Recent progress in the partial-wave analysis of the $\pi^-\pi^+\pi^-$ final state at COMPASS

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on behalf of the COMPASS collaboration

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and Fundamental Symmetries
—
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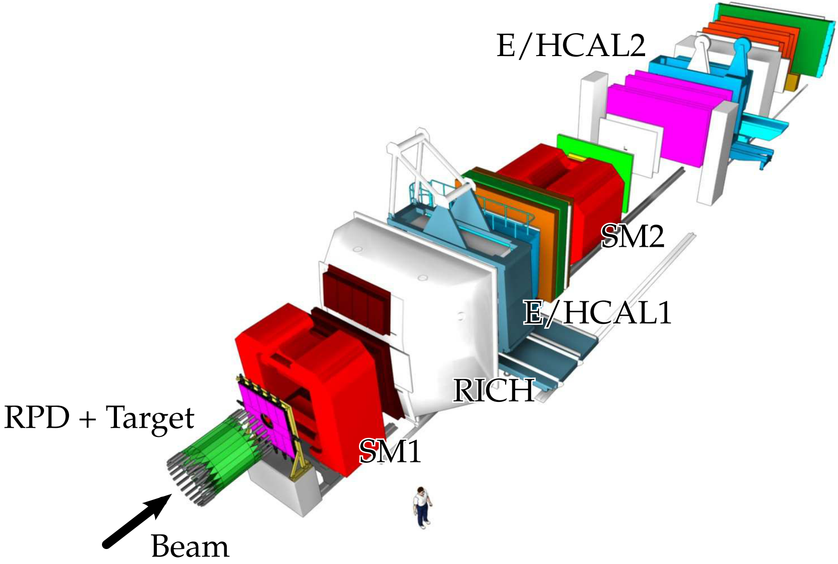
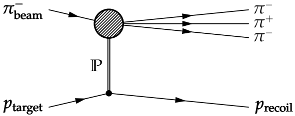
Jun 11th 2018 – Kraków



E/HCAL2
SM2
E/HCAL1
RICH
RPD + Target
SM1
Beam

Diffractive 3π production

- COMPASS: Very large data set for the diffractive process
  $\pi^-_{\mathrm{beam}} + p \rightarrow \pi^-\pi^+\pi^- + p$



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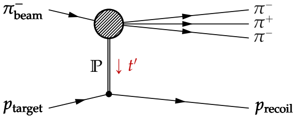

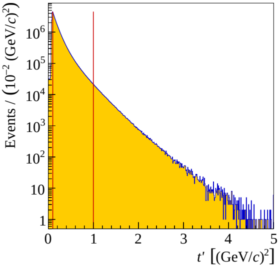

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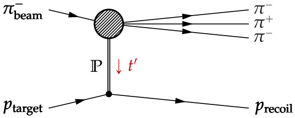

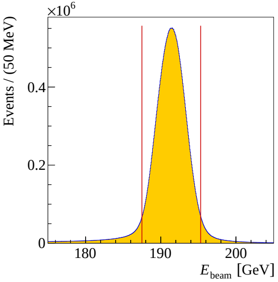

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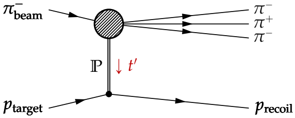

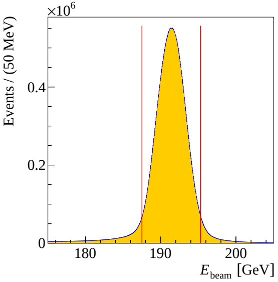

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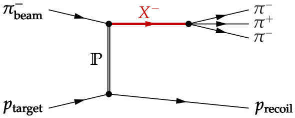

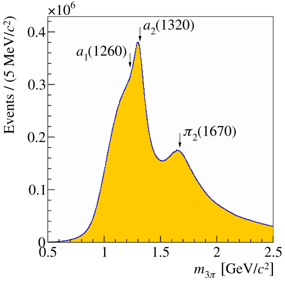

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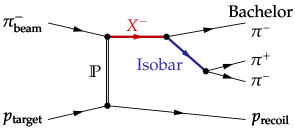
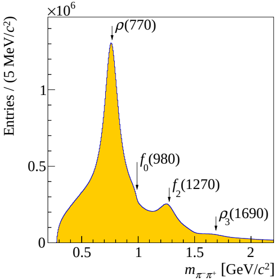
- Also structure in $\pi^+\pi^-$ subsystem: Intermediary states ξ (isobar)





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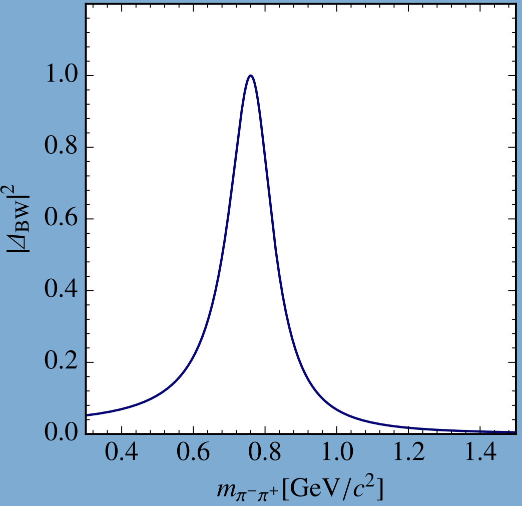
Dynamic isobar amplitude: $\rho(770)$, $J^{PC} = 1^{--}$



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- Physical dynamic isobar amplitudes may differ from the model
- Free parameters in dynamic isobar amplitudes computationally unfeasible

Freed-isobar method

Step-like isobar amplitudes

- Total intensity in one $(m_{3\pi}, t')$ bin as function of phase-space variables $\vec{\tau}$:

$$\mathcal{I}(\vec{\tau}) = \left|\sum_i^{\text{waves}} \mathcal{T}_i \left[\psi_i\left(\vec{\tau}\right)\Delta_i\left(m_{\pi^-\pi^+}\right) + \text{Bose sym.}\right]\right|^2$$

Fit parameters: Transition amplitudes $\mathcal{T}_i$

Fixed: Angular amplitudes $\psi_i\left(\vec{\tau}\right)$, dynamic isobar amplitudes $\Delta_i\left(m_{\pi^-\pi^+}\right)$

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$$\Delta_i(m_{\pi^-\pi^+}) \rightarrow \sum_{\text{bins}} \mathscr{T}_i^{\text{bin}} \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) \equiv [\pi\pi]_{J^{PC}}$$

$$\Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) = \begin{cases} 1, & \text{if } m_{\pi^-\pi^+} \text{ in the bin.} \\ 0, & \text{otherwise.} \end{cases}$$

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- Each $m_{\pi^-\pi^+}$ bin behaves like an independent partial wave with $\mathcal{T}_i^{\text{bin}} = \mathcal{T}_i \mathscr{T}_i^{\text{bin}}$:

$$\mathcal{I}(\vec{\tau}) = \left| \sum_{i}^{\text{waves}} \sum_{\text{bin}}^{\text{bins}} \mathcal{T}_i^{\text{bin}} \left[\psi_i(\vec{\tau}) \Delta_i^{\text{bin}}(m_{\pi^-\pi^+}) + \text{Bose sym.} \right] \right|^2$$

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Fit paramet…

Fixed: Angu… $\Delta_i\left(m_{\pi^-\pi^+}\right)$

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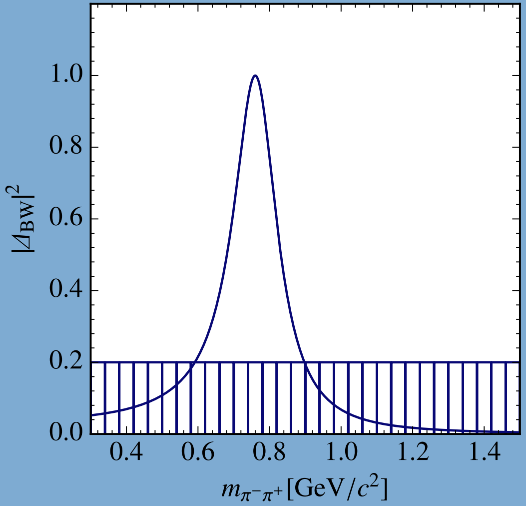
$\mathcal{I}(\vec{\tau}$ … m.$]\Big|^2$

