}He: two-body breakup

$$\pi^- + {}^3\mathrm{He} \rightarrow \gamma + d + n$$

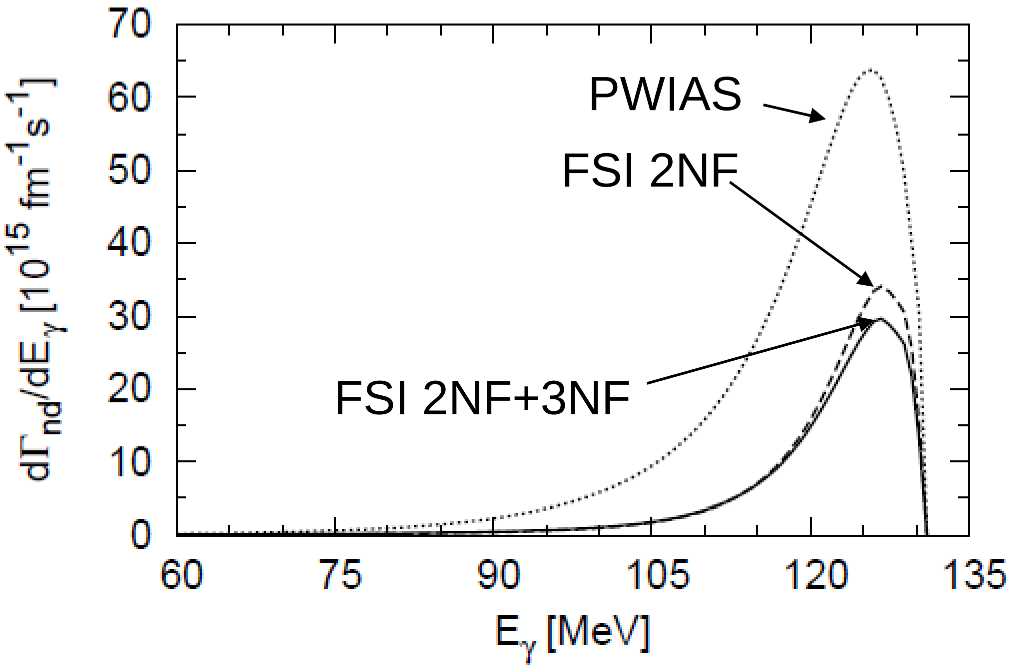


kinematically allowed regions



Radiative pion capture on ^{3}He: two-body breakup

$$\pi^- + {}^3\mathrm{He} \rightarrow \gamma + d + n$$

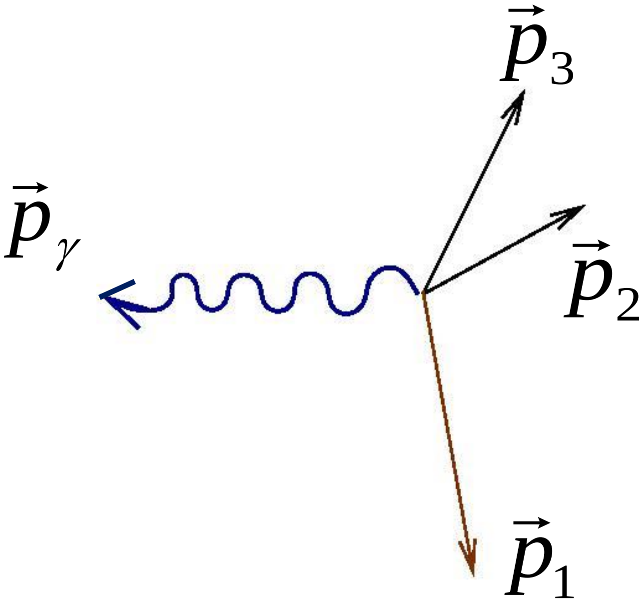


$\Gamma_{nd} = 5.201 \times 10^{15}$ 1/s (PWIAS)
$\Gamma_{nd} = 2.013 \times 10^{15}$ 1/s (FSI 2NF)
$\Gamma_{nd} = 1.840 \times 10^{15}$ 1/s (FSI 2NF+3NF)

