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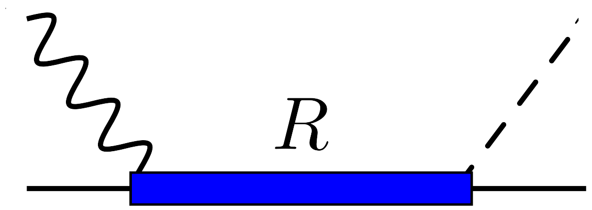
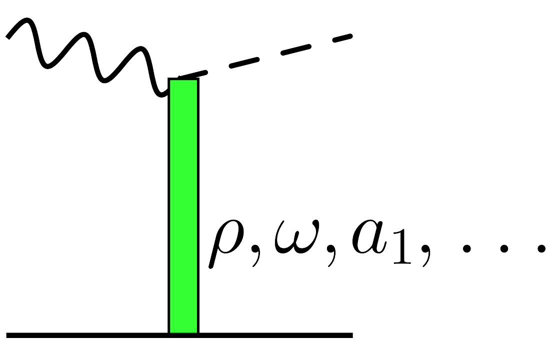




Unitarity, Analyticity and Duality constraints in π,η photoproduction

MESON2018, Jagiellonian University Krakow, June 7-12, 2018

OUTLINE

Motivation: QCD + baryon spectroscopy

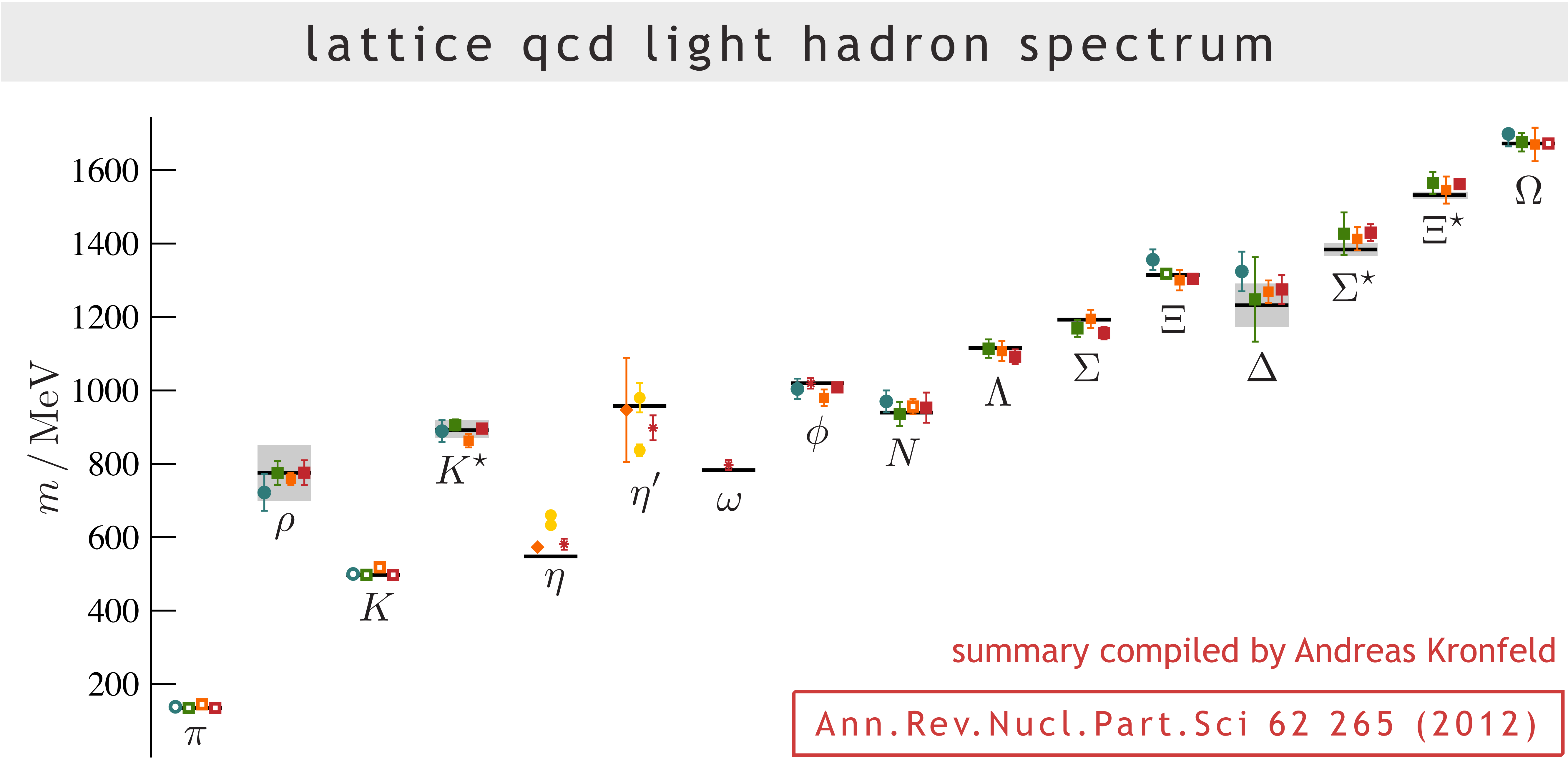
Duality in meson production

Dual parametrization

What do we know about the hadron spectrum?

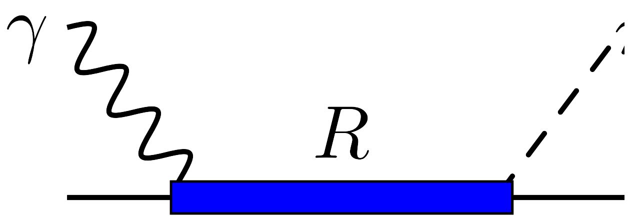
Cannot solve QCD analytically;

Test numeric solution - lattice QCD;



How do we study the hadron spectrum?

