Various types of theoretical uncertainties by the examples of the elastic nucleon-deuteron scattering observables

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Introduction

- Theoretical uncertainties in few-nucleon sector
- The OPE-Gaussian potential

Results

- The OPE-Gausian vs the AV18
- The OPE-Gausian: propagation of statistical errors to 3N observables
- Comparison of statistical errors with other theoretical uncertainties

Summary

Theoretical uncertainties in few-nucleon sector

- Arising from our ignorance of (effective) NN interaction. Various models like AV18, CD-Bonn, chiral models, ...
- Arising from uncertainty of free parameters of NN forces
  - origin in uncertainties of NN data (statistical errors)
- Chiral models
  - truncation errors
  - regularization dependence
- Theoretical methods introduce their own uncertainties (small in the Faddeev approach for Nd scattering but important e.g. in the No-Core Shell Models used for nuclear structure calculations) and finite computational accuracy.

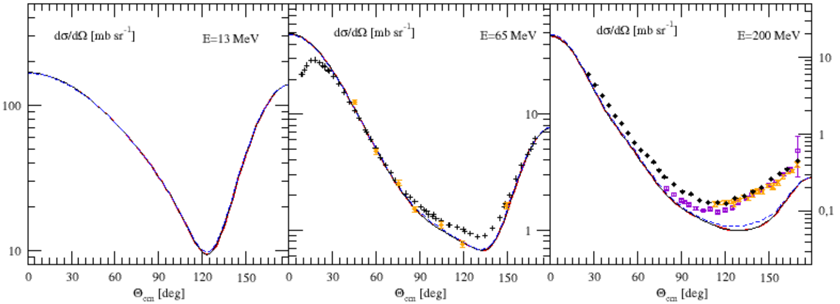
The OPE-Gaussian potential

R. Navarro Pérez, J. E. Amaro, and E. Ruiz Arriola, Phys. Rev. C 89 (2014) 064006

- The OPE-Gaussian interaction can be decomposed as
$$V(\vec{r}) = V_{short}(\vec{r})\theta(r_c - r) + V_{long}(\vec{r})\theta(r - r_c)$$
where r_c = 3 fm.
- The long range part has two parts: OPE part and electromagnetic corrections
$$V_{long}(\vec{r}) = V_{OPE}(\vec{r}) + V_{em}(\vec{r})$$
- The short range part is
$$V_{short}(\vec{r}) = \sum_{n=1}^{18} \hat{O}_n \left[\sum_{i=1}^{4} V_{i,n} F_i(r)\right], \; F_i(r) = e^{-r^2/(2a_i^2)}$$
where $O_{i,n}$ are the same operators as in the AV18 + three additional operators; $V_{i,n}$ and a_i are unknown coefficients to be determined from NN data, F_i are radial Gaussian functions.
- The OPE-Gaussian force can be seen as a remastered the AV18 interaction.
- **42 parameters and their covariance matrix** are fixed from "3σ self-consistent" database.
- We sample 50 sets of parameters and use Faddeev formalism to calculate observables.

Propagation of statistical errors for the differential cross section

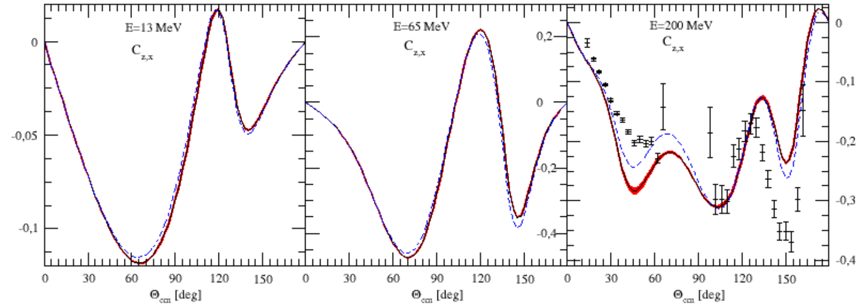
- Knowing central values and covariance matrix of potential parameters, we sample sets of parameters and estimate uncertainties in 3N calculations.
  - Black solid curve : OPE-Gaussian (central values of parameters)
  - Blue dashed curve : AV18
  - Red band : 50 predictions with sampled potential parameters



- The resulting uncertainty is very small. There are only small differences between the OPE-Gaussian and the AV18 predictions.
- The observed discrepancy with the data is mainly due to three-nucleon force neglected in our calculations.

