R. Pavao, S. Sakai and E. Oset

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Study of the possible role of triangle singularities in $B^- \to D^{*0} \pi^- \pi^0 \eta$ and $B^- \to D^{*0} \pi^- \pi^+ \pi^-$

CSIC
Consejo Superior de Investigaciones Científicas

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Vniversitat de València

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Institut de Física Corpuscular

Motivation

1. Hadronic spectrum:
    1. Quark model
    2. Molecular states
    3. Multiquark states
    4. **Triangle Singularities (TS)**

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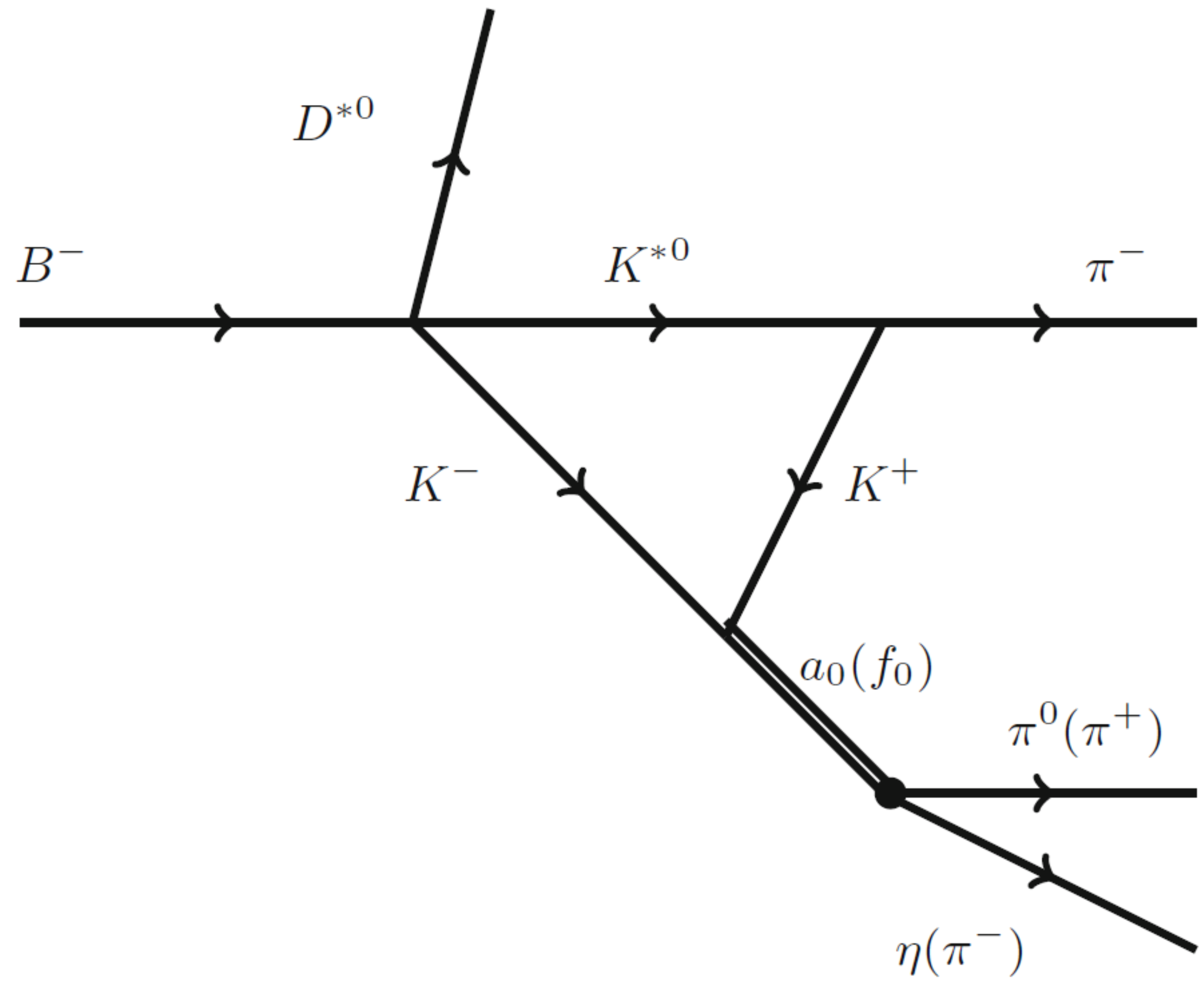
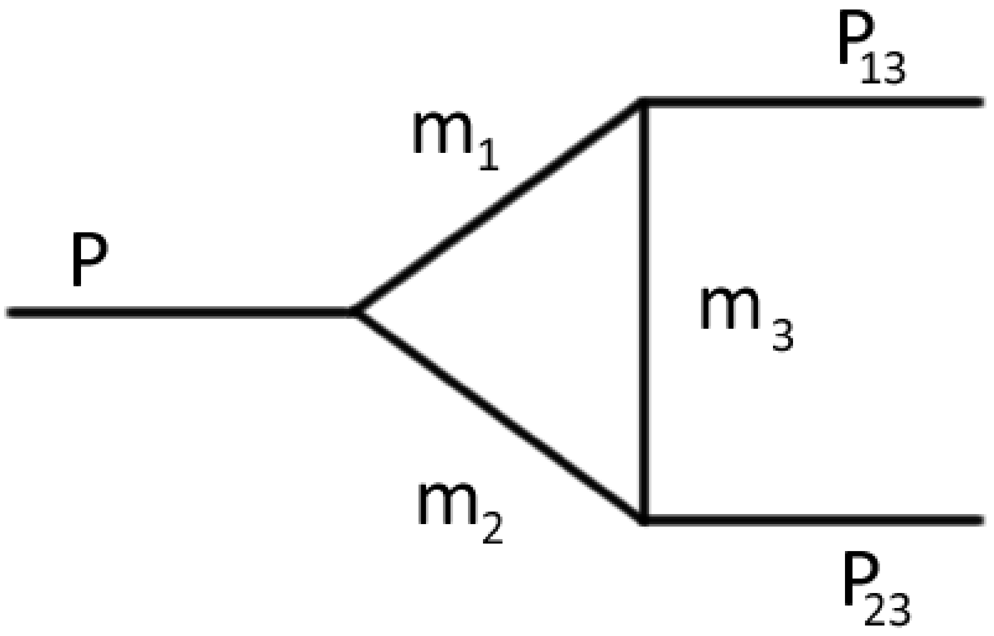
$B^- \rightarrow D^{*0} \pi^- \pi^0 \eta$ and **$B^- \rightarrow D^{*0} \pi^- \pi^+ \pi^-$**

X.H. Liu, M. Oka, Q. Zhao, Phys. Lett. B **753** (2016) 297


D^{*0}
B^-
K^{*0}
π^-
K^-
K^+
$a_0(f_0)$
$\pi^0(\pi^+)$
$\eta(\pi^-)$

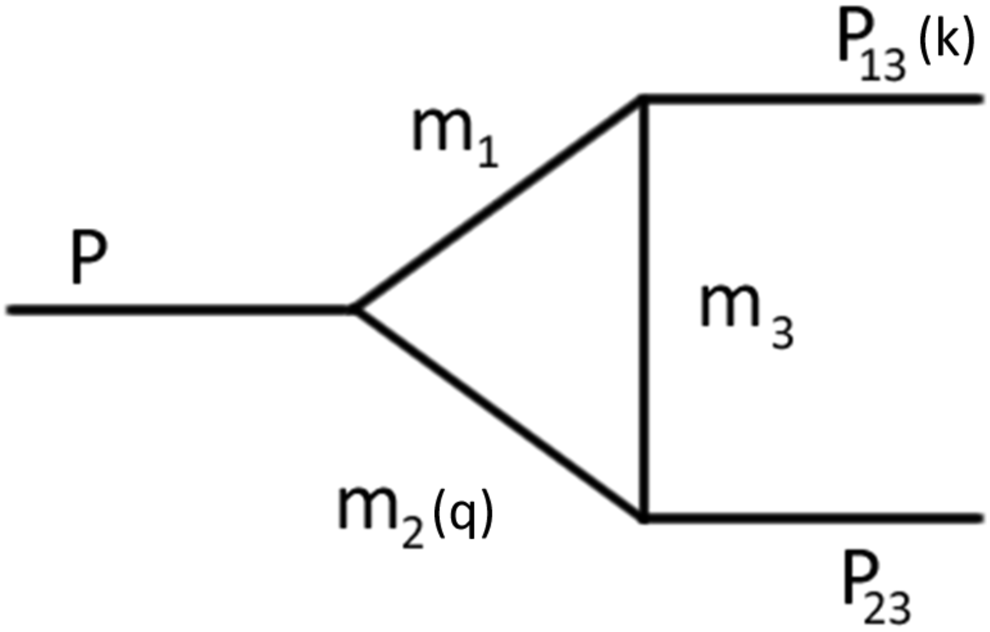
Formalism

Triangle Singularities



L.D.Landau, **Nucl. Phys. 13 (1959) 181**

Triangle Singularities



$$I = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_2^2 + i\epsilon)[(P-q)^2 - m_1^2 + i\epsilon][(P-q-k)^2 - m_3^2 + i\epsilon]}$$

$$I = \int \frac{d^3q}{(2\pi)^3} \frac{1}{8\omega^*\omega\omega'} \frac{1}{k^0 - \omega' - \omega^*} \frac{1}{P^0 + \omega + \omega' - k^0} \frac{1}{P^0 - \omega - \omega' - k^0 + i\epsilon} \times$$
$$\times \frac{\{2P^0\omega + 2k^0\omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]\}}{P^0 - \omega^* - \omega + i\epsilon}$$

$$\omega^* = \sqrt{m_1^2 + |\vec{q}|^2}$$

$$\omega = \sqrt{m_2^2 + |\vec{q}|^2}$$

$$\omega = \sqrt{m_3^2 + |\vec{q} + \vec{k}|^2}$$

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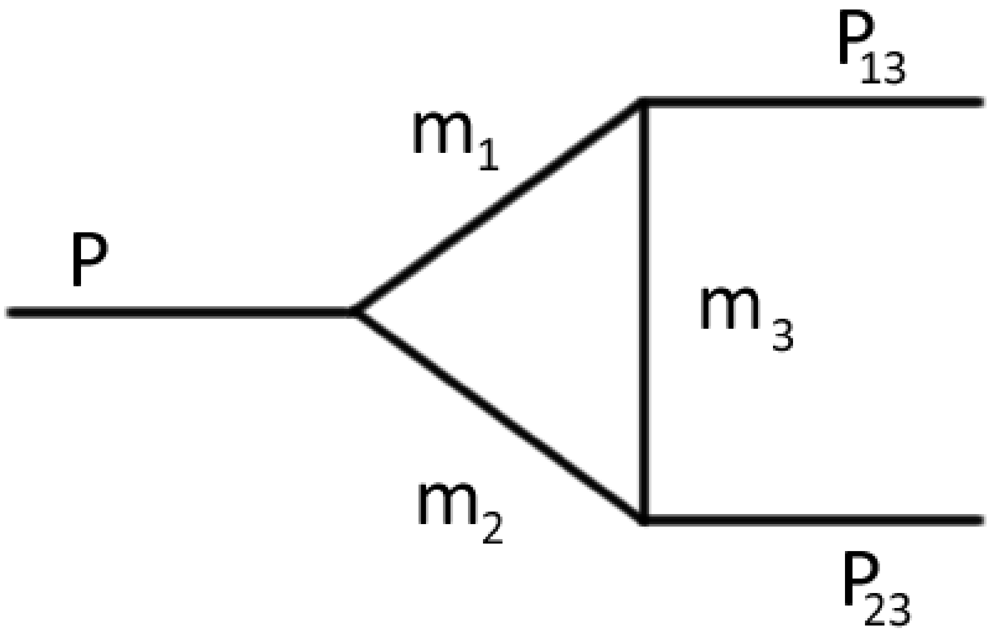
$$\omega = \sqrt{m_3^2 + |\vec{q} + \vec{k}|^2}$$

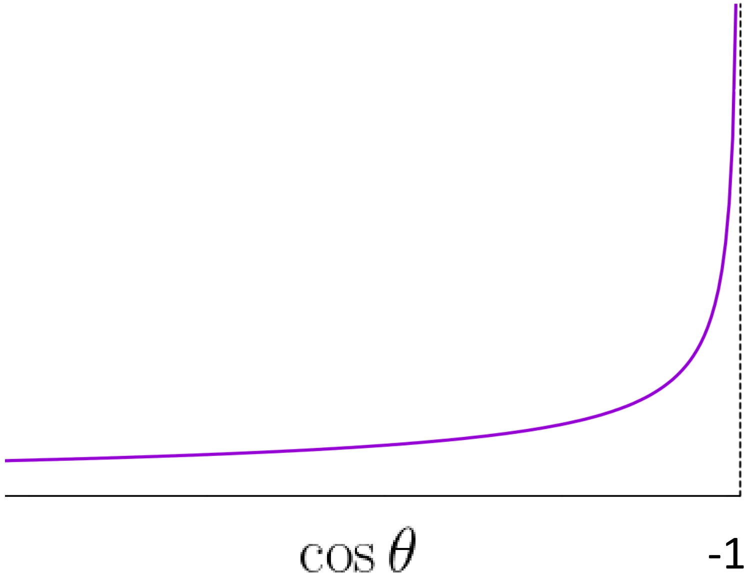
$$P^0 - \omega^*(\vec{q}) - \omega(\vec{q}) = 0 \rightarrow q_{\text{on}} + i\epsilon$$

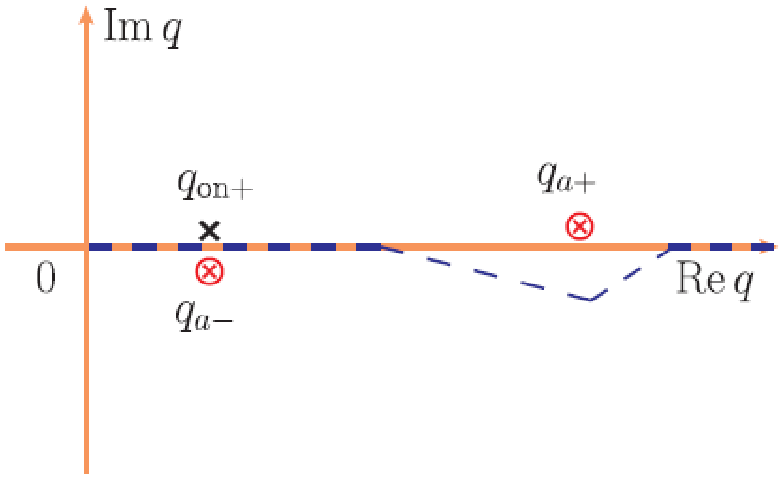
$$P^0 - k^0 - \omega(\vec{q}) - \omega'(\vec{q} + \vec{k}) = 0 \rightarrow q_{a-} - i\epsilon$$

M. Bayar, F. Aceti, F-K Guo and E. Oset, **Phys. Rev. D 94 (2016) 074039**

Triangle Singularities







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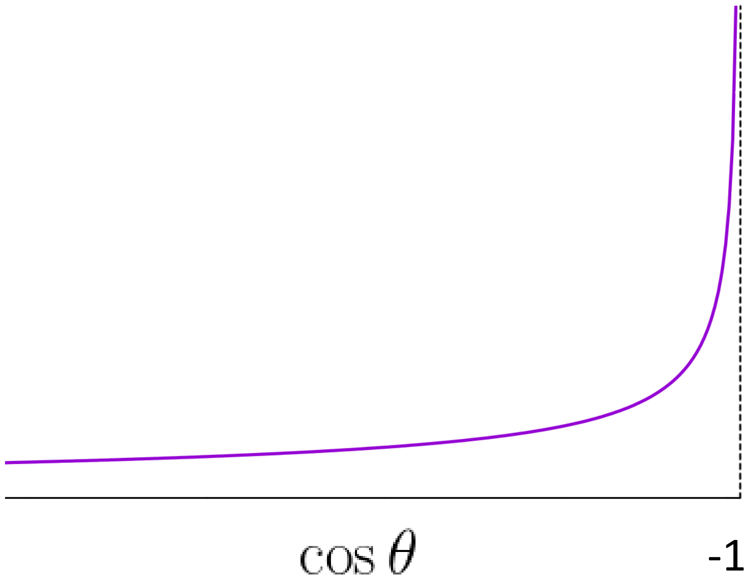

