Covariant Vector meson-Vector meson Interactions and Dynamically Generated Resonances

Dilege Gülmez

HISKP, Universität Bonn

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Overview

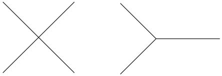
1. Formalism
2. $\rho\rho$ Interactions
   - The on-shell Bethe-Salpeter Equation (BSE)
   - The N/D method
3. The extension to $SU(3)$
   - The BSE method in coupled channels
   - Single channels
   - Coupled Channels
4. Results and Summary

The Hidden Gauge Formalism Lagrangian & On-Shell BSE

☐ Vector-Meson Interactions:

$$\mathcal{L}_{4V} = \frac{g^2}{2}\langle V_\mu V_\nu V^\mu V^\nu - V_\mu V^\mu V_\nu V^\nu \rangle$$

$$\mathcal{L}_{3V} = ig\langle (\partial_\mu V_\nu - \partial_\nu V_\mu) V^\mu V^\nu \rangle$$

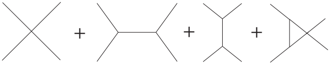


(4 V) (3 V)

☐ The on-shell BSE:

$$T(s) = [1 - V(s) \cdot G(s)]^{-1} \cdot V(s)$$

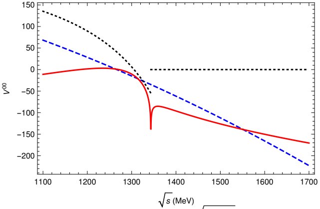
$$G_{ii}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - M_1^2 + i\epsilon)((P-q)^2 - M_2^2 + i\epsilon)}$$

$V=$ 

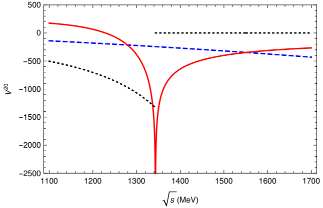
Partial-wave Interaction Kernels

DG, Meißner, Oller, Eur. Phys. J. C **77** (2017) no.7, arXiv:1611.00168 [hep-ph]

Molina, Nicmorus, Oset, Phys. Rev. D **78** (2008) 114018, arXiv:0809.2233 [hep-ph]



- ▷ The LHC starts at $\sqrt{3m_\rho^2} = 1343$ MeV.
- ▷ A pole is found at 1467 (1491) MeV.
- ▷ It is associated with the $f_0(1370)$.
- ▷ The width: $200 - 500$ MeV in PDG.



- ▷ No/A pole is found (at 1255 MeV).
- ▷ It is associated with the $f_2(1270)$.
- ▷ $f_2(1270)$ fits very well within the ideal $P-$wave $q\bar{q}$ nonet (analyses of the high-statistics Belle data[a], the Regge theory[b]).
- ▷ Far away from the $\rho\rho$ threshold (1551 MeV).

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[a] Dai, Pennington, Phys. Rev. D **90** (2014) 036004, arXiv:1404.7524 [hep-ph]

[b] Ananthanarayan, Colangelo, Gasser, Leutwyler, Phys. Rept. **353** (2001) 207, arXiv:0005297 [hep-ph]

Partial-wave Interaction Kernels

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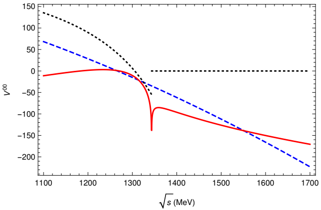

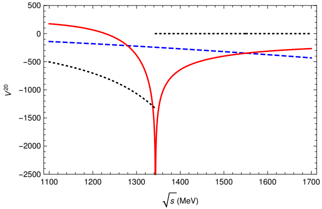

Importance of LHCs:

- ▷ The NN scattering[a]: One pion exchange starts at $p^2 = -m_\pi^2 \to s < 4(m_N^2 - m_\pi^2/4)$ and two pion exchange starts at $p^2 = -4m_\pi^2 \to s < 4(m_N^2 - m_\pi^2)$.
- ▷ Apply this to the $\rho\rho$ scattering (m_π and m_N are to be replaced by m_ρ).
- ▷ Ignoring the $\rho\rho$ exchange means ignoring LHCs of OPE like term of the interaction.
- ▷ The dynamics of low-energy NN scattering has the highest contribution from OPE.