Since the 50's accelerators produced beams of particles of higher and higher energy -> inelastic scattering -> look for resonances



$$\sigma \sim \frac{M_R \Gamma_R}{(W^2 - M_R^2) + M_R^2 \Gamma_R^2}$$

Breit-Wigner: position + width

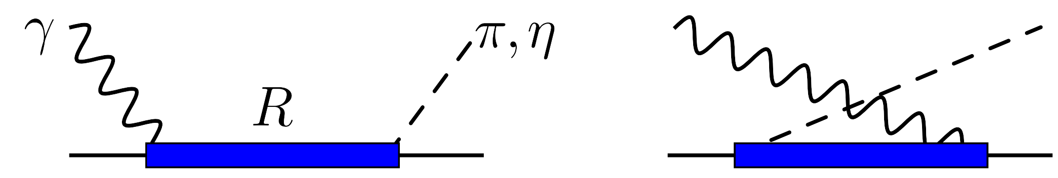
Works great for strong and narrow lone resonances (ρ, Δ)

Each resonance has definite quantum numbers ->
only contributes to one partial wave (P33, S11 etc.) ->
identify q.n. from angular distributions;
Scan all available energies; study all channels; done?

How is a simplistic analysis w. Breit-Wigner related to QCD?

QCD and QED are gauge field theories ->
amplitudes for processes with e.-m. and strong interaction
possess symmetries, analyticity, unitarity

CP conservation -> crossing symmetry
resonance in the crossed channel, too!

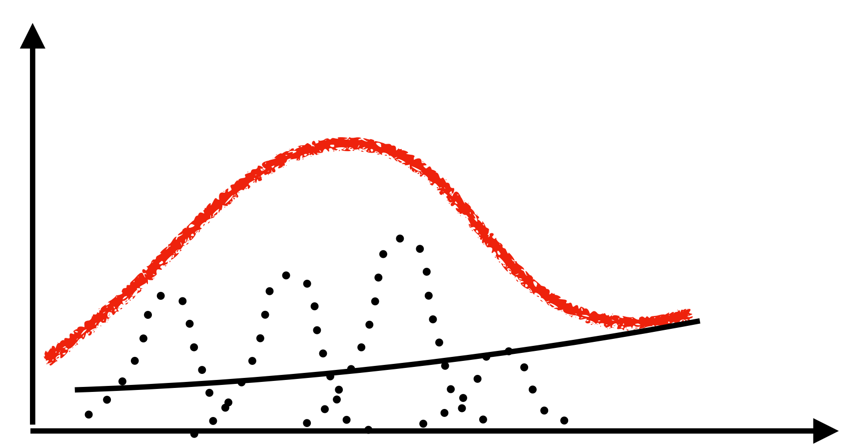


Crossing destroys the simple picture 1 resonance -> 1 partial wave

$$A_{\rm direct} \sim \frac{1}{W^2 - M_R^2} \rightarrow A_{\rm crossed} \sim \frac{1}{u(W,\theta) - M_R^2}$$

Angle-dependent crossed term -> all partial waves
(only one will be resonant; will give background in others)

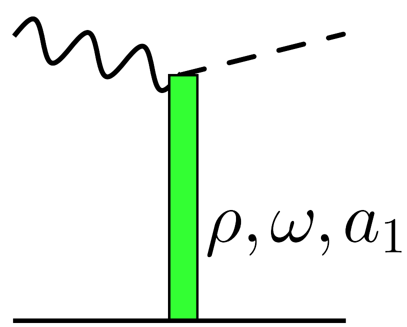
Overlapping resonances,
significant non-resonant background,
production thresholds
Breit-Wigner analysis is not viable

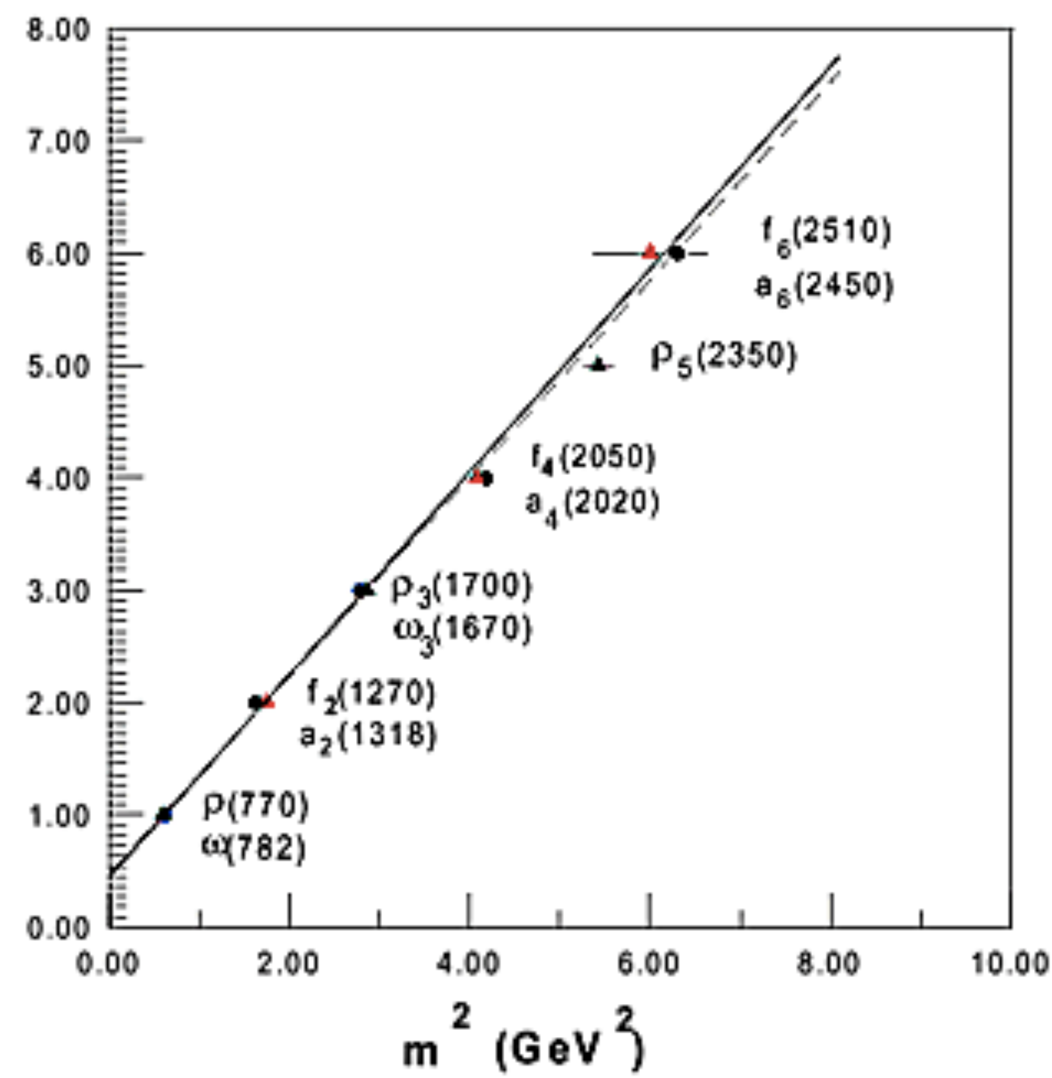


PWA (SAID, MAID, Bonn-Gatchina, KSU, ...) - talks by Andrei, Lothar

CQM and LQCD predict essentially infinite number of states
Will we be able to observe all these states?