Illustration



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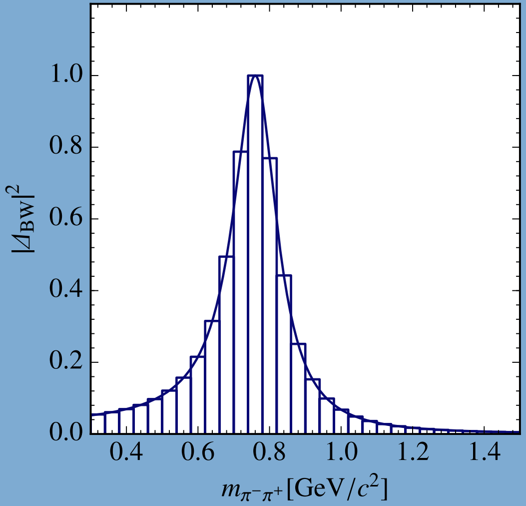

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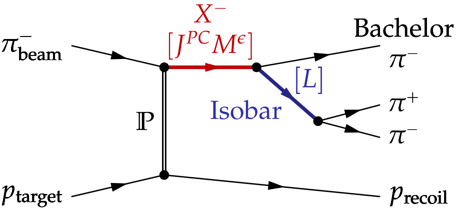
Waves defined by:

$$J^{PC} M^{\varepsilon} \xi \pi L$$



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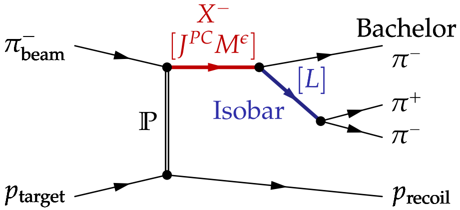

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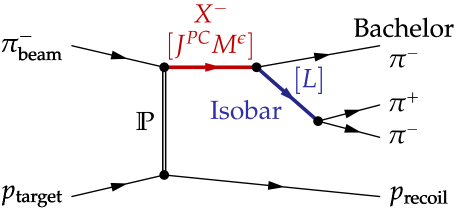

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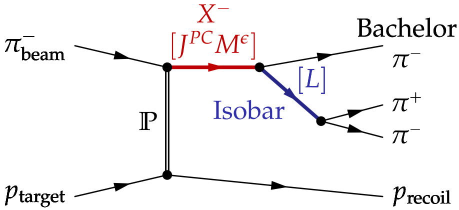

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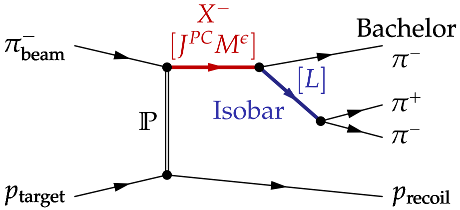

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Partial-Wave Analysis

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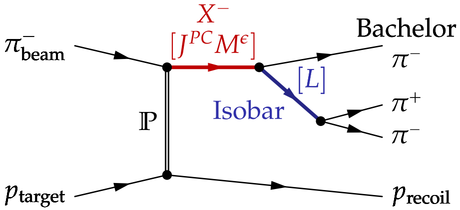

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Spin-exotic wave

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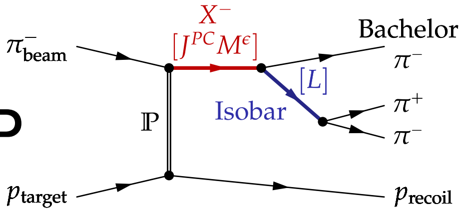

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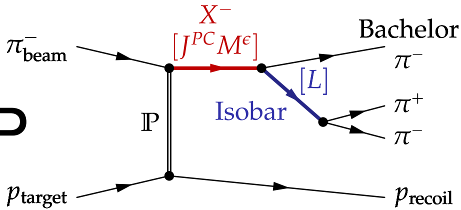

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- 50 bins in $m_{3\pi}$ from 0.5 to 2.5 GeV, four bins in t' from 0.1 to 1.0 $(\mathrm{GeV}/c)^2$
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- 12 waves freed (72 remaining waves still with fixed isobars):

$0^{-+}0^{+}[\pi\pi]_{0^{++}}\pi S$	$2^{-+}0^{+}[\pi\pi]_{0^{++}}\pi D$	$1^{++}1^{+}[\pi\pi]_{1^{--}}\pi S$
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- 40 MeV $m_{\pi^-\pi^+}$ bins for freed waves, finer binnings in regions of known resonances: $f_0(980)$, $\rho(770)$, $f_2(1270)$
- Depending on $m_{3\pi}$ and wave, up to 62 $m_{\pi^-\pi^+}$ bins per freed wave

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- Spin-exotic wave:

$$\psi\left(\vec{\tau}\right)\Delta^0\left(m_{\pi^-\pi^+}\right) + \mathrm{Bose\ sym.} = 0$$

at **every point** $\vec{\tau}$ in phase space

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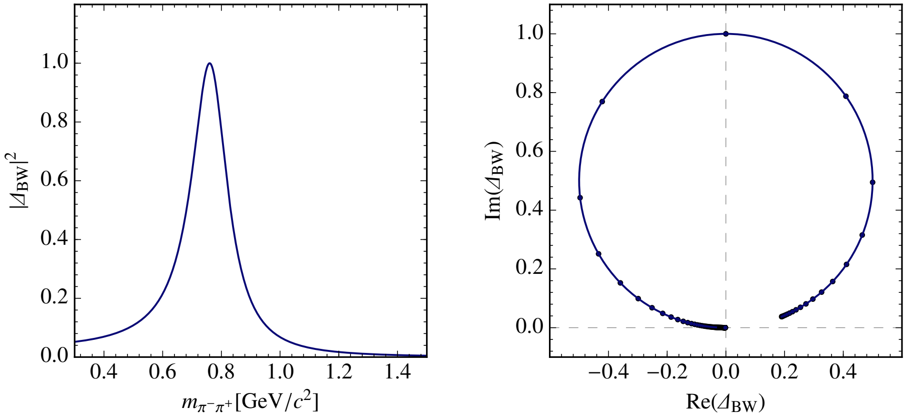
- Dynamic isobar amplitudes result in the same intensity, independent of the complex-valued $\mathcal{C}$:

$$\Delta^{\mathrm{meas}}\left(m_\xi\right) = \Delta^{\mathrm{phys}}\left(m_\xi\right) + \mathcal{C}\Delta^0\left(m_\xi\right)$$

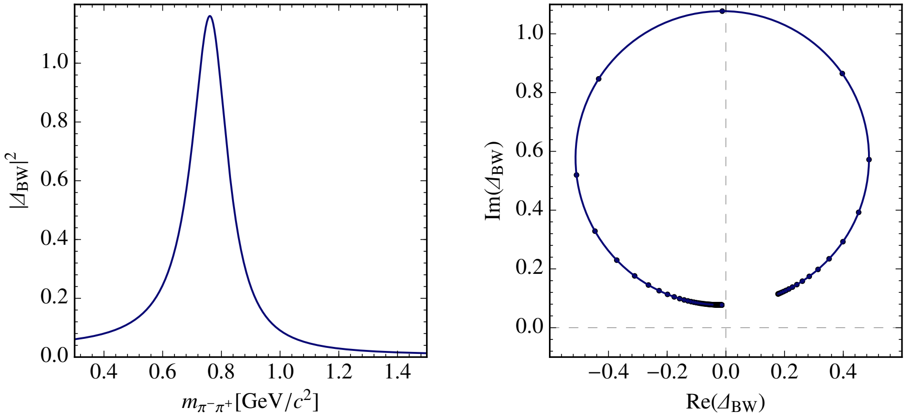
FK, D. Greenwald, D. Ryabchikov, B. Grube, S. Paul, *Phys. Rev.* **D97**, (2018) 114008

