Scalar Dipole Dynamical Polarizabilities from proton Real Compton Scattering data

University of Pavia
&
INFN, Pavia (Italy)

Outline

- RCS: theoretical framework
- Static polarizabilities
- Low Energy expansion
- Dynamical polarizabilities

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- Very high correlations
- Too much parameters
- New approach: simplex + bootstrap

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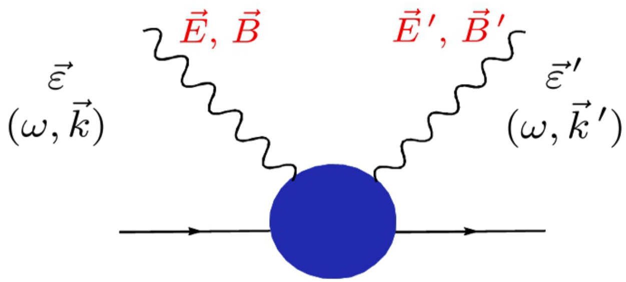
- Static polarizabilities: cross check
- Systematical errors
- Dynamical polarizabilities from data
- Conclusions and future perspectives

RCS amplitudes and Dispersion Relations



$$A_i(\nu, t) = A_i^s(\nu, 0) + A_i^t(0, t) + A_i(0, 0)$$

D. Drechsel, M. Gorchtein, B. Pasquini, M. Vanderhaeghen, Phys.Rev. C61 (1999) 015204
B. Pasquini , D. Drechsel M. Vanderhaeghen, Phys.Rev. C76 (2007) 015203

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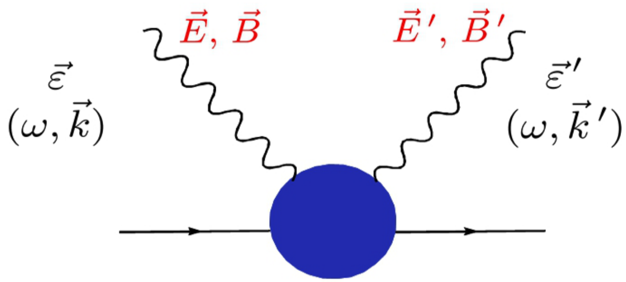

$A_i(0,0) = a_i$ → Static polarizabilities

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Subtracted Dispersion Relations (*s*-channel)

$$A_i^s(\nu, 0) \;=\; \frac{2}{\pi}\, \nu^2\, \mathrm{P} \int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\; A_i(\nu', t)\; \frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

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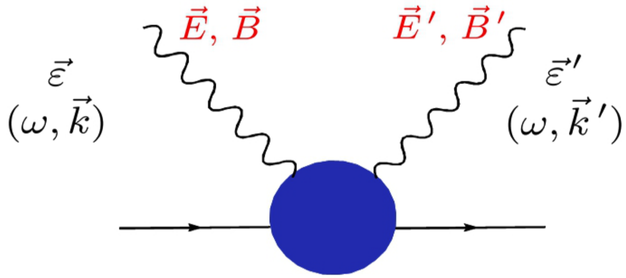

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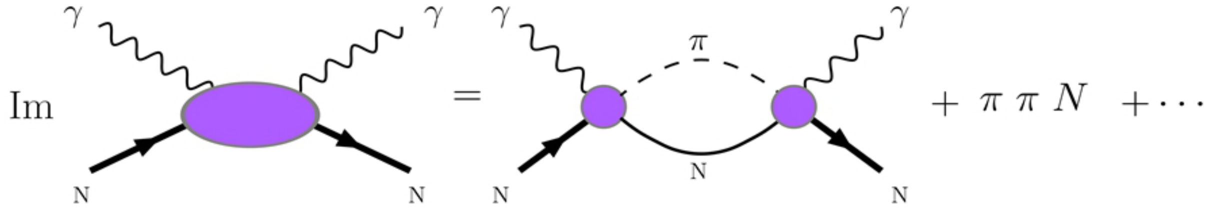
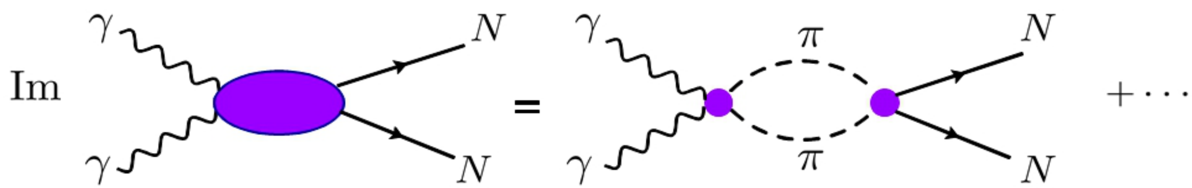
s-CHANNEL



t-CHANNEL



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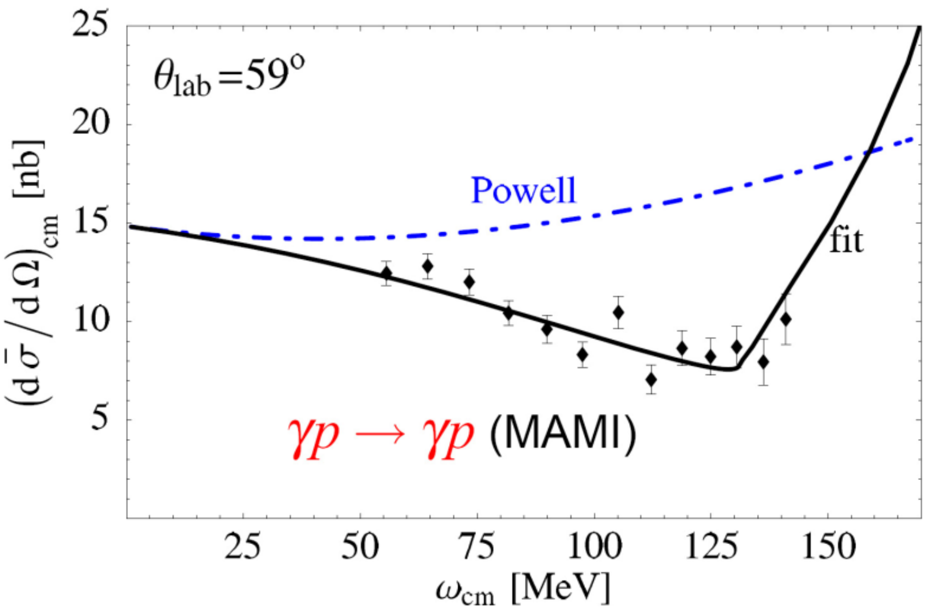

Powell cross section: point-like nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

A.I. L'vov, V.A. Petrun'kin, Martin Schumacher. Phys.Rev. C55 (1997) 359-377
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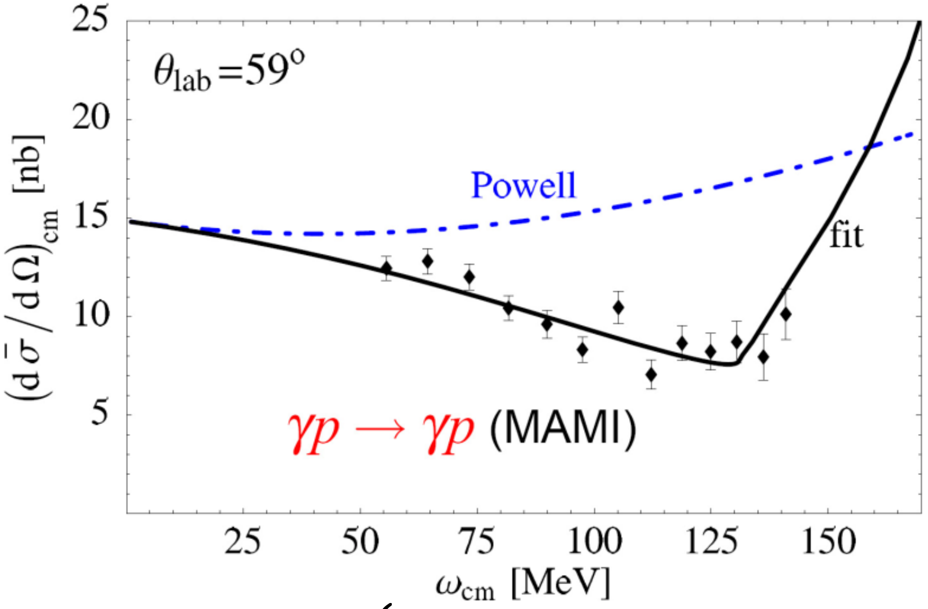

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$$H_{\rm eff}^{\rm pol} = -2\pi \left\{ \omega^2 \left[\alpha_{E1} \vec{E}^2 + \beta_{M1} \vec{B}^2 \right] \right.$$

spin-independent dipole

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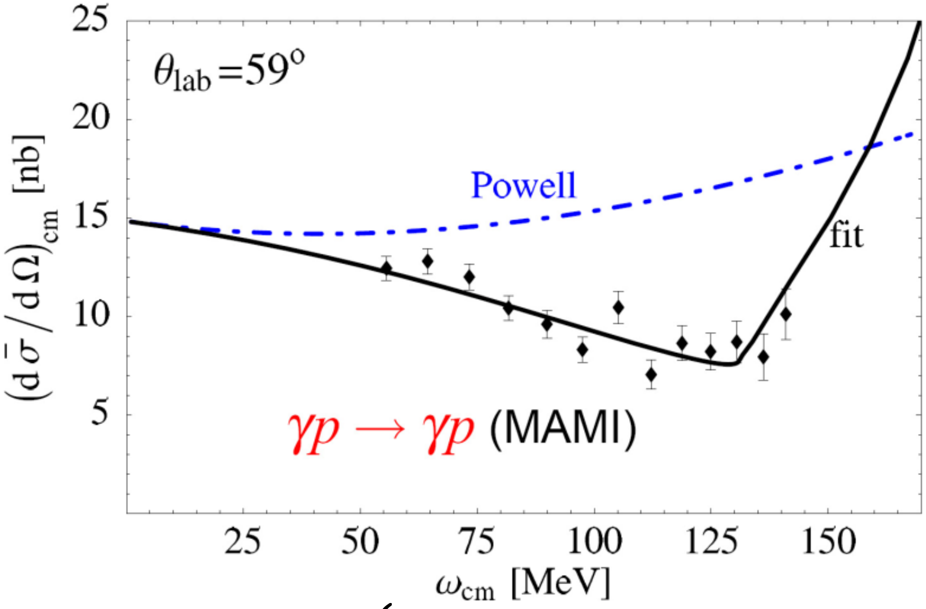