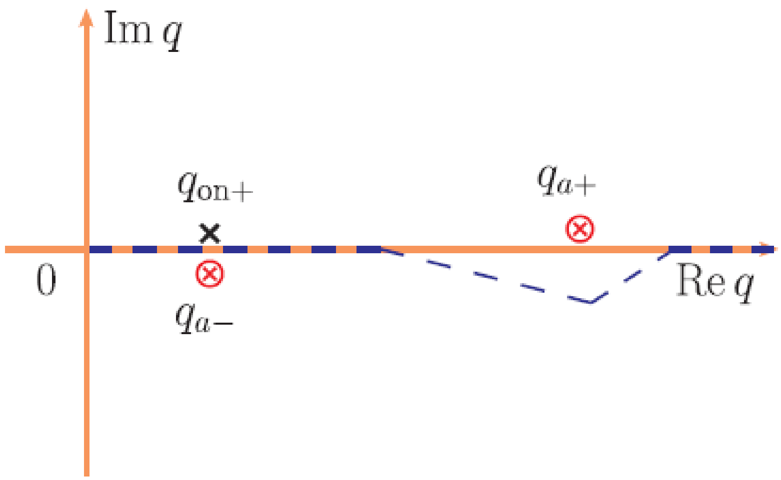

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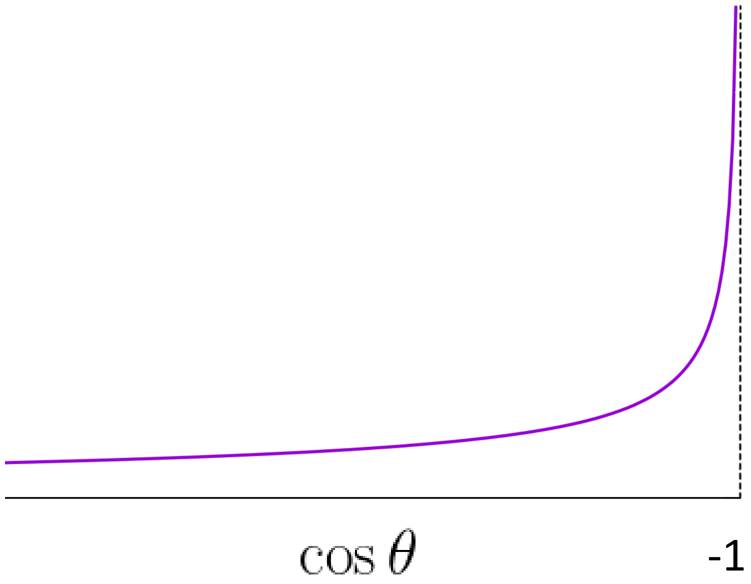
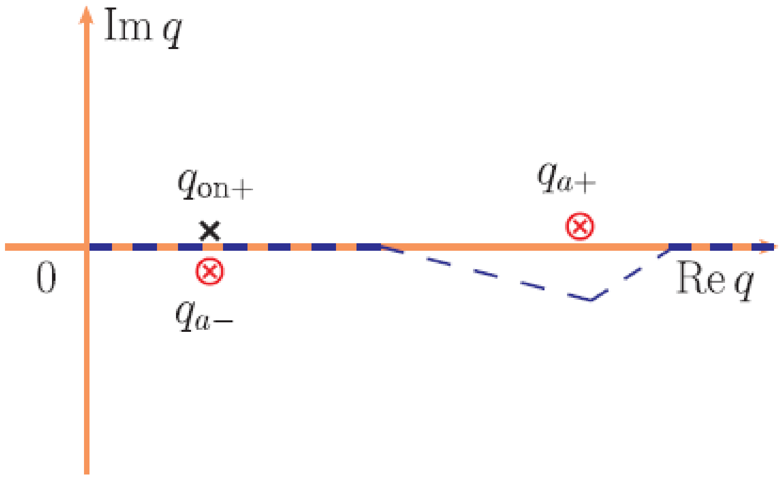
$$\lim_{\epsilon\rightarrow 0} (q_{\rm on+} - q_{a-}) = 0$$

M. Bayar, F. Aceti, F-K Guo and E. Oset, **Phys. Rev. D 94 (2016) 074039**





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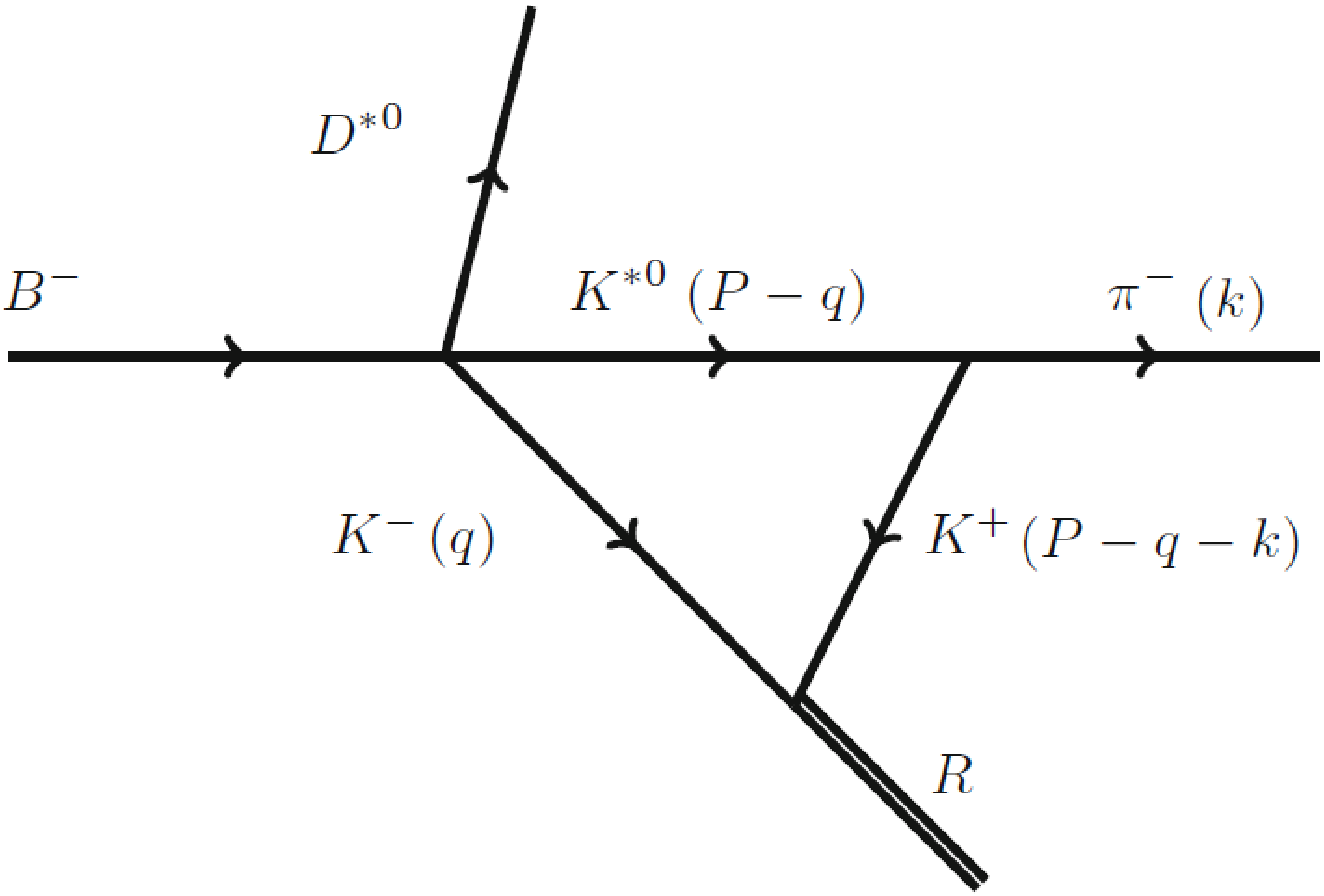
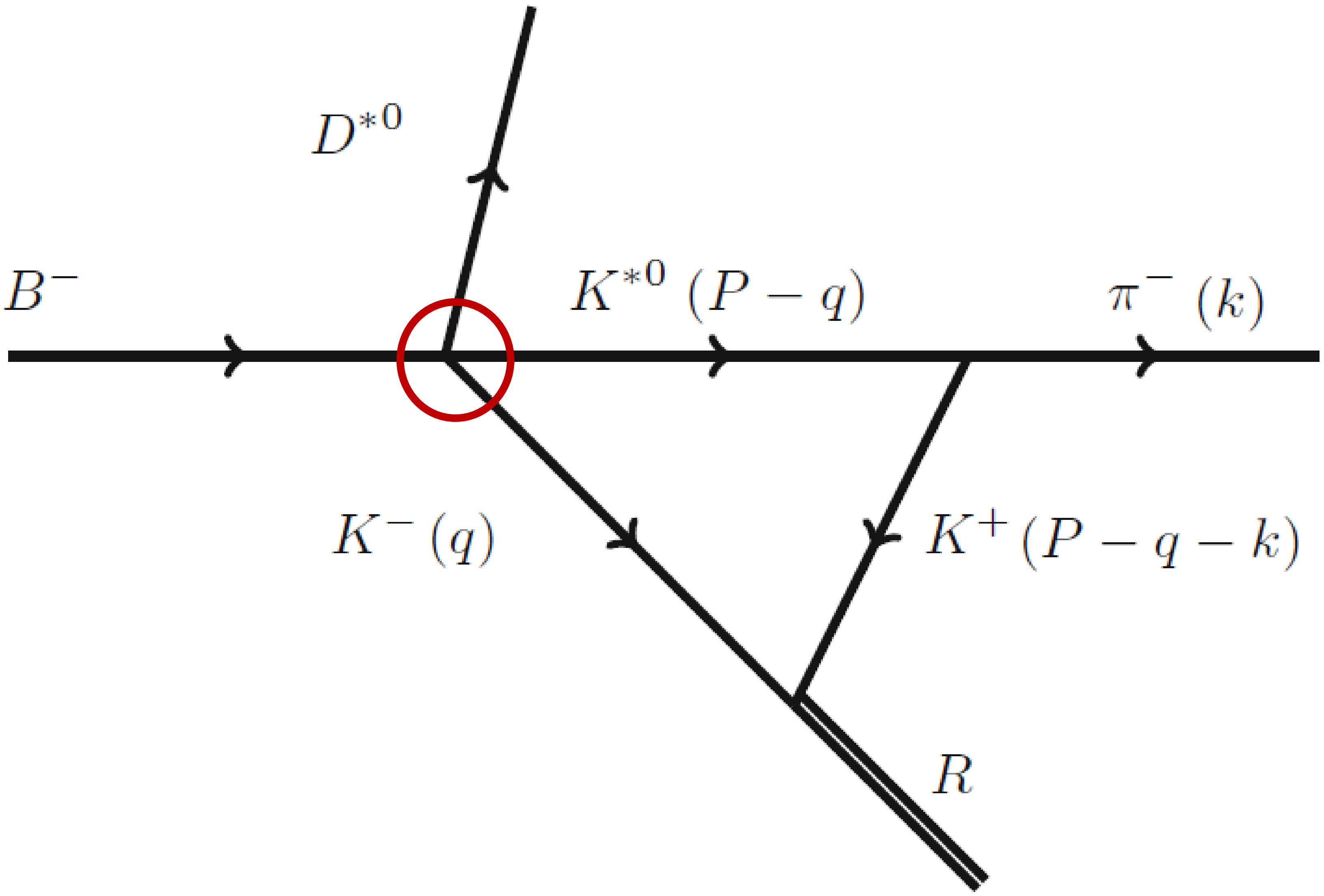
TS for $\boldsymbol{M_{inv}(\pi R)}$=**1418 MeV**

M. Bayar, F. Aceti, F-K Guo and E. Oset, **Phys. Rev. D 94 (2016) 074039**


D^{*0}
B^-
$K^{*0}\,(P-q)$
$\pi^-\,(k)$
$K^-\,(q)$
$K^+\,(P-q-k)$
R


D^{*0}
B^-
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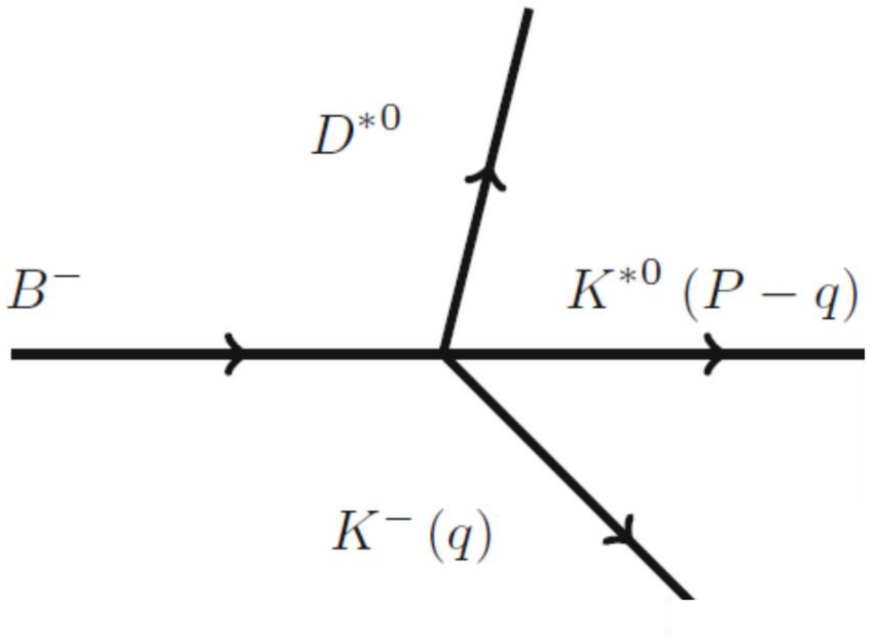
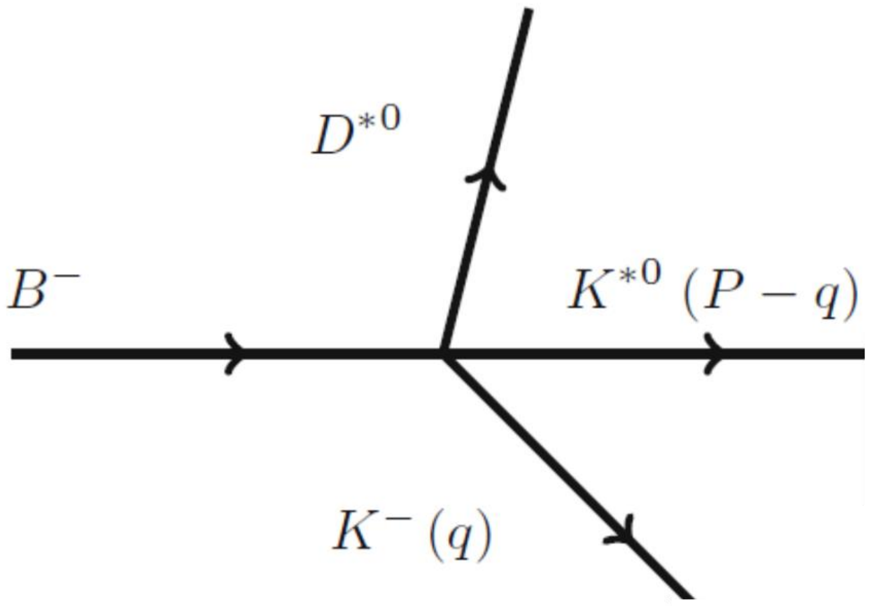
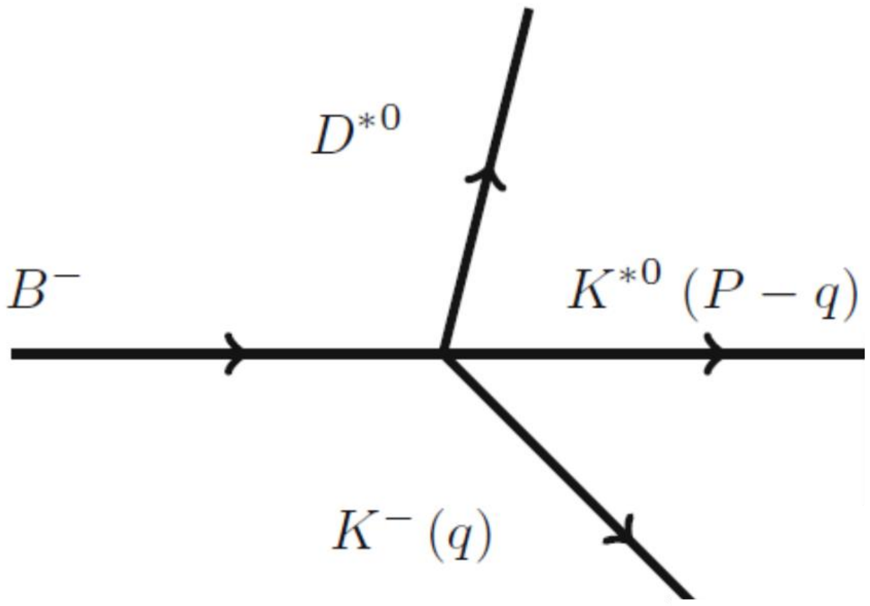
R