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = 0.00 + 0.00i$$



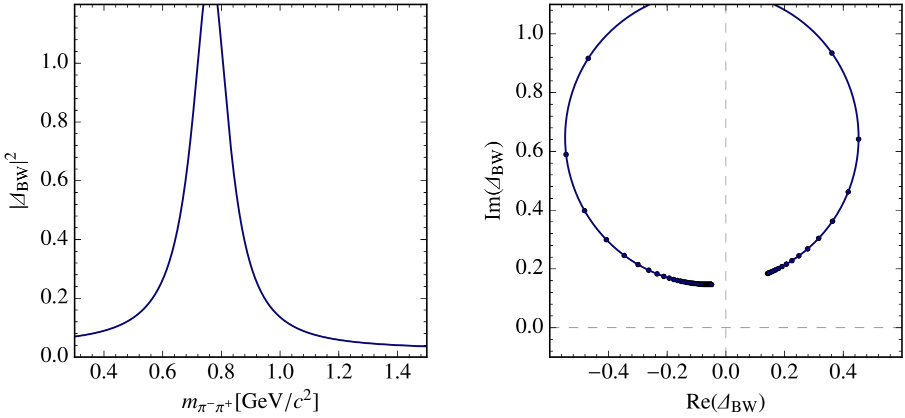
All describe the same total 3π decay amplitude

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.01 + 0.08i$$



All describe the same total 3π decay amplitude

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.05 + 0.15i$$

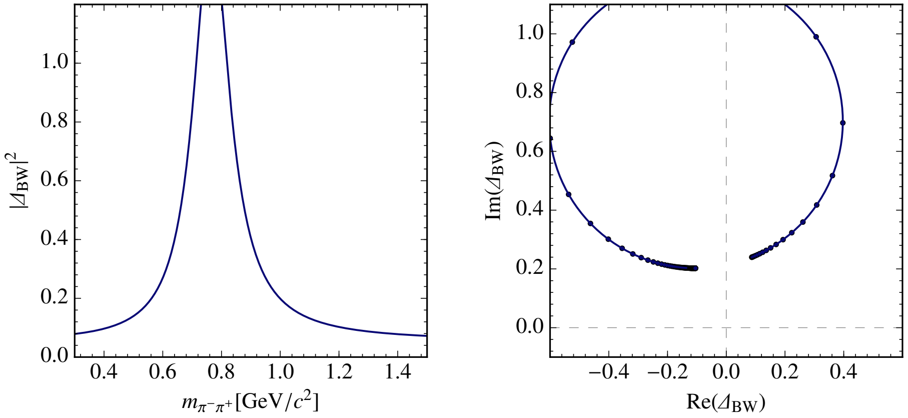


All describe the same total 3π decay amplitude

Zero mode in the spin-exotic wave

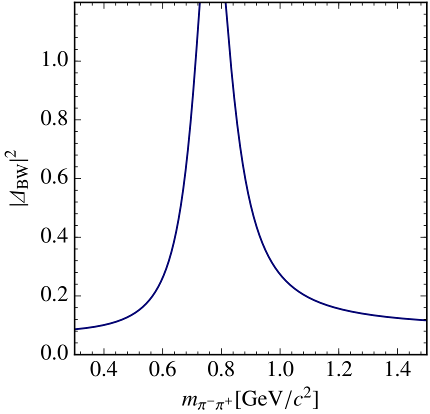
Effects on dynamic isobar amplitudes

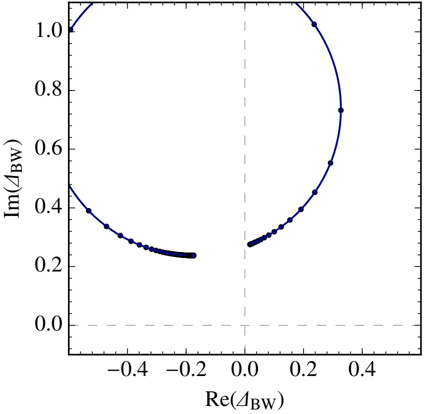
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.10 + 0.20i$$



All describe the same total 3π decay amplitude

$$\Delta_{\text{BW}}(m_{\pi^-\pi^+}) + \mathcal{C}\Delta^0(m_{\pi^-\pi^+})$$
$$\mathcal{C} = -0.17 + 0.24i$$

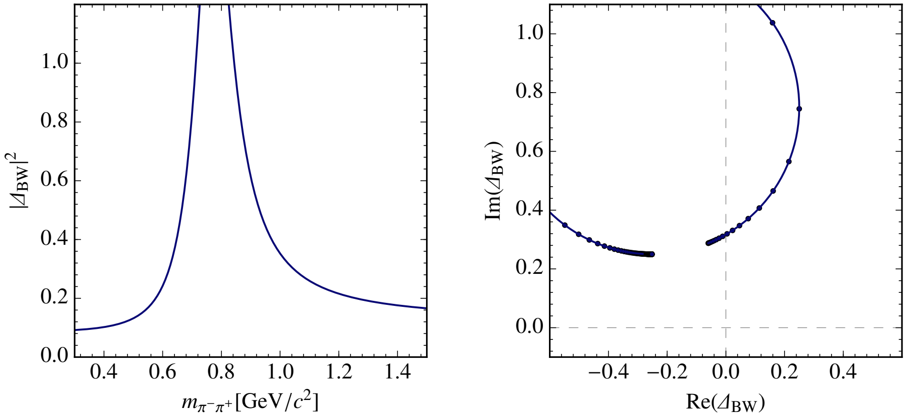




All describe the same total 3π decay amplitude

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$

$$\mathcal{C} = -0.25 + 0.25i$$



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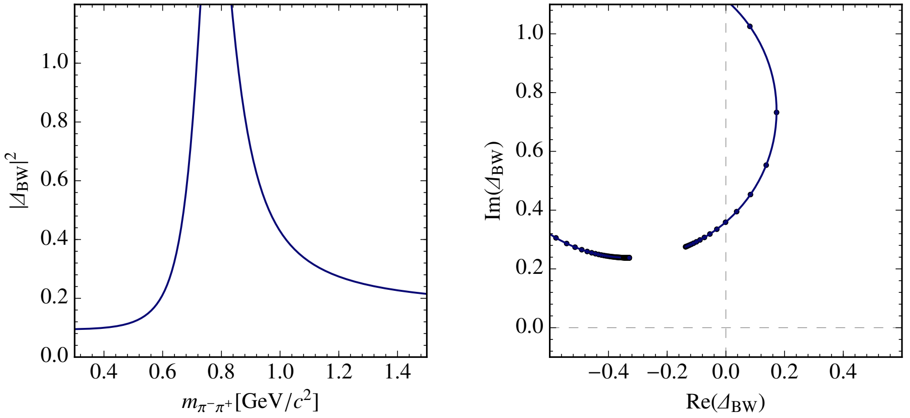
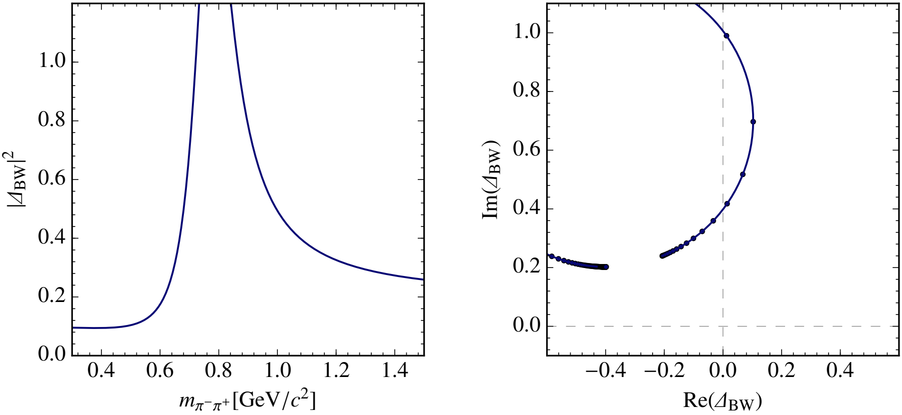


All describe the same total 3π decay amplitude

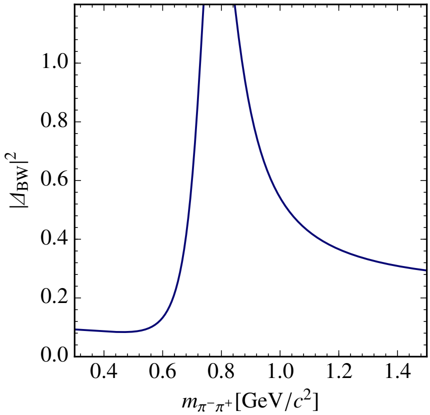
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$