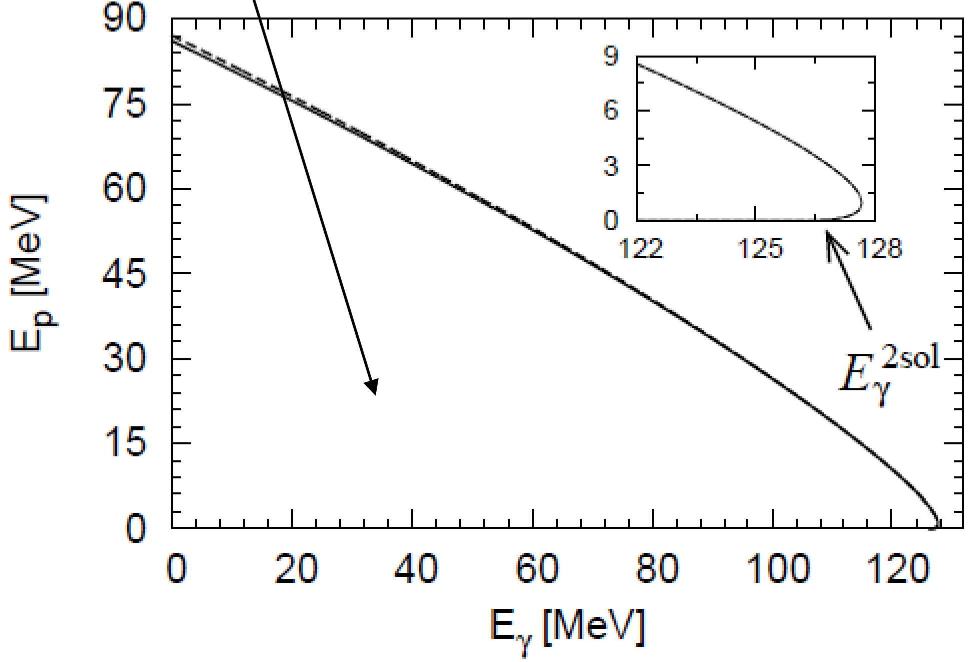
crucial importance of FSI !