D^{*0}
B^-
$K^{*0}\,(P-q)$
$K^-\,(q)$



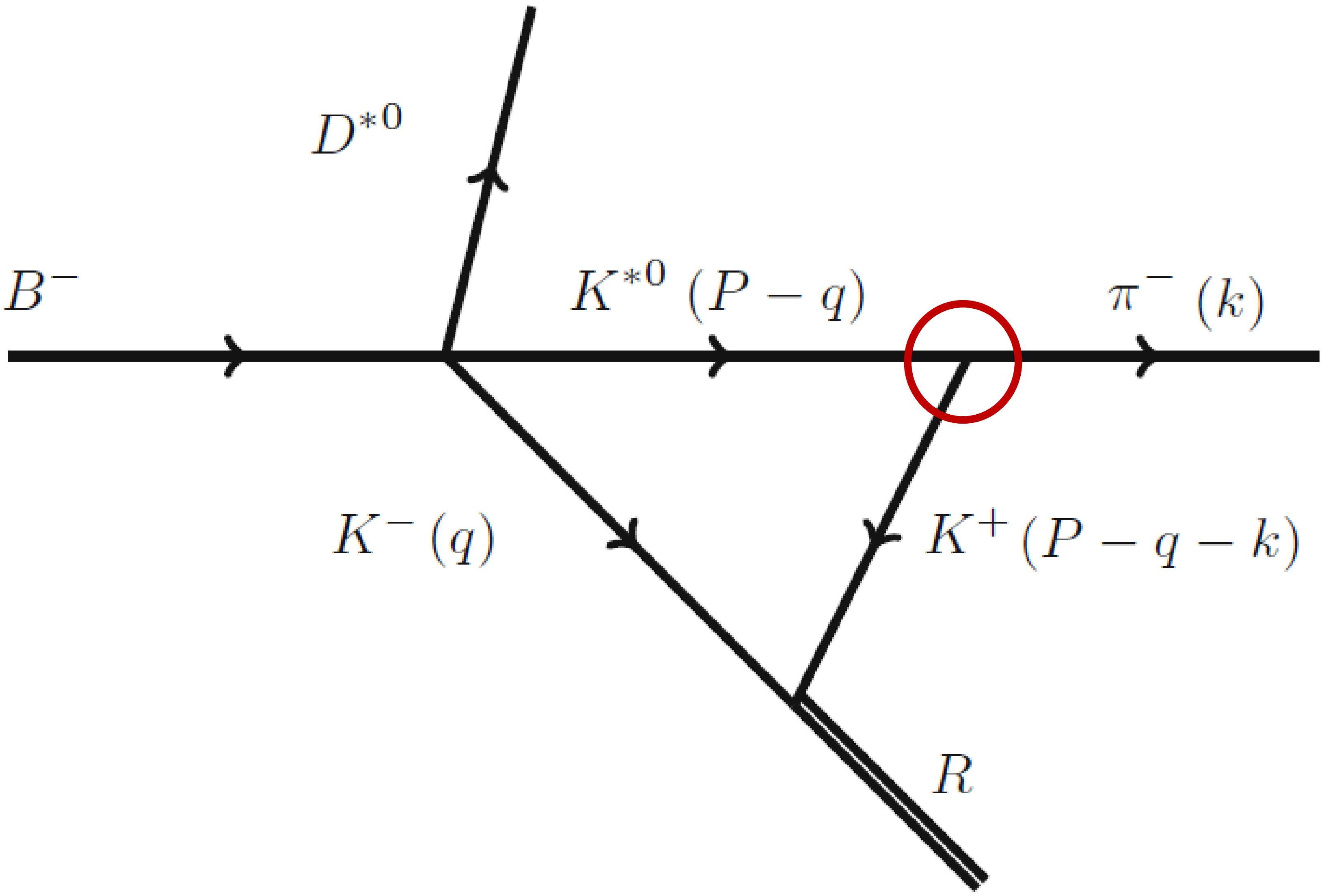
$$t_{B^- \to D^{*0} K^{*0} K^-} = C\ \epsilon_\mu(K^*)\epsilon^\mu(D^*)$$



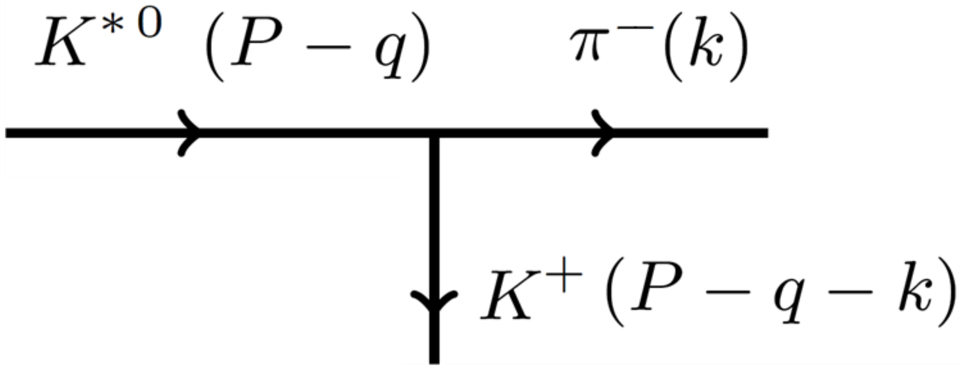
PDG

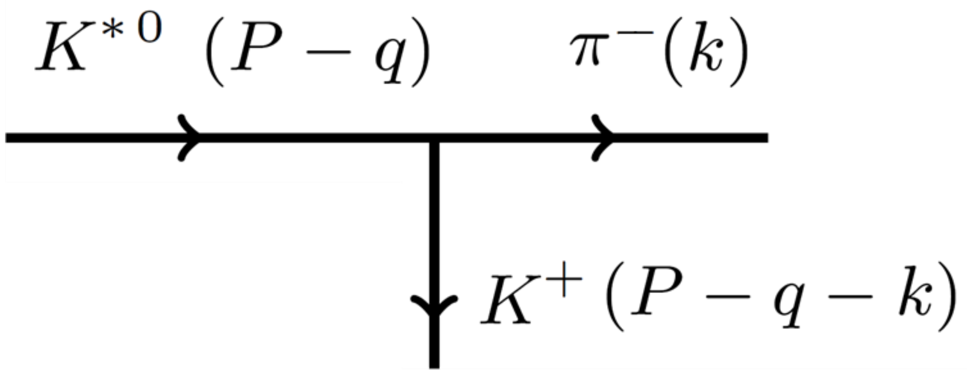
$$\mathrm{Br}(B^- \to D^{*0} K^{*0} K^-) = (1.5 \pm 0.4) \times 10^{-3}$$

$$t_{B^- \to D^{*0} K^{*0} K^-} = C\, \epsilon_\mu(K^*) \epsilon^\mu(D^*)$$



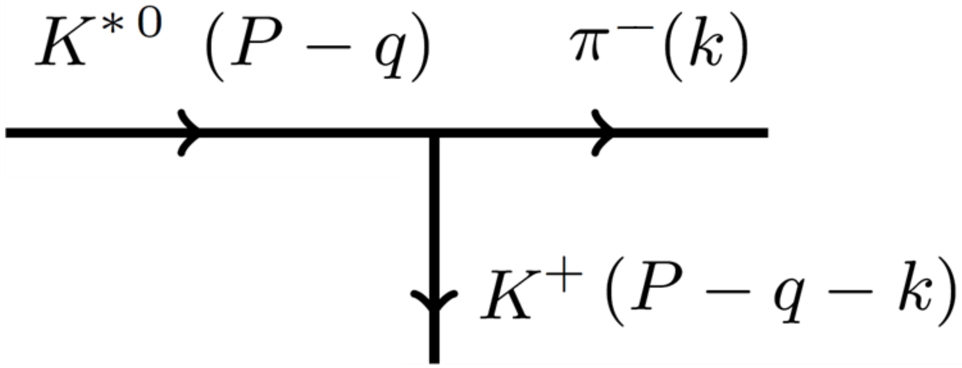
D^{*0}
B^-
$K^{*0}\ (P-q)$
$\pi^-\ (k)$
$K^-\ (q)$
$K^+(P-q-k)$
R


K^{*0} $(P-q)$
$\pi^-(k)$
$K^+(P-q-k)$



Local Hidden Gauge

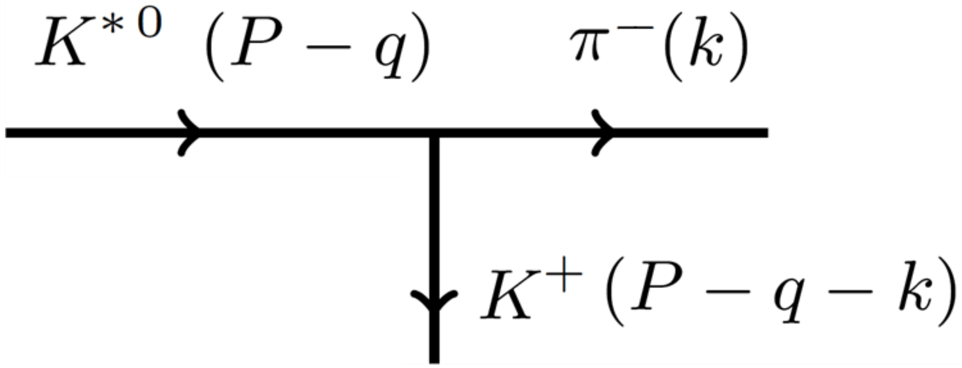
$$\mathcal{L}_{VPP} = -ig\,\langle [\Phi, \partial_\mu \Phi]\, V^\mu \rangle$$



Local Hidden Gauge

$$\mathcal{L}_{VPP} = -ig\,\langle [\Phi, \partial_\mu \Phi]\, V^\mu \rangle$$

$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}$$



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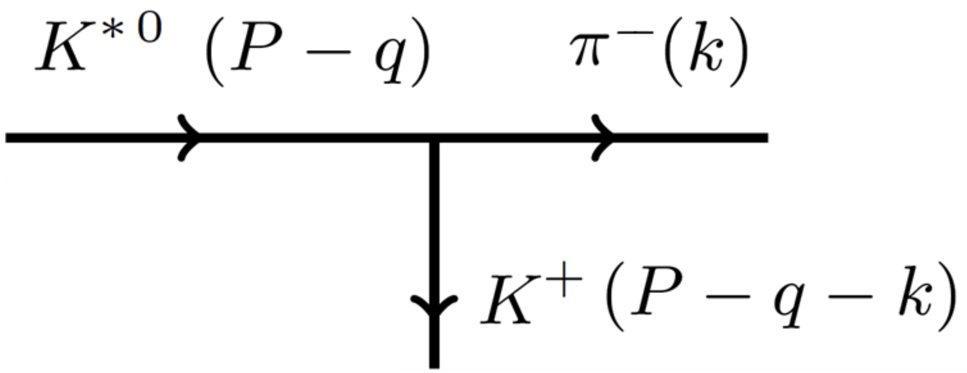
$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & -\frac{1}{\sqrt{2}}\rho^0_\mu + \frac{1}{\sqrt{2}}\omega_\mu & K^{*0}_\mu \\ K^{*-}_\mu & \bar{K}^{*0}_\mu & \phi_\mu \end{pmatrix}$$



$$-it_{K^*K^+\pi^-} \simeq -ig\vec{\epsilon}_{K^*}\cdot(\vec{\tilde{p}}_\pi - \vec{\tilde{p}}_{K^+})$$

