Near threshold kaon-kaon interaction in the reactions $e^+ e^- \to K^+K^- \gamma$ and $e^+ e^- \to K^0 \bar{K}^0 \gamma$

L. Leśniak, F. Morawski,
M. Silarski and F. Sobczuk

Institute of Physics, Jagiellonian University, Kraków

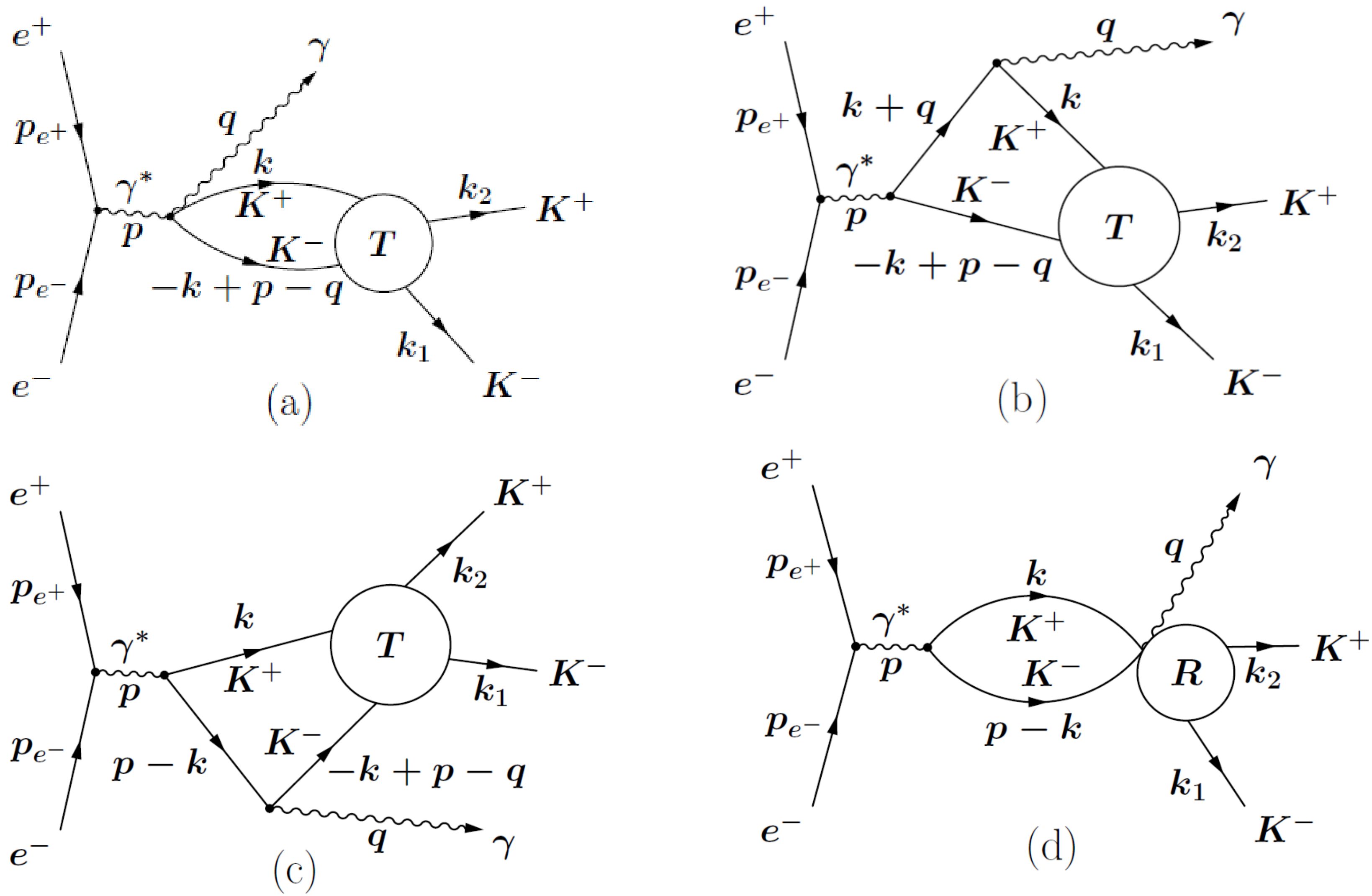
Meson2018

June 7. 2018

Motivation

1. The kaon-kaon interaction near the $K\bar{K}$ threshold is unknown.
2. The parameters of the scalar resonances **$f_0(980)$** and **$a_0(980)$** are still imprecise.
3. The ϕ decay reactions into $\pi^+\pi^-\gamma$, $\pi^0\pi^0\gamma$ and $\pi^0\eta\gamma$ have been measured, for the ϕ transition into $K^0\bar{K}^0\gamma$ only the upper limit $1.9\ 10^{-8}$ is known but there are no data for the $\phi \to K^+K^-\gamma$.
4. A general **theoretical model** of the radiative ϕ decays has to be formulated. It should allow for a coupled channel analysis of the amplitudes describing interactions of different meson-meson pairs in the final state. With a help of such a unitary model one can obtain more information on the threshold kaon-kaon scattering provided the relevant data are available.