Radiative pion capture on ^{3}He: three-body breakup

$$\pi^- + {}^3\text{He} \rightarrow \gamma + p + n + n$$

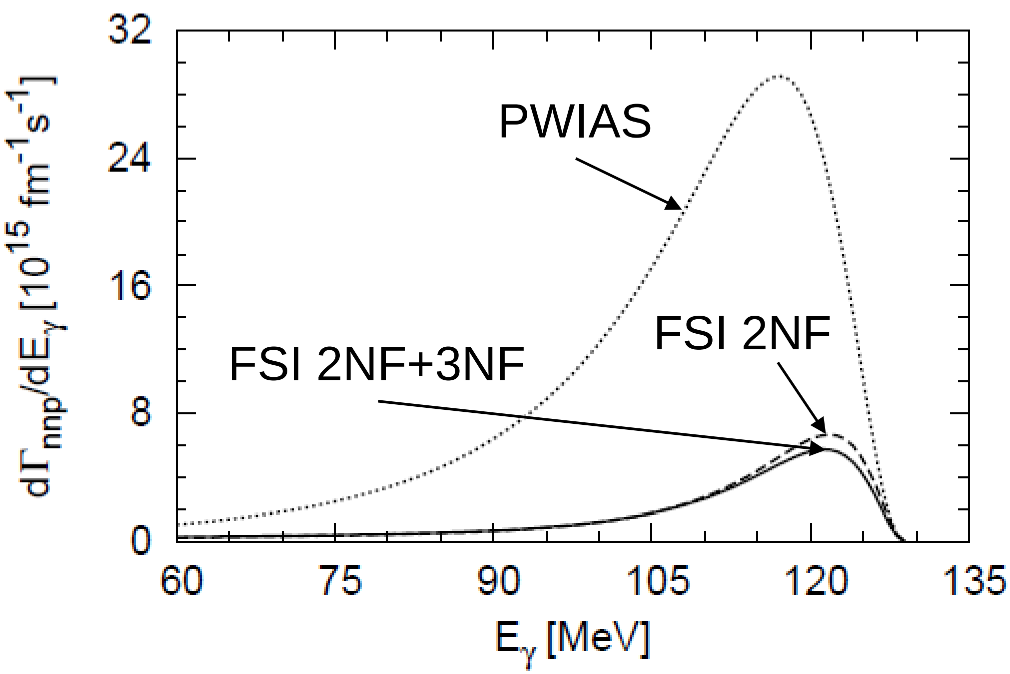


kinematically allowed region



Radiative pion capture on ^{3}He: three-body breakup

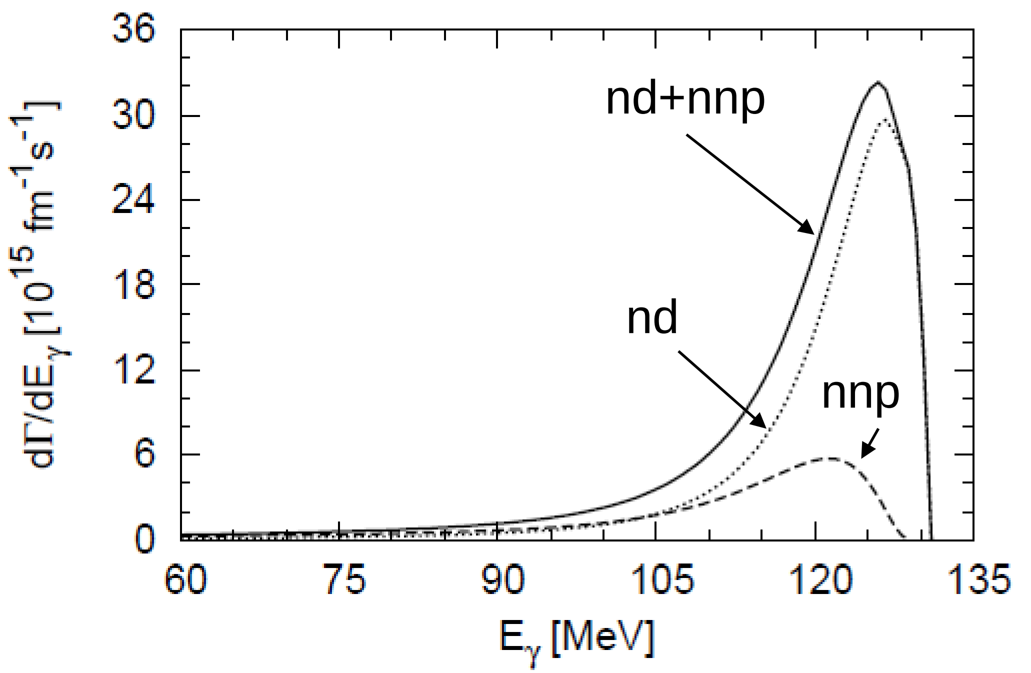
$$\pi^- + {}^3\text{He} \to \gamma + p + n + n$$



$\Gamma_{nnp} = 3.816 \times 10^{15}$ 1/s (PWIAS)
$\Gamma_{nnp} = 0.659 \times 10^{15}$ 1/s (FSI 2NF)
$\Gamma_{nnp} = 0.615 \times 10^{15}$ 1/s (FSI 2NF+3NF)

crucial importance of FSI !

Radiative pion capture on ^{3}He: comparison of two- and three-body breakup with best dynamics



$\Gamma_{nd} = 1.840 \times 10^{15}$ 1/s

$\Gamma_{nnp} = 0.615 \times 10^{15}$ 1/s

$\Gamma_{nd+nnp} = 2.455 \times 10^{15}$ 1/s

two-body breakup dominates !

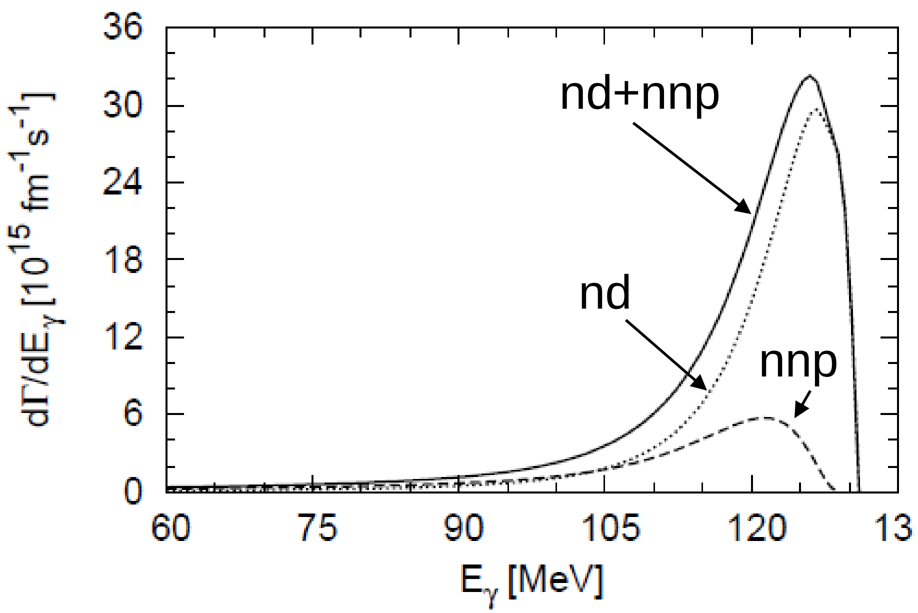
$\Gamma_{nd+nnp} = 9.017 \times 10^{15}$ 1/s (PWIAS)

$\Gamma_{nd+nnp} = 2.672 \times 10^{15}$ 1/s (FSI 2NF)

$\Gamma_{nd+nnp} = 2.455 \times 10^{15}$ 1/s (FSI 2NF+3NF)

crucial importance of FSI !