Empirical observation: above W=2 GeV s-channel resonances stop being the most prominent feature of the cross section
High energies - dominated by t-channel exchanges





Regge - exchange a tower of states
Spectrum: $J = J_0 + \alpha' (M_J^2 - M_0^2)$
One coupling per trajectory

$$\sum_{\mathrm{Res}_t}^{\infty} A^t(s,t,u) \sim s^{\alpha(t)}$$

How is it related to baryon spectrum?

Duality: a full theory knows all its states and their properties

Algebraic models (van Hove, Veneziano) - duality is trivial:
spectra and couplings are exactly known

$$A(s,t,u) = \sum_{\mathrm{Res}_s}^{\infty} A^s(s,t,u) = \sum_{\mathrm{Res}_t}^{\infty} A^t(s,t,u)$$

Regge - an example of an algebraic model (but incomplete)
We know the full amplitude at high energy

Cannot directly test duality as written above.
But the strength of low-lying resonances and Regge related!
This correspondence is addressed by finite energy sum rules

FESR for π,η photoproduction

$$\gamma(k) + N(p) \to PS(q) + N'(p')$$

Mandelstam scalars

$$s = (p+k)^2, \;\; u = (p-q)^2, \;\; t = (k-q)^2$$

Crossing-odd variable

$$\nu = \frac{s-u}{4M}$$

CGLN decomposition:
invariant amplitudes

$$T_{fi} = \sum_i \bar{u}(p') M_i u(p) A_i(\nu, t)$$

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu}\,,$$

$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu}\,, \qquad P = (p+p')/2$$

$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu}\,,$$

$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}$$

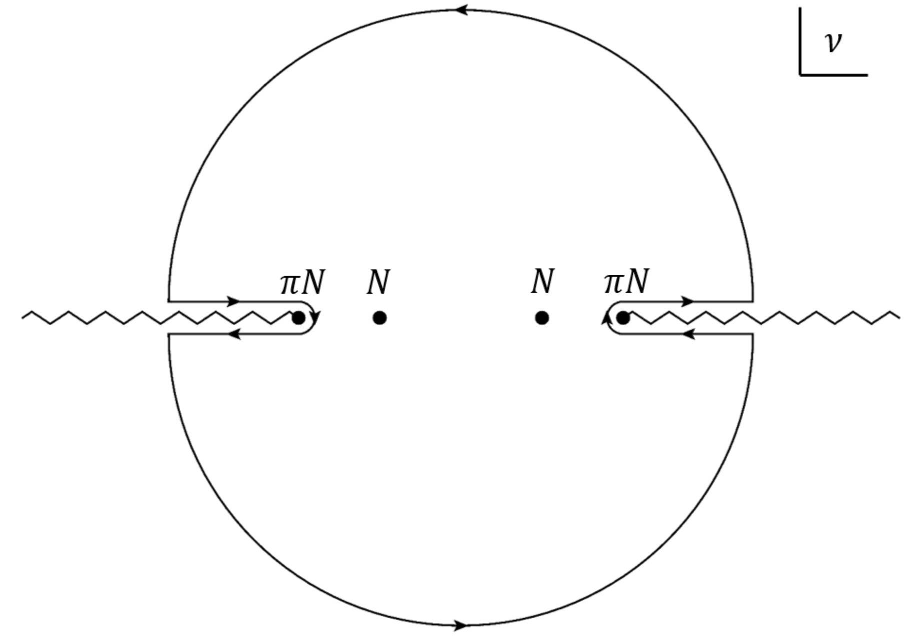
Crossing + isospin

$$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$$

σ = (+0,-) for pion, (s,v) for eta

$$\xi_1 = \xi_2 = -\xi_3 = \xi_4 = 1$$

Fixed-t dispersion relation



$$\mathrm{Re}\,A_i^I(\nu,t) = B_i^I\left[\frac{1}{\nu-\nu_N} + \xi_i^I\frac{1}{-\nu-\nu_N}\right] + \frac{\pi B_i^{(-)}}{t-m_\pi^2} + \frac{1}{\pi}\mathcal{P}\int_{\nu_\pi}^{\infty} d\nu'\left[\frac{1}{\nu'-\nu} + \xi_i^I\frac{1}{\nu'+\nu}\right]\mathrm{Im}\,A_i^I(\nu',t)$$

Above scale N: Regge dominates

Vector and axial vector: $$R_i^I(\nu,t) = \beta_i^I(t)\frac{\pi\alpha'}{2}\frac{e^{-i\pi\alpha_i^I(t)}-1}{\sin[\pi\alpha_i^I(t)]\Gamma[\alpha_i^I(t)]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Resembles V, A meson exchanges: $$R_i^I(\nu,t\to m_{V,A}^2) \to R_i^I(\nu,t)\big|_{\alpha_i^I=1} = \frac{\beta_i^I(m_{V,A}^2)}{t-m_{V,A}^2}$$

Regge amplitude obeys DR

$$\mathrm{Re}\,R_i^I(\nu,t) = \frac{1}{\pi}\mathcal{P}\int_0^{\infty} d\nu'\left[\frac{1}{\nu'-\nu} + \xi_i^I\frac{1}{\nu'+\nu}\right]\mathrm{Im}\,R_i^I(\nu',t)$$

Match full ampl. on Regge at ν > N

$$\mathrm{Re},\ \mathrm{Im}\, A_i^I(\nu,t) = \mathrm{Re},\ \mathrm{Im}\, R_i^I(\nu,t) \ \text{ for } \ \nu > N$$

Sidenote: A and R are analytical in slightly different regions of the nu-plane; mathematically still viable. But the two are only "identical" within finite experimental errors, not in exact sense.

$$\frac{\tilde{B}_i^I(t)}{N}\left(\frac{\nu_N}{N}\right)^k + \int_{\nu_\pi}^{N} \frac{d\nu'}{N}\left(\frac{\nu'}{N}\right)^k \mathrm{Im}\, A_i^I(\nu',t) = \frac{\beta_i^I(t)\pi\alpha'}{2(\alpha(t)+k)\Gamma[\alpha(t)]}\left(\frac{N}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Regge parameters: HE fit

Nys et al [JPAC] arXiv:1611.0465
Kashevarov, Ostrick, Tiator arXiv:1706.07376

LHS of FESR - fit to resonance data

FESR - a powerful tool for constraining resonance parameters by imposing duality, fixed-t analyticity, crossing etc.