Diagrams for the reaction $e^+ e^- \to K^+K^-\gamma$



T(k)=<K$^-$(k$_1$) K$^+$(k$_2$)|Ť| K$^-$(-k+p-q)K$^+$(k)>
>

R ~ T(k-q) - T(k)

Amplitudes for the reaction $e^+ e^- \to K^+K^-\gamma$

The total amplitude $A = A_a + A_b + A_c + A_d$ is gauge invariant.

$$A_a = 2i \int \frac{d^4k}{(2\pi)^4} \frac{J_\nu \, \varepsilon^{\nu *} \, T(k)}{D(k)\, D(-k+p-q)}$$

$\varepsilon =$ photon polarization

$$A_b = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu \, \varepsilon^{\nu *} k_\nu (k_\mu + q_\mu) T(k)}{D(k+q) D(k)\, D(-k+p-q)}$$

$p = p_{e^+} + p_{e^-}$

$$A_c = -4i \int \frac{d^4k}{(2\pi)^4} \frac{J_\mu \, \varepsilon^{\nu *} (k_\nu - p_\nu) k_\mu \, T(k)}{D(p-k) D(k)\, D(-k+p-q)}$$

$$A_d = -2i \int \frac{d^4k}{(2\pi)^4} \frac{J \cdot k \, \varepsilon^* \cdot \tilde{k}}{D(k)\, D(p-k)} \frac{[T(k-q) - T(k)]}{q \cdot \tilde{k}};$$

$\tilde{k} = (0, \hat{k}); \; \hat{k} = \vec{k} / |\vec{k}|$

$$J_\mu = \frac{e^3}{s} F_K(s) v(p_{e^+}) \gamma_\mu u(p_{e^-}) \; ; \; s = (p_{e^+} + p_{e^-})^2$$

$F_K(s)$ – kaon form factor

$$D(k) = k^2 - m_K^2$$

$D(k)$ – inverse of the kaon propagator

Definitions and approximations

$\mathrm{m} = \mathrm{K^+K^-}$ effective mass; $\quad m^2 = (k_1 + k_2)^2$

half – off – shell amplitude: $T(k) = \langle K^-(k_1)\, K^+(k_2) \,|\, \tilde{T}(m) \,|\, K^-(-k+p-q)\, K^+(k) \rangle; \quad k_1^2 = k_2^2 = m_K^2$

on – shell amplitude: $T_{K^+K^-}(m) = \langle K^-(k_1)\, K^+(k_2) \,|\, \tilde{T}(m) \,|\, K^-(k_1)\, K^+(k_2) \rangle$

K^+K^- center-of-mass frame: $\quad \vec{k}_1 + \vec{k}_2 = 0; \quad \vec{p} = \vec{q}; \quad$ photon energy $\quad \omega = |\vec{q}\,|$

relative kaon momentum in the final state: $\quad \vec{k}_f = \frac{1}{2}(\vec{k}_2 - \vec{k}_1)$

relative kaon momentum in the initial state: $\quad \vec{k}$

Approximation:

$$T(k) \approx g(|\vec{k}\,|)\, T_{K^+K^-}(m)$$

Condition:

$$g(|\vec{k}_f\,|) = 1$$

$s = m_\phi^2; \quad \omega \le 32\ \mathrm{MeV},$ soft photon

Next approximation:

$$\frac{[T(k\text{ - }q)\text{ - }T(k)]}{q \cdot \tilde{k}} \approx \frac{g(|\vec{k} - \vec{q}\,|) - g(|\vec{k}\,|)}{-\vec{q} \cdot \hat{k}} T_{K^+K^-}(m) \approx \frac{dg(|\vec{k}\,|)}{d\,|\vec{k}\,|} T_{K^+K^-}(m)$$

Approximations, part 2

Dominance of the terms with the **positive** kaon energy $E_k \approx \frac{m}{2}; \quad E_k = \sqrt{|\vec{k}|^2 + m_K^2}$

$$A_a + A_b + A_c \approx \vec{J} \cdot \vec{\varepsilon}^{\,*} T_{K^+K^-}(m)\, I(m)$$

$$I(m) = -2\int \frac{d^3k}{(2\pi)^3} \frac{g(|\vec{k}|)}{2E_k m(m-2E_k)}[1 - 2\frac{|\vec{k}|^2 - (\vec{k}\cdot\hat{q})^2}{2p_0E_k - s + 2\vec{k}\cdot\vec{q}}]; \quad p_0 = m + \omega$$