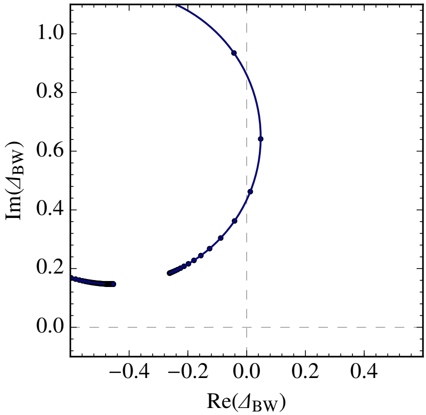
$$\mathcal{C} = -0.40 + 0.20i$$



All describe the same total 3π decay amplitude

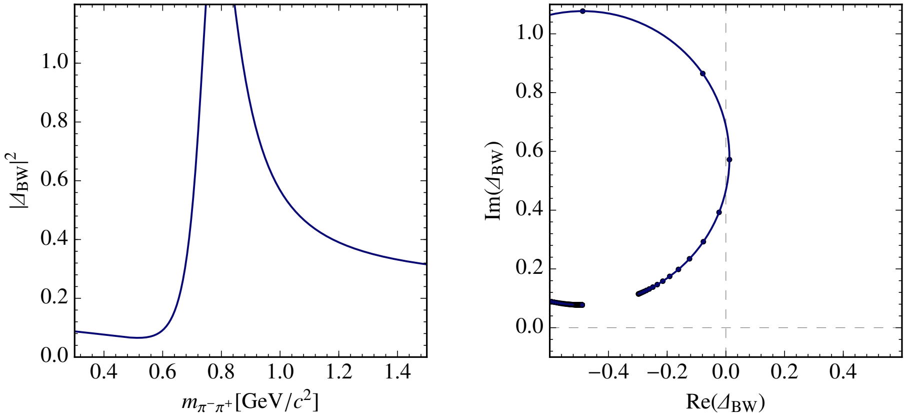
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.45 + 0.15i$$





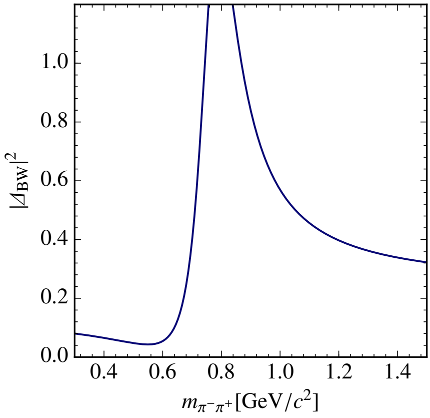
All describe the same total 3π decay amplitude

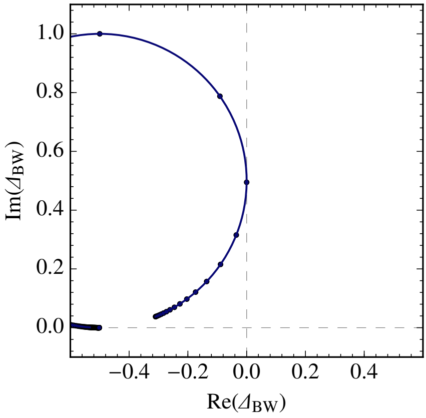
$$\Delta_{\text{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.49 + 0.08i$$



All describe the same total 3π decay amplitude

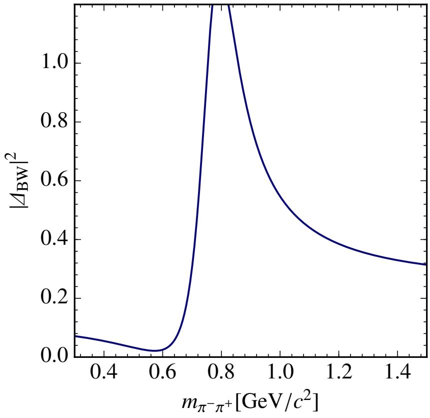
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.50 + 0.00i$$

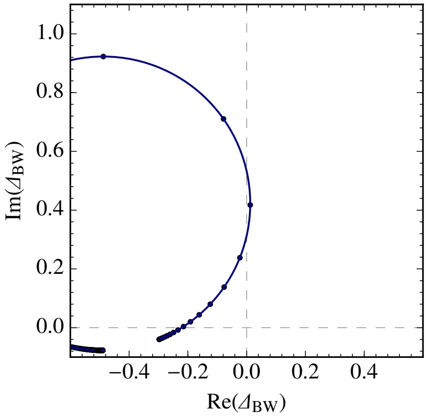




All describe the same total 3π decay amplitude

$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = -0.49 - 0.08i$$





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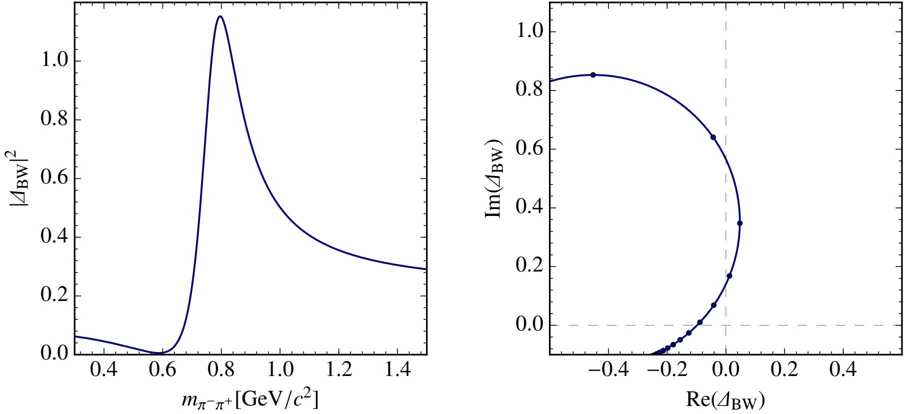

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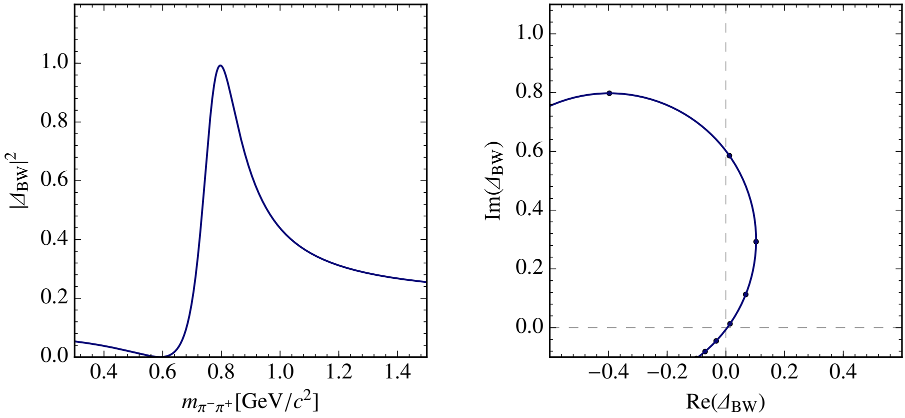

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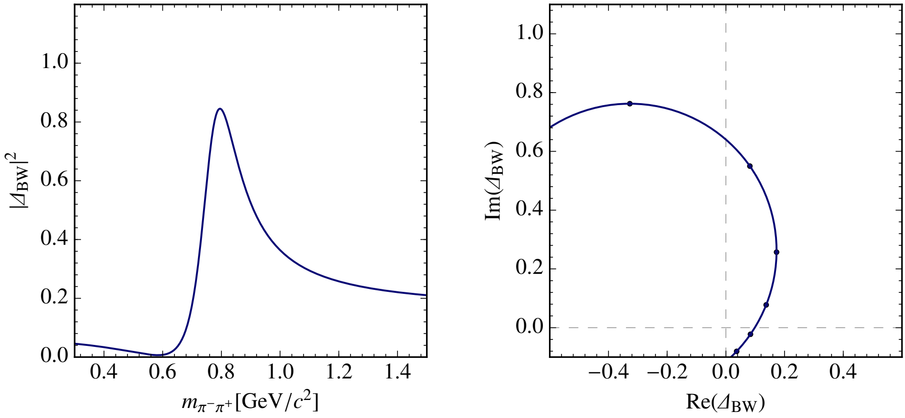