Numerical implementation can be tricky: strong cancellation

Historically, models used in the resonance region and at high energy are completely different (VM exchanges <—> Regge exchanges)

Matching is plagued by a potentially significant systematic uncertainty; May still depend on the matching point N

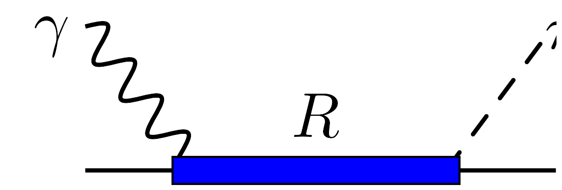
Use Regge amplitude modified in the resonance region joining smoothly onto Regge

η photoproduction - start with isobar model

$$t_{\gamma,\eta}(W) = t^{bg}_{\gamma,\eta}(W) + t^{Res}_{\gamma,\eta}(W)$$

Resonance
direct channel

$$t^{\alpha,Res}_{\gamma,\eta}(W) = \sum_{j=1}^{N_\alpha} t^{\alpha,BW,j}_{\gamma,\eta}(W) e^{i\Phi^\alpha_j}$$

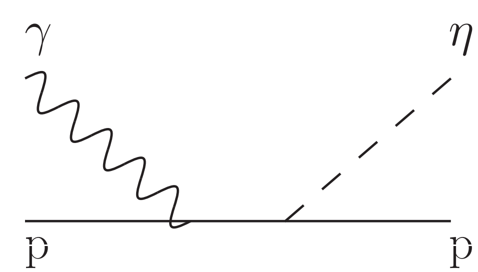


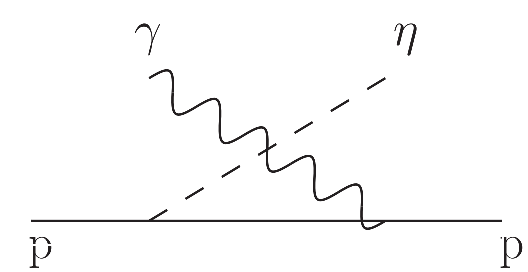
BW parametrization
direct channel

$$\mathcal{M}_{\ell\pm}(W) = \bar{\mathcal{M}}_{\ell\pm} \frac{f_{\gamma N}(W)\, M_R \tilde{f}_{\eta N}(W)}{M_R^2 - W^2 - i M_R \Gamma_{\rm tot}(W)} C_{\eta N}$$

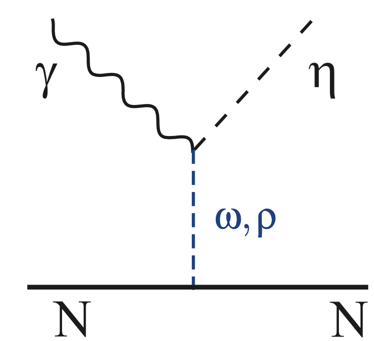
Background (no multipole decomposition needed)

s, u-channel Born





t-channel exchanges



Pure spin-1 exchange leads to unphysical rise at high energy;
Usually suppressed by phenomenological from factors
Matching onto Regge is delicate - opt for a Regge-based background

Exploit duality for extracting few low-lying resonances

$$A(s,t,u)=\sum_{\mathrm{Res}_s}^{\infty}A^s(s,t,u)=\sum_{\mathrm{Res}_t}^{\infty}A^t(s,t,u)$$

Remove part of the strength of Regge in the resonance region to leave space for resonances

$$A(s,t,u)=\sum_{\mathrm{Res}_s=1}^{N}A^{\mathrm{Res}}(s,t,u)+\sum_{\mathrm{Res}_t}^{\infty}A^t(s,t,u)-\sum_{\mathrm{Res}_s=1}^{N}A^{\mathrm{Res}}(s,t,u)$$
$$\approx\sum_{\mathrm{Res}_s=1}^{N}A^{\mathrm{Res}}(s,t,u)+DF(W)\times A^{\mathrm{Regge}}(s,t,u)$$

Damping factor removes double counting:
DF(W) -> 0 at threshold;
DF(W) -> 1 at high energy

The form of DF is unknown; we chose $DF(W)=1-e^{-\frac{W^2-W_{thr}^2}{\Lambda^2}}$

Λ - fit parameter (of order 1 GeV)