Limit of vanishing photon energy: $\omega \to 0$, $m \to m_\phi$

$$A_d(0) = -[A_a(0) + A_b(0) + A_c(0)] \qquad \text{if} \quad |\vec{k}|\, g(|\vec{k}|) \to 0 \ \text{at}\ |\vec{k}| \to \infty$$

A_d (ω) depends very weakly on ω: $A_d(\omega) \approx A_d(0)$

The full amplitude

$$A(m) \approx \vec{J} \cdot \vec{\varepsilon}^{\,*} T_{K^+K^-}(m)[I(m) - I(m_\phi)]$$

Comparison with two other approaches

1. K^+K^-system as a quasi-bound state (kaons as extended objects)

F. Close, N. Isgur, S. Kumano, Nucl. Phys. B 389 (1993) 513

$$g(|\vec{k}|) = \frac{\mu^4}{(\mu^2 + |\vec{k}|^2)^2}; \quad \mu = 141 \text{ MeV}; \quad \text{notice that } g(|\vec{k}| = 0) = 1$$

2. Point-like K^+K^- system

N.N. Achasov, V.V. Gubin, Phys. Rev. D 64 (2001) 094016

$$g(|\vec{k}|) \equiv 1$$

Resonant K^+K amplitude:
$R=f_0(980)$ or a(980)

$$T_{res} = \frac{(c_{RK^+K^-})^2}{D_R(m)}$$

c = coupling constant
D_R = inverse of the propagator R

Remark: **both approaches** can be treated **as special cases** of the present model.

Elastic and transition K $\bar{\text{K}}$ amplitudes

Relations to the S-matrix elements:

$$T_{K^+K^-}(m) = \frac{4\pi m}{i\,k_f}\left(S_{K^+K^-} - 1\right) \qquad T_{K^+K^-\to K^0\bar{K}^0}(m) = \frac{4\pi m}{i\,k_f} S_{K^+K^-\to K^0\bar{K}^0}$$

Isospin decomposition: index 0 or 1

$$T_{K^+K^-}(m) = \frac{1}{2}[t_0(m) + t_1(m)] \qquad T_{K^+K^-\to K^0\bar{K}^0}(m) = \frac{1}{2}[t_0(m) - t_1(m)]$$

Example: **separable K $\bar{\text{K}}$ interactions in the S-wave**

$$\langle k_f \,|V|\, k_i\rangle = \lambda G(k_f)G(k)$$

λ=potential strength

β=range parameter

G(k)= Yamaguchi form factor

$$G(k) = \sqrt{\frac{4\pi}{m_K}}\frac{1}{k^2+\beta^2}$$

Separable amplitudes:

off-shell :

$$T_{off} = \langle k_f \,|T_{sep}|\, k\rangle = G(k_f)\tau(m)G(k)$$

on-shell :

$$T_{on} = \langle k_f \,|T_{sep}|\, k_f\rangle = G(k_f)\tau(m)G(k_f)$$

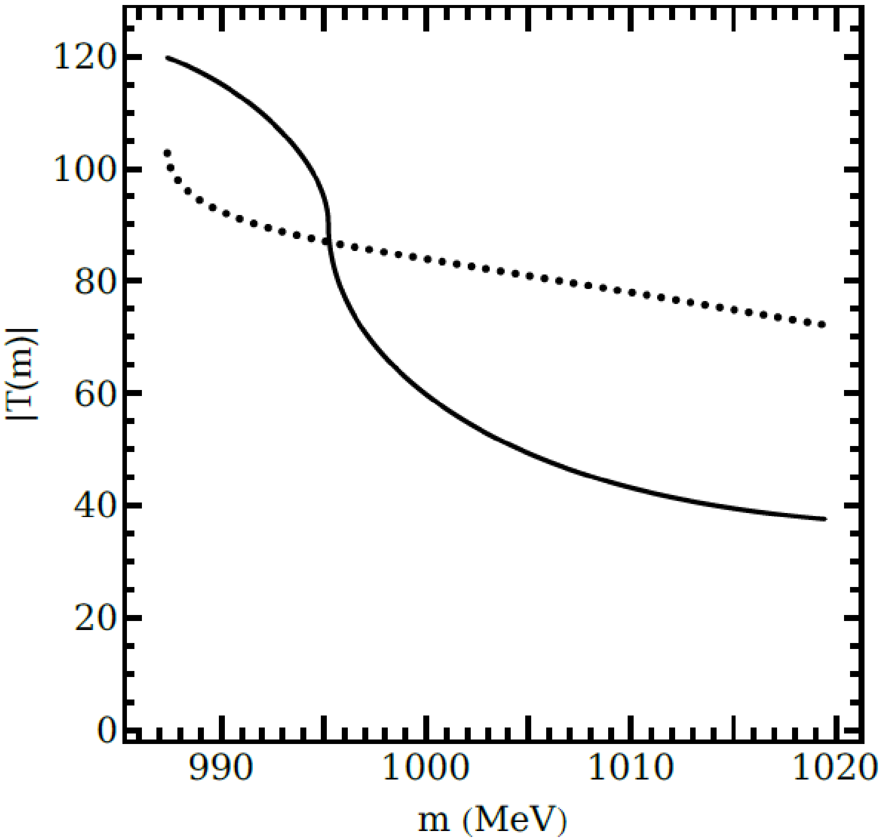
The function g(k)