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$$+\,\omega^3\left[\gamma_{E1E1}\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}})+\gamma_{M1M1}\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})\right.$$

spin-dependent dipole

$$\left.-2\,\gamma_{M1E2}\,\sigma_i B_j E_{ij}+2\,\gamma_{E1M2}\,\sigma_i E_j B_{ij}\right]+\,\mathcal{O}(\omega^3)\Big\}$$

spin-dependent dipole-quadrupole

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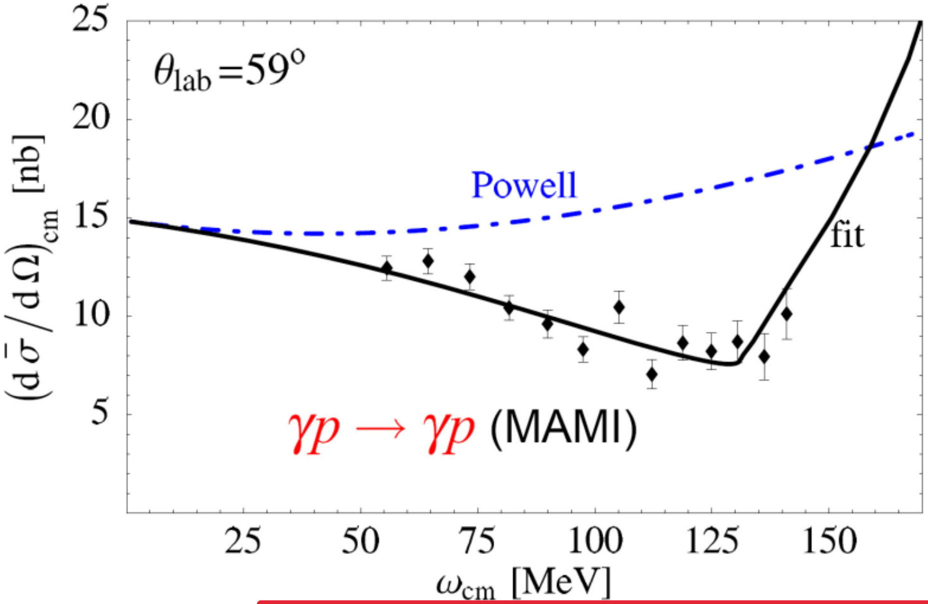

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Low Energy Expansion (LEX) (I)

$$
\begin{aligned}
A_i(\nu,t) &= A_i(\nu,t)|_{(0,0)} + \left.\frac{\partial A_i(\nu,t)}{\partial \nu^2}\right|_{(0,0)} \nu^2 + \left.\frac{\partial A_i(\nu,t)}{\partial t}\right|_{(0,0)} t \\
&+ \frac{1}{2}\left(\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^4}\right|_{(0,0)} \nu^4 + \left.\frac{\partial^2 A_i(\nu,t)}{\partial t}^2\right|_{(0,0)} t^2 + 2\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^2 \partial t}\right|_{(0,0)} \nu^2 t \right)
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\end{aligned}
$$

ν and *t* as independent variables

Lorentz invariant amplitudes

Need to choose a ref-frame: CM

} **R_i (A_i)** (ready for multipole expansion)

D. Babusci, G. Giordano, A.I. L'vov, G. Matone, A.M. Nathan. Phys.Rev.C58 (1998) 1013-1041

Multipole expansion and DYNAMICAL polarizabilities

$$R_1 = \sum_{l\geq 1}\{[(l+1)f_{EE}^{l+} + l f_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + l f_{MM}^{l-}]P_l''\}$$

$$R_2 = \sum_{l\geq 1}\left\{[(l+1)f_{MM}^{l+} + l f_{MM}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{EE}^{l+} + l f_{EE}^{l-}]P_l''\right\}$$

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DYNAMICAL POLARIZABILITIES

$$\alpha_{El} = a(l)\frac{(l+1)f_{EE}^{l+} + l f_{EE}^{l-}}{\omega^{2l}} \quad \beta_{Ml} = a(l)\frac{(l+1)f_{MM}^{l+} + l f_{MM}^{l-}}{\omega^{2l}}$$

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DIPOLE DYNAMICAL POLARIZABILITIES
(DDPs)

$\alpha_{E1}(\omega)$ $\beta_{M1}(\omega)$

D. Babusci, G. Giordano, A.I. L'vov, G. Matone, A.M. Nathan.Phys.Rev. C58 (1998) 1013-1041
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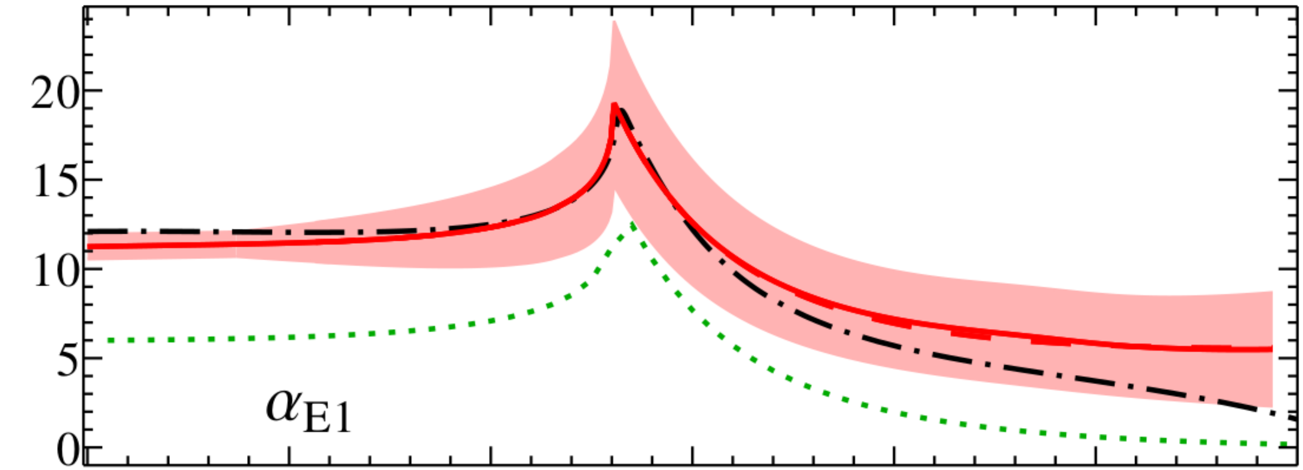
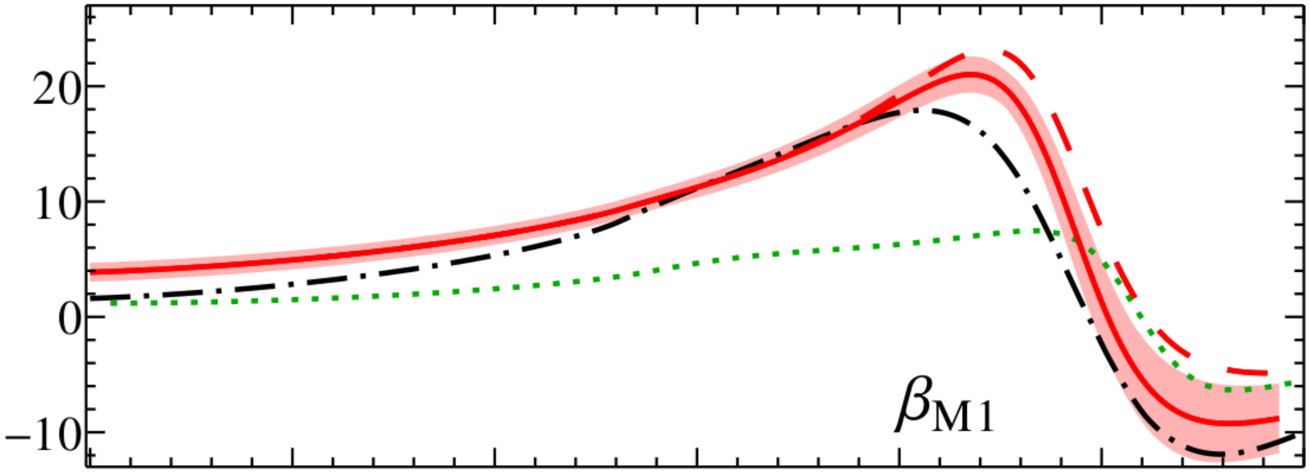
DDPs: physical meaning