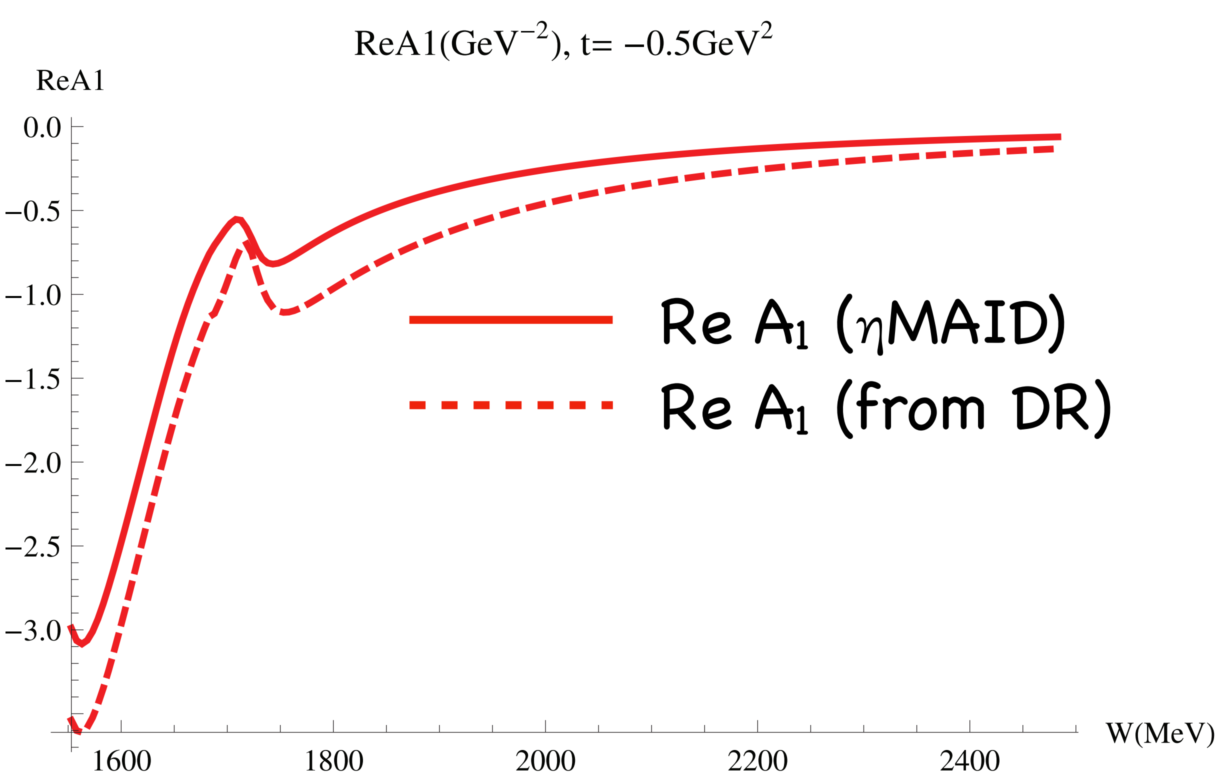
Eta photoproduction with dispersion relations:

1. Isobar model fit: Born + Regge x DF + Resonances
2. Obtain Re, Im parts of the amplitudes
3. Use Im part in a dispersion relation
4. Obtain Re part
5. Reiterate

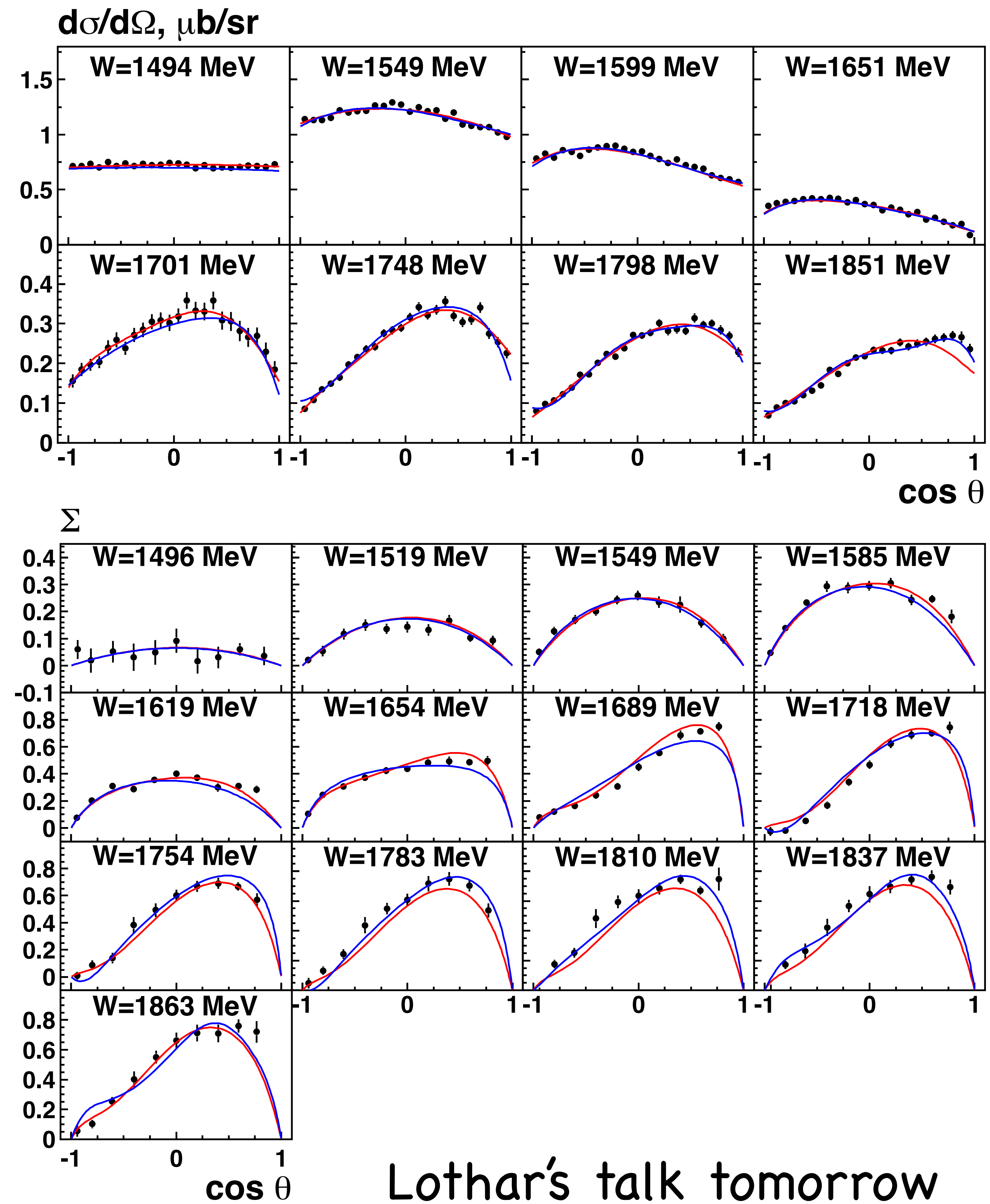
Fit: DR vs. Isobar model

$\chi^2_{IB}/N_{dof} = 1.61 \quad \chi^2_{DR}/N_{dof} = 1.61$

$\gamma p \to \eta p$	Observable	χ^2_{IB}	χ^2_{DR}	Number of points
MAMI	$d\sigma/d\Omega$	3448	3388	2544
A2 MAMI	T	456	423	144
A2 MAMI	F	318	426	144
GRAAL	Σ	323	353	130
CLAS	E	38	31	42
DESY,Wilson,Daresbury,CEA	$d\sigma/d\Omega$	11	13	52
Daresbury	Σ	7	13	12
Daresbury	T	1	2	3