$$g(k) = \frac{T_{off}}{T_{on}} = \frac{k_f^2+\beta^2}{k^2+\beta^2}$$

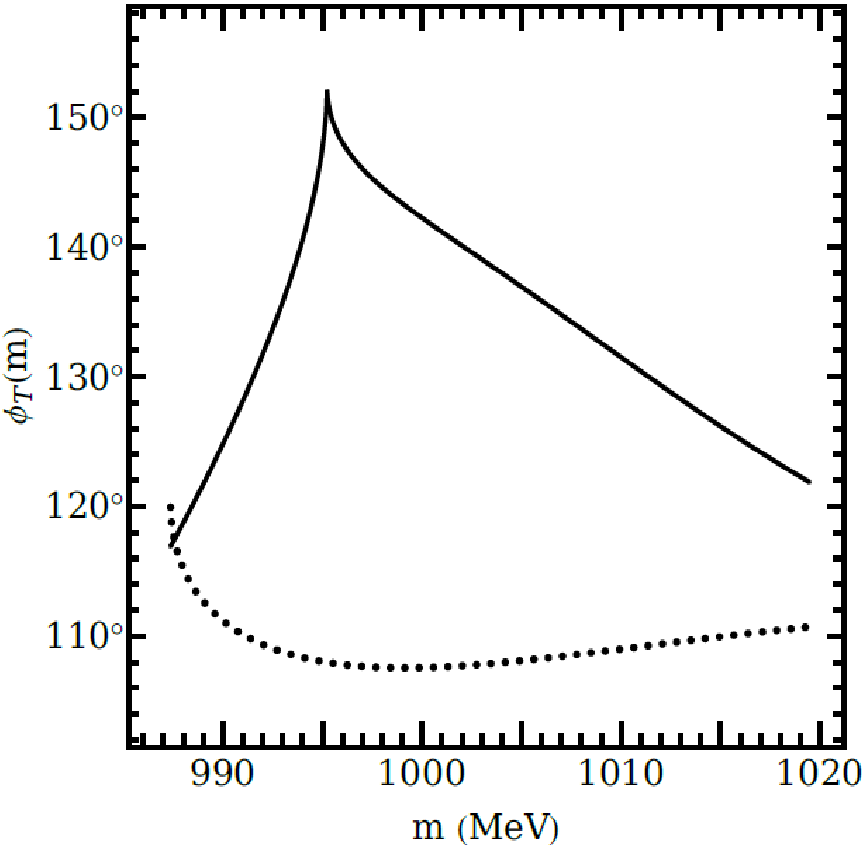
β= 1.5 GeV for isospin 0

K^+K^- scattering and transition amplitudes

modulus



phase

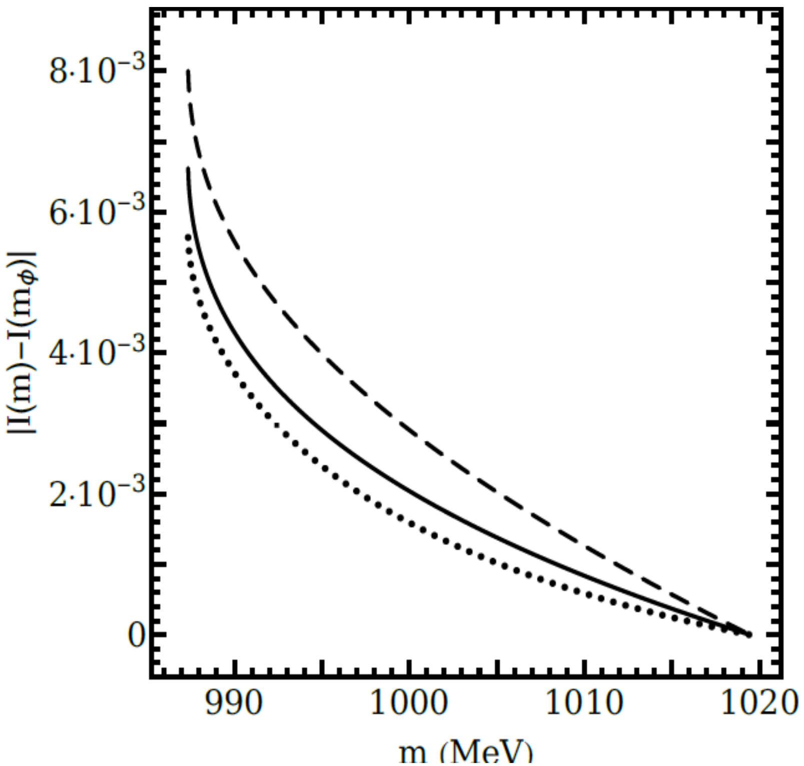


dotted line : $K^+K^- \to K^+K^-$ elastic amplitude

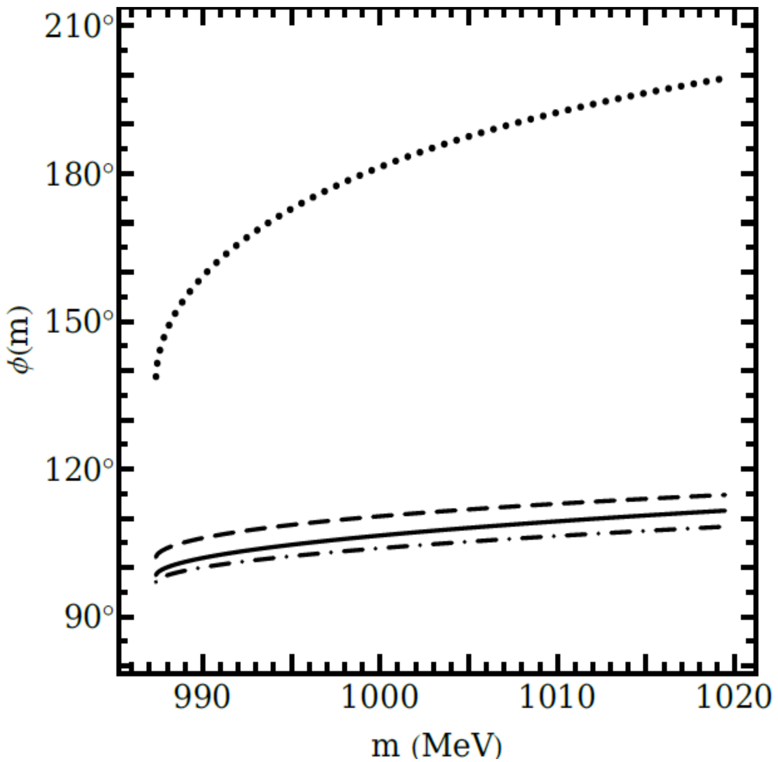
solid line : $K^+K^- \to K^0\overline{K}^0$ transition amplitude

The kaon loop function $I(m)-I(m_\phi)$

modulus



phase



solid line: $g(k)=\dfrac{k_f^2+\beta^2}{k^2+\beta^2}$ and $k_{cut}=1\,\mathrm{GeV}$

dashed line: $g(k)=\dfrac{(k_f^2+\mu^2)^2}{(k^2+\mu^2)^2}$, $\mu=141\,\mathrm{MeV}$

dotted line: $g(k)=\dfrac{\mu^4}{(k^2+\mu^2)^2}$, $\mu=141\,\mathrm{MeV}$

dashed-dotted line: $g(k)\equiv 1$ (point-like case)

Physical observables

Matrix elements squared for the processes $e^+ e^- \to K^+ K^- \gamma$ and $e^+ e^- \to K^0_S K^0_S \gamma$ are proportional to:

$$U(m) = (\frac{e^3}{s})^2 \, | F_K(s) |^2 \, | I(m) - I(m_\phi) |^2$$

Photon angular distribution in the $e^+ e^-$ center-of-mass frame:

$$\frac{d\sigma}{d\cos\theta_\gamma} \propto (1 + \cos^2 \theta_\gamma)$$

Effective mass distribution for the process $\mathbf{e^+ e^- \to K^+ K^- \gamma}$

$$\frac{d\sigma}{dm} \propto m^2 k_f U(m) \, | T_{K^+K^-}(m) |^2$$

Effective mass distribution for the process $\mathbf{e^+ e^- \to K^0_S K^0_S \gamma}$