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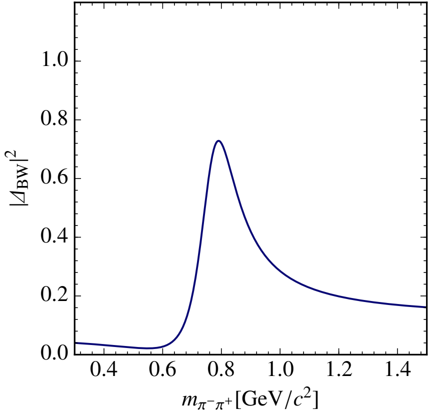

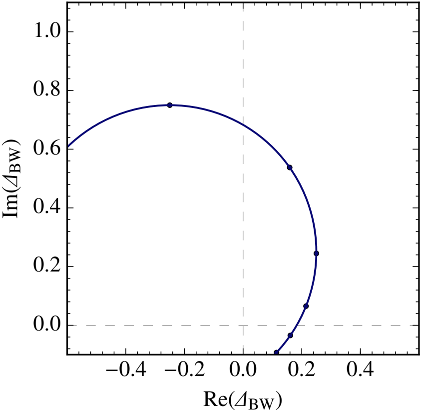

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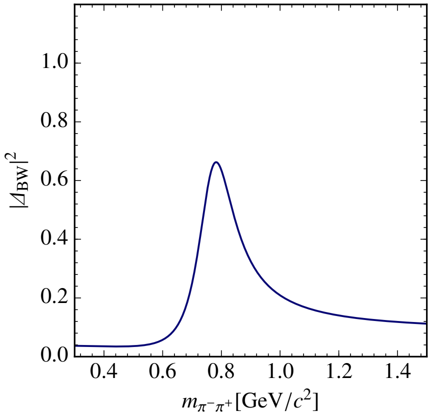

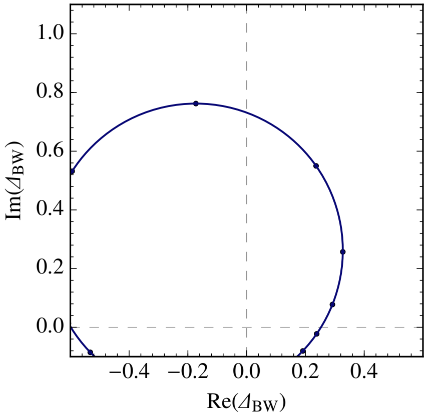

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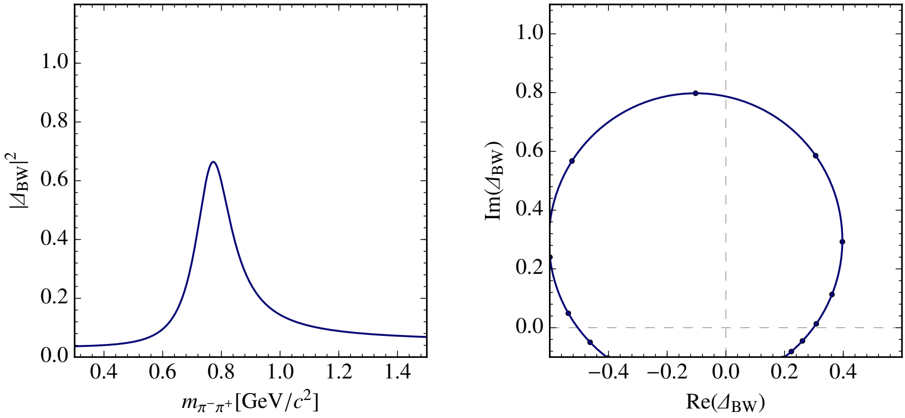

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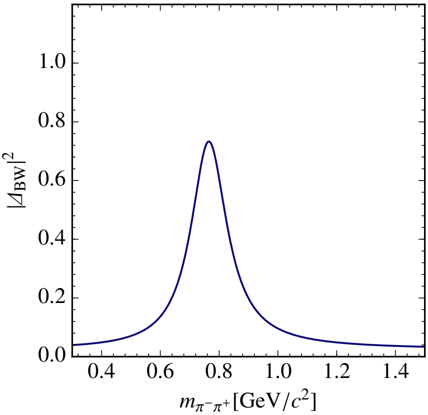

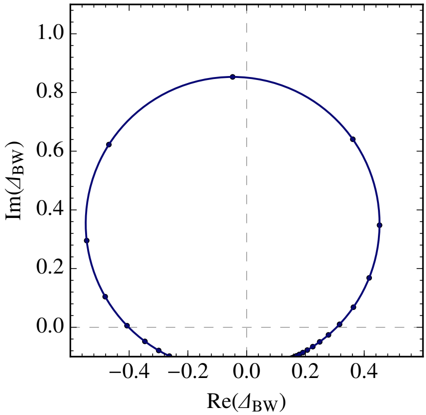

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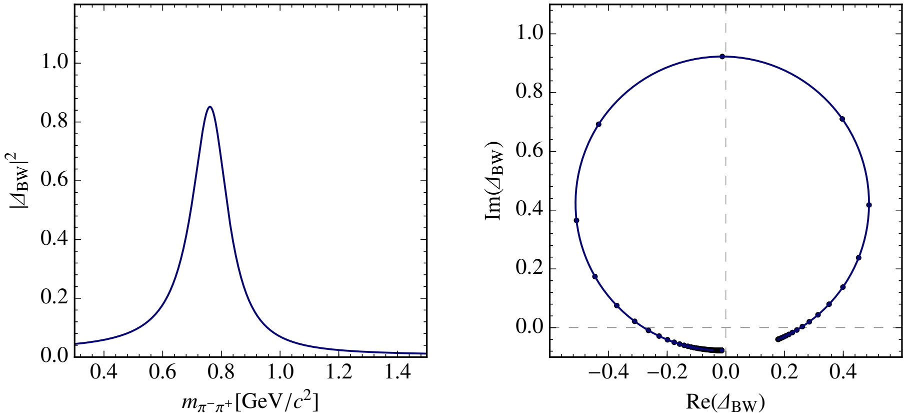

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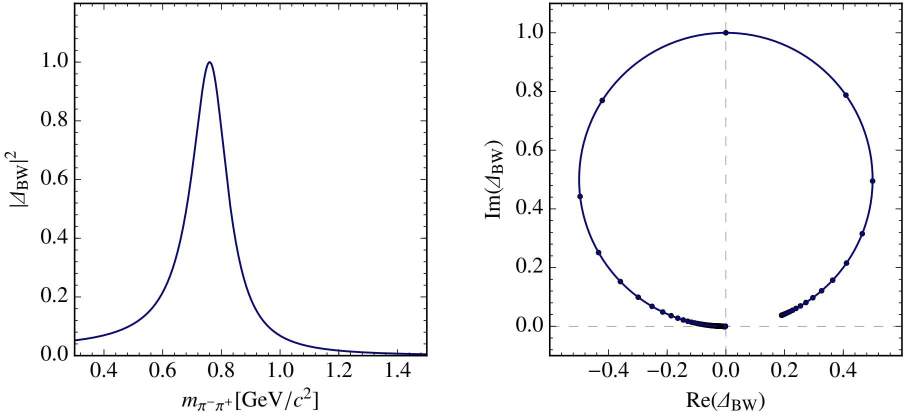
$$\Delta_{\mathrm{BW}}\left(m_{\pi^-\pi^+}\right) + \mathcal{C}\Delta^0\left(m_{\pi^-\pi^+}\right)$$
$$\mathcal{C} = 0.00 + 0.00i$$



All describe the same total 3π decay amplitude

- Now for $m_{\pi^-\pi^+}$ bins: $\vec{\mathcal{T}}^0 = \left\{\Delta^0\left(m_{\mathrm{bin}}\right)\right\}$ for all $m_{\pi^-\pi^+}$bins

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  - use the Breit-Wigner for the $\rho(770)$ resonance with fixed parameters as in the fixed-isobar analysis
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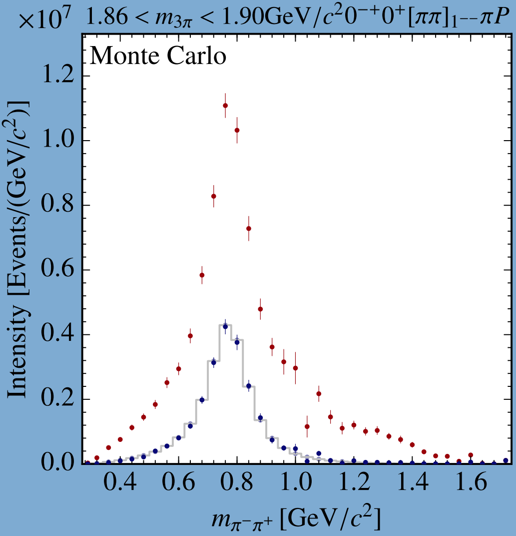
- **Note:** Resolving the ambiguity fixes only a single complex-valued degree of freedom. $n_{\text{bins}} - 1$ complex-valued degrees of freedom remain free.