Next step: FESR - work in progress



Lothar's talk tomorrow

Application to pion production - work in progress

Similar in spirit - but there's a new constraint: Watson's theorem

Strong rescattering in each partial wave:

phase of γN->πN amplitude equals that of πN->πN amplitude

Now multipole decomposition is needed for the full amplitude, not only resonance part

Follow MAID approach: $t_{\gamma\pi}(W,\theta) = t_{\gamma\pi}^{Bg}(W,\theta) + t_{\gamma\pi}^{Res}(W,\theta)$

Resonance contributions - $BW_{\ell,\alpha}^{Res}(W) \to BW_{\ell,\alpha}^{Res}(W) e^{i\phi_\alpha^{Res}(W)}$

Background = Born + Regge x DF

Born $t_{\gamma\pi}^{B}(W,\theta) = \sum_\ell \mathcal{M}_{\ell,\alpha}^{B}(W)\,\{P_\ell(\cos\theta)\}$

Unitarization (K-m.) $\mathcal{M}_{\ell,\alpha}^{B}(W) \to \mathcal{M}_{\ell,\alpha}^{B}(W)[1 + i t_{\pi N}^{\alpha}]$ $\quad t_{\pi N}^{\alpha} = \dfrac{\eta_\alpha e^{2i\delta_\alpha^{\pi N}} - 1}{2i}$

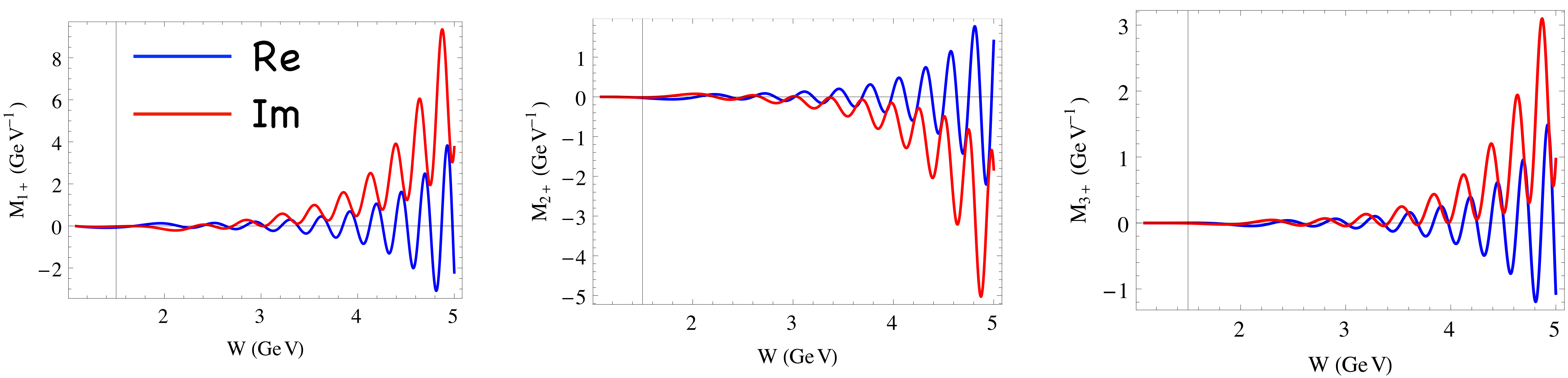
Pi-N phases and inelasticities - e.g. from GWU analysis, W < 2 GeV

Multipole decomposition of Regge amplitude

$$R_i^I(\nu,t) = \beta_i^I(t)\frac{\pi\alpha'}{2}\frac{e^{-i\pi\alpha_i^I(t)}-1}{\sin[\pi\alpha_i^I(t)]\Gamma[\alpha_i^I(t)]}\left(\frac{\nu}{\nu_0}\right)^{\alpha_i^I(t)-1}$$

Several possibilities:
-> unitarize few lowest partial waves, match them back to Regge at HE;
-> match the one's favorite low-energy multipoles onto Regge multipoles above resonance region

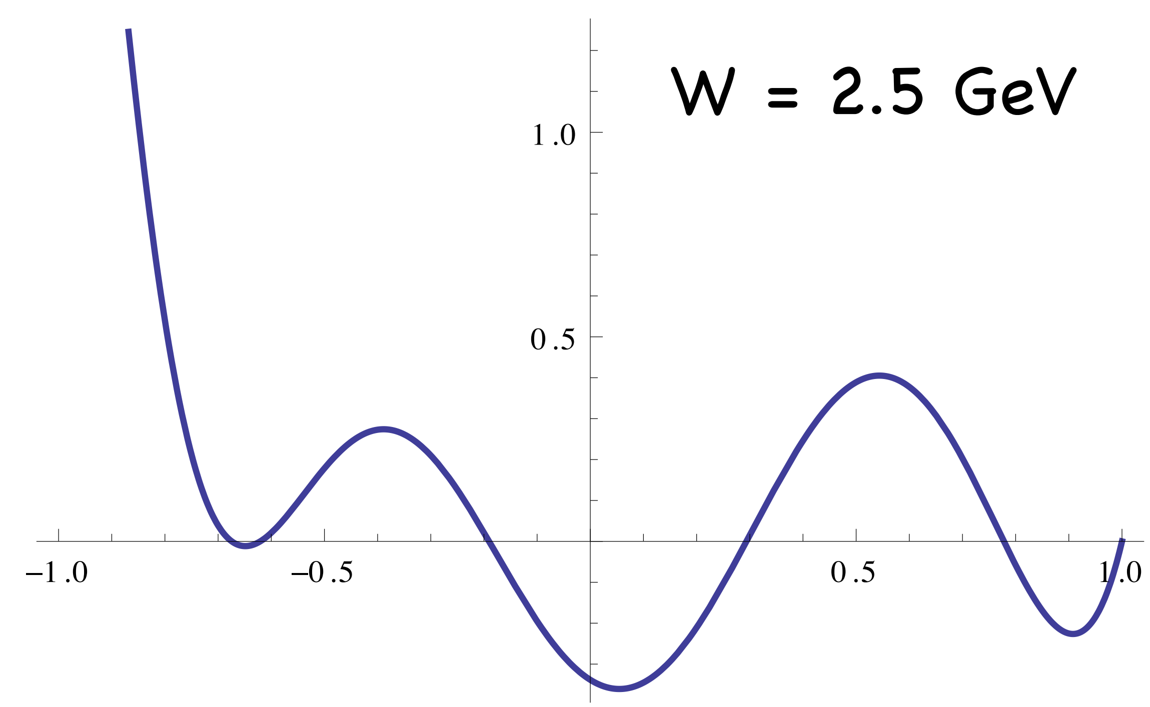
Example: ρ-exchange in $\pi^+ n$ channel
Vector coupling to the nucleon; M_{l+} multipoles



Oscillations observed: make matching impossible!