F. Hagelstein, R. Miskimen, V. Pascalutsa. Prog.Part.Nucl.Phys. 88 (2016) 29-97

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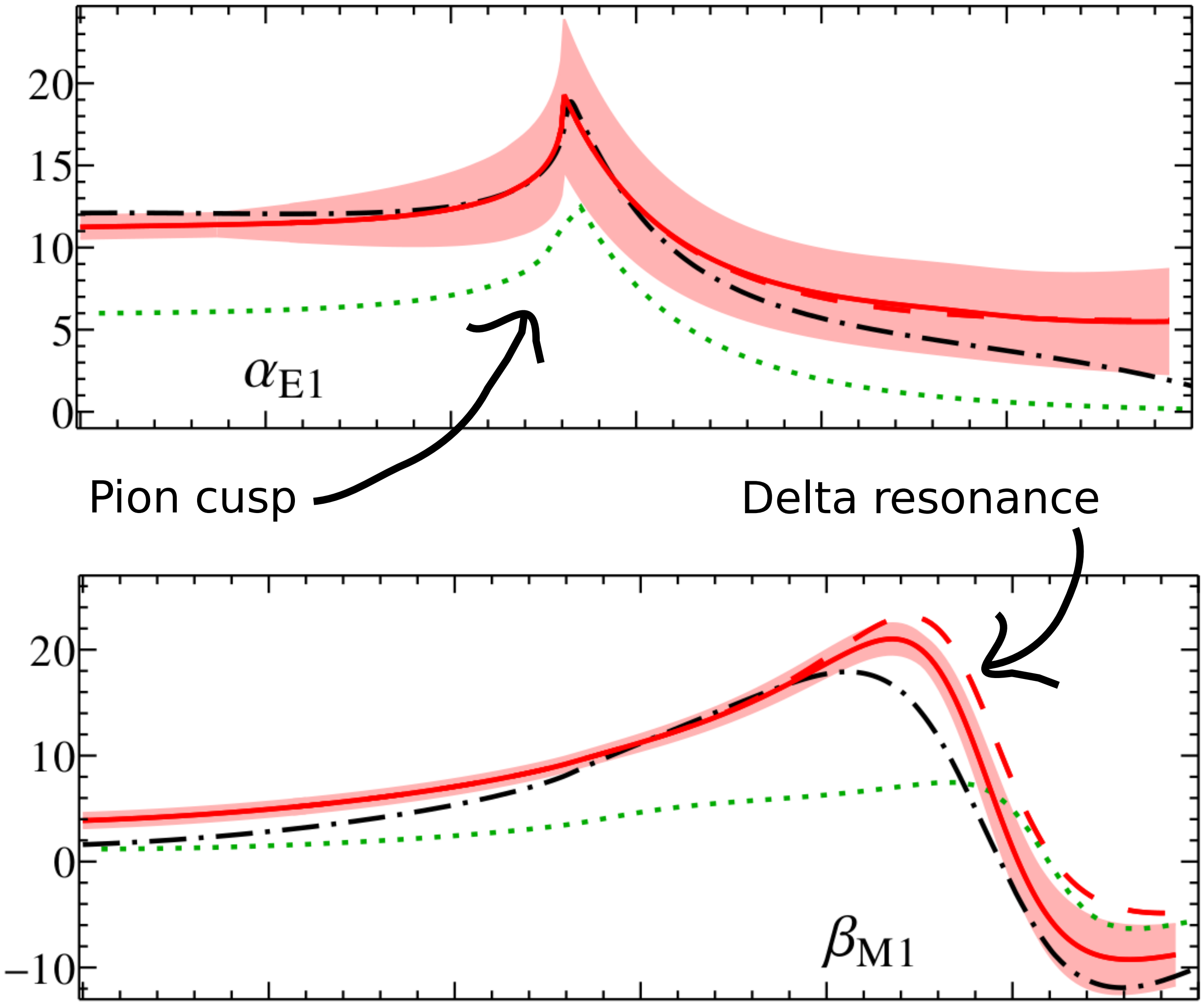

DDPs:
response of the internal nucleon degrees of freedom to an electric and magnetic field with and explicit dependence on energy

F. Hagelstein, R. Miskimen, V. Pascalutsa. Prog.Part.Nucl.Phys. 88 (2016) 29-97

Low Energy Expansion (LEX) (II)

$$\alpha_{E1}(\omega) = \alpha_{E10} + \alpha_{E11}\,\omega + \alpha_{E12}\,\omega^2 + \alpha_{E13}\,\omega^3 + \alpha_{E14}\,\omega^4 + \alpha_{E15}\,\omega^5$$

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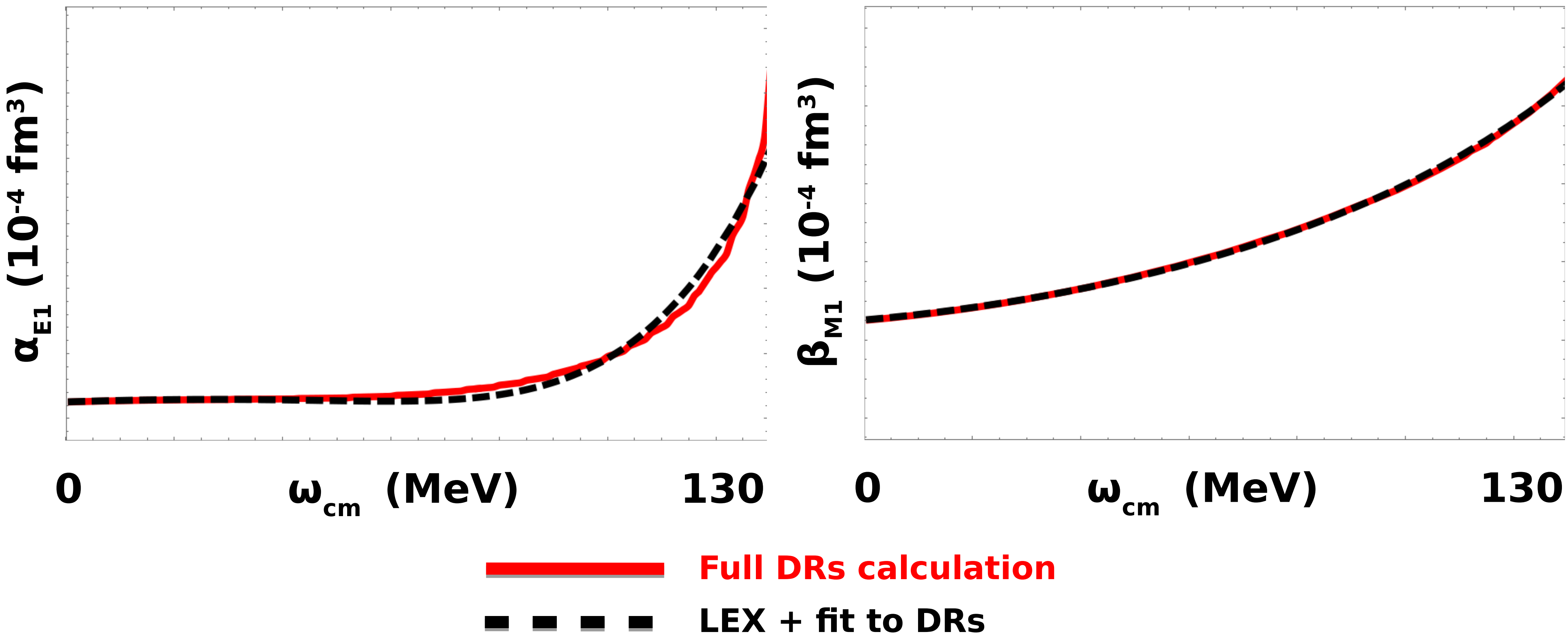
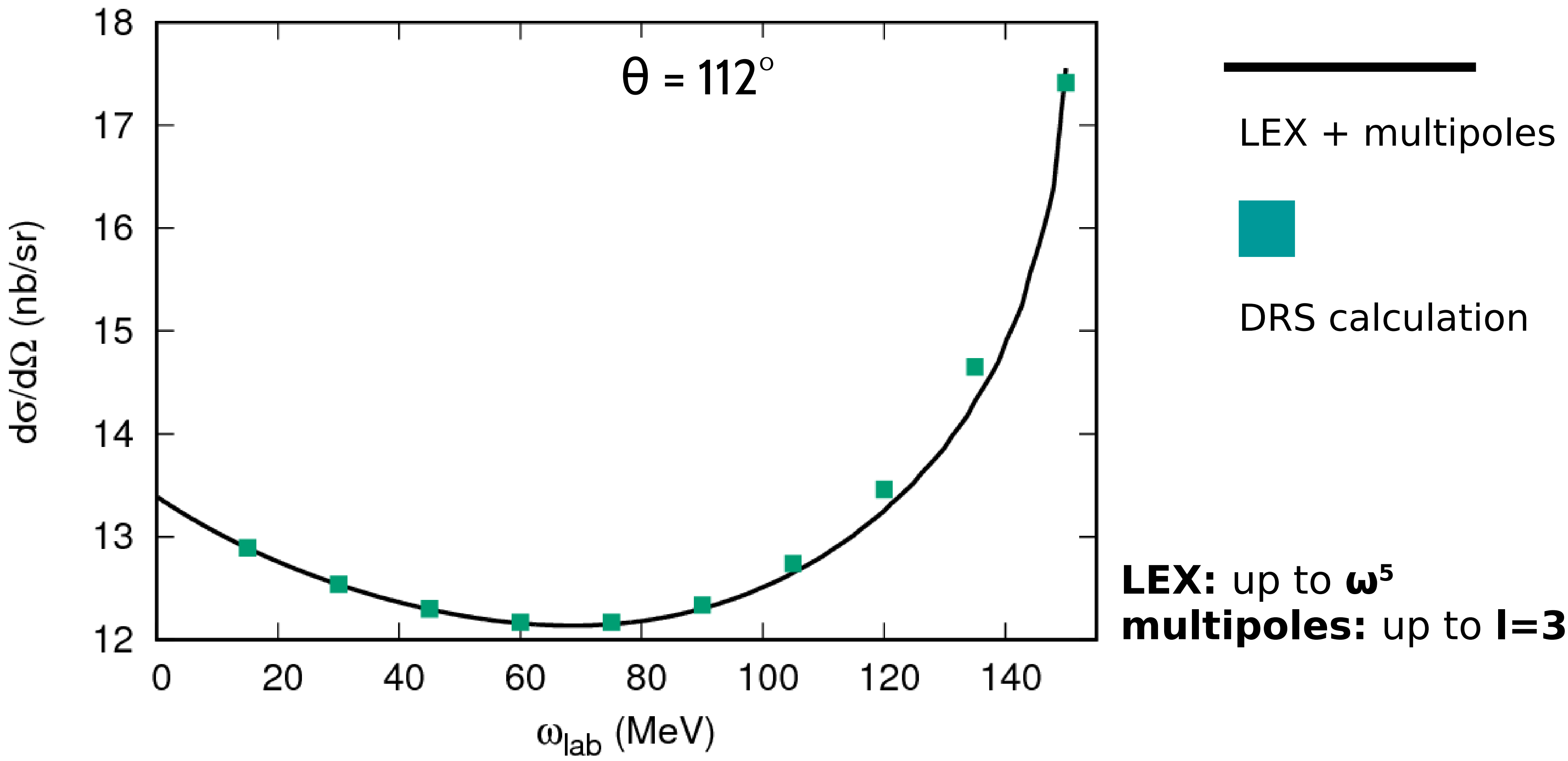


DRs vs (LEX + multipoles)



LEX: up to $\boldsymbol{\omega^5}$
multipoles: up to **l=3**

The GOAL

Extract scalar

Dipole Dynamical Polarizabilites (DDPs)

from RCS data

Complications

Gradient method to find the χ^2 minimum

VERY **high correlations** between parameters!