Radiative pion capture on ^{3}He: other predictions and data for breakup channels



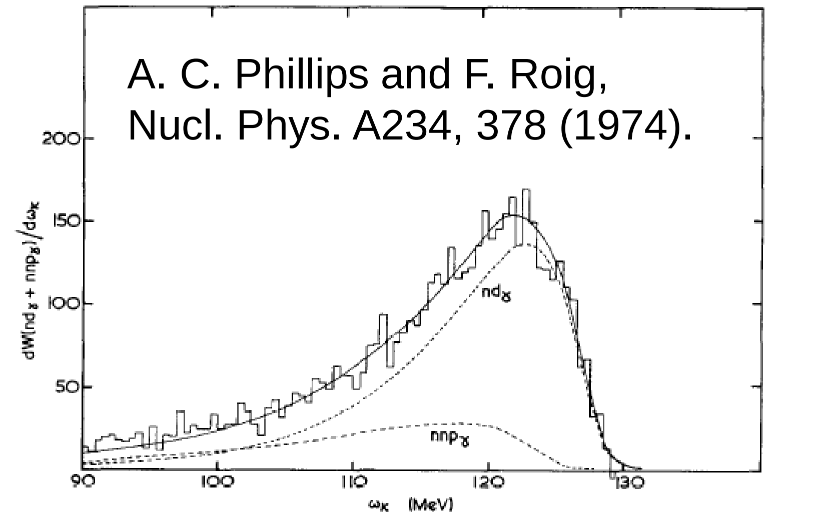


Fig. 1. Comparison of the theoretical and experimental photon spectra for ^{3}He(π^-, γ). The final-state interactions were calculated using the Amado model. The contributions of the ndγ and nnpγ channels are shown.