What's the reason for these oscillations?
Consider the integrand of R -> M_{1+} conversion
Strong backward peak plus oscillations between
But one expects t-channel Regge exchanges
to dominate forward angles

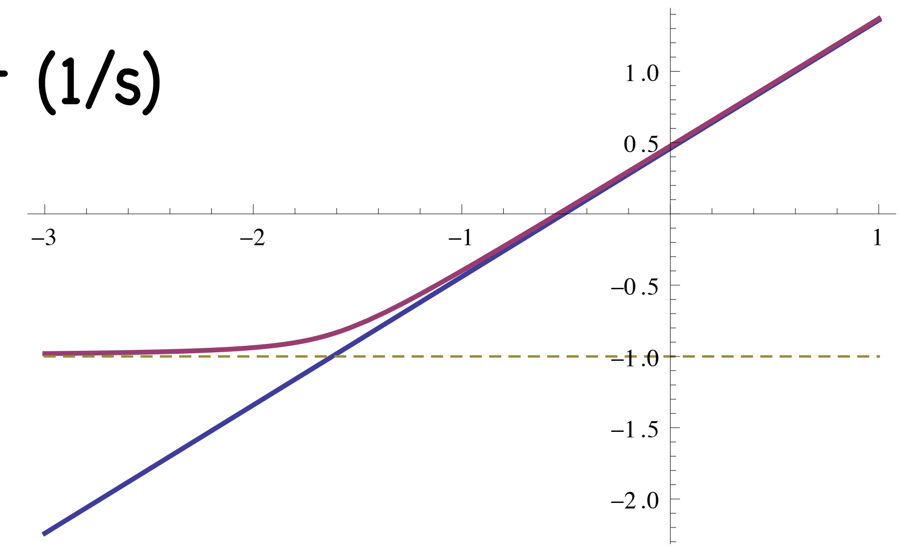
Combination of two factors: ν decreases for -t>>
Oscillations - $1/\Gamma[\alpha(t)]$ for large negative t
Gamma-fn. removes unphysical poles at t = -1, -3, ...



Saturated Regge trajectory

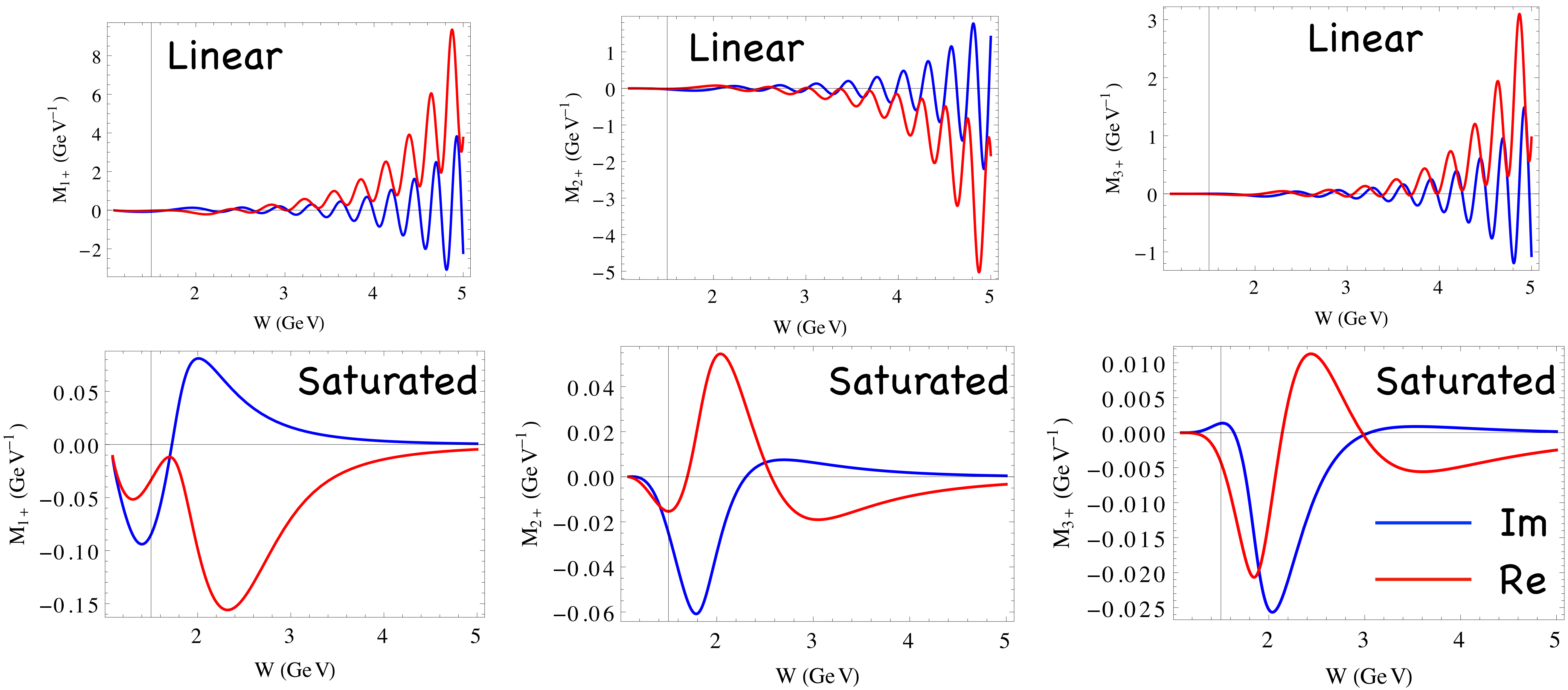
$\alpha(t)$ - linear at positive t (Frautschi plot, meson poles)

- at large $|t|\sim s$: pQCD quark exchange - expect 1/t (1/s)