Propagation of statistical errors for the spin correlation coefficients

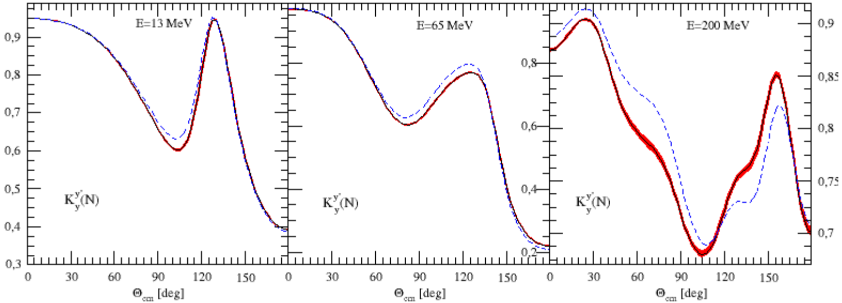
- Black solid curve : OPE-Gaussian (central values of parameters)
- Blue dashed curve : AV18
- Red band : 50 predictions with sampled potential parameters



- The resulting uncertainty becomes bigger at higher energies. Discrepancy between the OPE-Gaussian and the AV18 predictions is observed.

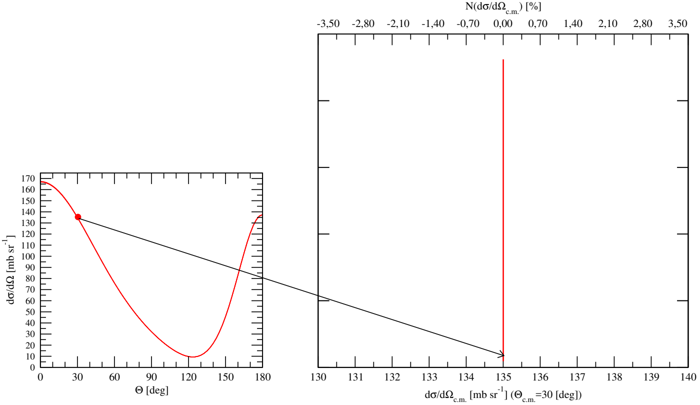
Propagation of statistical errors for the spin transfer coefficients

- Black solid curve : OPE-Gaussian (central values of parameters)
- Blue dashed curve : AV18
- Red band : 50 predictions with sampled potential parameters



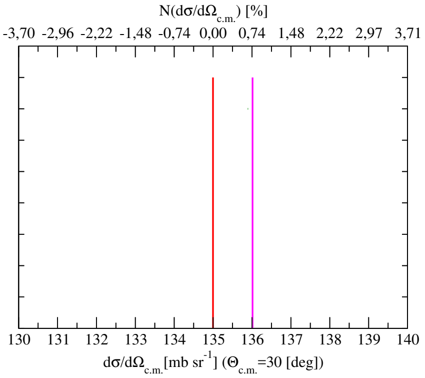
- The resulting uncertainty becomes bigger at higher energies. Discrepancy between the OPE-Gaussian and the AV18 predictions is observed.

Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{o}$ and at $E = 13$ MeV



The predicted value of the differential cross section is given at the horizontal bottom axis and the vertical line show position of prediction based on the OPE-Gaussian force at central values of parameters (the length of this line has no meaning). The top horizontal axis shows the relative difference (expressed in percents) in respect to the OPE-Gaussian prediction.

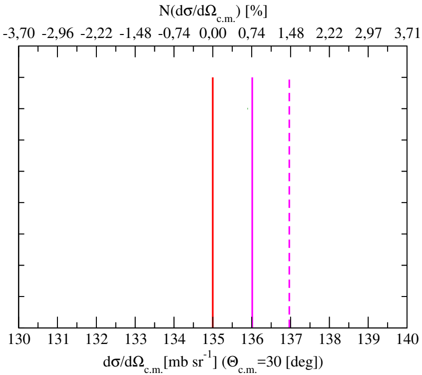
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\circ}$ and at $E = 13$ MeV



E_{lab} = 13 MeV, $\Theta_{c.m.}$ = 30°

The OPE-Gaussian result are represented by vertical red line, the Bochum-Bonn (chiral approach at the N^4LO) R = 0.9 fm result.