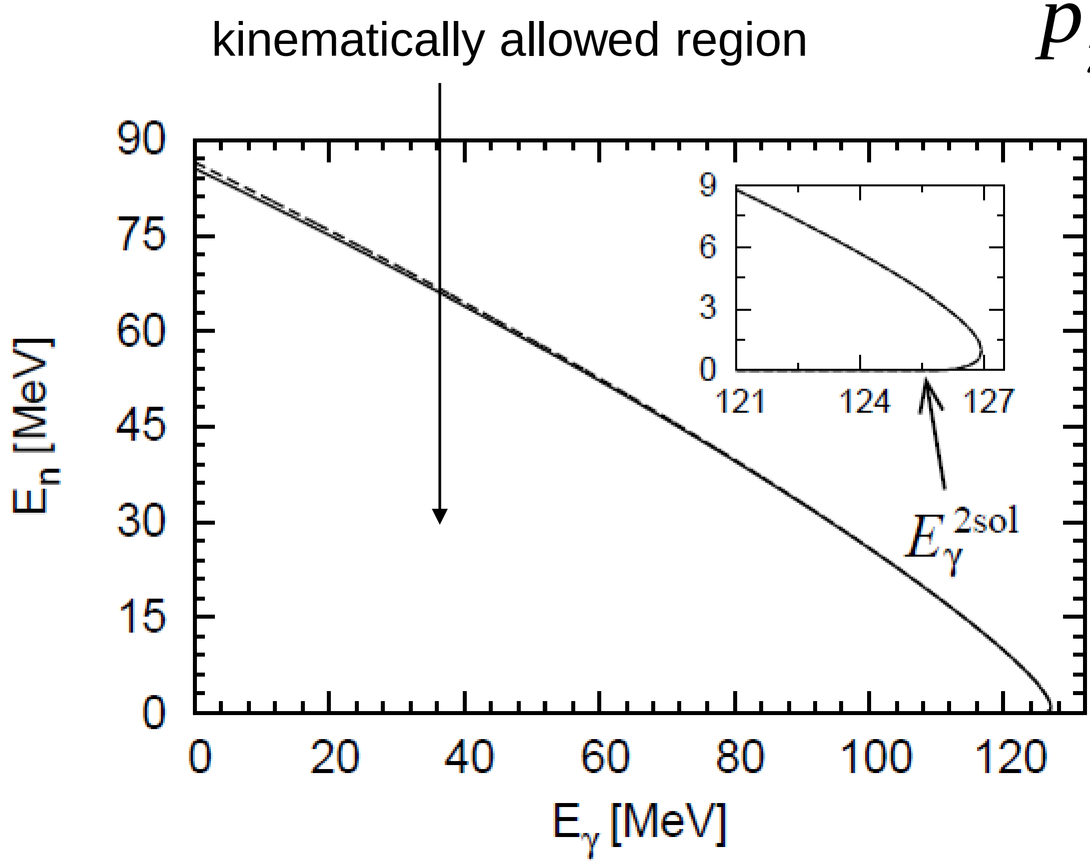
$\Gamma_{nd+nnp}/\Gamma_{3H}$=1.2

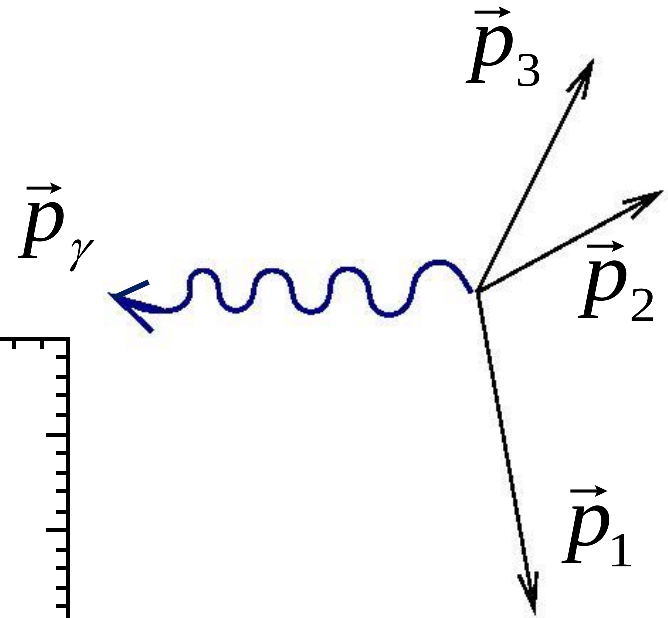
THEORY: $\Gamma_{nd+nnp}/\Gamma_{3H}$ in (0.84 -1.27)

P. Truöl et al., Phys. Rev. Lett. 32, 1268 (1974)
EXPERIMENT: (1.12 ± 0.05)

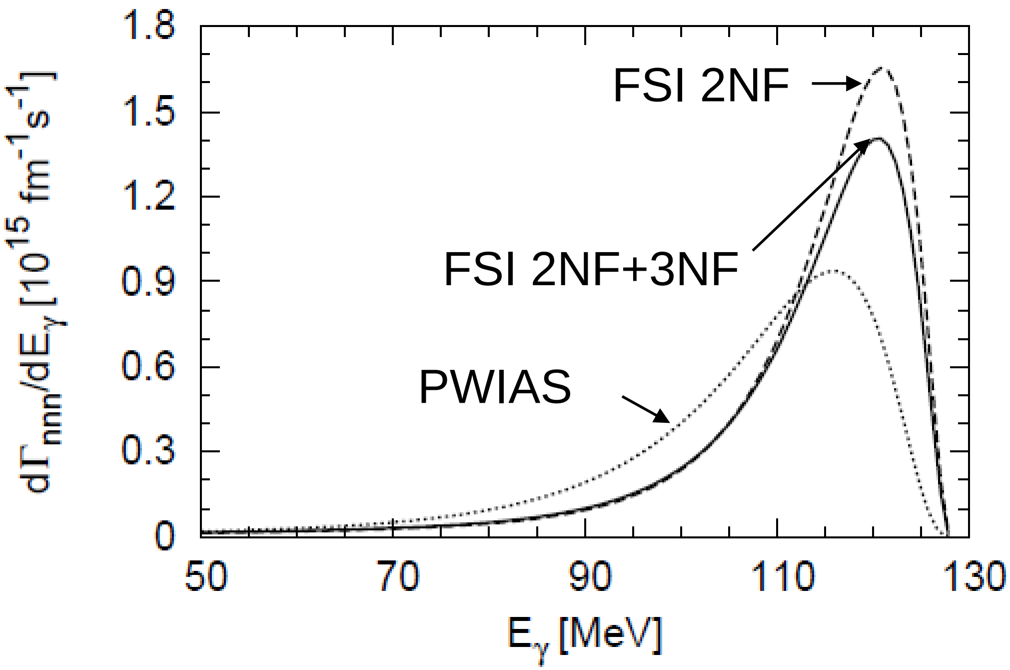
Radiative pion capture on ^{3}H: three-neutron breakup