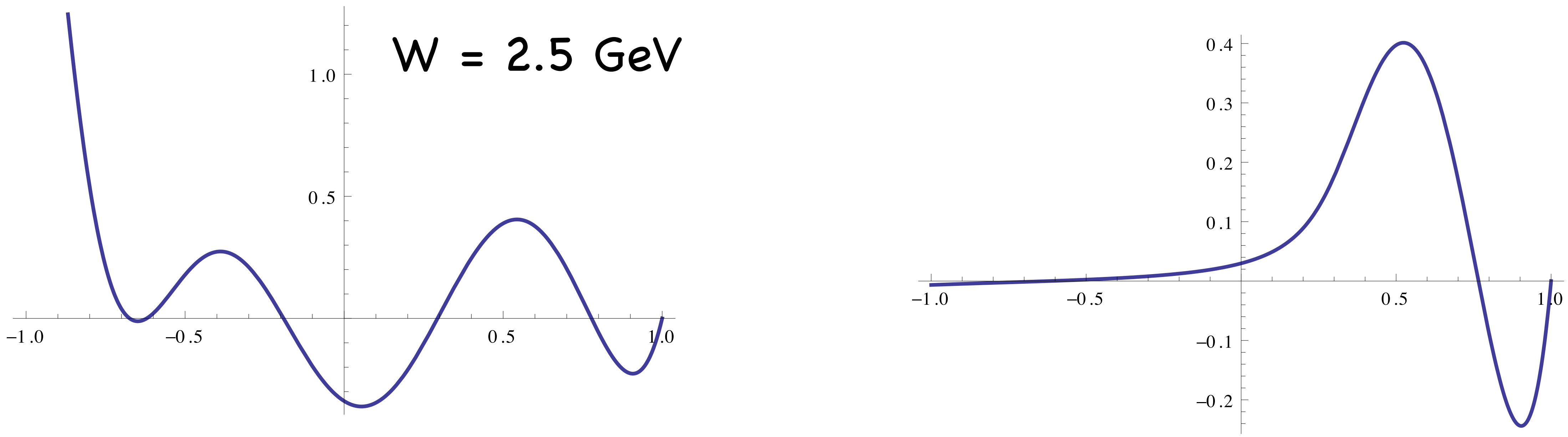


$$\tilde{\alpha}(t) = \frac{\alpha(t)-1}{2} + \frac{1}{2}\sqrt{(\alpha(t)+1)^2 + 4\lambda^2}$$

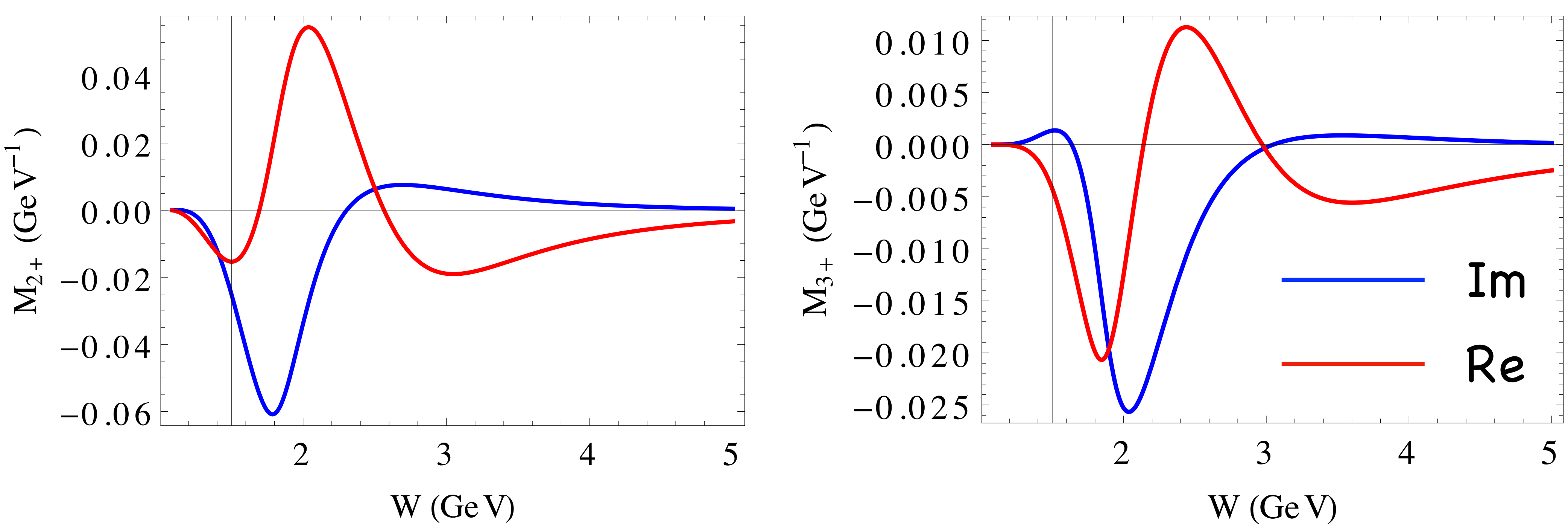
Transition: λ



Saturated Regge trajectory
Eliminates backward structure and unphysical oscillations!



Regge amplitude generates resonance-like structures - that's why need to take double counting seriously



Unitarize the lowest PW of Regge background for W < 2 GeV

For W > 2.5 GeV - pure Regge $\mathcal{M}^R_\alpha = |\mathcal{M}^R_\alpha| e^{i\phi^R_\alpha}$

For W < 2 GeV - pi-N phase $t^{R,\alpha}_{\gamma\pi} = \mathcal{M}^R_\alpha e^{-i\phi^R_\alpha}\left[1 + i t^\alpha_{\pi N}\right]$

Smooth interpolation in between

Or use more conventional MAID parametrization in the resonance region and match low multipoles onto Regge multipoles

WORK IN PROGRESS

t-channel Regge exchanges: correct physics input at forward angles;
Saturated Regge removes artifacts from the backward angles;

To provide a more complete description
– desirable to include baryon Regge exchanges, as well
not trivial: formal problems - parity doubling; fixed-t DR don't work at large negative t;

Ideally, a combination of fixed-t DR at backward angles
and interior DR at backward angles

Conclusions and Outlook

FESR is a powerful tool to constrain resonance contributions

To gain most of FESR - ensure matching between LE and HE

May or may not use modified Regge as the background in the resonance region in place of vector meson exchanges

To cure many problems that appear on the intersection of SE PWA and fixed-t DR - use saturated Regge

Projection of saturated Regge onto multipoles is meaningful -> allows for matching lowest multipoles (with phase) to Regge

Future steps will involve u-channel Regge exchanges in a combination with interior DR