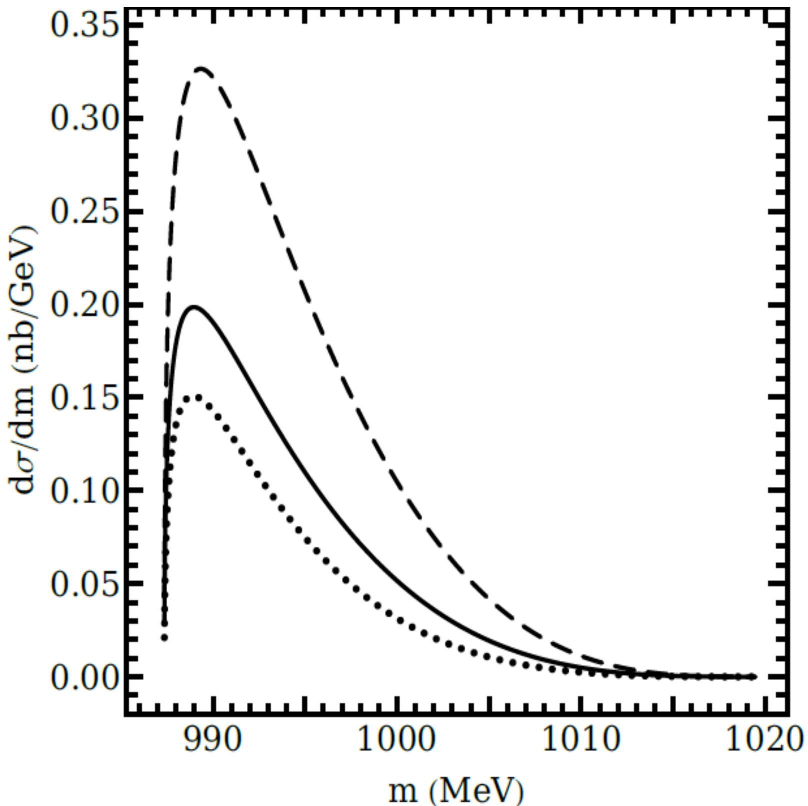
$$\frac{d\sigma}{dm} \propto m^2 k_f U(m) \, | T_{K^+K^- \to K^0 \overline{K}^0}(m) |^2$$

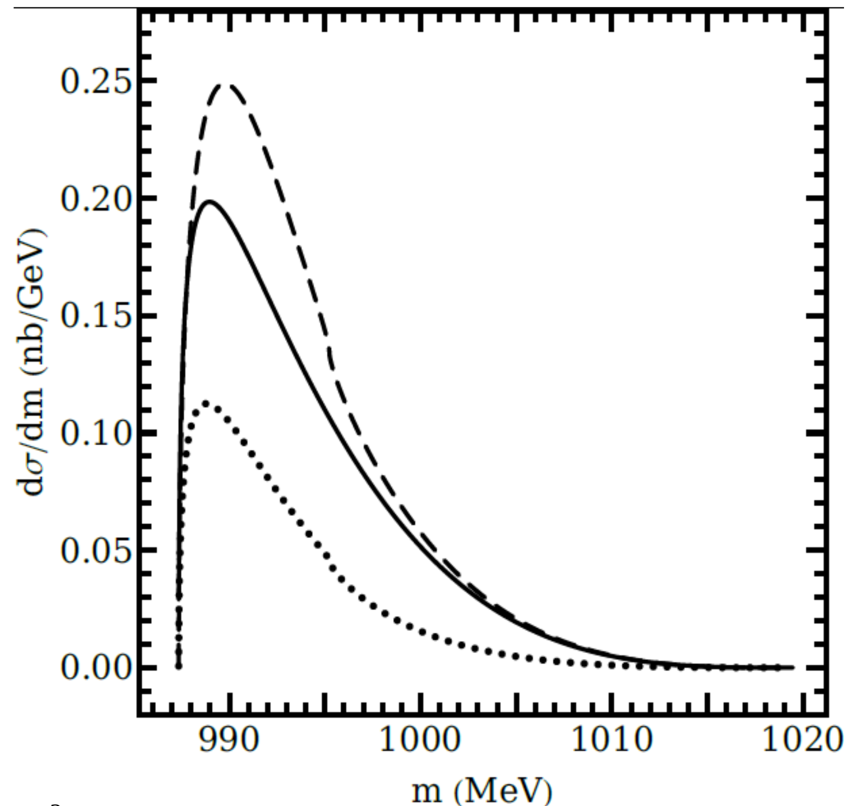
Branching fractions

$$Br(\phi \to K^+ K^- \gamma) = \sigma(e^+ e^- \to K^+ K^- \gamma, \ \mathrm{s} \approx m_\phi^2) / \sigma(e^+ e^- \to \phi)$$

$$Br(\phi \to K^0 \overline{K}^0 \gamma) = \sigma(e^+ e^- \to K^0 \overline{K}^0 \gamma, \ \mathrm{s} \approx m_\phi^2) / \sigma(e^+ e^- \to \phi)$$