Different constraints

- Now fo[...] s
- The fitti[...] om the physical solution
- Obtain [...] e resulting dynami[...]
- In the c[...]
  - use[...] parameters as in the[...]
  - lim[...] m possible excited ρ s[...]
- **Note:** F[...] ex-valued degree of freed[...] n remain free.



- Now fo[...]
- The fitti[...] om the physical solution[...]
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  - use[...] parameters as in the[...]
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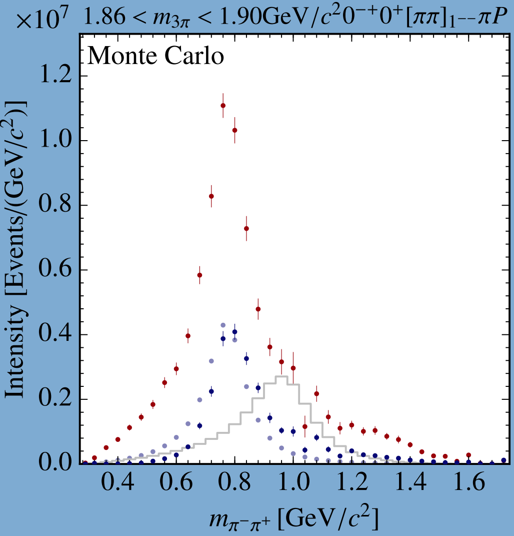
Different constraints

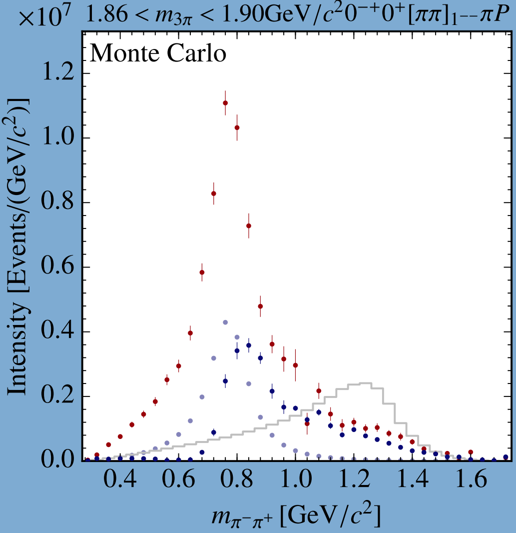


Zero mode in the spin-exotic wave

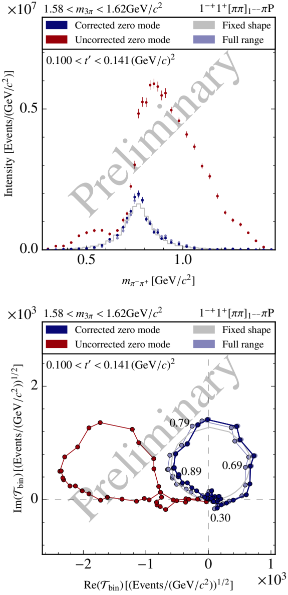
Resolving the ambiguity

- Now fo s
- The fitti om the physical solution
- Obtain e resulting dynami
- In the c
  - use parameters as in the
  - lim m possible excited ρ s
- **Note:** ex-valued degree of freed n remain free.

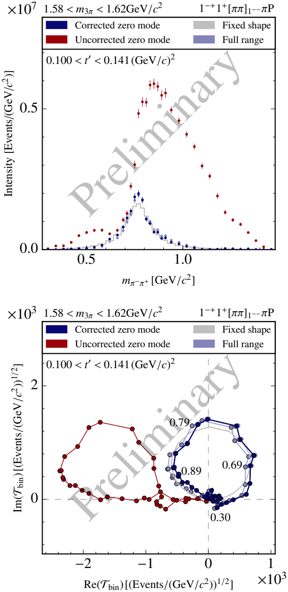
Different constraints

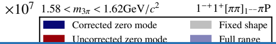


- Dynamic isobar amplitude for the spin-exotic wave obtained via freed-isobar PWA

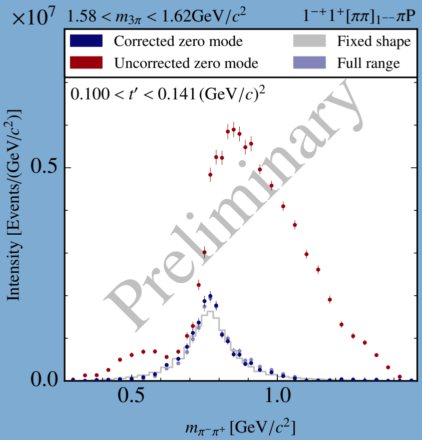


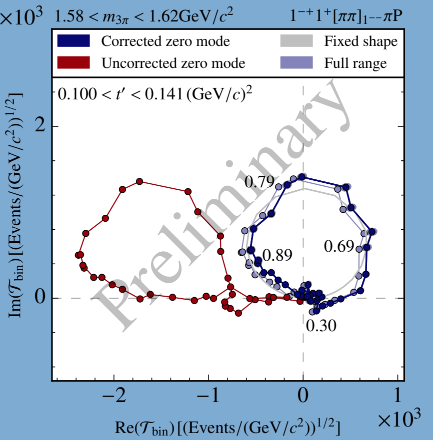
- Dynamic isobar amplitude for the spin-exotic wave obtained via freed-isobar PWA
- Zero-mode ambiguity resolved with $\rho\,(770)$ as constraint

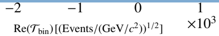




Freed-isobar resuld

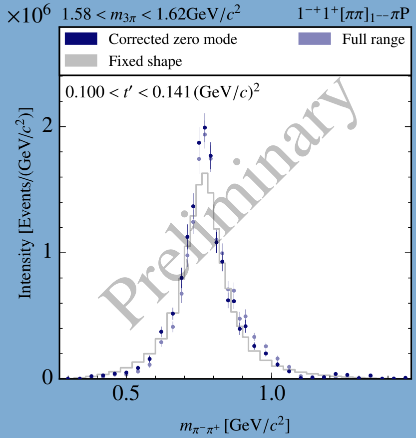


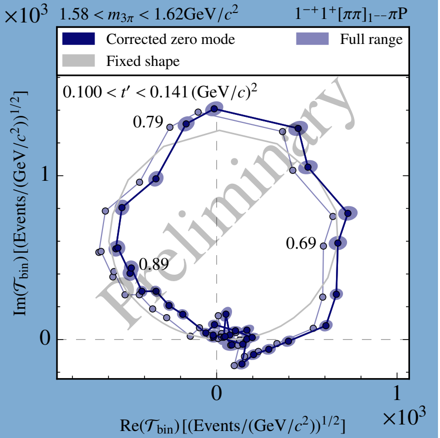




$\times 10^6$ $1.58 < m_{3\pi} < 1.62\text{GeV}/c^2$ $1^{-+}1^+[\pi\pi]_1$--πP
Corrected zero mode Full range
Fixed shape