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```
MINUIT WARNING IN HESSE
 =============== MATRIX FORCED POS-DEF BY ADDING
0.13727E-01 TO DIAGONAL.
```

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VERY **low sensitivity** of the data to dynamical polarizabilities

NO WAY to find the “right” minimum and to define “right” errors on fit parameters

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Combination of **SIMPLEX** method and **BOOTSTRAP** technique

The DATA set

Half of the Spartans
that King Leonidas led
to the Battle of
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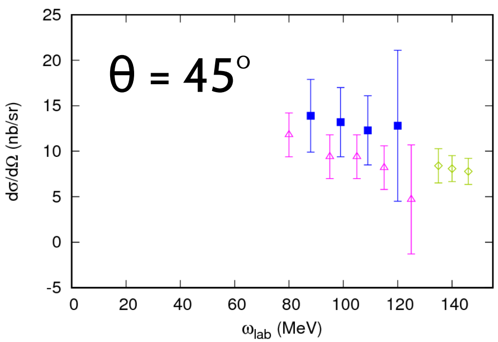
The DATA set



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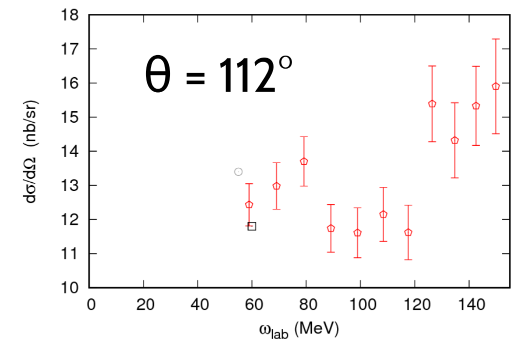
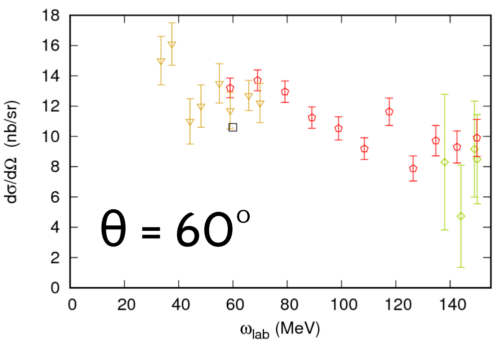
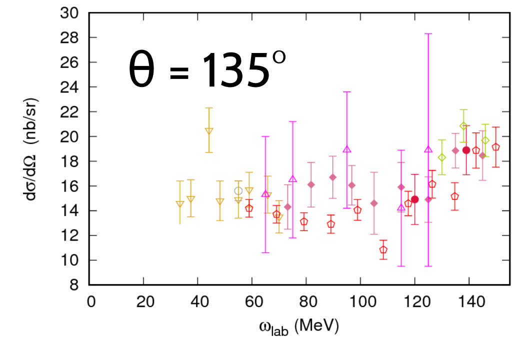
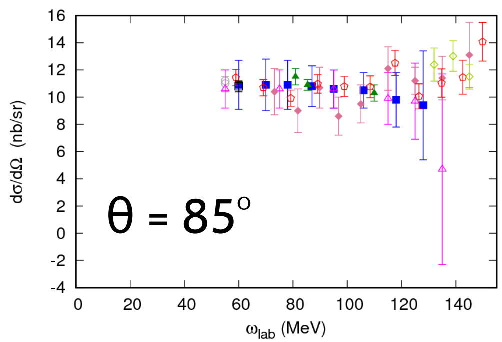
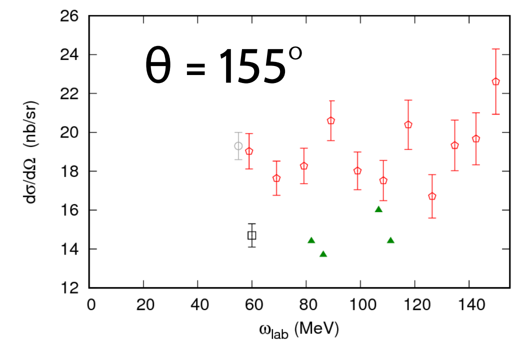




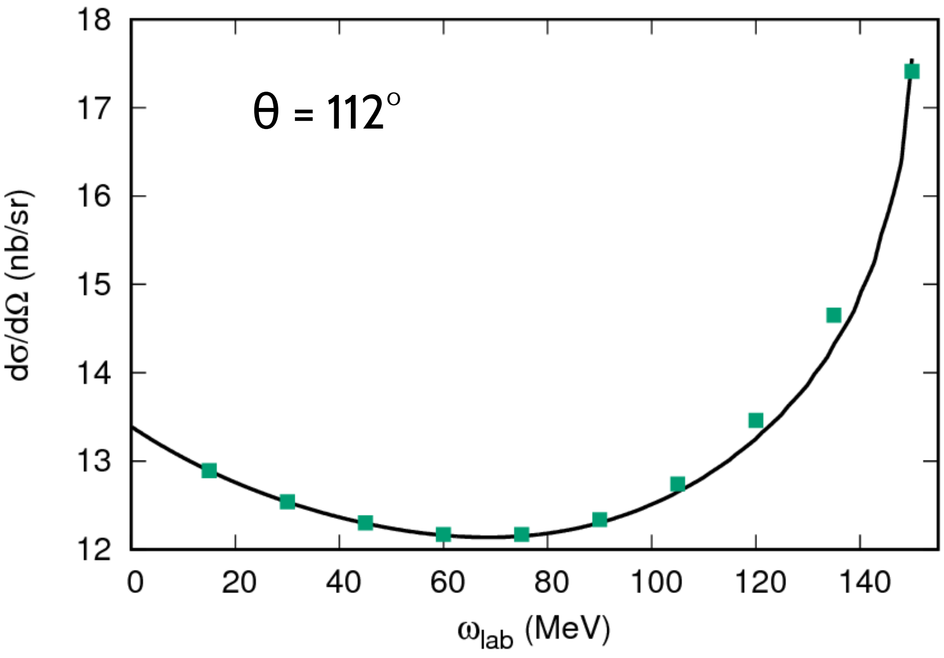
Symbol	Set	Ref
	GOL 60	Goldansky et al.
	OdL 01	Olmos de León
	HAL 93	Hallin et al.
	HYM 59	Hyman et al.
	PUG 67	Pugh et al.
	FED 91	Federspiel et al.
	BER 61	Bernardini et al.
	BAR 74	Baranov et al.
	OXL 58	Oxley
	MAC 95	MacGibbon et al.







DRs vs (LEX + multipoles)



FIRST cross check:
comparison with