Effective mass distributions for $e^+ e^- \rightarrow K^+K^-\gamma$





solid line: $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and $k_{cut} = 1\,\mathrm{GeV}$

dashed line: $g(k) = \dfrac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$

dotted line: $g(k) = \dfrac{\mu^4}{(k^2 + \mu^2)^2}$, $\mu = 141\,\mathrm{MeV}$

dashed line: no-structure model of Isidori et al. (2006)

dotted line: kaon-loop model of Achasov and Gubin (2001)

Transitions into pseudoscalar meson pairs P_1 P_2

Generalization of the model for the process $e^+ e^- \to K^+ K^- \gamma$ to $\mathbf{e^+ e^- \to P_1 P_2 \gamma}$

Inelastic **$K^+ K^- \to P_1 P_2$ amplitude:**

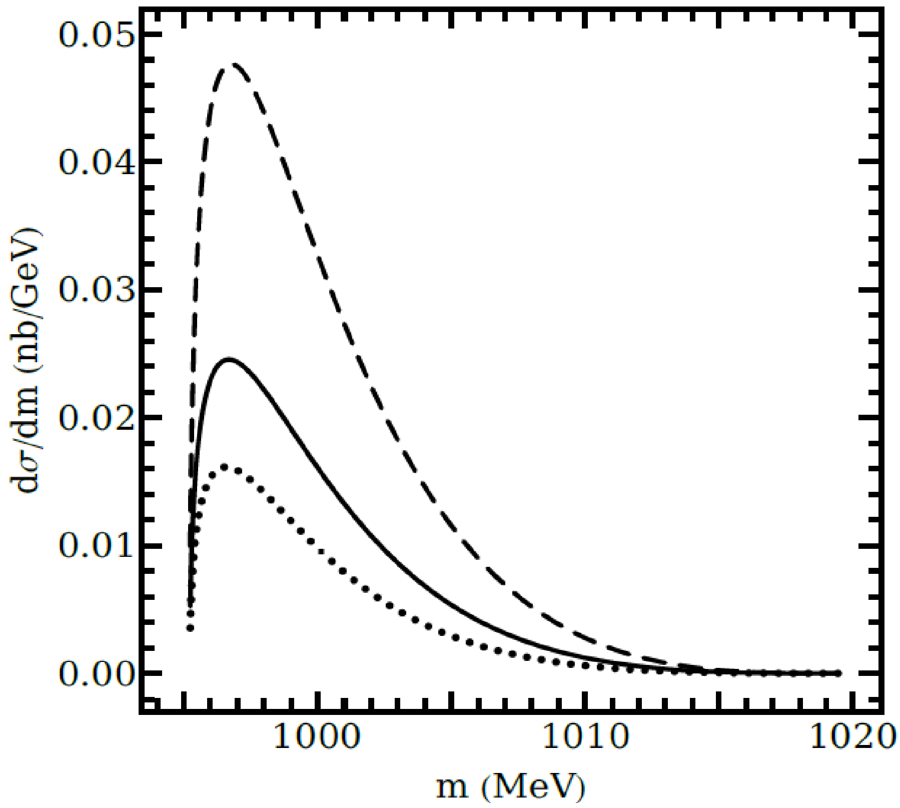
$$T_{K^+K^- \to P_1 P_2 \gamma} = \frac{4\pi\, m}{i\sqrt{k_f k_{12}}}\left(S_{K^+K^- \to P_1 P_2 \gamma} - \delta_{K^+K^-,\, P_1 P_2}\right)$$

k_{12}= relative P_1P_2 momentum

Application of the **unitary S- matrix** to several coupled channels is possible:

1. $e^+e^- \to \pi^+\pi^- \gamma$
2. $e^+e^- \to \pi^0\pi^0 \gamma$
3. $e^+e^- \to \pi^0 \eta \gamma$
4. $e^+e^- \to K^0 \overline{K}^0 \gamma$
5. $e^+e^- \to K^+ K^- \gamma$