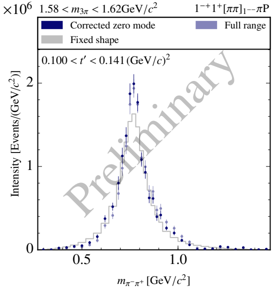
Freed-isobar resuld

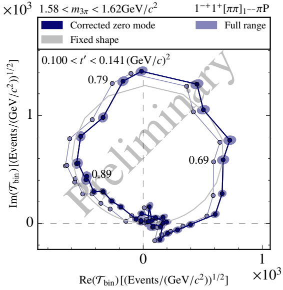




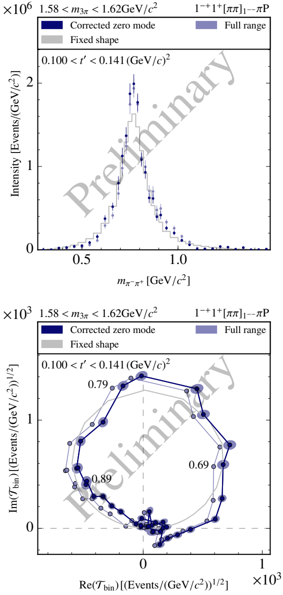
0 1
Re($\mathcal{T}_{\text{bin}}$) [(Events/(GeV/c^2))$^{1/2}$] $\times 10^3$

- Dynamic isobar amplitude for the spin-exotic wave obtained via freed-isobar PWA
- Zero-mode ambiguity resolved with $\rho(770)$ as constraint
- Dynamic isobar amplitude dominated by $\rho(770)$

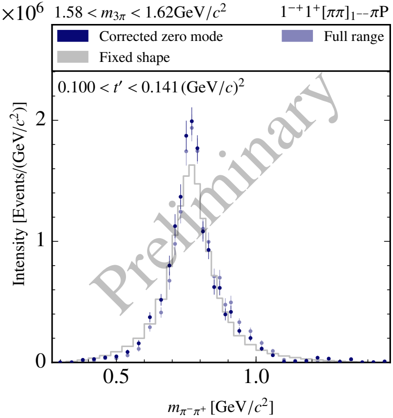




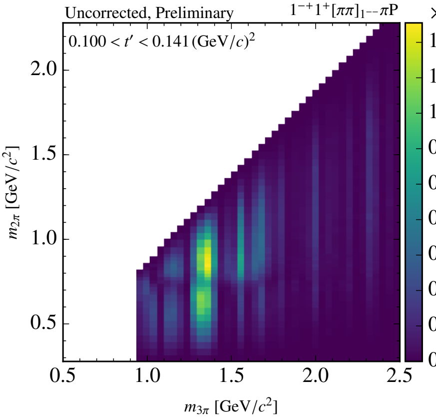
- Dynamic isobar amplitude for the spin-exotic wave obtained via freed-isobar PWA
- Zero-mode ambiguity resolved with $\rho\,(770)$ as constraint
- Dynamic isobar amplitude dominated by $\rho\,(770)$
- Significant deviations from a pure Breit-Wigner shape



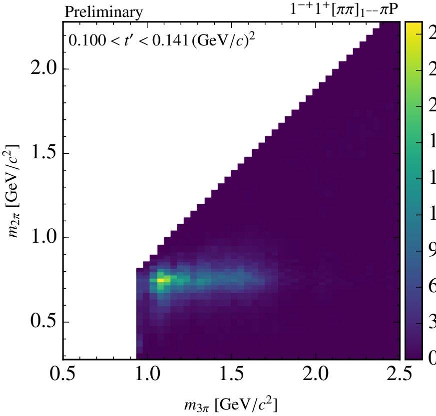
- Results for all bins in $m_{3\pi}$



- Results for all bins in $m_{3\pi}$
- Two-dimensional intensity distribution



- Results for all bins in $m_{3\pi}$
- Two-dimensional intensity distribution
- Zero-mode ambiguity resolved in every $m_{3\pi}$ bin individually



- Results for all bins in $m_{3\pi}$
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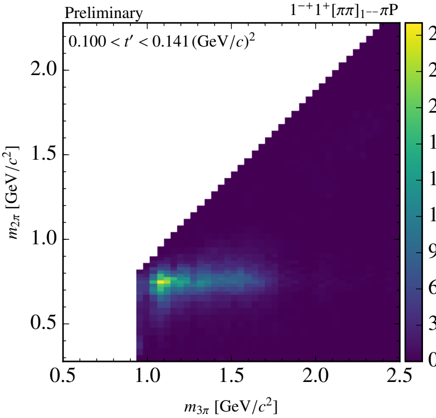
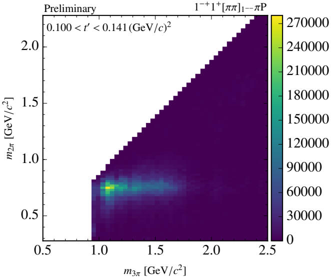
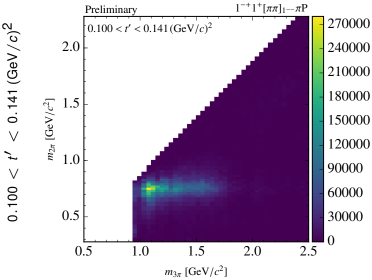
- Zero-mode ambiguity resolved in every $m_{3\pi}$ bin individually
- $m_{3\pi}$ dependence:
  - Spin-exotic π_1 resonance
  - Non-resonant effects

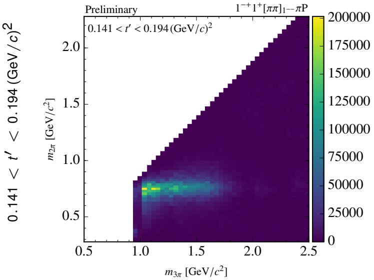


$0.100 < t' < 0.141\ (\text{GeV}/c)^2$





$0.100 < t' < 0.141$ (GeV/c)^2
Preliminary
$1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$
$0.100 < t' < 0.141\,(\mathrm{GeV}/c)^2$
$m_{2\pi}$ [GeV/c^2]
$m_{3\pi}$ [GeV/c^2]
0.5
1.0
1.5
2.0
2.5
0
30000
60000
90000
120000
150000
180000
210000
240000
270000

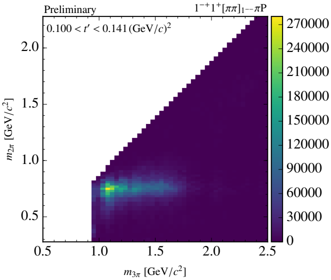


$0.141 < t' < 0.194$ (GeV/c)^2
Preliminary
$1^{-+}1^{+}[\pi\pi]_{1^{--}}\pi P$
$0.141 < t' < 0.194\,(\mathrm{GeV}/c)^2$
$m_{2\pi}$ [GeV/c^2]
$m_{3\pi}$ [GeV/c^2]
0.5
1.0
1.5
2.0
2.5
0
25000
50000
75000
100000
125000
150000
175000
200000

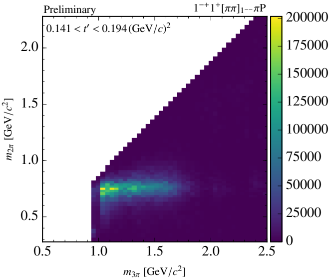
The spin-exotic wave

Results — Different bins in t'