LEX + multipoles
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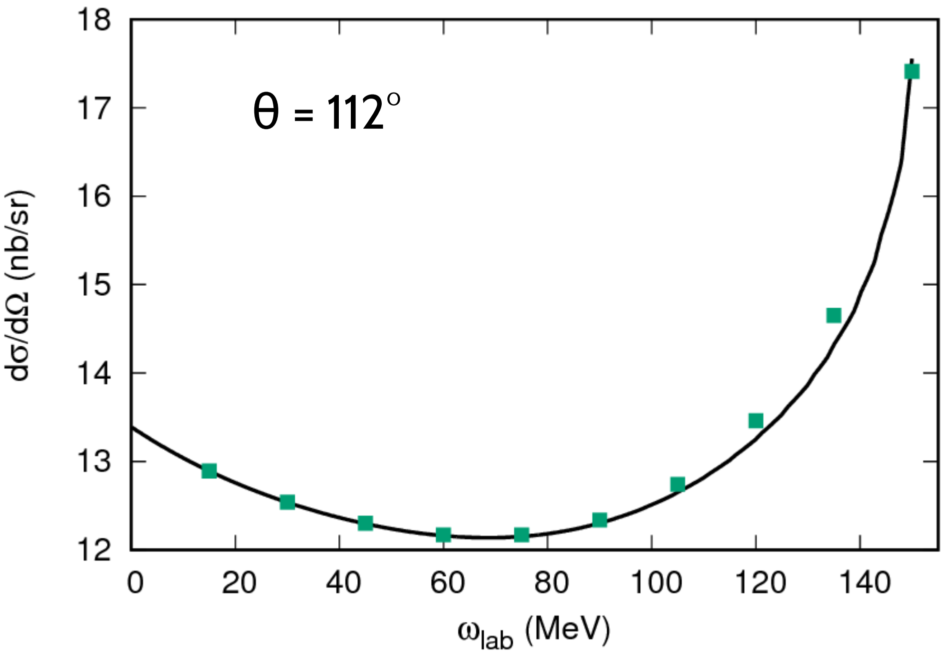
DRs vs (*LEX + multipoles*)



FIRST cross check: comparison with ***LEX + multipoles*** and ***DRs***

	α_{E1} (10^{-4} fm^3)	β_{M1} (10^{-4} fm^3)
DRs	**11.9 ± 0.2**	**1.9 ± 0.2**
LEX + MULTIPOLES	**11.8 ± 0.2**	**2.0 ± 0.2**

B. Pasquini, P. Pedroni, S. S., arXiv:1711.07401v1

Bootstrap sampling and systematics

$$S_{i,exp}^{boot} = S_{i,exp} \pm \gamma\, \sigma_{i,exp}$$ Gaussian distributed

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How can we include systematical errors?

$$\chi^2_{mod} = \sum_{i=1}^{N_{tot}} \left[\frac{\mathcal{N} \mathcal{S}_{i,exp} - \mathcal{S}_{i,theory}}{\mathcal{N} \sigma_{i,exp}} \right]^2 + \left(\frac{\mathcal{N} - 1}{\sigma_{i,sys}} \right)^2$$

...one normalization factor per data set is needed!

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$$S_{i,exp}^{boot} = \xi_i \left[S_{i,exp} \pm \gamma\, \sigma_{i,exp} \right]$$

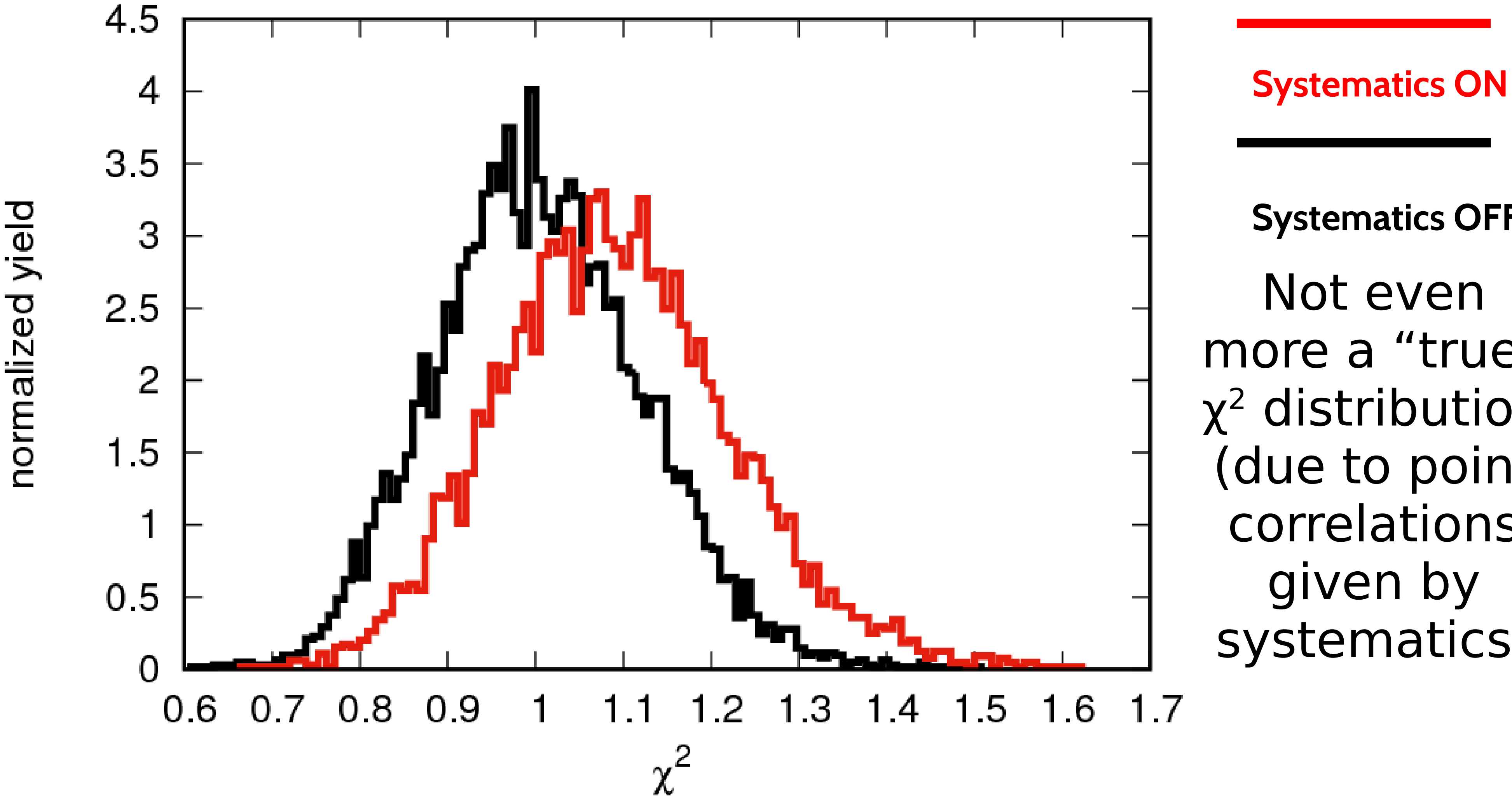
At every bootstrap cycle the systematical errors for each set can vary independently!

Bootstrap vs *Gradient*: systematics ON

	α_{E1}	β_{M1}
BOOTSTRAP	11.8 ± 0.2	2.0 ± 0.2
LEX + MULTIPOLES	11.8 ± 0.2	2.0 ± 0.2
BOOTSTRAP SYS ON	11.8 ± 0.3	2.0 ± 0.3