Local Hidden Gauge

$$\mathcal{L}_{VPP} = -ig\,\langle[\Phi, \partial_\mu\Phi]\,V^\mu\rangle$$

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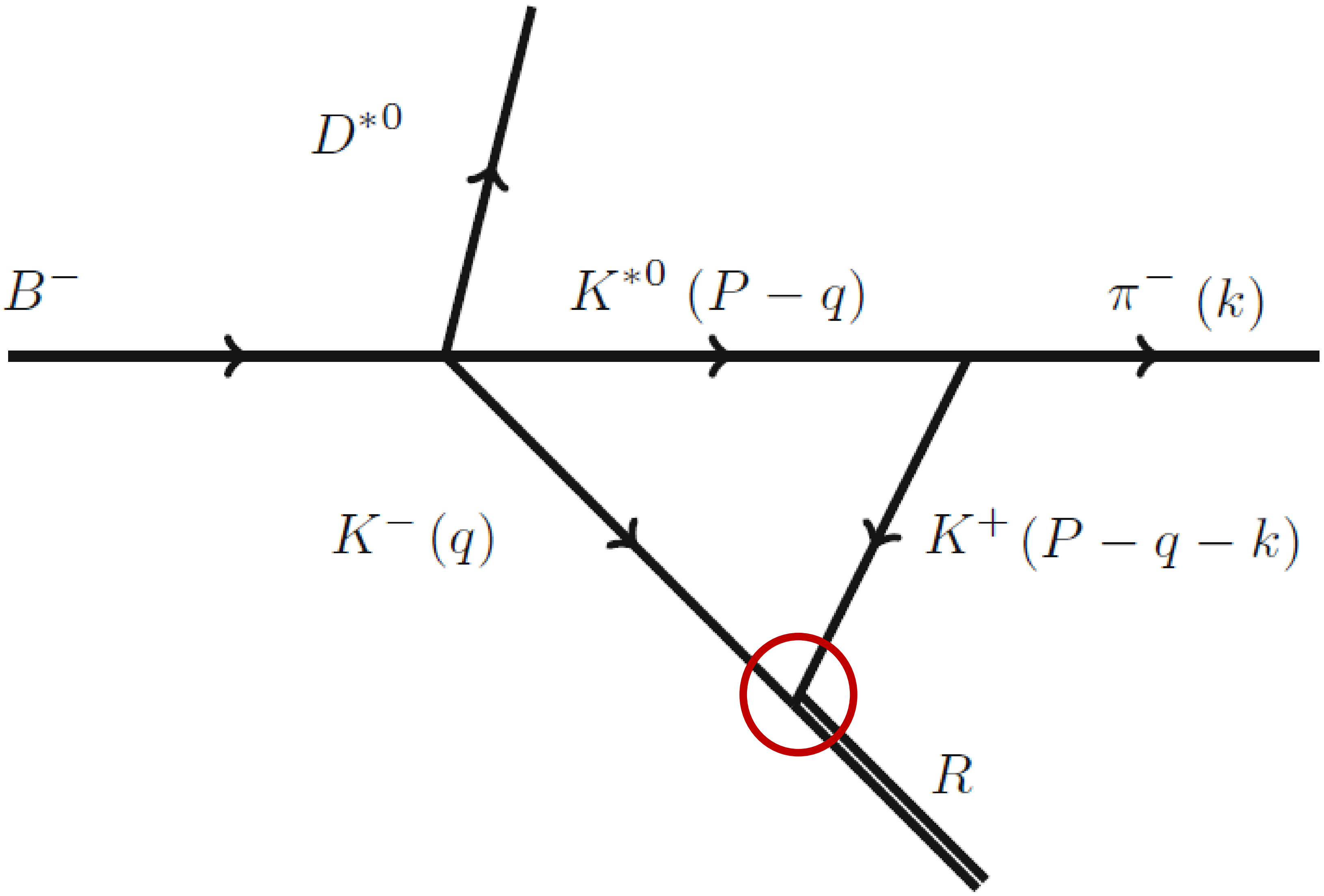
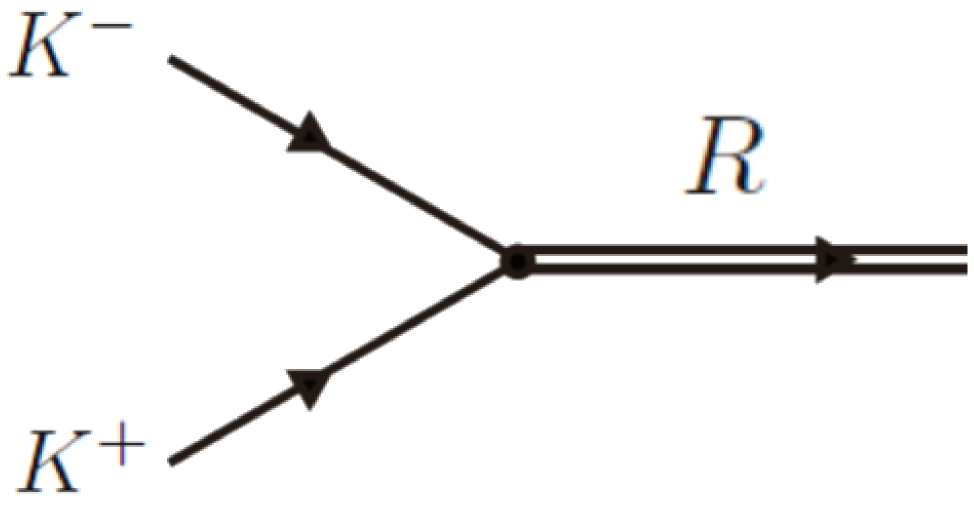
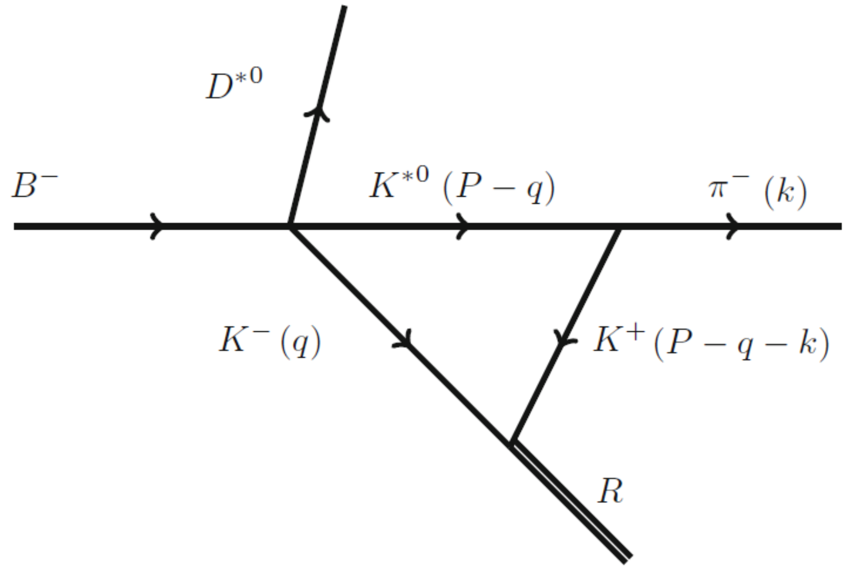


D^{*0}
B^-
$K^{*0}\ (P-q)$
$\pi^-\ (k)$
$K^-\ (q)$
$K^+(P-q-k)$
R



$$t_{K^+K^-,R} = g_{K^-K^+,R}$$


D^{*0}
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$K^{*0}\,(P-q)$
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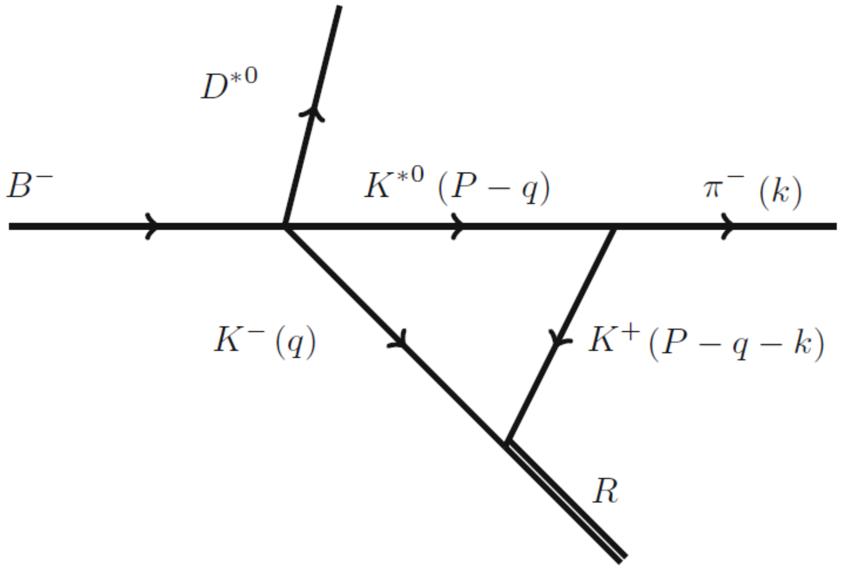
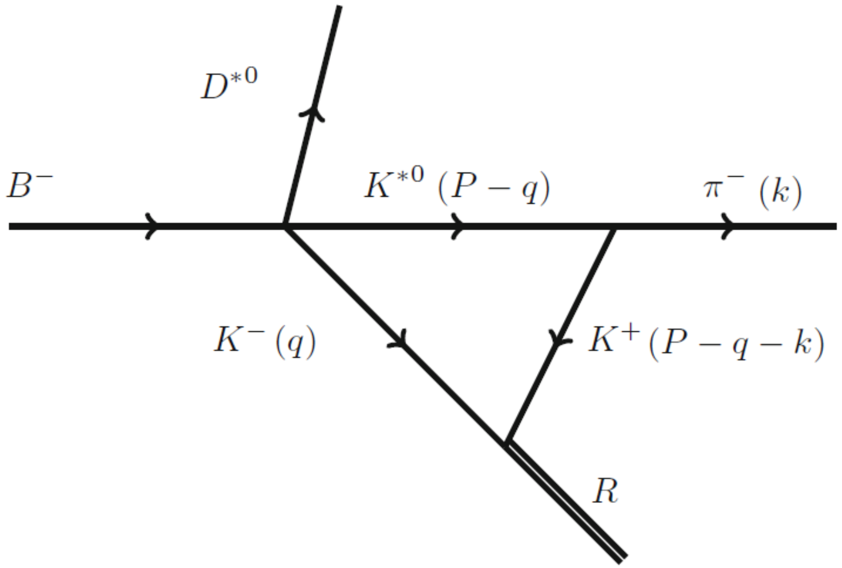
R



$$\int \frac{d^4q}{(2\pi)^4} \frac{\vec{\epsilon}_{D^*} \cdot \vec{\epsilon}_{K^*}}{q^2 - m_\pi^2 + i\epsilon} \frac{\vec{\epsilon}_{K^*} \cdot (\vec{\tilde{p}}_\pi - \vec{\tilde{p}}_{K^+})}{(P-q)^2 - m_{K^*}^2 + i\epsilon} \frac{1}{(P-q-k)^2 - m_\pi^2 + i\epsilon}$$



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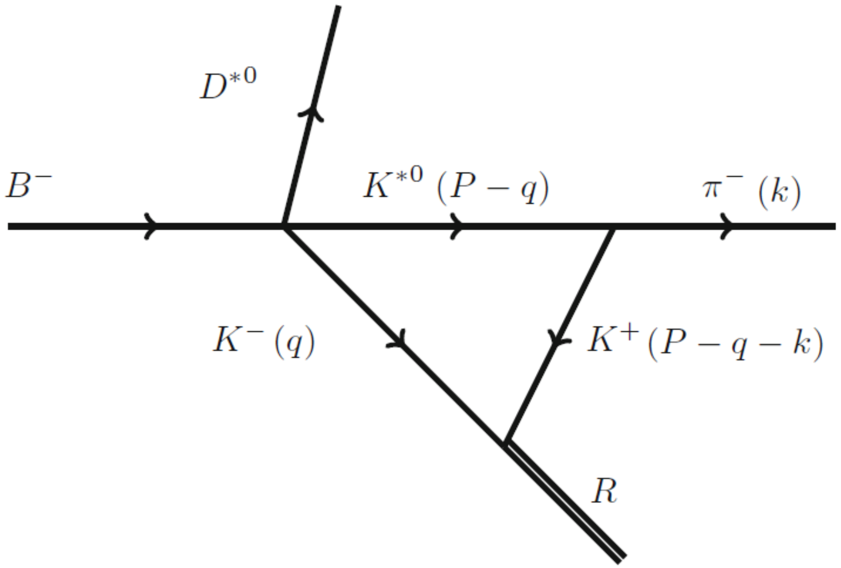
$$-\vec{\epsilon}_{D^*} \cdot \vec{k}\; t_T = i \int \frac{d^4q}{(2\pi)^4} \left(2 + \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2}\right) \frac{-\vec{\epsilon}_{D^*} \cdot \vec{k}}{q^2 - M_\Sigma^2 + i\epsilon} \frac{1}{(P-q)^2 - m_{K^*}^2 + i\epsilon} \frac{1}{(P-q-k)^2 - m_\pi^2 + i\epsilon}.$$



$$\int \frac{d^4q}{(2\pi)^4} \frac{\vec{\epsilon}_{D^*} \cdot \vec{\epsilon}_{K^*}}{q^2 - m_\pi^2 + i\epsilon} \frac{\vec{\epsilon}_{K^*} \cdot (\vec{\tilde{p}}_\pi - \vec{\tilde{p}}_{K^+})}{(P-q)^2 - m_{K^*}^2 + i\epsilon} \frac{1}{(P-q-k)^2 - m_\pi^2 + i\epsilon}$$

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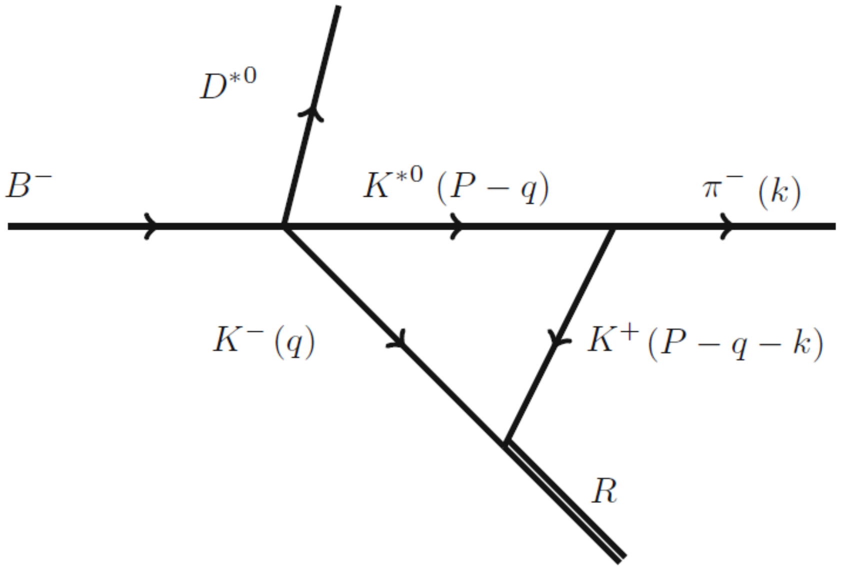
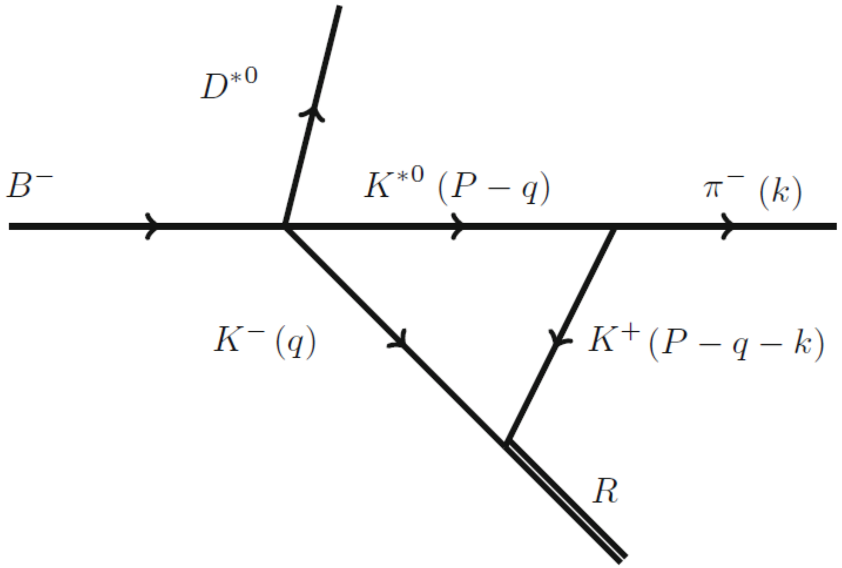
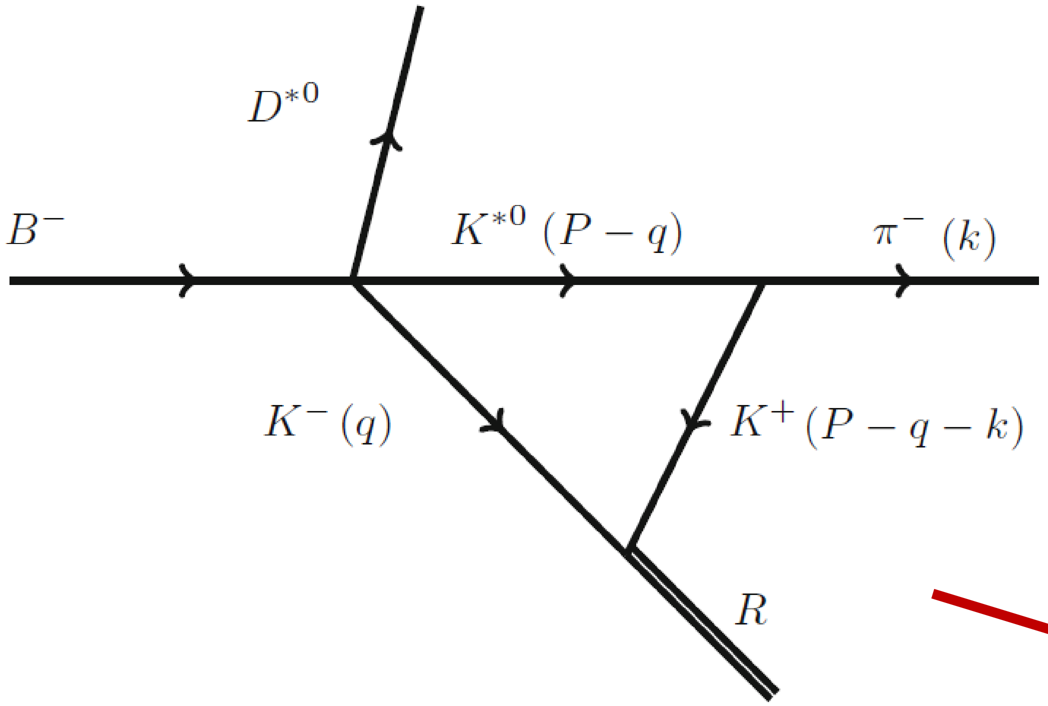


$$t_{B^- \to D^* \pi R} = -\vec{\epsilon}_{D^*} \cdot \vec{k}\; g_{K^-K^+,R}\; gC\; t_T$$



$$t_{B^-\to D^*\pi R} = -\vec{\epsilon}_{D^*}\cdot\vec{k}\; g_{K^-K^+,R}\; gC\; t_T$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}M_{\mathrm{inv}}(\pi R)} = \frac{1}{(2\pi)^3}\frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_{\pi}|}{4M_B^2}\sum_{\mathrm{pol.}}|t_{B^-\to D^*\pi R}|^2$$