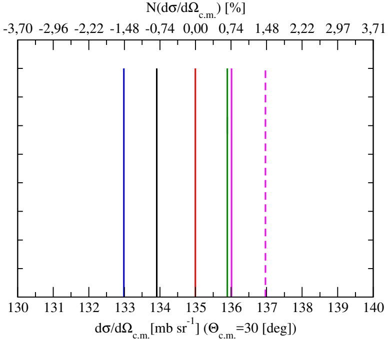
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\mathrm{o}}$ and at $E = 13$ MeV



$E_{lab} = 13$ MeV, $\Theta_{c.m.} = 30^{\circ}$

The vertical lines shows positions of the OPE-Gaussian result, the Bochum-Bonn N^4LO $R = 0.9$ fm and the Bochum-Bonn N^4LO $R = 1.0$ fm results.

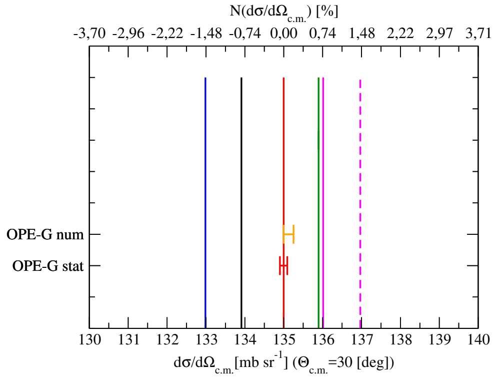
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\mathrm{o}}$ and at $E = 13$ MeV



$E_{lab} = 13$ MeV, $\Theta_{c.m.} = 30^{\circ}$

The vertical lines shows positions of the cross section obtained with the OPE-Gaussian potential, the Bochum-Bonn at N^4LO $R = 0.9$ fm force ones by the magenta solid line and the Bochum-Bonn N^4LO $R = 1.0$ fm force ones by the magenta dashed line, Idaho $\Lambda = 500$ MeV, the AV18 force and the CD-Bonn potential.

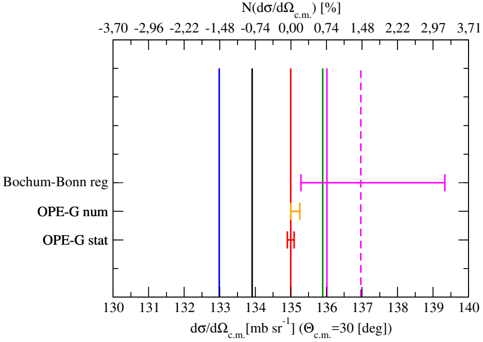
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\circ}$ and at E = 13 MeV



E_{lab} = 13 MeV, $\Theta_{c.m.}$ = 30°

The horizontal lines represent magnitudes of various theoretical uncertainties starting from the bottom: statistical error for the OPE-Gaussian model, numerical error is a difference between the OPE-Gaussian predictions based on the j_{max} = 5 and the j_{max} = 4 calculations.

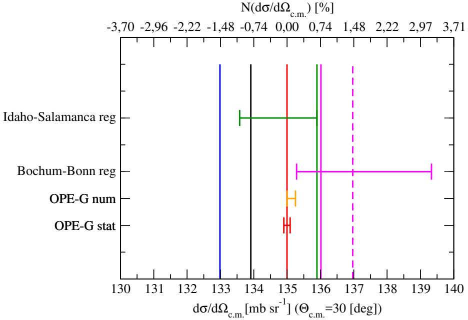
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\circ}$ and at E = 13 MeV