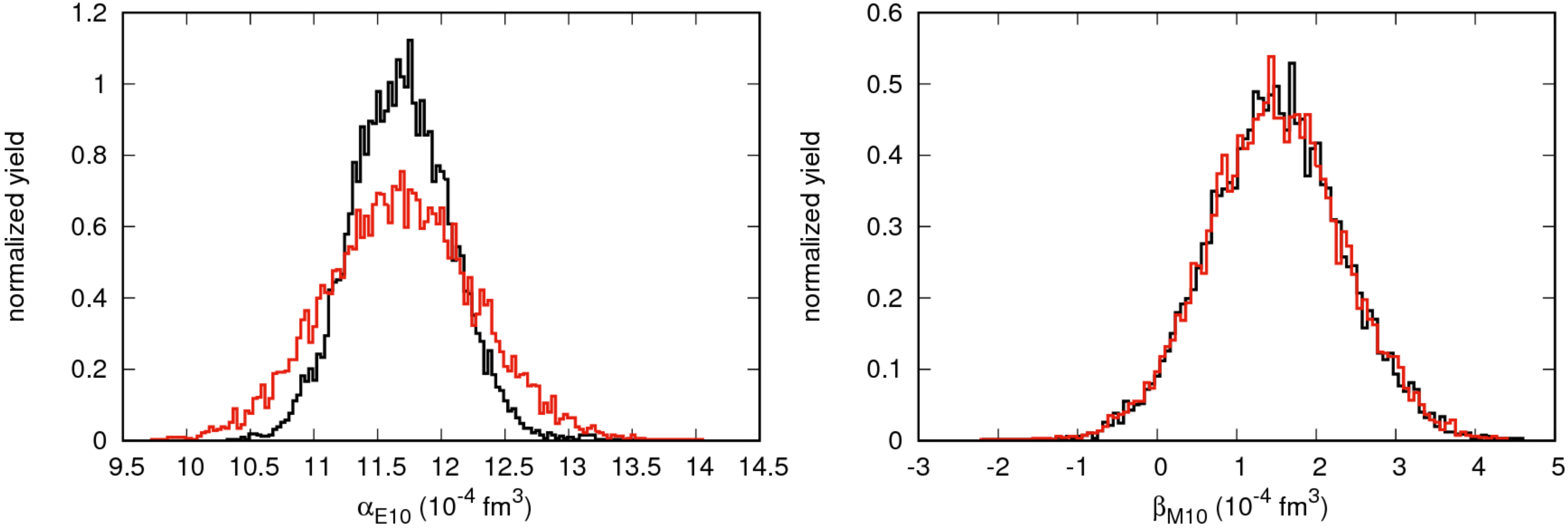
Systematical errors enlarge the error band of polarizabilities!

"χ^2" probability distribution in bootstrap framework (static pol.)



Not even more a "true" χ^2 distribution (due to point correlations given by systematics)

The effect of systematics (static spin-independent polarizabilities)



Systematics ON

Systematics OFF

Expected Gaussian shape + systematics enlarging

B. Pasquini, P. Pedroni, S. S., in preparation

Fit conditions

A. M. Baldin, Nucl. Phys. 18, 310 (1960)
V. Olmos de León, Eur. Phys. J. A 10, 207–215 (2001)

Fit conditions

- ✓ Baldin's sum rule
- ✓ Systematical errors ON
- ✓ FULL data set (150 data)
- ✓ TAPS data set (55 data)
- ✓ Errors on Baldin's sum rule and γ_π included in the procedure

A. M. Baldin, Nucl. Phys. 18, 310 (1960)
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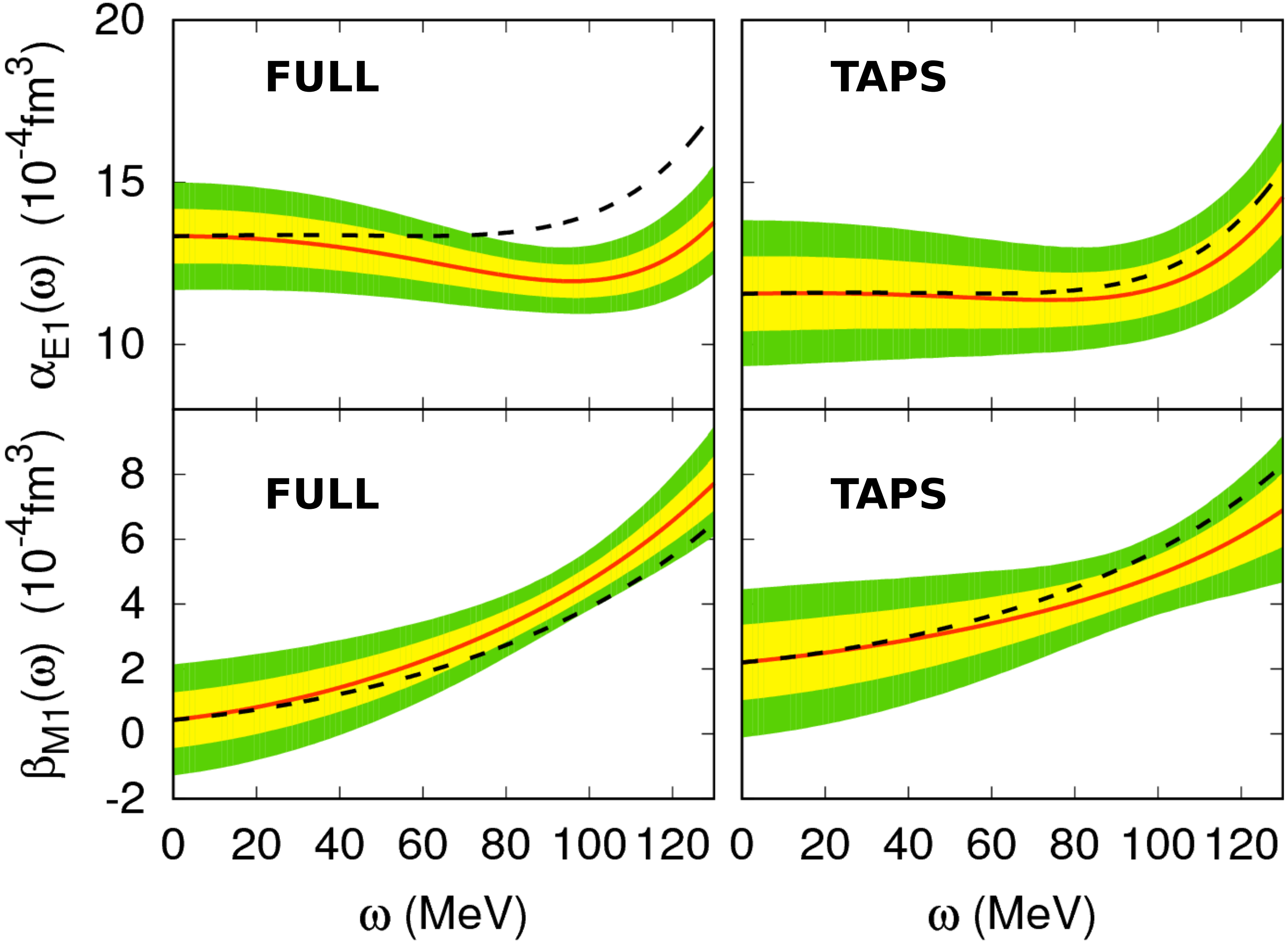
DDPs from the fit (II)



From FIT

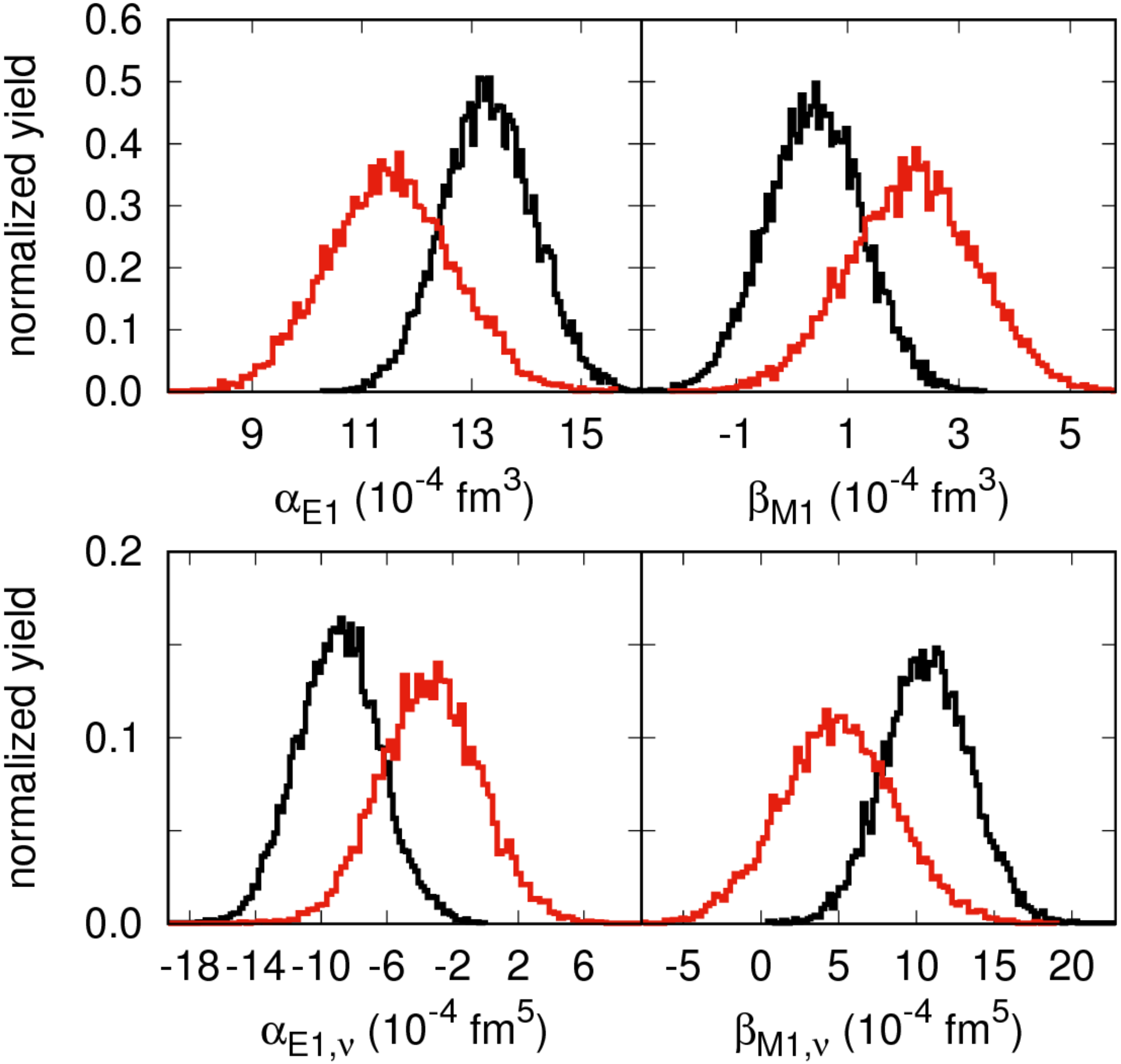
DRs calculation

68% CL band

95 % CL band

B. Pasquini, P. Pedroni, S. S., arXiv:1711.07401v1

DDPs from the fit: probability distributions



TAPS data set

FULL data set

Probability distributions given by our technique (**not** a priori assumed)

Global results: numerical values