D^{*0}
B^-
$K^{*0}\ (P-q)$
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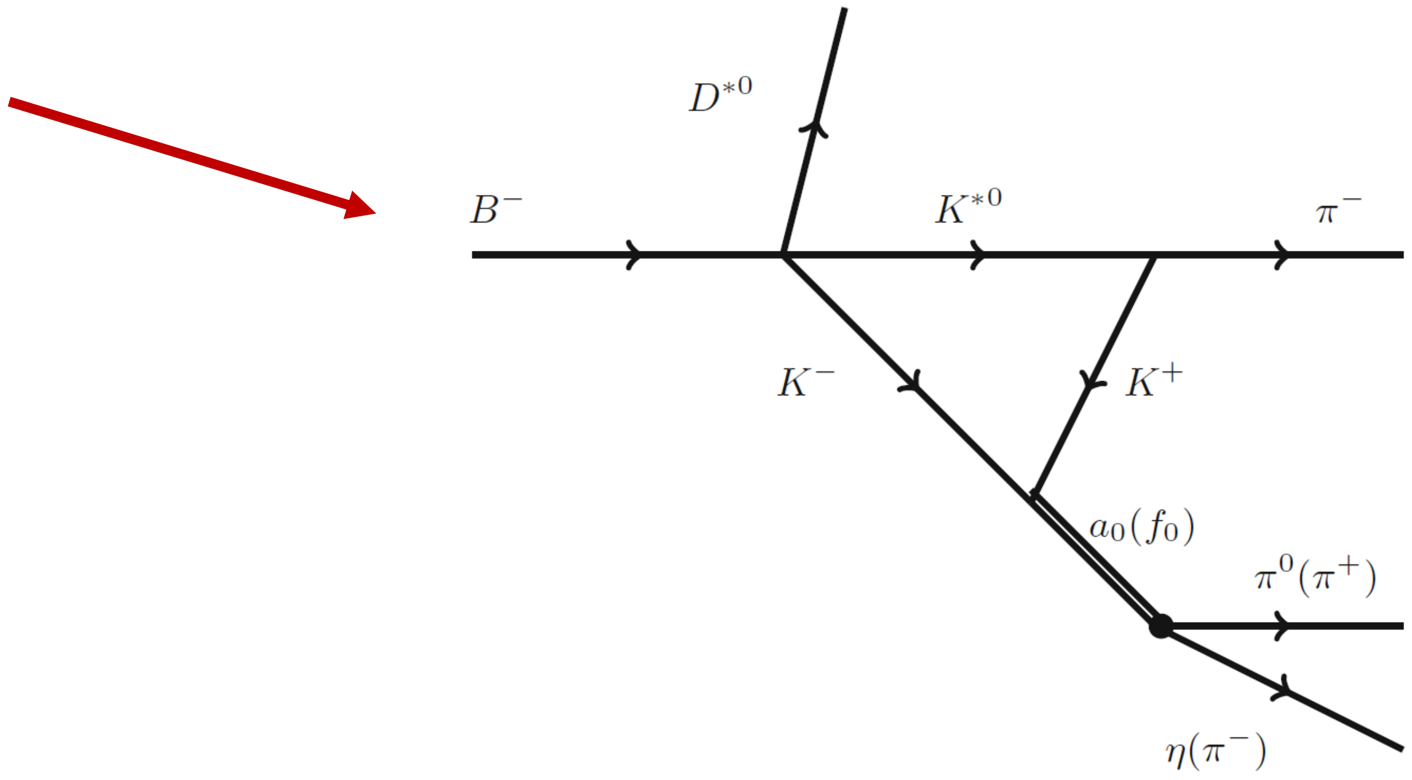
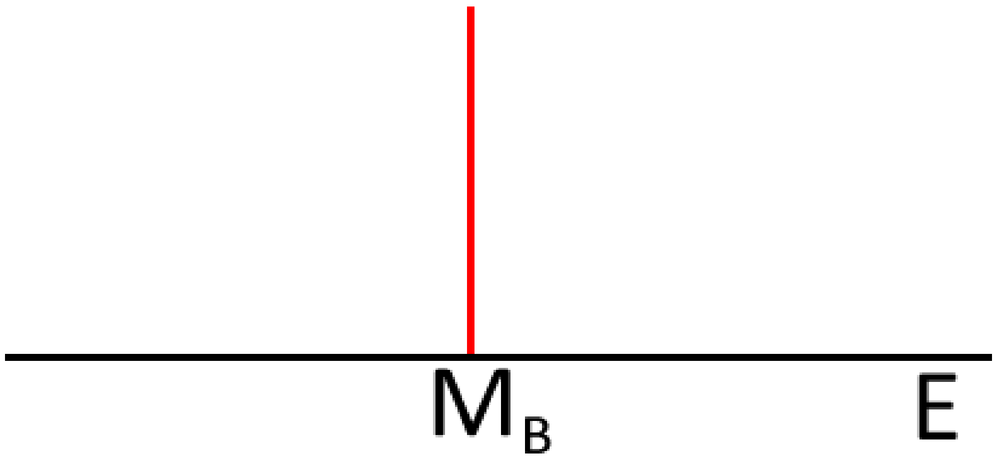
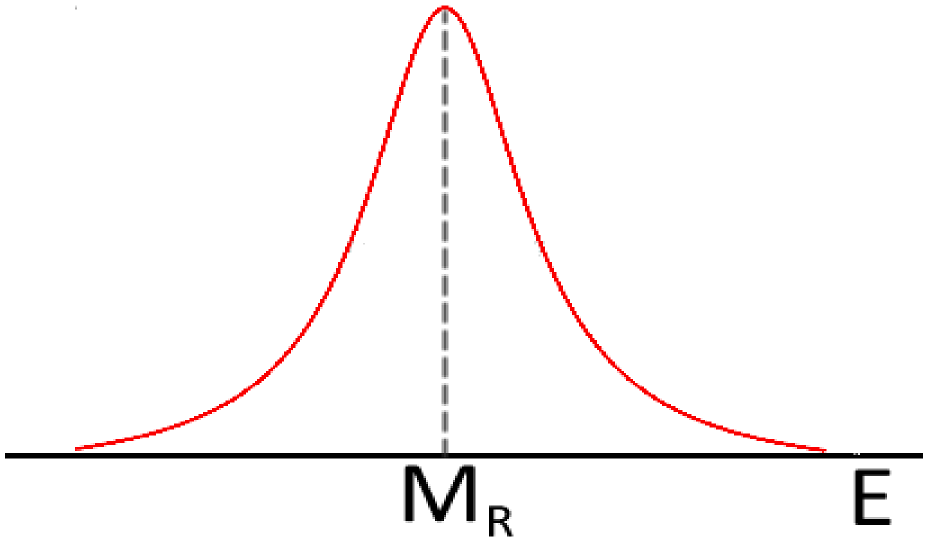
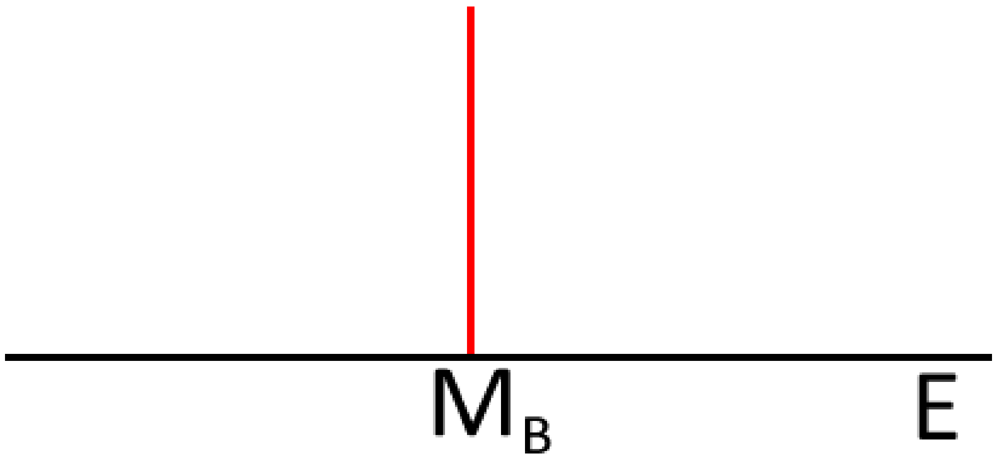
R

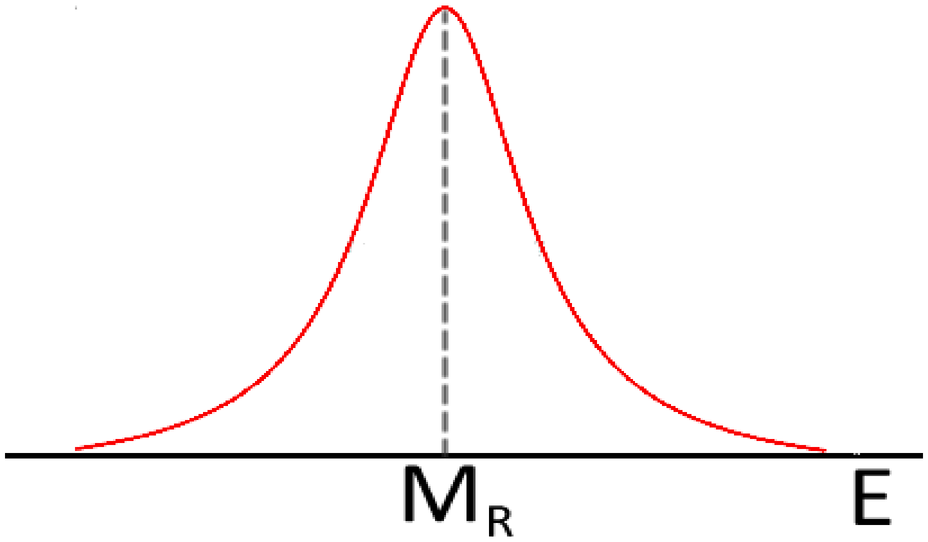

D^{*0}
B^-
K^{*0}
π^-
K^-
K^+
$a_0(f_0)$
$\pi^0(\pi^+)$
$\eta(\pi^-)$



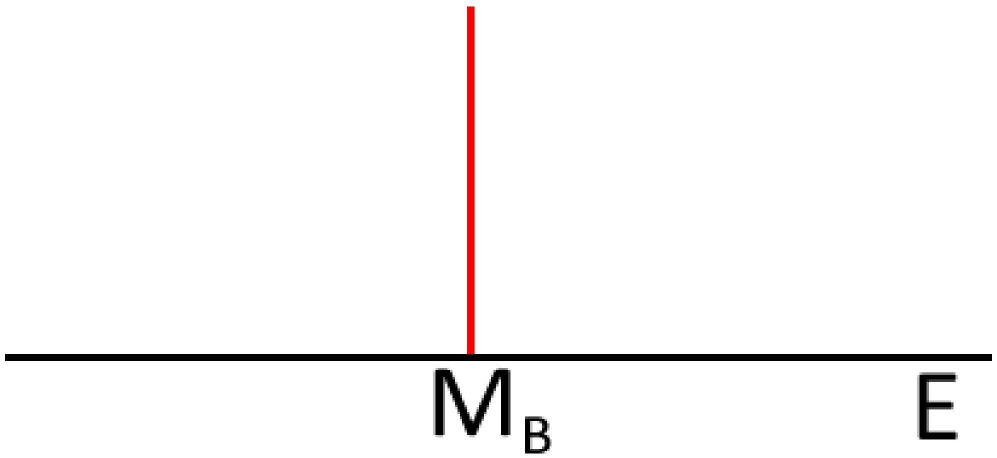


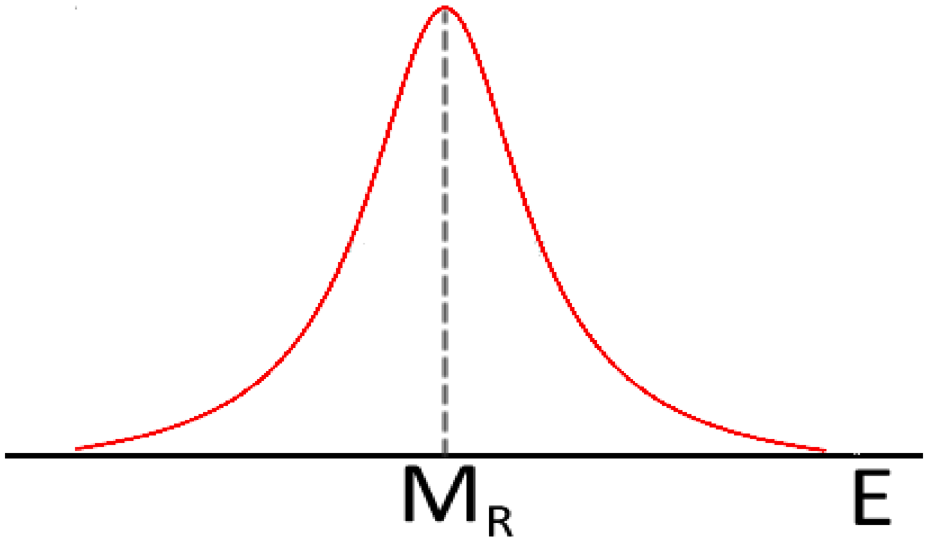
$$\frac{d\Gamma}{dM_{\rm inv}(\pi R)} = \frac{1}{(2\pi)^3}\int dM^2_{\rm inv}(R)\left(-\frac{1}{\pi}{\rm Im}D\right)|g_{K^-K^+,R}|^2\frac{|\vec{p}_{D^*}||\tilde{\vec{p}}_\pi|}{4M_B^2}\overline{\sum}\sum|\tilde{t}_{B^-,D^*\pi R}|^2$$





$$\frac{d\Gamma}{dM_{\rm inv}(\pi R)} = \frac{1}{(2\pi)^3} \int dM^2_{\rm inv}(R) \left(-\frac{1}{\pi}{\rm Im}D\right) |g_{K^-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_\pi|}{4M_B^2} \overline{\sum}\sum |\tilde{t}_{B^-,D^*\pi R}|^2$$





$$\frac{d\Gamma}{dM_{\mathrm{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \int dM^2_{\mathrm{inv}}(R) \left(-\frac{1}{\pi}\mathrm{Im}D\right) |g_{K^-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\tilde{\vec{p}}_\pi|}{4M_B^2} \overline{\sum}\sum |\tilde{t}_{B^-,D^*\pi R}|^2$$

$$D = \frac{1}{M^2_{\mathrm{inv}}(R) - M_R^2 + i M_R \Gamma_R}$$

$$\frac{d\Gamma}{dM_{\rm inv}(\pi R)} = \frac{1}{(2\pi)^3} \int dM^2_{\rm inv}(R)\, \frac{1}{\pi}\, |g_{K^-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_\pi|}{4M_B^2} \overline{\sum}\sum \left|\tilde{t}_{B^-,D^*\pi R}\right|^2 \frac{M_R\Gamma_R}{(M^2_{\rm inv}(R)-M_R^2)^2+(M_R\Gamma_R)^2}$$

Rafael Pavao

$$\frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \int dM_{\text{inv}}^2(R)\, \frac{1}{\pi}\, |g_{K^-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_\pi|}{4M_B^2} \overline{\sum}\sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{M_R\Gamma_R}{(M_{\text{inv}}^2(R) - M_R^2)^2 + (M_R\Gamma_R)^2}$$

$$a_0 \to \pi^0\eta$$

$$\Gamma_{a_0} = \frac{1}{8\pi} \frac{|g_{a_0\to\pi^0\eta}|^2}{M_{\text{inv}}^2(\pi^0\eta)} |\vec{\tilde{q}}_\eta|$$

Rafael Pavao

$$\frac{d\Gamma}{dM_{\rm inv}(\pi R)} = \frac{1}{(2\pi)^3} \int dM_{\rm inv}^2(R)\, \frac{1}{\pi}\, |g_{K^-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_\pi|}{4M_B^2} \overline{\sum}\sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{M_R\Gamma_R}{(M_{\rm inv}^2(R) - M_R^2)^2 + (M_R\Gamma_R)^2}$$