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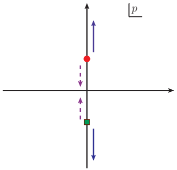
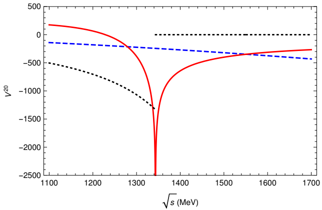
[a] Guo, Oller, Ríos, Phys. Rev. C **89** (2014) 014002, arXiv:1305.5790 [hep-th]

Partial-wave Interaction Kernels



Red Circle: Bound State ($a > 0$), Green Square: Virtual state ($a < 0$)



- ▷ The effective range is determined by: $T(s) = \dfrac{8\pi\sqrt{s}}{-1/a + r_0\mathbf{p}^2 - i\mathbf{p}}$.
- ▷ r_0 is proportional to $d(\sqrt{s}T^{-1})/dE\Big|_{E=\sqrt{s_{th}}}$.
- ▷ As the slope of the potential increases, r_0 decreases.
- ▷ Therefore, the effective range for the scalar sector is much larger than the tensor sector.
- ▷ $\mathbf{p} = \begin{cases} \frac{i}{a} + \frac{ir_0}{a^2} + \dots \\ \frac{i}{r_0} - \frac{i}{a} - \frac{ir_0}{a^2} + \dots \end{cases}$
- ▷ In the non-relativistic approach, both sectors have the same energy-dependence, thus, positive effective range (follows blue solid lines: $r_0 a > 0$).
- ▷ In the covariant form, the tensor sector has a negative effective range (follows purple dashed lines: $r_0 a < 0$).

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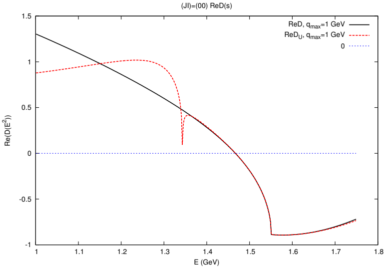
The N/D Method

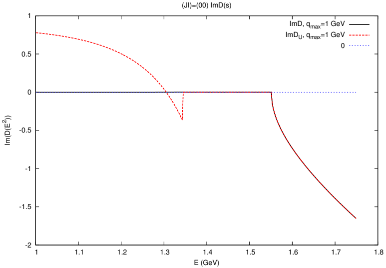
- $T = N(s)/D(s)$.
- N has only LHCs and D has only RHCs.
- $D = 0$ corresponds to resonances or bound-states.
- $N(s) = V(s)$ (first iterated solution).
- $D(s)$ diverges as s^2. Therefore, three subtractions in the dispersion relation for D(s):

$$D(s) = \gamma_0 + \gamma_1 (s - s_{th}) + \frac{1}{2}\gamma_2 (s - s_{th})^2 + \frac{(s - s_{th})s^2}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') V^{(JI)}(s')}{(s' - s_{th})(s' - s)(s')^2} .$$

- Matching condition in the threshold region up to $\mathcal{O}(s^3)$:

$$\gamma_0 + \gamma_1 (s - s_{th}) + \frac{1}{2}\gamma_2 (s - s_{s_{th}})^2 = 1 - V(s) G_c(s) - \frac{(s - s_{th})s^2}{\pi} \int_{s_{th}}^{\infty} ds' \frac{\rho(s') V(s')}{(s' - s_{th})(s' - s)(s')^2} .$$





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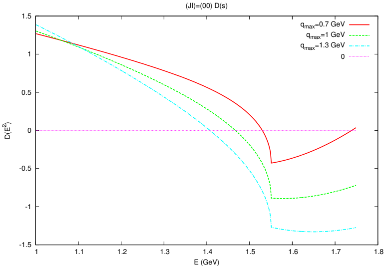
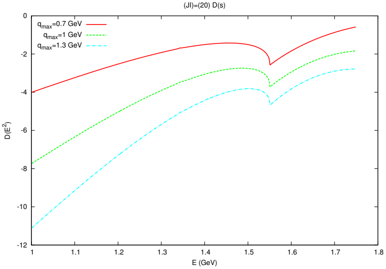