		FULL	TAPS
α_{E1}	(10^{-4}fm^3)	13.3 ± 0.8	11.6 ± 1.1
$\alpha_{E1,\nu}$	(10^{-4}fm^5)	-8.8 ± 2.5	-3.2 ± 3.1
β_{M1}	(10^{-4}fm^3)	0.4 ∓ 0.9	2.2 ∓ 1.1
$\beta_{M1,\nu}$	(10^{-4}fm^5)	10.8 ± 2.8	5.1 ± 3.7

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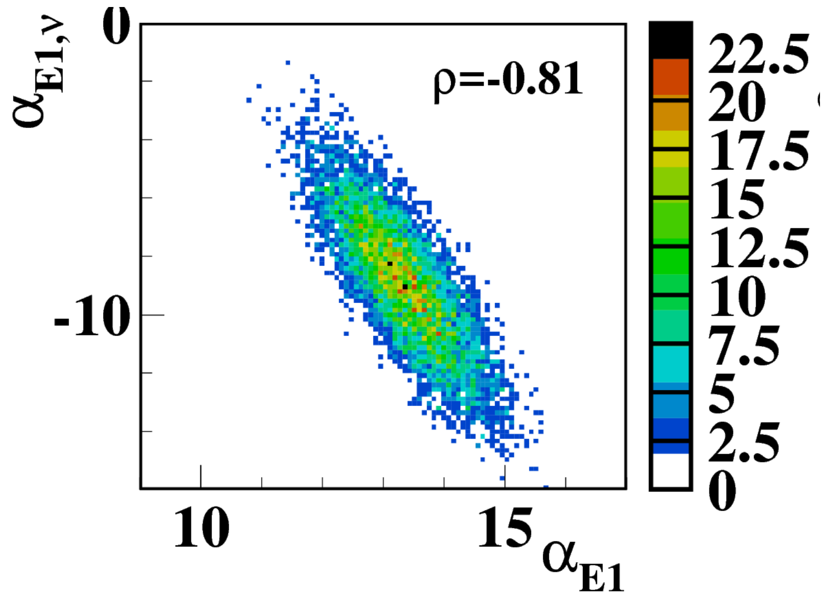
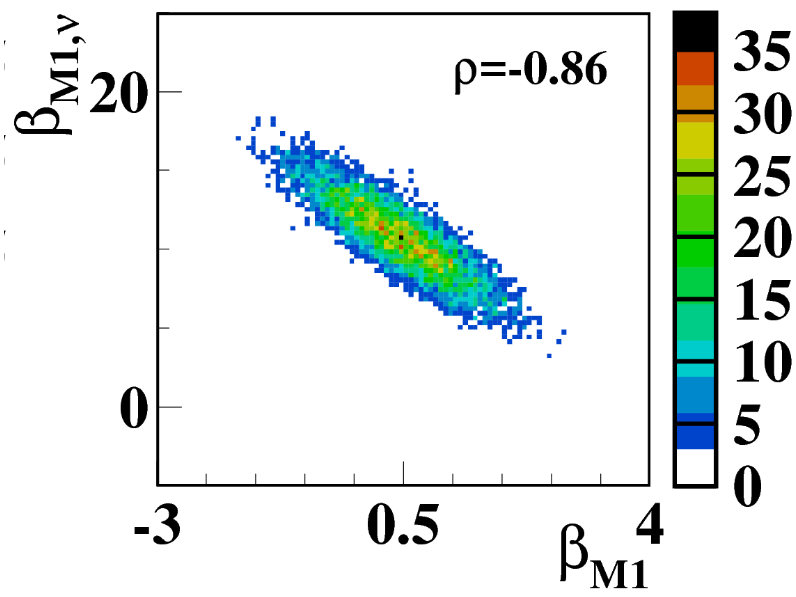
$\alpha_{E1,\nu}$	(10^{-4}fm^5)	-8.8 ± 2.5	-3.2 ± 3.1
β_{M1}	(10^{-4}fm^3)	0.4 ∓ 0.9	2.2 ∓ 1.1
$\beta_{M1,\nu}$	(10^{-4}fm^5)	10.8 ± 2.8	5.1 ± 3.7

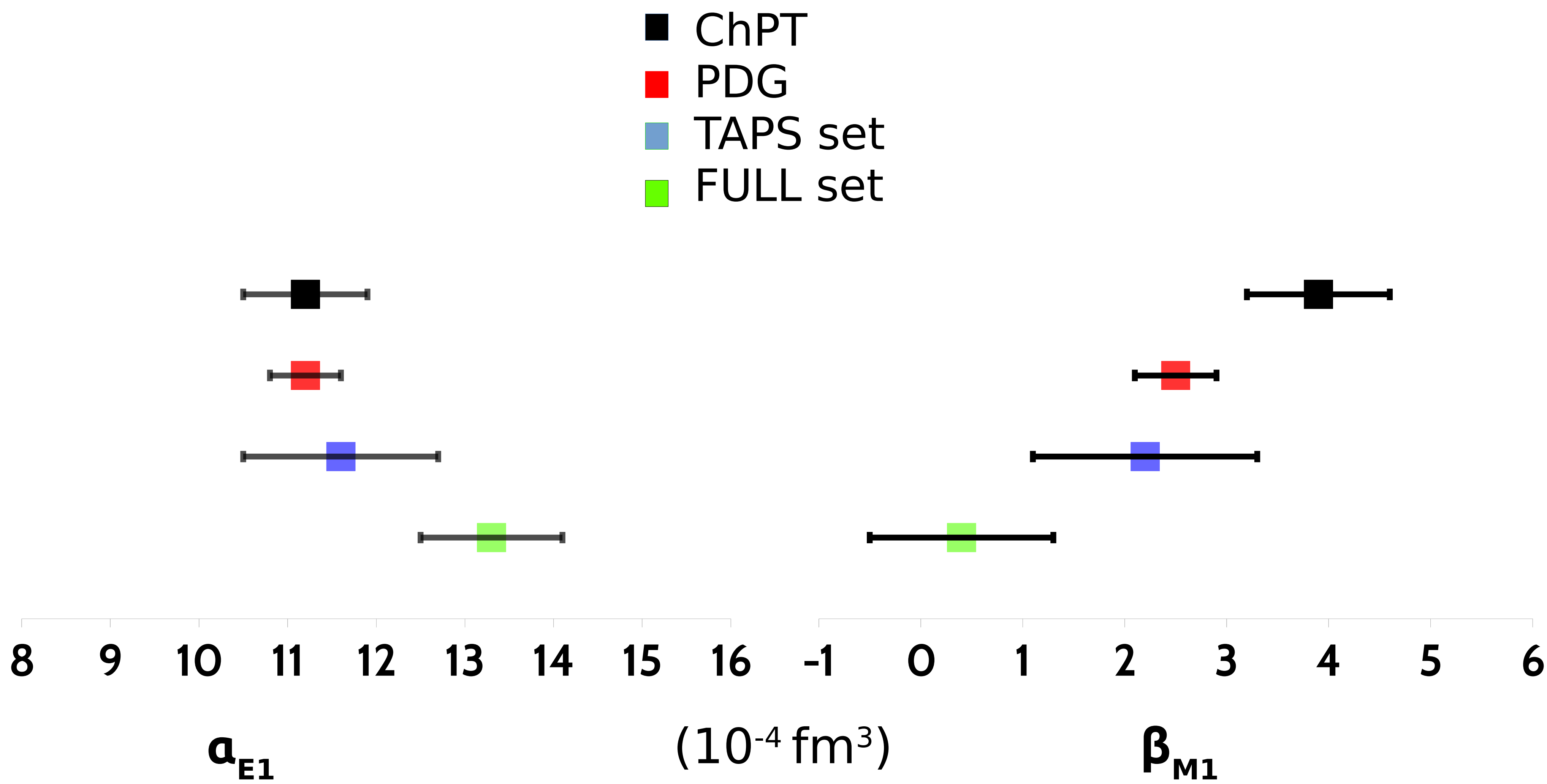
Very STRONG dependence on data set (maybe due to different angular regions...)

Very HIGH correlations among parameters





Global results: α_{E1} and β_{M1}



C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update
V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)

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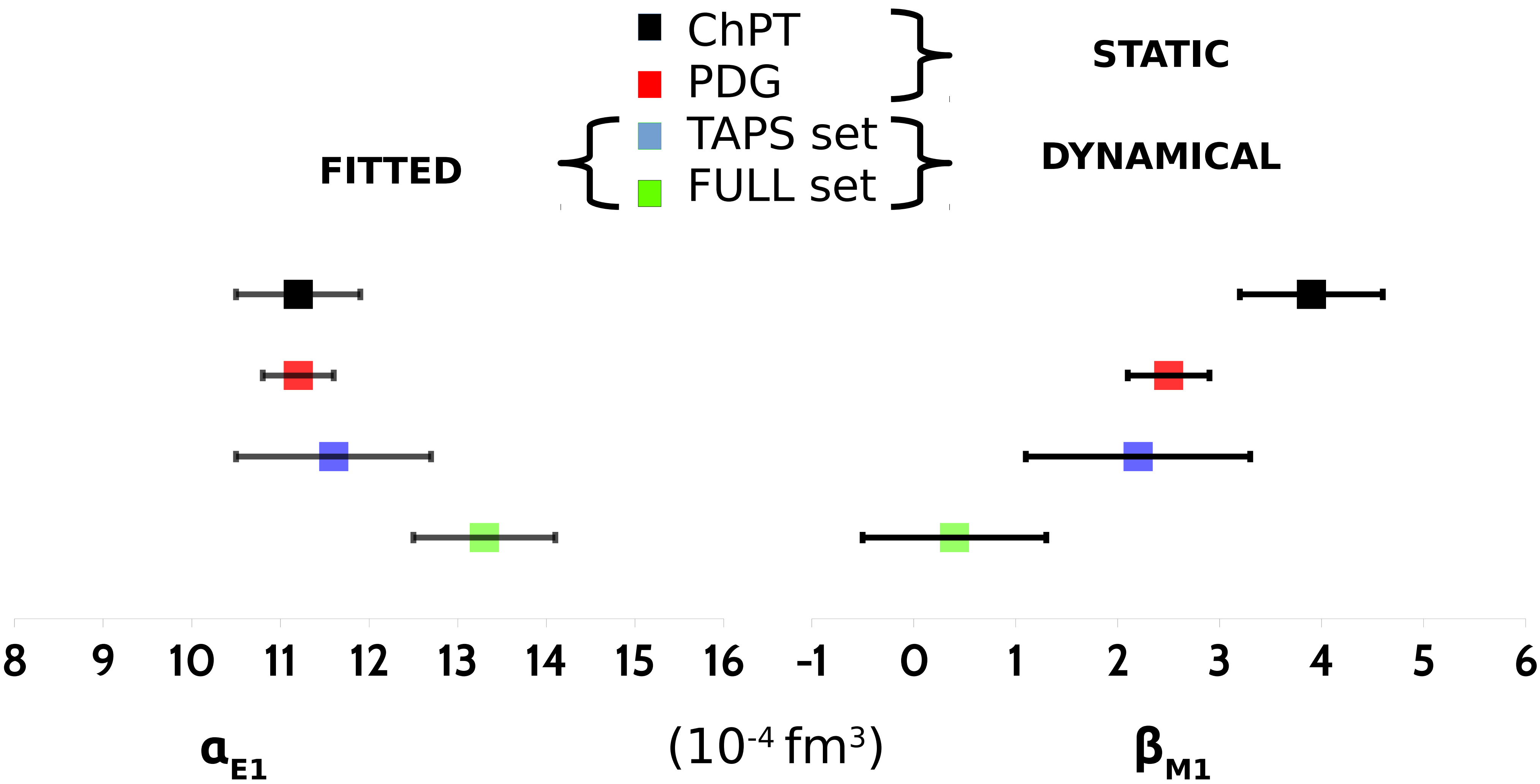

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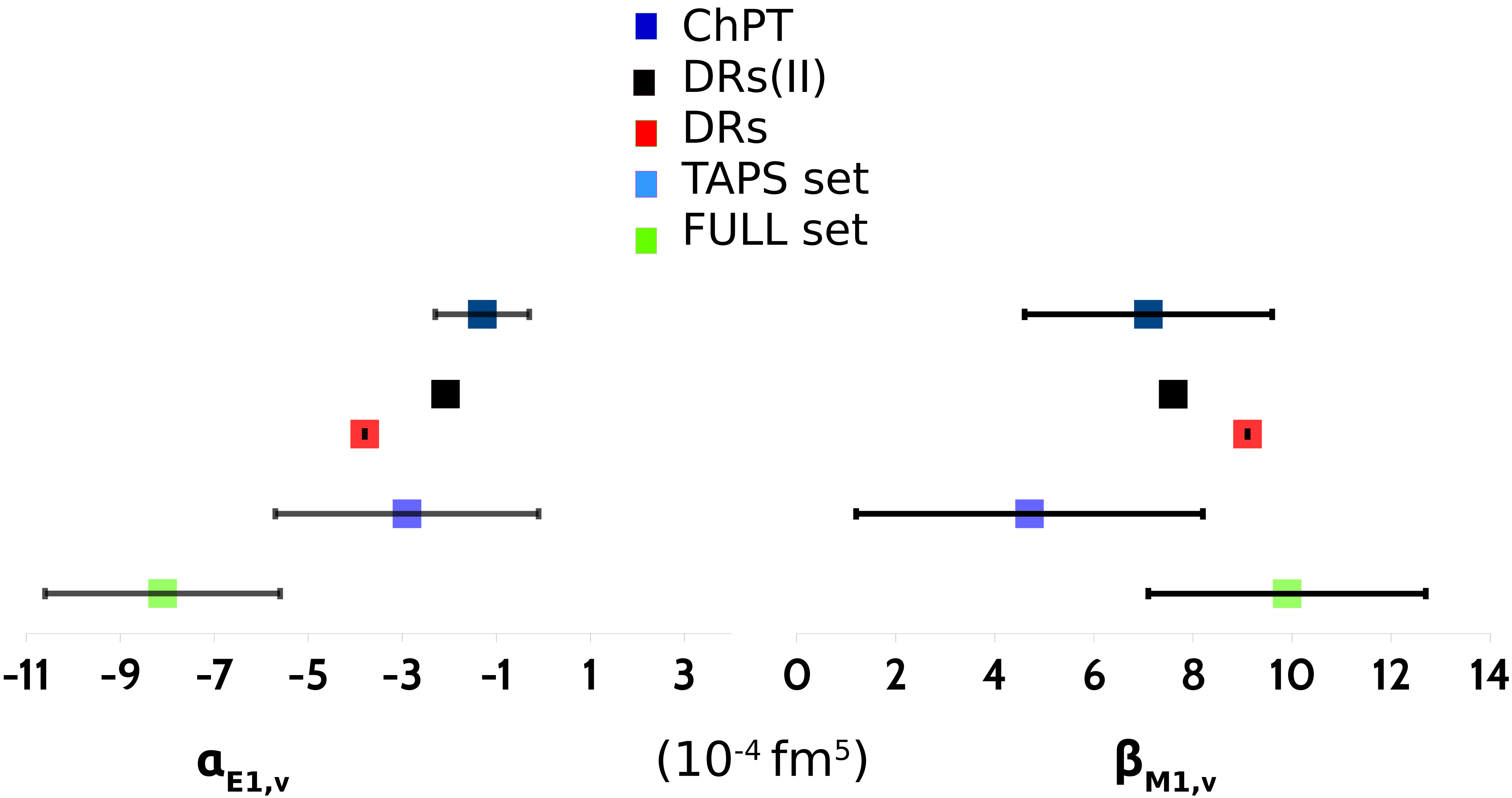

B. R. Holstein, D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rev. C61 (2000) 034316
B. Pasquini, D. Drechsel, M. Vanderhaeghen. Phys.Rev. C76 (2007) 015203
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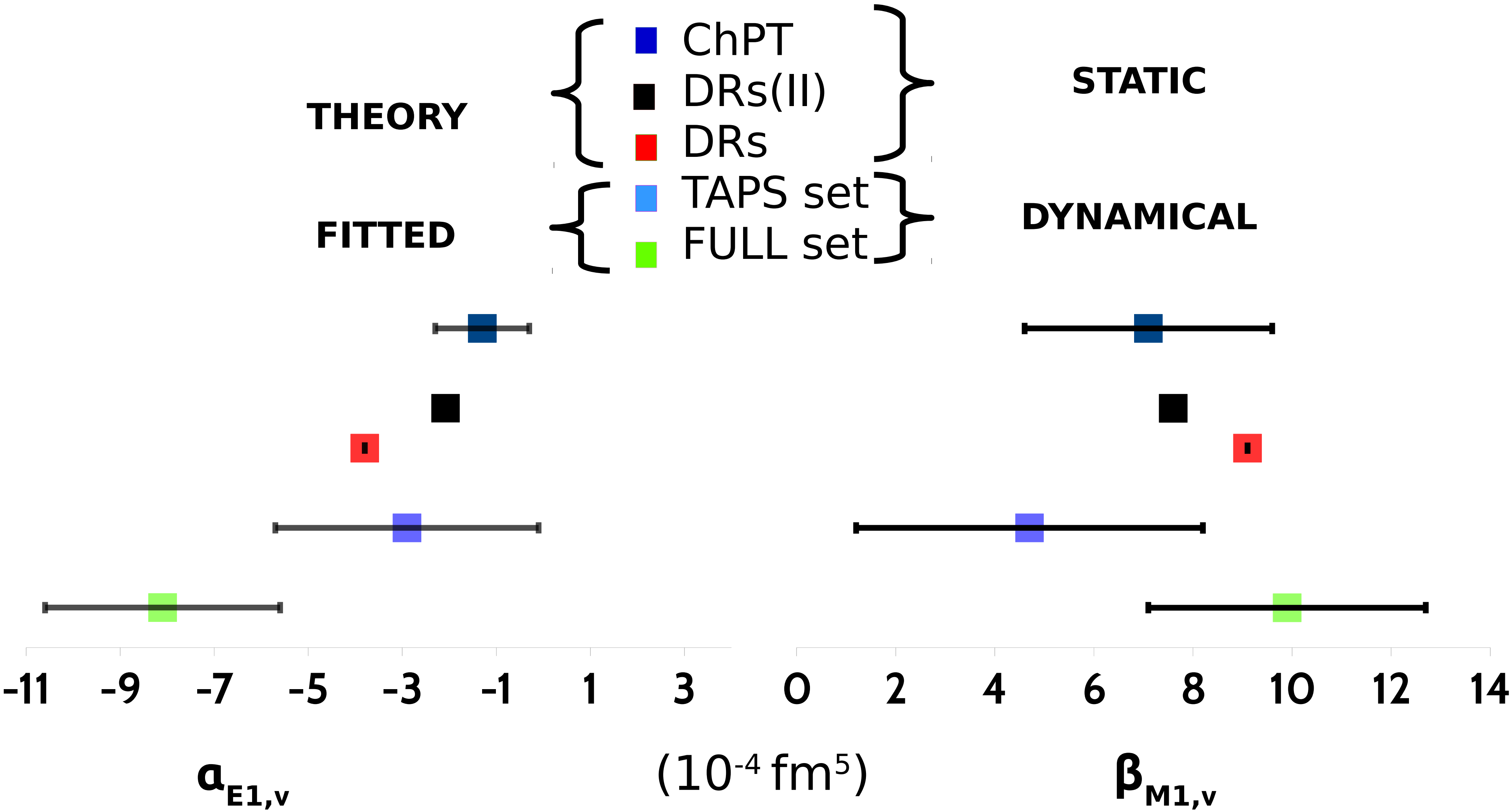