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$$\frac{d\Gamma}{dM_{\rm inv}(\pi a_0)} = \frac{1}{(2\pi)^3} \int dM_{\rm inv}^2(\pi^0\eta)\, \frac{|\vec{p}_{D^*}||\vec{\tilde{p}}_\pi|}{4M_B^2} \overline{\sum}\sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{M_{a_0}|g_{a_0\to\pi^0\eta}|^2|g_{K^-K^+\to a_0}|^2}{(M_{\rm inv}^2(\pi^0\eta) - M_{a_0}^2)^2 + (M_{a_0}\Gamma_{a_0})^2} \frac{1}{8\pi^2} \frac{|\vec{\tilde{q}}_\eta|}{M_{\rm inv}^2(\pi^0\eta)}$$

Rafael Pavao

$$\frac{d\Gamma}{dM_{\rm inv}(\pi R)} = \frac{1}{(2\pi)^3}\int dM^2_{\rm inv}(R)\,\frac{1}{\pi}\,|g_{K^-K^+,R}|^2\,\frac{|\vec{p}_{D^*}||\tilde{\vec{p}}_\pi|}{4M_B^2}\,\overline{\sum}\sum|\tilde{t}_{B^-,D^*\pi R}|^2\,\frac{M_R\Gamma_R}{(M^2_{\rm inv}(R)-M_R^2)^2+(M_R\Gamma_R)^2}$$

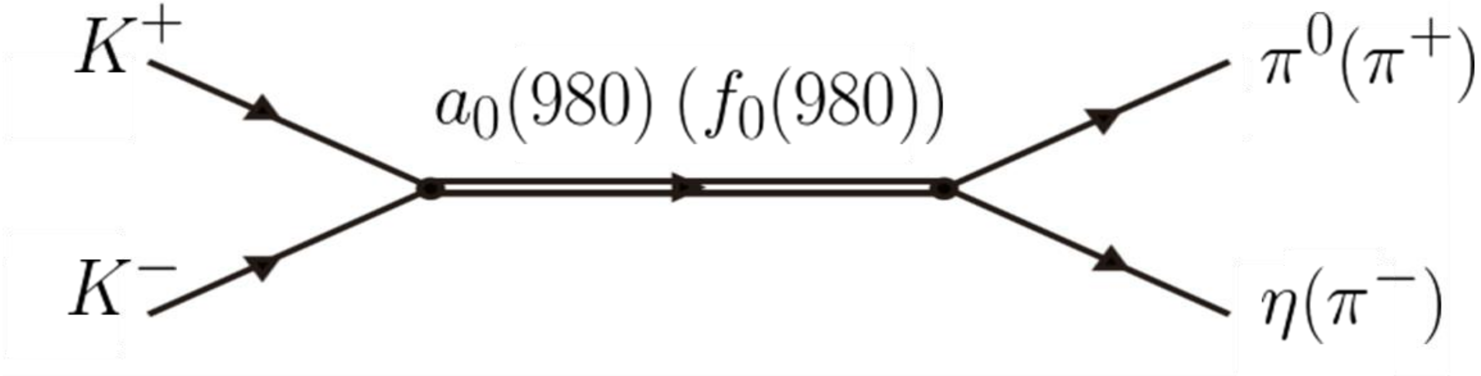
$$a_0 \rightarrow \pi^0\eta$$

$$\Gamma_{a_0} = \frac{1}{8\pi}\,\frac{|g_{a_0\rightarrow\pi^0\eta}|^2}{M^2_{\rm inv}(\pi^0\eta)}\,|\tilde{\vec{q}}_\eta|$$

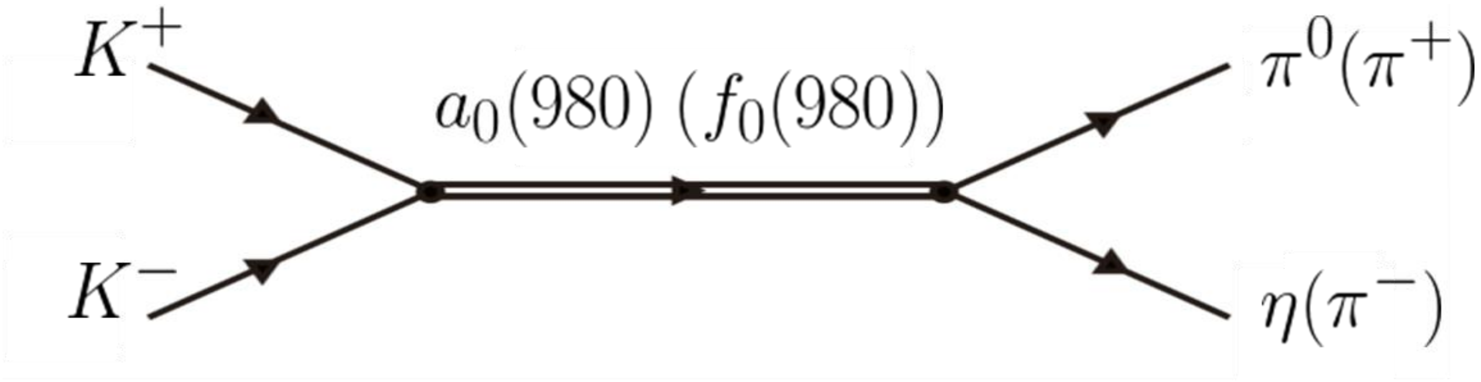
$$\frac{d\Gamma}{dM_{\rm inv}(\pi a_0)} = \frac{1}{(2\pi)^3}\int dM^2_{\rm inv}(\pi^0\eta)\,\frac{|\vec{p}_{D^*}||\tilde{\vec{p}}_\pi|}{4M_B^2}\,\overline{\sum}\sum|\tilde{t}_{B^-,D^*\pi R}|^2\,\frac{M_{a_0}|g_{a_0\rightarrow\pi^0\eta}|^2|g_{K^-K^+\rightarrow a_0}|^2}{(M^2_{\rm inv}(\pi^0\eta)-M^2_{a_0})^2+(M_{a_0}\Gamma_{a_0})^2}\,\frac{1}{8\pi^2}\,\frac{|\tilde{\vec{q}}_\eta|}{M^2_{\rm inv}(\pi^0\eta)}$$

$$\frac{|g_{a_0\rightarrow\pi^0\eta}|^2|g_{K^-K^+\rightarrow a_0}|^2}{(M^2_{\rm inv}(\pi^0\eta)-M^2_{a_0})^2+(M_{a_0}\Gamma_{a_0})^2} = |t_{K^+K^-\rightarrow\pi^0\eta}|^2$$

Rafael Pavao



K^+
K^-
$a_0(980)\ (f_0(980))$
$\pi^0(\pi^+)$
$\eta(\pi^-)$



$$t = [1 - VG]^{-1}V$$

$a_0(980)$: πη, $K\bar{K}$

$f_0(980)$: ππ , $K\bar{K}$, ηη

$$\mathfrak{L} = \frac{1}{12f^2}\left\langle \left(\partial_\mu \Phi \Phi - \Phi \partial_\mu \Phi\right)^2 + M\Phi^4 \right\rangle$$

W.H. Liang, E. Oset, Phys. Lett. B **737**, 70 (2014).

J.J. Xie, L.R. Dai, E. Oset, Phys. Lett. B **742**, 363 (2015).

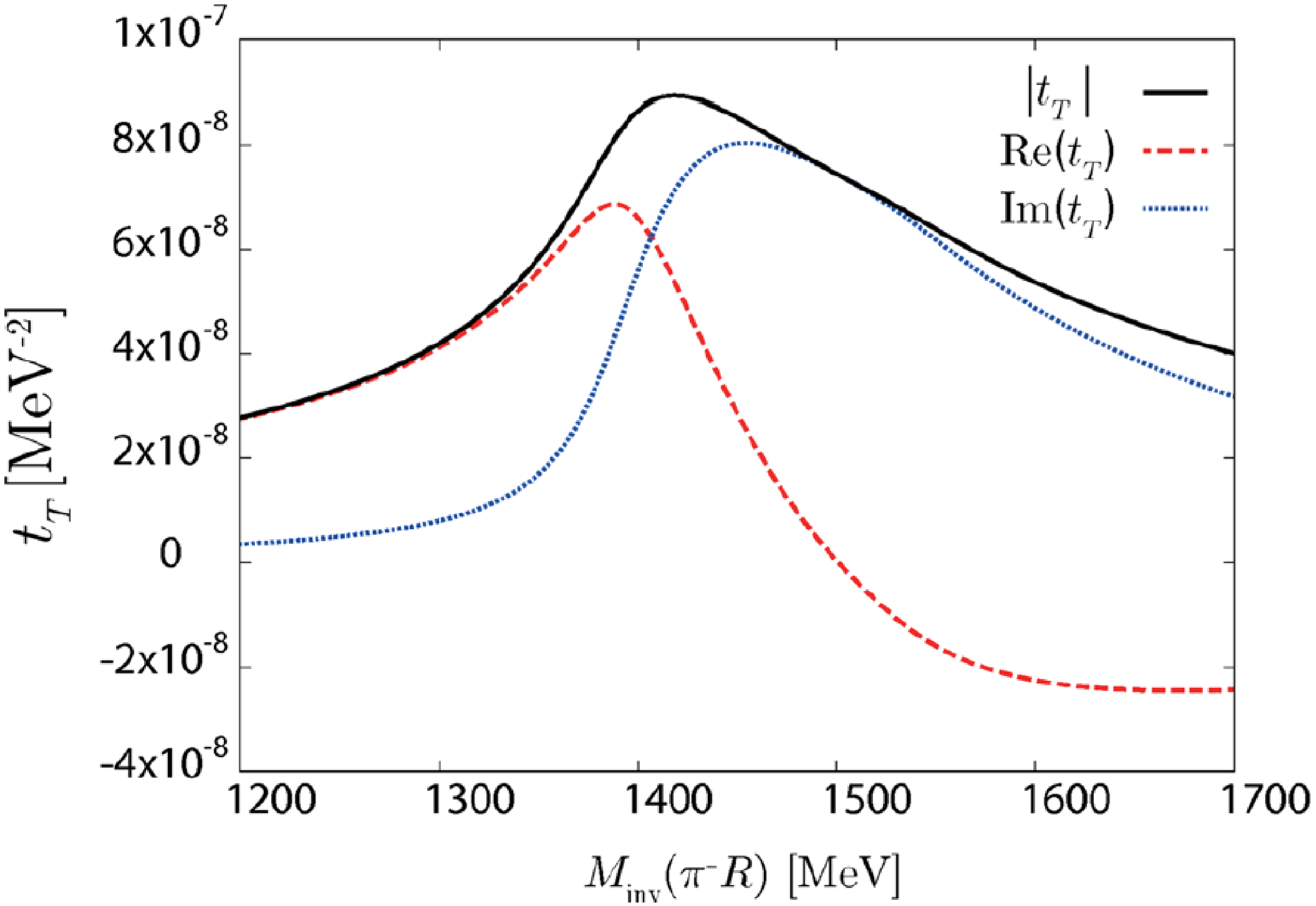
$$\frac{d^2\Gamma}{dM_{\rm inv}(\pi a_0)dM_{\rm inv}(\pi^0\eta)} = \frac{1}{(2\pi)^5}\frac{|\vec{p}_{D^*}||\vec{k}||\vec{\tilde{q}}_\eta|}{4M_B^2}\,\overline{\sum}\sum\left|\tilde{t}_{B^-,D^*\pi R}\times t_{K^+K^-\to\pi^0\eta}\right|^2$$

$$\frac{d^2\Gamma}{dM_{\rm inv}(\pi a_0)\, dM_{\rm inv}(\pi^0\eta)} = \frac{1}{(2\pi)^5} \frac{|\vec{p}_{D^*}||\vec{k}||\vec{\tilde{q}}_\eta|}{4M_B^2} \overline{\sum}\sum \left|\tilde{t}_{B^-,D^*\pi R} \times t_{K^+K^-\to\pi^0\eta}\right|^2$$

$$\frac{d^2\Gamma}{dM_{\rm inv}(\pi f_0)\, dM_{\rm inv}(\pi^+\pi^-)} = \frac{1}{(2\pi)^5} \frac{|\vec{p}_{D^*}||\vec{k}||\vec{\tilde{q}}_\eta|}{4M_B^2} \overline{\sum}\sum \left|\tilde{t}_{B^-,D^*\pi R} \times t_{K^+K^-\to\pi^+\pi^-}\right|^2$$