E_{lab} = 13 MeV, $\Theta_{c.m.}$ = 30°

The horizontal lines represent magnitudes of various theoretical uncertainties starting from the bottom: **statistical error** for the OPE-Gaussian model, **numerical error** is difference between the OPE-Gaussian predictions based on the j_{max} = 5 and the j_{max} = 4 calculations, regulator dependence for **the Bochum-Bonn N^4LO force in the range R = 0.8 - 1.2 fm .**

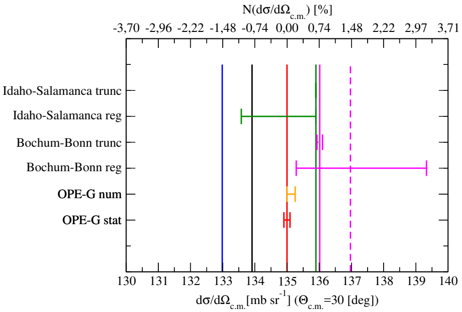
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{o}$ and at E = 13 MeV



E_{lab} = 13 MeV, $\Theta_{c.m.}$ = 30°

The magnitudes of various theoretical uncertainties: **statistical error** for the OPE-Gaussian model, **numerical error** is difference between the OPE-Gaussian predictions based on the j_{max} = 5 and the j_{max} = 4 calculations (**orange line**) , regulator dependence for **the Bochum-Bonn N^4LO force in the range R = 0.9 - 1.2 fm** , regulator dependence for **the Idaho-Salamanca N^4LO force in the range Λ = 450 MeV - 550 MeV potential.**

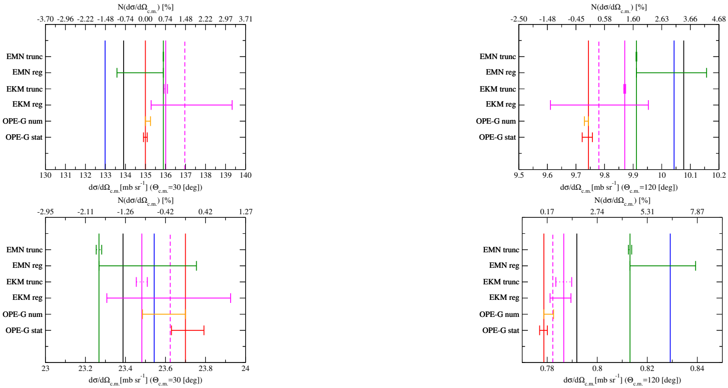
Theoretical uncertainties for the differential cross section at given scattering angle $\theta = 30^{\mathrm{o}}$ and at $E = 13$ MeV



$E_{lab} = 13$ MeV, $\Theta_{c.m.} = 30^{\circ}$

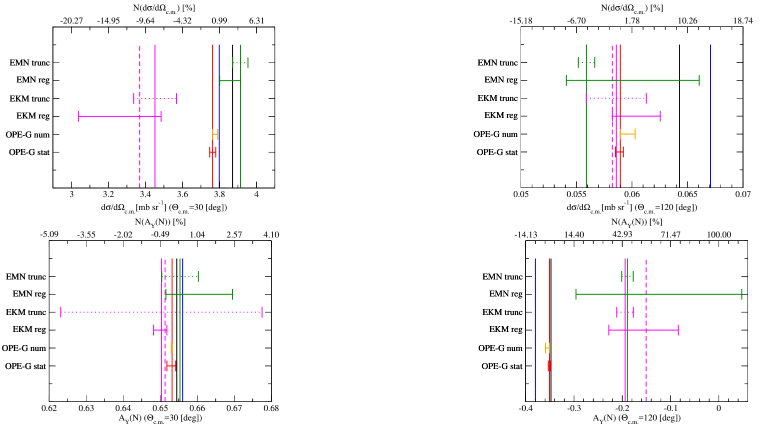
The magnitudes of various theoretical uncertainties: **statistical error** for the OPE-Gaussian model, **numerical error** is difference between the OPE-Gaussian predictions based on the $j_{max} = 5$ and the $j_{max} = 4$ calculations, the regulator dependences for **the Bochum-Bonn N^4LO force in the range R = 0.8 - 1.2 fm** and for **the Idaho-Salamanca N^4LO force in the range Λ = 450 MeV - 550 MeV potential** , the truncation errors for **the Bochum-Bonn N^4LO at R = 0.9 fm (the dashed line)** and for **the Idaho-Salamanca N^4LO at Λ = 500 MeV.**