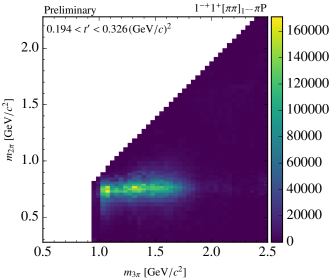
$0.100 < t' < 0.141$ (GeV/c)^2

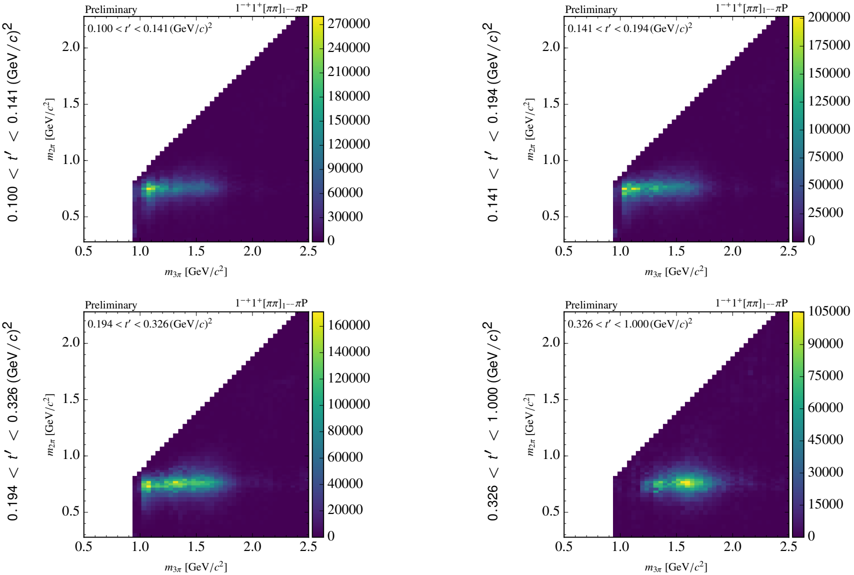


$0.141 < t' < 0.194$ (GeV/c)^2



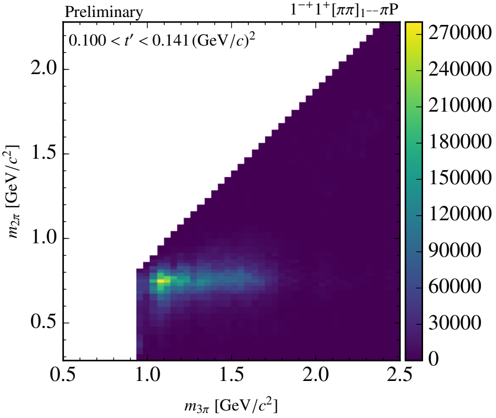
$0.194 < t' < 0.326$ (GeV/c)^2





0.100 < t′ < 0.141 (GeV/c)²
Preliminary
1⁻⁺1⁺[ππ]₁⁻⁻πP
0.100 < t′ < 0.141 (GeV/c)²
0.141 < t′ < 0.194 (GeV/c)²
Preliminary
1⁻⁺1⁺[ππ]₁⁻⁻πP
0.141 < t′ < 0.194 (GeV/c)²
0.194 < t′ < 0.326 (GeV/c)²
Preliminary
1⁻⁺1⁺[ππ]₁⁻⁻πP
0.194 < t′ < 0.326 (GeV/c)²
0.326 < t′ < 1.000 (GeV/c)²
Preliminary
1⁻⁺1⁺[ππ]₁⁻⁻πP
0.326 < t′ < 1.000 (GeV/c)²
m2π [GeV/c²]
m3π [GeV/c²]

- Coherently sum up all $m_{\pi^-\pi^+}$ bins to obtain $m_{3\pi}$ spectra



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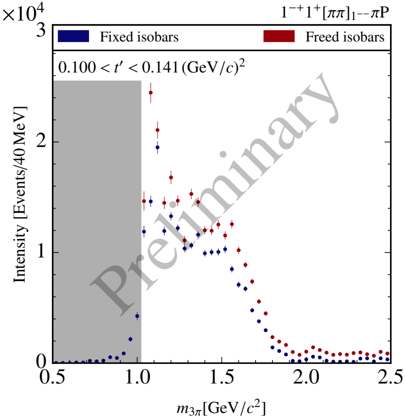

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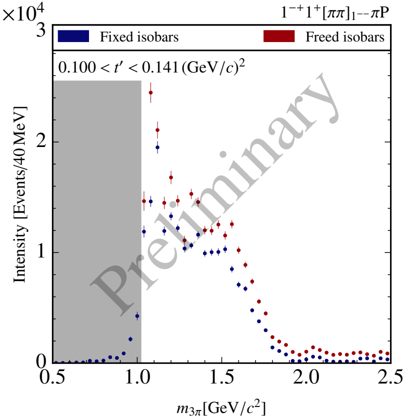
- Compare to results from conventional PWA



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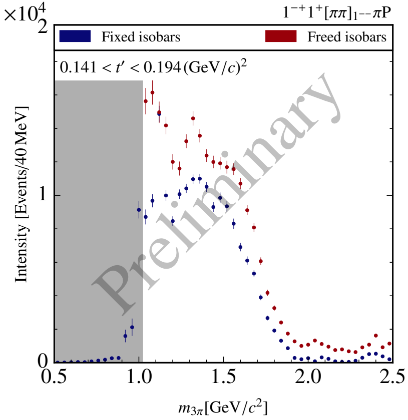
- In all t' bins:



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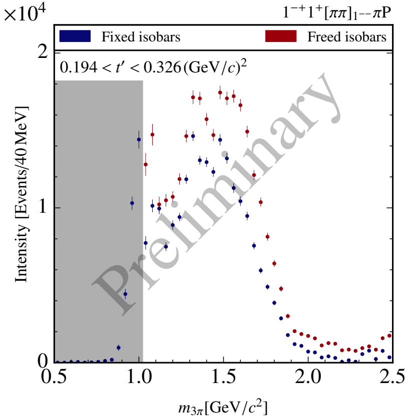

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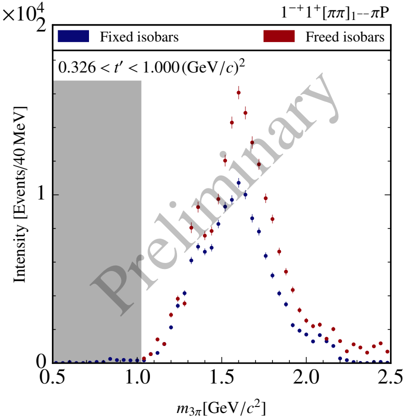
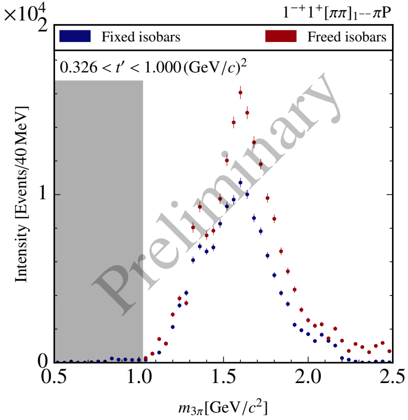


- Coherently sum up all $m_{\pi^-\pi^+}$ bins to obtain $m_{3\pi}$ spectra
- Compare to results from conventional PWA
- In all t' bins:
  - Main features reproduced



- Coherently sum up all $m_{\pi^-\pi^+}$ bins to obtain $m_{3\pi}$ spectra
- Compare to results from conventional PWA
- In all t' bins:
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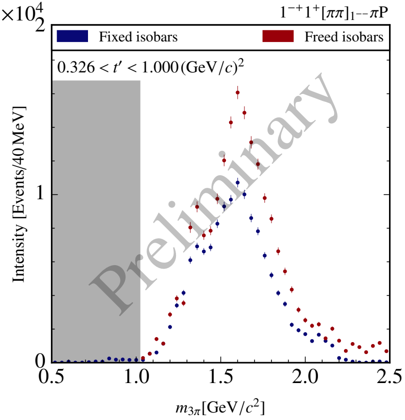
  - General t' dependence matches



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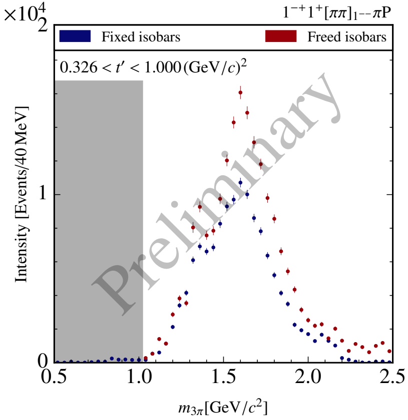
  - higher intensity in the freed-isobar result



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- Compare to results from conventional PWA
- In all t' bins:
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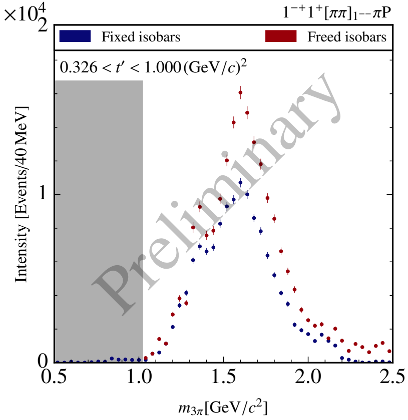
  - General t' dependence matches
  - higher intensity in the freed-isobar result
  - at high t': π_1 peak reproduced



Conclusions & Outlook

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- Apply method to other channels: e.g. $K\pi\pi$, heavy mesons