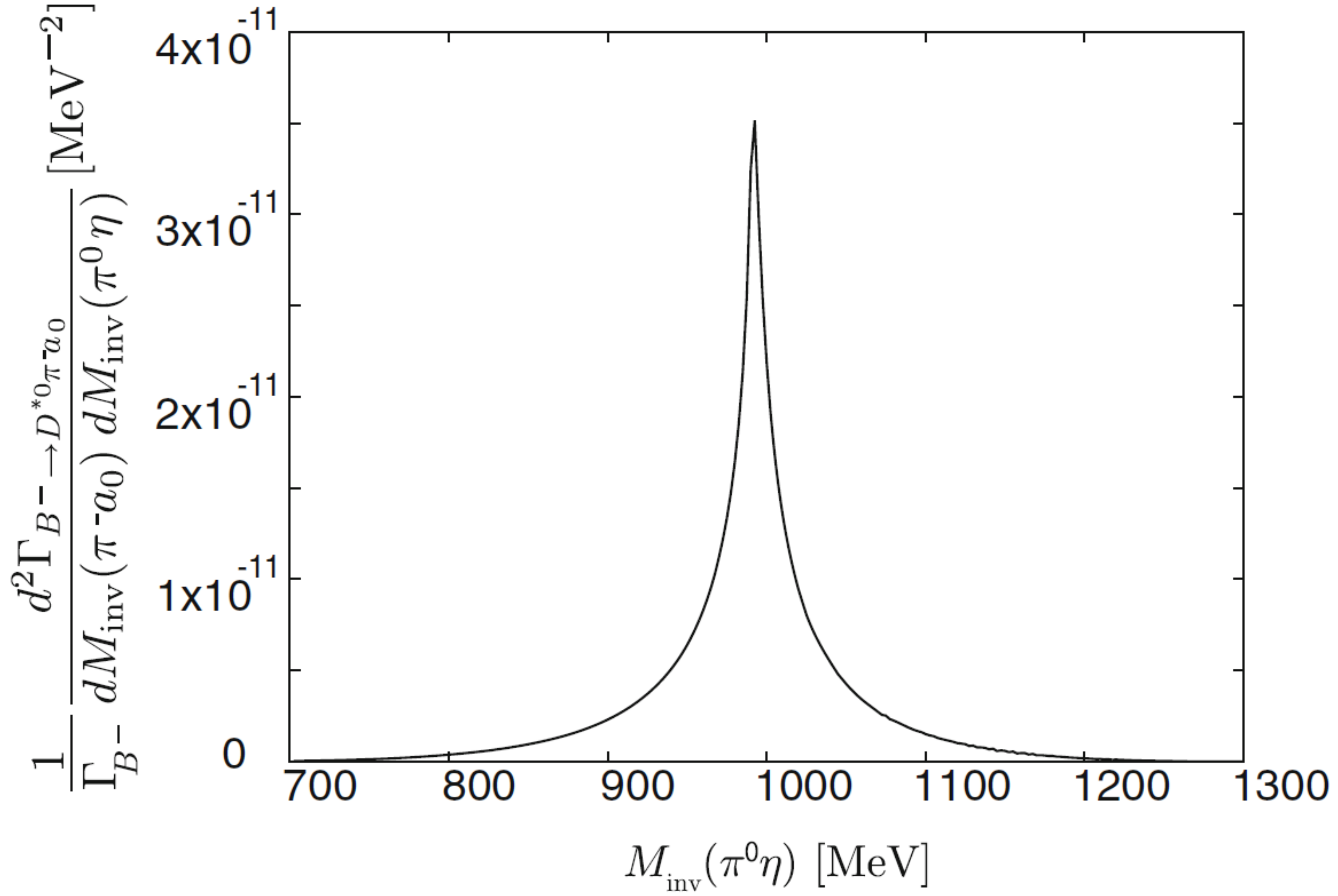
Results

t_T



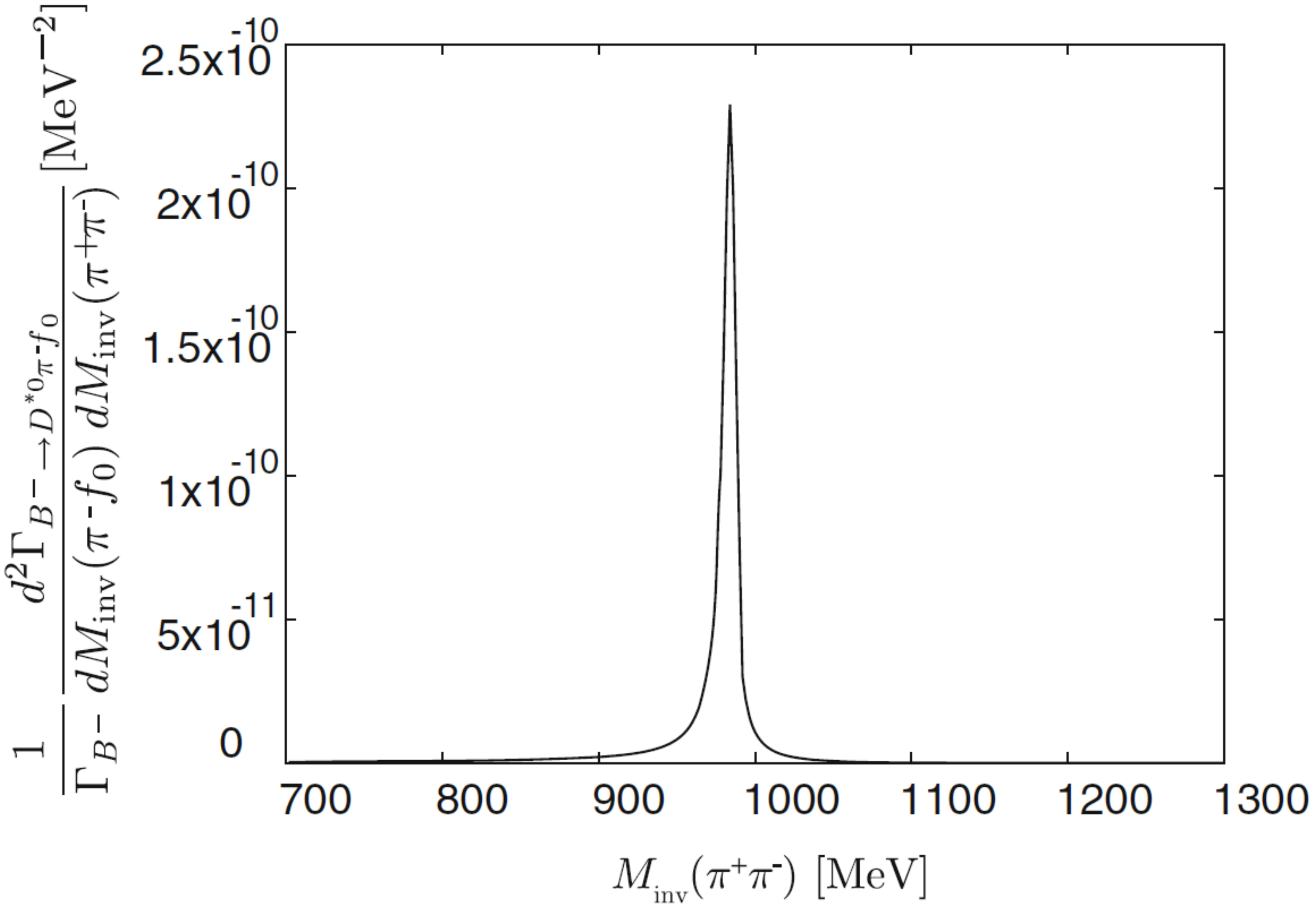
1x10-7
8x10-8
6x10-8
4x10-8
2x10-8
0
-2x10-8
-4x10-8
1200
1300
1400
1500
1600
1700
t_T [MeV-2]
$M_{inv}(\pi^- R)$ [MeV]
$|t_T|$
Re(t_T)
Im(t_T)

a_0

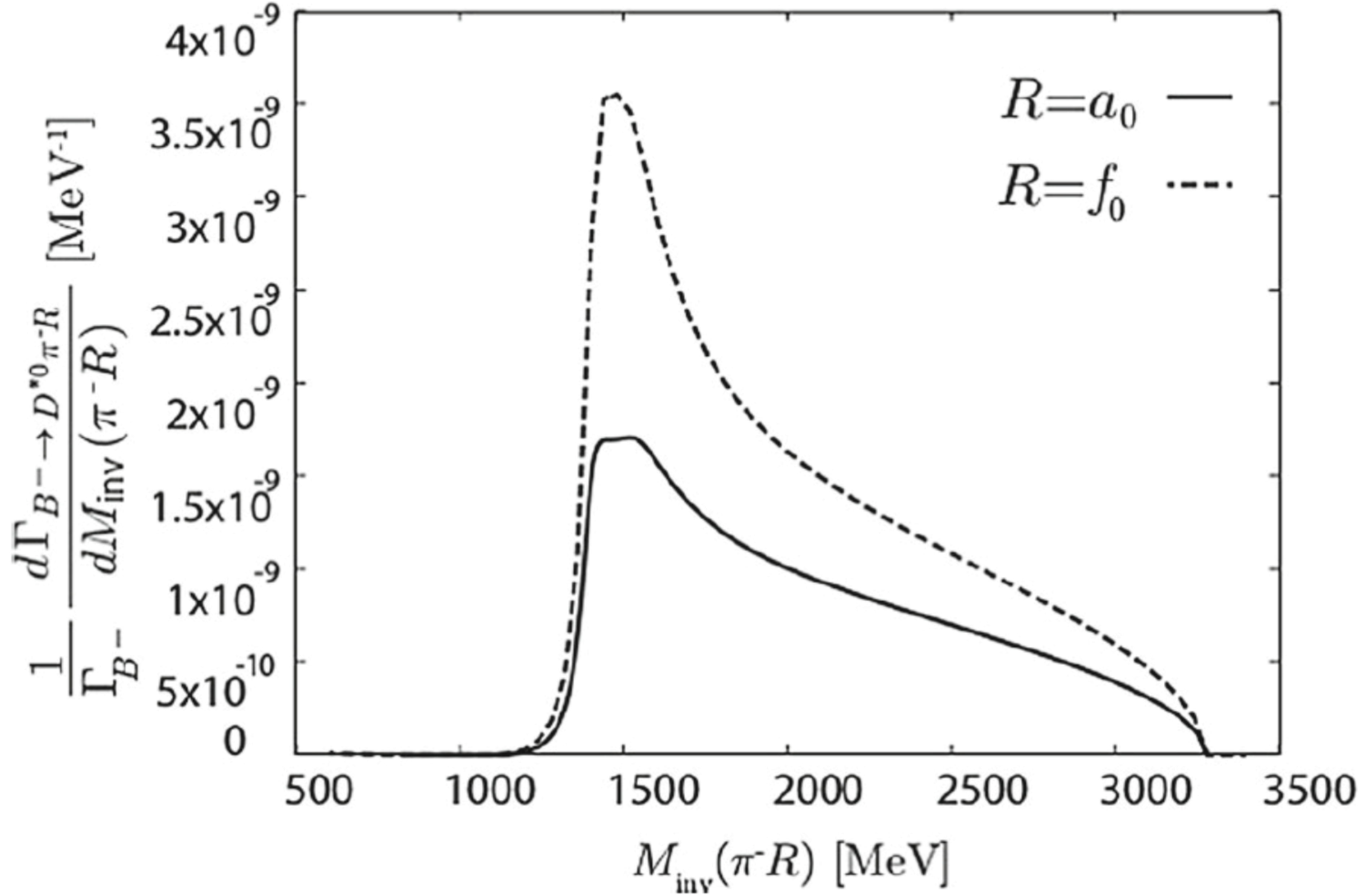


$\frac{1}{\Gamma_{B^-}} \frac{d^2\Gamma_{B^- \to D^{*0}\pi^- a_0}}{dM_{\rm inv}(\pi^- a_0)\, dM_{\rm inv}(\pi^0\eta)}$ [MeV^{-2}]
4x10-11
3x10-11
2x10-11
1x10-11
0
700
800
900
1000
1100
1200
1300
$M_{\rm inv}(\pi^0\eta)$ [MeV]

f_0



$\frac{1}{\Gamma_{B^-}} \frac{d^2\Gamma_{B^- \to D^{*0}\pi^- f_0}}{dM_{\rm inv}(\pi^- f_0)\, dM_{\rm inv}(\pi^+\pi^-)}$ [MeV^{-2}]
2.5x10^{-10}
2x10^{-10}
1.5x10^{-10}
1x10^{-10}
5x10^{-11}
0
700
800
900
1000
1100
1200
1300
$M_{\rm inv}(\pi^+\pi^-)$ [MeV]



$\frac{1}{\Gamma_{B^-}} \frac{d\Gamma_{B^- \to D^{*0}\pi^- R}}{dM_{\rm inv}(\pi^- R)}$ [MeV^{-1}]
4x10^{-9}
3.5x10^{-9}
3x10^{-9}
2.5x10^{-9}
2x10^{-9}
1.5x10^{-9}
1x10^{-9}
5x10^{-10}
0
500
1000
1500
2000
2500
3000
3500
$M_{\rm inv}(\pi^- R)$ [MeV]
$R{=}a_0$
$R{=}f_0$

$$t_{B^- \to D^* \pi R} = -\vec{\epsilon}_{D^*} \cdot \vec{k}\, g_{K^-K^+,R}\, gC\, t_T$$

$$\frac{d^2\Gamma}{dM_{\rm inv}(\pi a_0)\, dM_{\rm inv}(\pi^0\eta)} = \frac{1}{(2\pi)^5} \frac{|\vec{p}_{D^*}||\vec{k}||\vec{\tilde{q}}_\eta|}{4M_B^2} \overline{\sum}\sum \left|\tilde{t}_{B^-,D^*\pi R} \times t_{K^+K^- \to \pi^0\eta}\right|^2$$

$$\mathrm{Br}(B^- \to D^{*0}\pi^- a_0;\, a_0 \to \pi^0\eta) = (1.66 \pm 0.45) \times 10^{-6}$$

$$\mathrm{Br}(B^- \to D^{*0}\pi^- f_0;\, f_0 \to \pi^+\pi^-) = (2.82 \pm 0.75) \times 10^{-6}$$

Conclusions

1. There is a peak of a TS around $\boldsymbol{M_{inv}(\pi R)}$**=1420 MeV**

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2. Clear effect on differential decay width

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3. Measurable Branching Ratios

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2. Clear effect on differential decay width

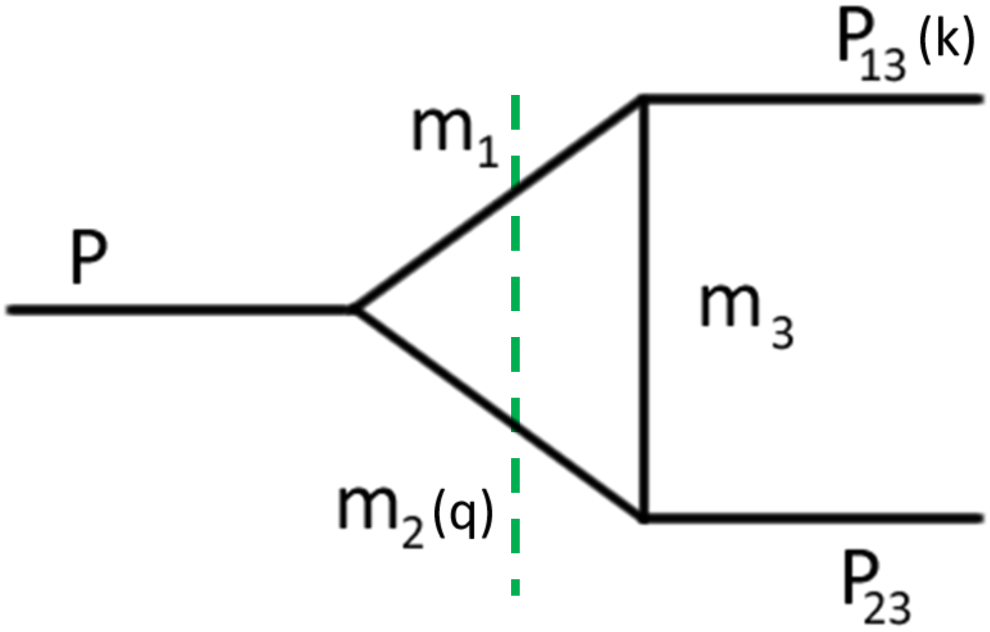
3. Measurable Branching Ratios

4. Interesting prediction for future experiment

Thank You!

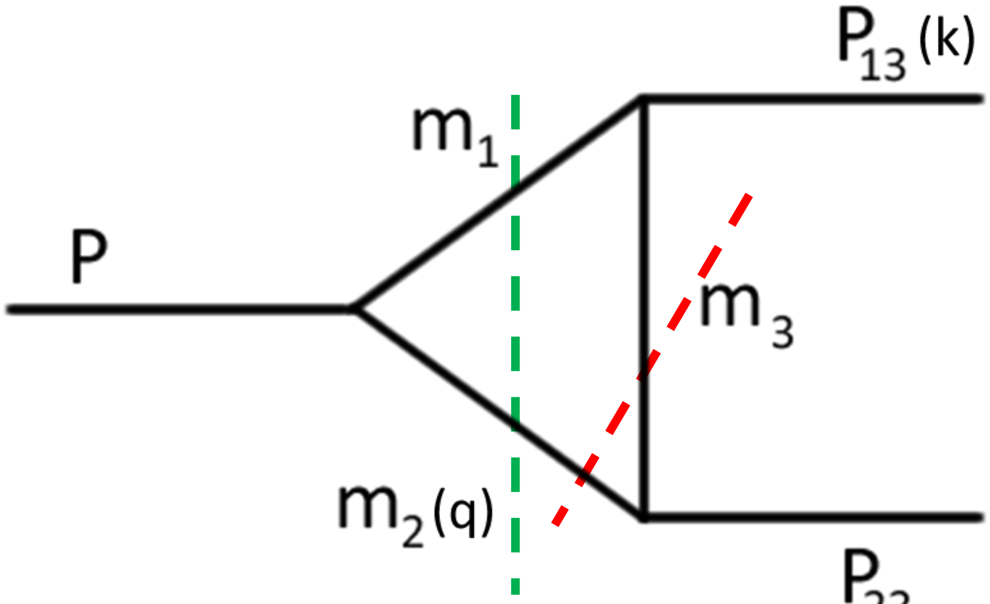
Backup Slides

Triangle Singularities



$$P^0 - \omega^*(\vec{q}) - \omega(\vec{q}) = 0$$

Triangle Singularities



$$P^0 - \omega^*(\vec{q}) - \omega(\vec{q}) = 0$$

$$P^0 - k^0 - \omega(\vec{q}) - \omega'(\vec{q} + \vec{k}) = 0$$

$$I = \int \frac{d^3q}{(2\pi)^3} \; \frac{1}{8\omega^*\omega\omega'} \frac{1}{k^0 - \omega' - \omega^*} \frac{1}{P^0 + \omega + \omega' - k^0} \frac{1}{P^0 - \omega - \omega' - k^0 + i\epsilon} \times$$
$$\times \; \frac{\{2P^0\omega + 2k^0\omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]\}}{P^0 - \omega^* - \omega + i\epsilon}$$

$$\omega^* = \sqrt{m_1^2 + |\vec{q}|^2}$$

$$\omega = \sqrt{m_2^2 + |\vec{q}|^2}$$

$$\omega = \sqrt{m_3^2 + |\vec{q} + \vec{k}|^2}$$

$$I = \int \frac{d^3q}{(2\pi)^3} \; \frac{1}{8\omega^*\omega\omega'} \frac{1}{k^0 - \omega' - \omega^*} \frac{1}{P^0 + \omega + \omega' - k^0} \frac{1}{P^0 - \omega - \omega' - k^0 + i\epsilon} \times$$

$$\times \frac{\{2P^0\omega + 2k^0\omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]\}}{P^0 - \omega^* - \omega + i\epsilon}$$

$$I = \int \frac{d^3q}{(2\pi)^3} \frac{1}{8\omega^*\omega\omega'} \frac{1}{k^0 - \omega' - \omega^*} \frac{1}{P^0 + \omega + \omega' - k^0} \frac{1}{P^0 - \omega - \omega' - k^0 + i\epsilon} \times \frac{\{2P^0\omega + 2k^0\omega' - 2[\omega + \omega'][\omega + \omega' + \omega^*]\}}{P^0 - \omega^* - \omega + i\epsilon}$$

$$\int d^3q \frac{1}{[P^0 - \omega^* - \omega + i\epsilon][P^0 - k^0 - \omega - \omega' + i\epsilon]} = 2\pi \int_0^\infty dq \frac{q^2}{P^0 - \omega^* - \omega + i\epsilon} f(q)$$

$$f(q) = \int_{-1}^{1} d\cos\theta \, \frac{1}{P^0 - k^0 - \sqrt{m_3^2 + q^2 + k^2 + 2qk\cos\theta} + i\epsilon}$$