Theoretical uncertainties for the differential cross section at given scattering angle θ = 30° and at E = 13 MeV



The *Nd* elastic scattering differential cross section and various theoretical uncertainties at scattering angles: $\theta_{c.m} = 30^\circ$ (1st column) , $\theta_{c.m} = 120^\circ$ (2st column); and at scattering energies: E = 13 MeV (the upper row), E = 65 MeV (the bottom row). The vertical lines shows position of the cross section obtained with **the AV18 force, the OPE-Gaussian force, the Idaho-Salamanca Λ = 500 *MeV* model, the CD-Bonn potential, the Bochum-Bonn N^4LO R = 0.9 fm force (solid line)** and **the Bochum-Bonn N^4LO R = 1.0 fm force (dashed line)** . The horizontal lines shows the width of the estimated errors for various the NN models.

Theoretical uncertainties for the differential cross section and the nucleon vector analyzing power at given scattering angle for E_{lab} = 200 MeV



The vertical lines shows position of the cross section (the upper row) and predictions of the neutron vector analyzing power (the bottom row) obtained with **the AV18 force, the OPE-Gaussian force, the Idaho-Salamanca Λ = 500 *MeV* model, the CD-Bonn potential, the Bochum-Bonn N^4LO R = 0.9 fm force (solid line)** , and **the Bochum-Bonn N^4LO R = 1.0 fm force (dashed line).** The horizontal lines shows the width of the estimated errors for various the NN models.

Conclusions for the theoretical uncertainties

- The dominant theoretical uncertainties arise from using various models of the NN interaction.
- For chiral models the dependence on regularization parameters dominates. Truncation errors became relatively big at higher energies.
- Two chiral models (the Bochum-Bonn and the Idaho-Salamanca) at N^4LO often disagree.
- All forces has the similar quality.
- In general, the theoretical uncertainties remain smaller than the experimental ones.

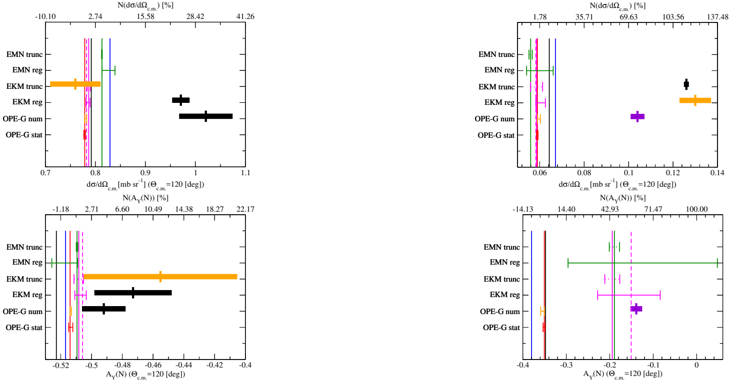
Summary

- The OPE-Gaussian force allows us to study a propagation of uncertainties of parameters of the NN interaction to 3N observables. We conclude that resulting uncertainty is smaller than dependence on interaction models both at low and at high energies.
- The OPE-Gaussian force delivers description of Nd elastic scattering of comparable quality as the AV18 model.
- The dominant uncertainty arises from using various models of nuclear interaction.
- The regulator dependence is relatively strong.
- Statistical and numerical uncertainties are less important.

Thanks to Dr R. Navarro Pérez and Prof. E. Ruiz Arriola for sets with potential parameters.

Thank you for your kind attention !

The differential cross section and the nucleon vector analyzing power compared with experimental results at given scattering angle for E_{lab} = 200 MeV



The vertical lines shows positions of predictions for the cross section (the upper row) and the neutron vector analyzing power (the bottom row) compared to the same models in the previous slides but supplemented by experimental points at angles near $\theta_{c.m.}$ = 120°. The vertical lines and thin horizontal lines are as in the previous slides, the thick boxes represent experimental data and their statistical errors.