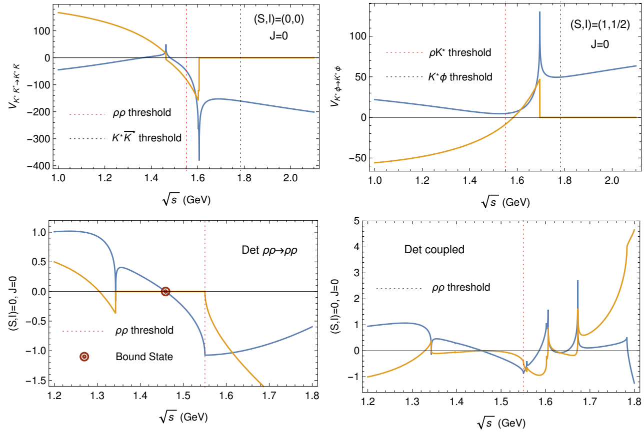
The BSE approach in coupled channels

DG, Du, Guo, Meißner, Wang, Preliminary Results

- ☐ In coupled channel calculations, g is evaluated with the average mass of vector mesons ($g = 4.596$).

- ☐ The LHC overlaps with the RHC.

- ☐ The comparison of the $\mathrm{Det}(I - V \cdot G)$ in the single and coupled channel isoscalar scalar channel:
- ☐ No pole is found in the coupled channel!
- ☐ The on-shell BSE method is not valid for coupled channels.

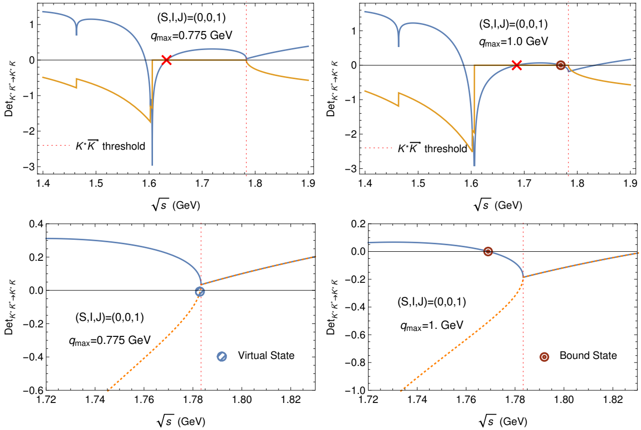


Single channels in $SU(3)$ (BSE)

- Single channels in $SU(3)$: $I = 2\,(\rho\rho)$, $I = 3/2\,(K^*K^*)$, $I = 1\,(K^*K^*)$, $I = 0\,(K^*K^*)$, and $(I, J) = (0, 1)\,(K^*\overline{K}^*)$.
- Poles: 1 pole in $(I, J) = (0, 1)\,(K^*K^*)$, and 2 poles in $(0, 1)\,(K^*\overline{K}^*)$.

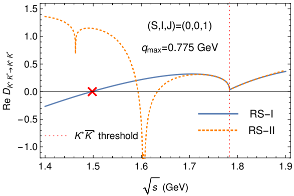
- The pole is an artefact of the LHC.
- It is the same scenario for $(0, 1)\,(K^*K^*)$.

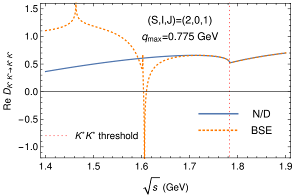
- The second pole is sensitive to the cutoff.
- For $g = M_\rho/f$, the pole is virtual for $q_{max} < 1.03$ GeV.