$$\pi^- + {}^3\mathrm{H} \rightarrow \gamma + n + n + n$$





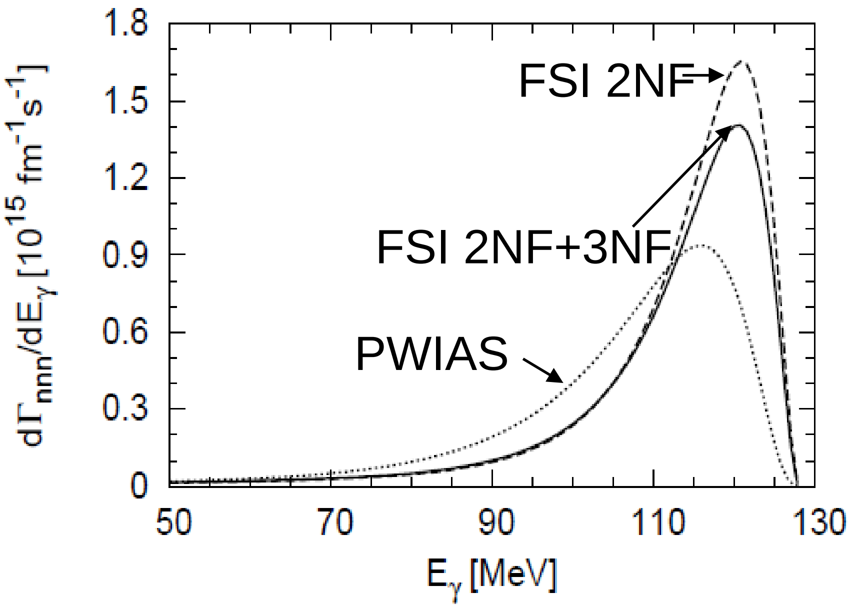
Radiative pion capture on ^{3}H: three-neutron breakup



Γ_{nnn} = 0.117 × 10^{15} 1/s (PWIAS)
Γ_{nnn} = 0.141 × 10^{15} 1/s (FSI 2NF)
Γ_{nnn} = 0.128 × 10^{15} 1/s (FSI 2NF+3NF)

FSI work differently than for ^{3}He !

Radiative pion capture on ^{3}H: other predictions and data



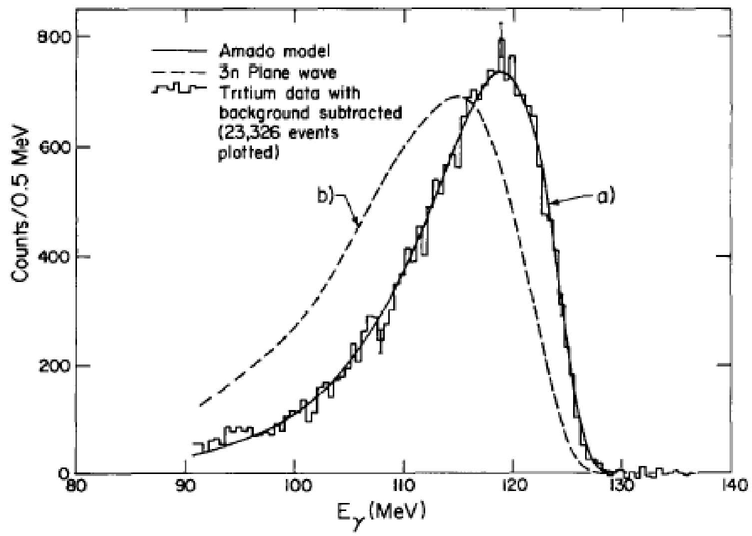


Fig. 4. Measured tritium spectrum with background subtracted. Curve a, Amado model and curve b, plane wave for 3n final state, from refs. [3,21]), folded with the experimental resolution and corrected for spectrometer acceptance.