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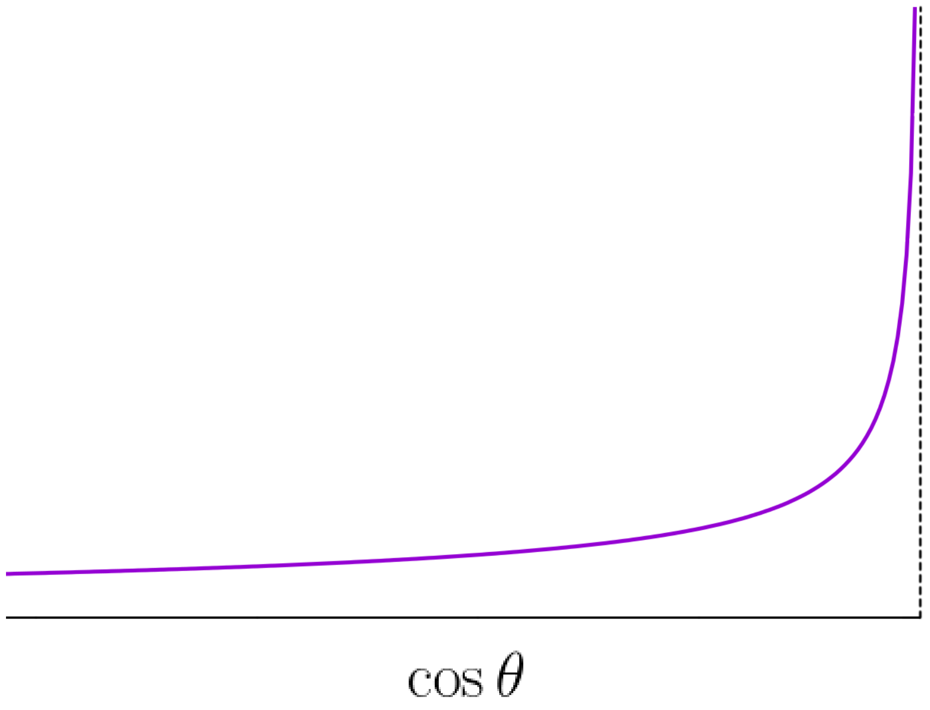
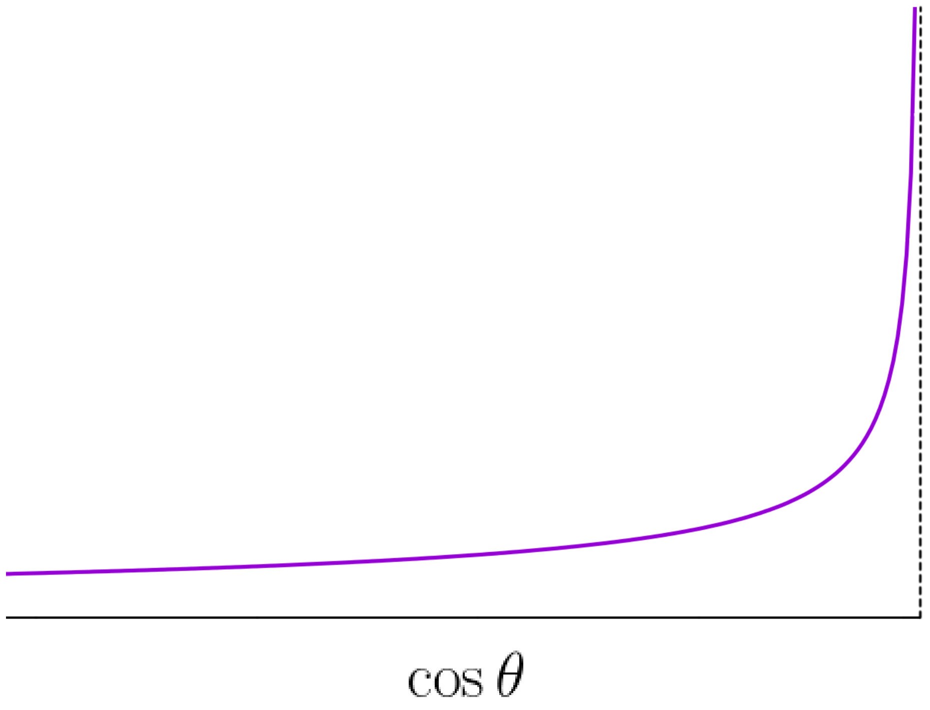
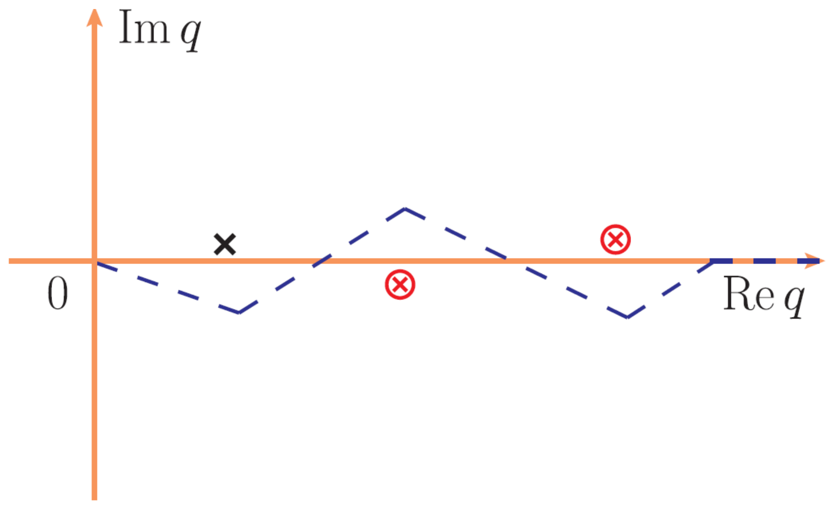


End point singularity

$$f(q) = \int_{-1}^{1} d\cos\theta \, \frac{1}{P^0 - k^0 - \sqrt{m_3^2 + q^2 + k^2 + 2qk\cos\theta} + i\epsilon}$$





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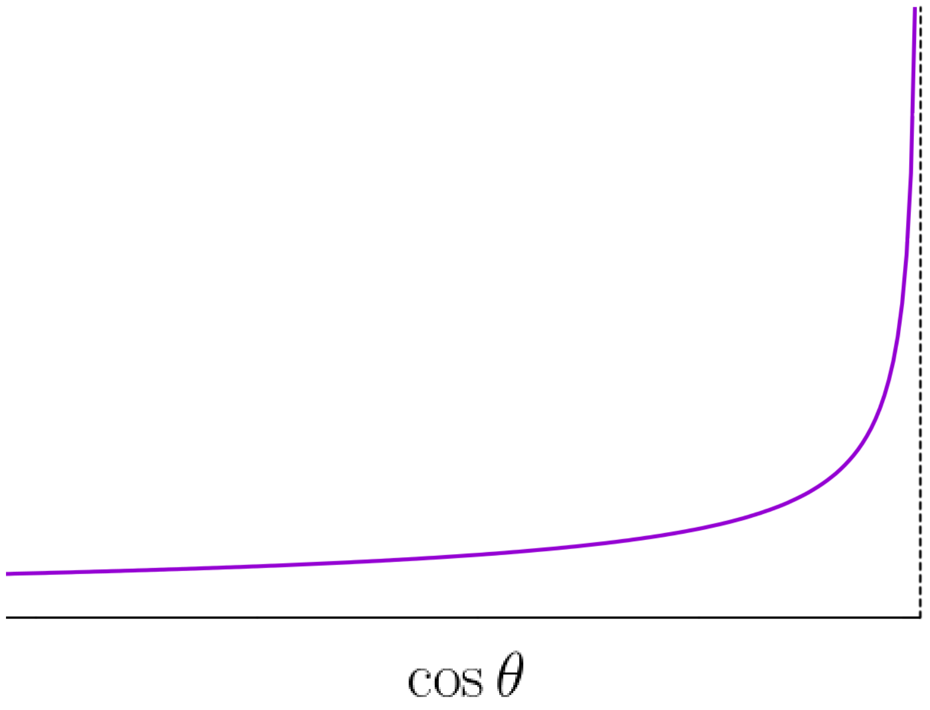
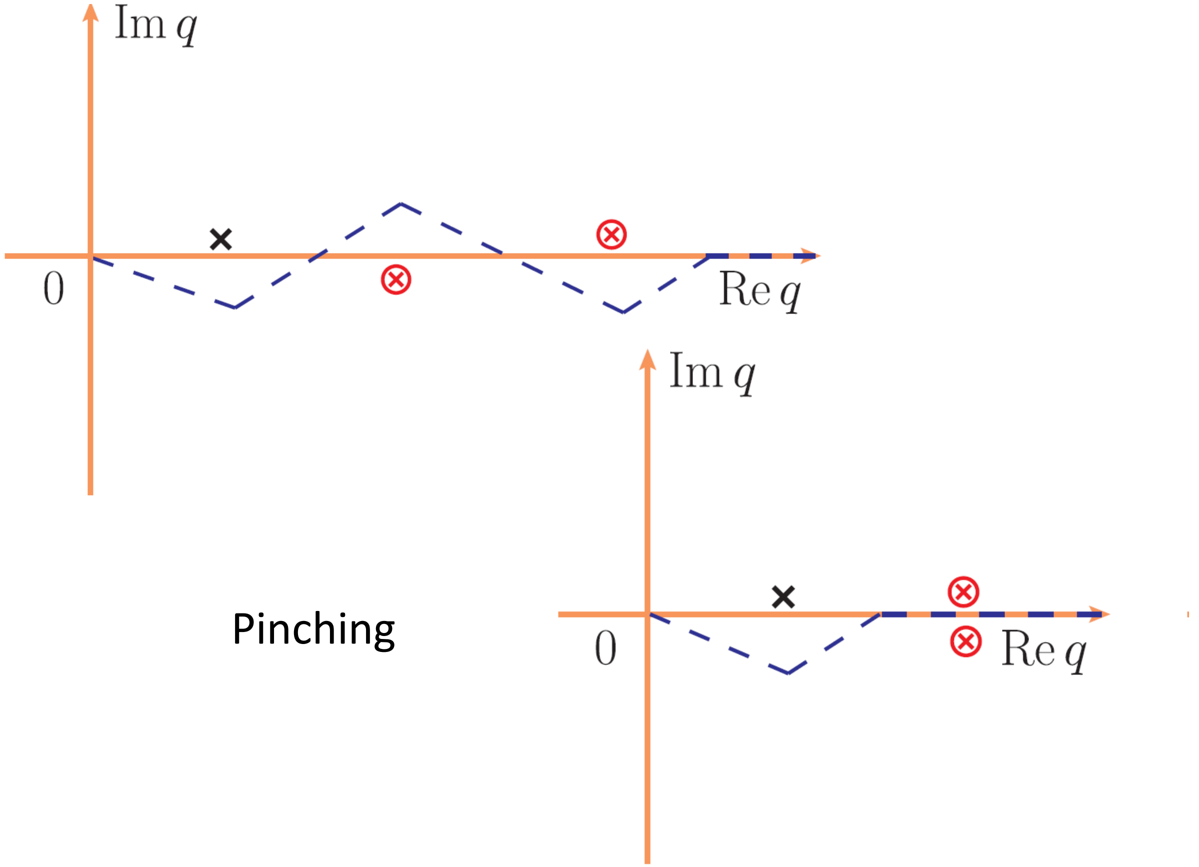
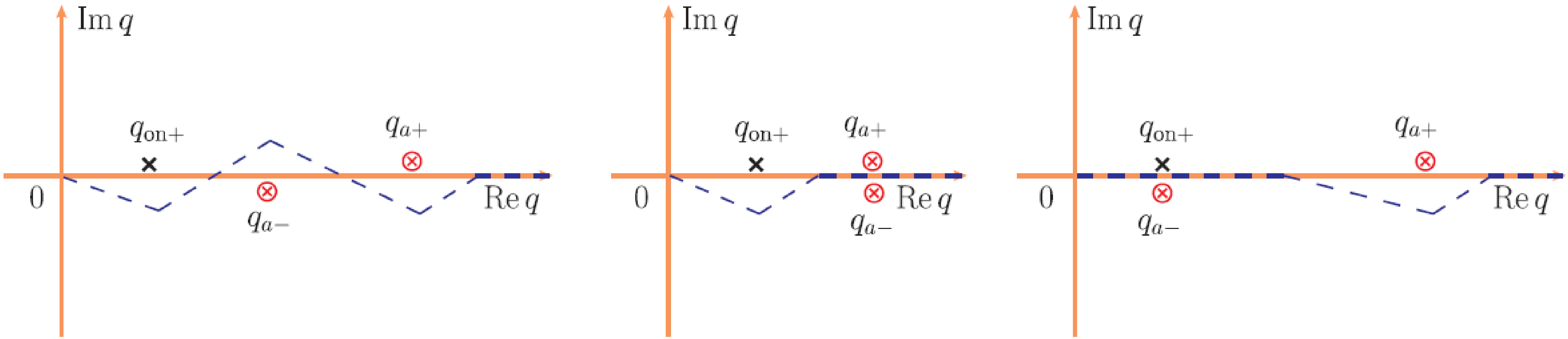
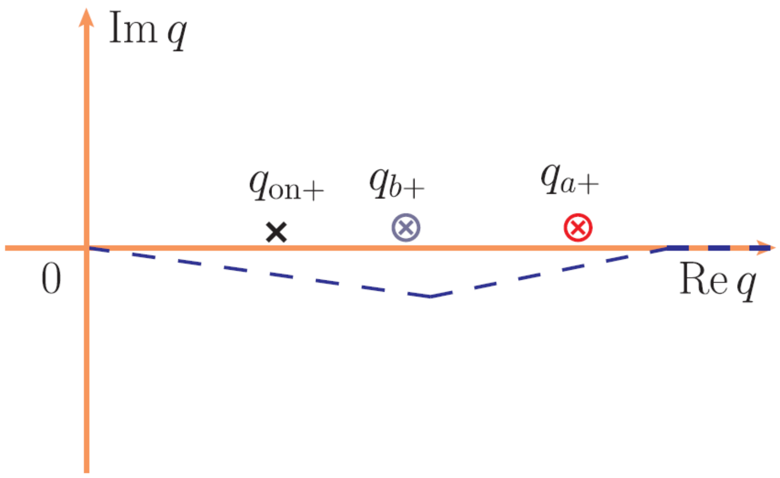


End point singularity



Pinching





$$P^0 - \omega^*(\vec{q}) - \omega(\vec{q}) = 0 \ \rightarrow q_{\text{on}} + i\epsilon$$

$$P^0 - k^0 - \omega(\vec{q}) - \omega'(\vec{q} + \vec{k}) = 0 \ \rightarrow q_{a-} - i\epsilon$$

M. Bayar, F. Aceti, F-K Guo and E. Oset, **Phys. Rev. D 94 (2016) 074039**