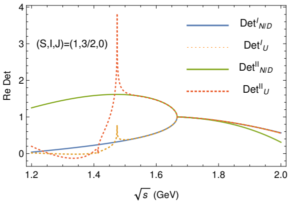
Single channels in $SU(3)$ (N/D)

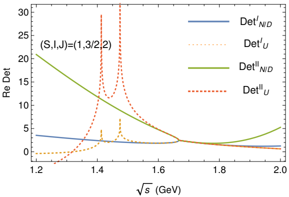
- ☐ The second pole behaviour, in $K^*\overline{K}^*$, does not change.
- ☐ One of the LHC artefacts disappears where the other moves deeper on the real axis.





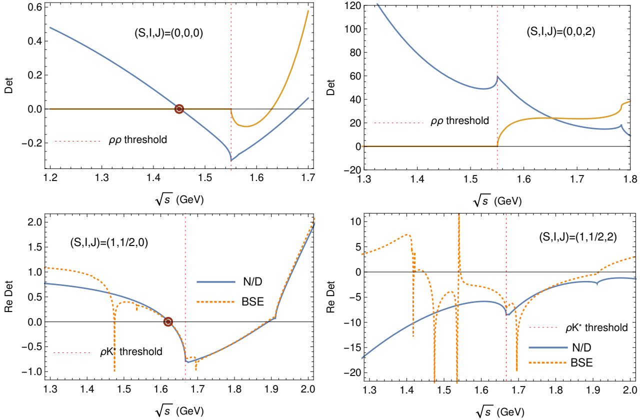
- ☐ We have found three resonance poles on the 2nd R.S. in (2,2), (3/2,2), (1,2).
- ☐ In the tensor sector, matching does not work well for the 2nd R.S. .





Coupled Channels (N/D)

- The (0,0) pole is re-established for the coupled channel.
- Far away from $K^*\overline{K}^*$ or $\phi\phi$ threshold? $\rho\rho$ dominates.
- A resonance pole is found at $1.68 \pm 0.2i$ GeV $\to f_0(1710)$. $K^*\overline{K}^*$ dominates.



- In conclusion, the results are reproducible close to the threshold.

Results and Summary

DG, Du, Guo, Meißner, Wang, Preliminary results

Geng, Oset, Phys. Rev. D **79** (2009) 074009, arXiv:0812.1199[hep-ph]

Major	$I^G(J^{PC})$	Pole positions [GeV]	Pole positions [GeV]	PDG	Mass[GeV]
$\rho\rho$	$0^+(0^{++})$	$[1.41 - 1.50]$	1.51	$f_0(1370)$	$[1.2 - 1.5]$
$K^*\overline{K}^*$	$0^+(0^{++})$	$[1.56 - 1.73]$	1.73	$f_0(1710)$	$[1.72 - 1.73]$
$K^*\overline{K}^*$	$0^-(1^{+-})$	$[1.77 - 1.78]$	1.80	–	–
$\rho\rho$	$0^+(2^{++})$	–	1.28	$f_2(1270)$	$[1.28]$
$K^*\overline{K}^*$	$0^+(2^{++})$	–	1.53	$f_2'(1525)$	$[1.52 - 1.53]$
$K^*\overline{K}^*$	$1^-(0^{++})$	–	1.78	–	–
$\rho\rho$	$1^+(1^{+-})$	$[1.44 - 1.50]$	1.68	–	–
$K^*\overline{K}^*$	$1^-(2^{++})$	–	1.57	–	–
ρK^*	$1/2(0^+)$	$[1.58 - 1.66]$	1.64	–	–
ρK^*	$1/2(1^+)$	$[1.86 - 1.92]$	1.74	$K_1(1650)$?	$[1.62 - 1.72]$
ρK^*	$1/2(2^+)$	–	1.43	$K_2^*(1430)$	$[1.42 - 1.43]$

- The on-shell BSE does not provide the correct analytic structure for the coupled channels.
- The difference between two methods is of order $\mathcal{O}((s - s_{\rm th})^3)$ $\Rightarrow$ Good agreement around the threshold.
- A more careful treatment is needed, especially away from the threshold.
- LHCs are treated perturbatively. A conclusive approach would be the full N/D method.

Thank you for your attention!