J. P. Miller et al., Nucl. Phys. A343, 347 (1980)

$\Gamma_{nnn} = 0.128 \times 10^{15}$ 1/s

Calculations from
A. C. Phillips and F. Roig,
AIP Conf. Proc. No. 26, (1975)

$\Gamma_{nnn} = 0.07 \times 10^{15}$ 1/s

Conclusions and outlook

- A very robust momentum space framework to deal with many electroweak processes has been applied to radiative pion capture processes
- First consistent results for ^{2}H, ^{3}He and ^{3}H with realistic 2N and 3N potentials have been obtained in impulse approximation
- Sensitivity to properties of neutron-neutron interaction in the ^{2}H case has been confirmed
- Comparisons with other theories yield a mixed picture
- Room for improvement: consistent 2N and 3N potentials as well as transition operators should be used for all radiative capture reactions
- New data are necessary to establish detailed relations among many processes (3N scattering, weak proton-proton capture, neutrino scattering, muon capture, pion absorption, pion photoproduction)

Investigations of double radiative pion capture are planned

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W. K. H. Panofsky et al., Phys. Rev. 81 (1951) 565
K. M. Watson and R. N. Stuart, Phys. Rev. 82, 738 (1951)
A. M. L. Messiah, Phys. Rev. 87, 639 (1952).
N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954)
K. McVoy, Phys. Rev. 121, 1401 (1961)
P. Truöl et al., Phys. Rev. Lett. 32, 1268 (1974)
A. C. Phillips and F. Roig, Nucl. Phys. A 234, 378 (1974)
J. A. Bistirlich et al., Phys. Rev. Lett. 36, 942 (1976)
H. W. Baer, K. M. Crowe, and P. Truöl, Adv. Nucl. Phys. 9, 177 (1977)
W. R. Gibbs, B. F. Gibson, and Q. J. Stephenson, Jr.,
Phys. Rev. C 11, 90 (1975); 12, 2130(E) (1975);
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Phys. Rev. C 16, 327 (1977); 17, 856(E) (1978);
Phys. Rev. C 18, 1761 (1978)

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G. F. de Teramond, Phys. Rev C 16, 1976 (1977); 36, 691 (1987)
J. P. Miller et al., Nucl. Phys. A343, 341 (1980)
V. Bernard, N. Kaiser, and U.-G. Meiβner, Phys. Lett. B 383, 116, (1996)
H. W. Fearing et al., Phys. Rev. C 62, 054006 (2000)
A. Gårdestig and D. R. Phillips, Phys. Rev. C 73, 014002 (2006)
A. Gårdestig, Phys. Rev. C 74, 017001 (2006)
Q. Chen et al., Phys. Rev. C 77, 054002 (2008)
A. Gårdestig, J. Phys. G: Nucl. Phys. 36, 053001 (2009).

J. Golak et al., Phys. Rept. 415, 89 (2005)
J. Golak et al., Phys. Rev. C 90, 024001 (2014)
J. Golak et al., Phys. Rev. C 94, 034002 (2016)

Thank you !

Auxiliary slides

Formalism *(cont.)*

$$N^{\alpha} = \left\langle \Psi_{f\,m_f} \middle| j^{\alpha} \middle| \Psi_{i\,m_i} \right\rangle$$

from *ab initio* calculations in momentum space

Dynamical ingredients (1): 2N and 3N Hamiltonians

$$H_{2N} = H_0^{2N} + V_{12}$$

$$H_{3N} = H_0^{3N} + V_{23} + V_{13} + V_{12} + V_{123} \equiv H_0^{3N} + V_1 + V_2 + V_3 + V_4$$

$$\equiv H_0^{3N} + V_1 + V_2 + V_3 + \underbrace{V_4^{(1)} + V_4^{(2)} + V_4^{(3)}}_{V_4}$$

used to generate nuclear bound and scattering states
contain 2N and 3N potentials

Formalism *(cont.)*

Dynamical ingredients (2): nuclear single-nucleon, 2N and 3N current operators

$$j_{2N} = j_1 + j_2 + j_{12}$$

$$j_{3N} = j_1 + j_2 + j_3 + j_{12} + j_{23} + j_{13} + j_{123}$$

$$\equiv j_1 + j_{23} + j_2 + j_{13} + j_3 + j_{12} + \underbrace{j_{123}^{(1)} + j_{123}^{(2)} + j_{123}^{(3)}}_{j_{123}}$$

$$\equiv \underbrace{j_1 + j_{23} + j_{123}^{(1)}}_{j(1)} + \underbrace{j_2 + j_{13} + j_{123}^{(2)}}_{j(2)} + \underbrace{j_3 + j_{12} + j_{123}^{(3)}}_{j(3)}$$

describe interactions of an external probe with a nuclear system

Formalism *(reactions with ^{2}H)*

$$H_{2N}\left|\psi_d\right\rangle = E_d\left|\psi_d\right\rangle$$ deuteron state with $E_d < 0$