Effective mass distributions for $e^+ e^- \to K^0 \bar{K}^0 \gamma$



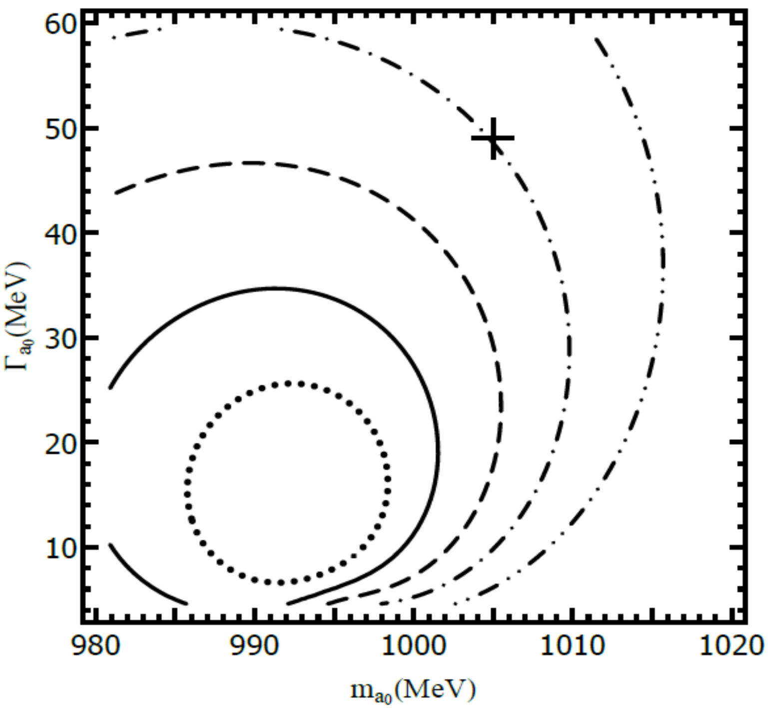
1. solid line: $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$, $\beta = 1.5\,\text{GeV}$ (for I = 0) and $k_{cut} = 1\,\text{GeV}$
2. dotted line: $g(k) = \dfrac{\mu^4}{(k^2 + \mu^2)^2}$, $\mu = 141\,\text{MeV}$, $g(0) = 1$
3. dashed line: $g(k) = \dfrac{(k_f^2 + \mu^2)^2}{(k^2 + \mu^2)^2}$, $\mu = 141\,\text{MeV}$, $g(k_f) = 1$

Total cross sections and branching fractions

	$\sigma_{e^+e^- \to K^+K^-\gamma}(pb)$	$\mathrm{Br}(\phi \to K^+K^-\gamma)$	
1.	1.85	$4.47 \cdot 10^{-7}$	
2.	1.29	$3.10 \cdot 10^{-7}$	
3.	3.37	$8.13 \cdot 10^{-7}$	
4.	2.29	$5.51 \cdot 10^{-7}$	no-structure model NS
5.	0.85	$2.05 \cdot 10^{-7}$	kaon-loop model KL

	$\sigma_{e^+e^- \to K^0\overline{K}^0\gamma}(pb)$	$\mathrm{Br}(\phi \to K^0\overline{K}^0\gamma)$
1.	0.167	$4.03 \cdot 10^{-8}$
2.	0.102	$2.46 \cdot 10^{-8}$
3.	0.338	$8.16 \cdot 10^{-8}$

The $\phi \to K^0 \bar{K}^0 \gamma$ branching fraction and the $a_0(980)$ pole position



Branching fraction of the decay $\phi \to K^0 \bar{K}^0 \gamma$

for $g(k) = \dfrac{k_f^2 + \beta^2}{k^2 + \beta^2}$ and $k_{cut} = 1\,\mathrm{GeV}$:

dotted curve	$1 \cdot 10^{-8}$,
solid curve (KLOE limit)	$1.9 \cdot 10^{-8}$,
dashed curve	$3. \cdot 10^{-8}$,
dotted - dashed curve	$4. \cdot 10^{-8}$,
double dotted - dashed curve	$5. \cdot 10^{-8}$

$\Gamma_{a_0(980)}$ = resonance width

$m_{a_0(980)}$ = resonance mass

The values of the branching fraction depend on the **resonance pole position**.

Conclusions

1. A theoretical model of the reactions $e^+ e^- \to K^+ K^- \gamma$ and $e^+ e^- \to K^0 \bar{K}^0 \gamma$ has been formulated.
2. The strong interaction between kaons is taken into account.
3. The elastic $K^+ K^-$ and the transition $K^+ K^- \to K^0 \bar{K}^0$ **amplitudes** in a **general form** can be used.
4. Numerical results for the $K \bar{K}$ effective mass distributions, for the total reaction cross sections and the radiative ϕ decays are given.
5. The model can be generalized to treat other coupled channel reactions with **two pseudoscalar mesons** in the final state.
6. Measurements of the $e^+ e^- \to K^+ K^- \gamma$ process can provide a valuable information about the pole positions of the $a_0(980)$ and $f_0(980)$ resonances.