$$N^\alpha \equiv \left\langle\psi'_d\right| j^\alpha \left|\psi_d\right\rangle$$ elastic channel

$$N^\alpha \equiv \left\langle\psi^{(-)}\right| j^\alpha_{2N} \left|\psi_d\right\rangle = {}_a\left\langle\vec{p}_o\right|\left(1+t_{12}G_0^{2N}\right) j^\alpha_{2N}\left|\psi_d\right\rangle$$ break-up channel

$$H_{2N}\left|\psi^{(-)}\right\rangle = E\left|\psi^{(-)}\right\rangle, \qquad E = \frac{p_0^2}{m} > 0$$

$$t_{12} = V_{12} + t_{12}\,G_0^{2N}\left(E + i\varepsilon\right)V_{12}$$ Lippmann-Schwinger equation

$$G_0^{2N}(E) \equiv \lim_{\varepsilon\to 0^+} \frac{1}{E + i\varepsilon - H_0^{2N}}$$ free 2N propagator

Formalism *(reactions with ^{3}He and ^{3}H)*

$$H_{3N}\left|\Psi\right\rangle = E_b\left|\Psi\right\rangle$$

3N bound state with $E_b < 0$
generated by the Faddeev equation

$$N^{\lambda} = \left\langle \Psi' \middle| j_{3N}^{\lambda} \middle| \Psi \right\rangle$$

elastic or quasielastic channel with
initial and final bound states

$$N^{\lambda} = \left\langle \Psi_f^{(-)} \middle| j_{3N}^{\lambda} \middle| \Psi_i \right\rangle$$

two-body or three-body
break-up channel with final
scattering states

$$\left|\Psi_f^{(-)}\right\rangle = \lim_{\varepsilon\to 0^+} \frac{-i\varepsilon}{E-i\varepsilon-H_{3N}}\left|\phi_f\right\rangle \longleftarrow$$

formal definition including
the channel state

Formalism *(reactions with ^{3}He and ^{3}H)*

Operators in 3N space:

(1) 3N force decomposed as $$V_4 = V_4^{(1)} + V_4^{(2)} + V_4^{(3)}$$

$V_4^{(i)}$ is symmetric under the exchange of nucleons j and k, i≠j≠k≠i

(2) free 3N propagator $$G_0^{3N}(E) \equiv \lim_{\varepsilon \to 0^+} \frac{1}{E + i\varepsilon - H_0^{3N}}$$

(3) 2N off-shell t-matrix generated via LSE: $$t_1 = V_1 + V_1 G_0^{3N} t_1$$

(4) permutation operator: $$P = P_{12} P_{23} + P_{13} P_{23}$$

Formalism *(reactions with ^{3}He and ^{3}H)*

Auxiliary equation for $\left|U^{\lambda}\right\rangle \equiv \left|U(j^{\lambda}, E_{c.m.}, Q)\right\rangle$

3N internal energy

magnitude of the three momentum transfer

$$\left|U^{\lambda}\right\rangle = \left\{ t_1 G_0^{3N} + \frac{1}{2}(1+P)V_4^{(1)} G_0^{3N}\left(1+t_1 G_0^{3N}\right)\right\}(1+P)\, j^{\lambda}(1)\left|\Psi_i\right\rangle$$

$$+\left\{ t_1 G_0^{3N} P + \frac{1}{2}(1+P)V_4^{(1)} G_0^{3N}\left(1+t_1 G_0^{3N}\right)P\right\}\left|U^{\lambda}\right\rangle$$

Formalism *(reactions with ^{3}He and ^{3}H)*

Quadratures

$$N_{Nd}^{\lambda} = \langle \phi_{Nd} | (1+P)\, j^{\lambda}(1) | \Psi_i \rangle + \langle \phi_{Nd} | P | U^{\lambda} \rangle$$

$$N_{3N}^{\lambda} = \langle \phi_{3N} | (1+P)\, j^{\lambda}(1) | \Psi_i \rangle + \langle \phi_{3N} | t_1 G_0^{3N} (1+P)\, j^{\lambda}(1) | \Psi_i \rangle$$
$$+ \langle \phi_{3N} | P | U^{\lambda} \rangle + \langle \phi_{3N} | t_1 G_0^{3N} P | U^{\lambda} \rangle$$

to obtain nuclear matrix elements for arbitrary exclusive kinematics !
Semi-exclusive and inclusive observables are calculated by suitable integrations over the phase space domains.