B. R. Holstein, D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rev. C61 (2000) 034316
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Conclusions & perspectives

Very useful and versatile technique for data analysis

Effect of systematic sources of uncertainties on the fitted parameters

Waiting for new data in order to reduce the uncertainties of the fitted parameters (MAMI)

E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Conclusions & perspectives

Very useful and versatile technique for data analysis

Effect of systematic sources of uncertainties on the fitted parameters

Waiting for new data in order to reduce the uncertainties of the fitted parameters (MAMI)

DDPs without LEX (double subtraction in DRs)

Fit of polarized observables in RCS with the same technique

E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

BACKUP SLIDES

Some comments on the data set

Differential cross section

TAPS vs FULL data set

LEX is very slow…

χ^2 curvature close to its minimum

Outliers identifications: rescaling of the errors

Some comments on the data set

Strong correlation between parameters

Reduction of parameters number thanks to sum rules

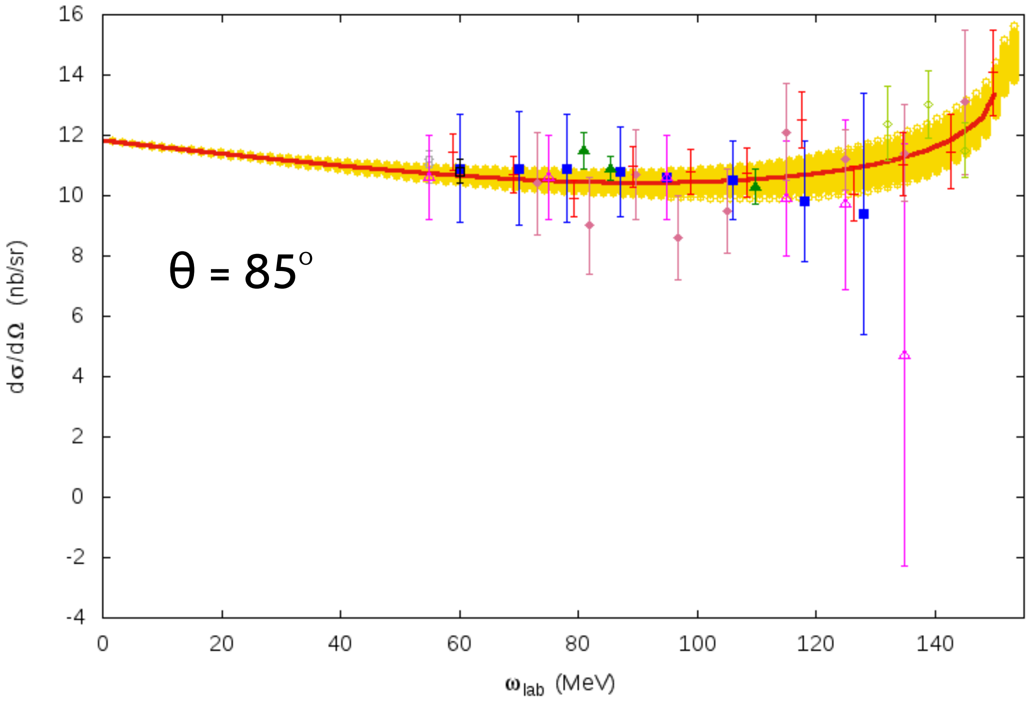
Identifications of the *outliers* (rescaling for statistic errors?)

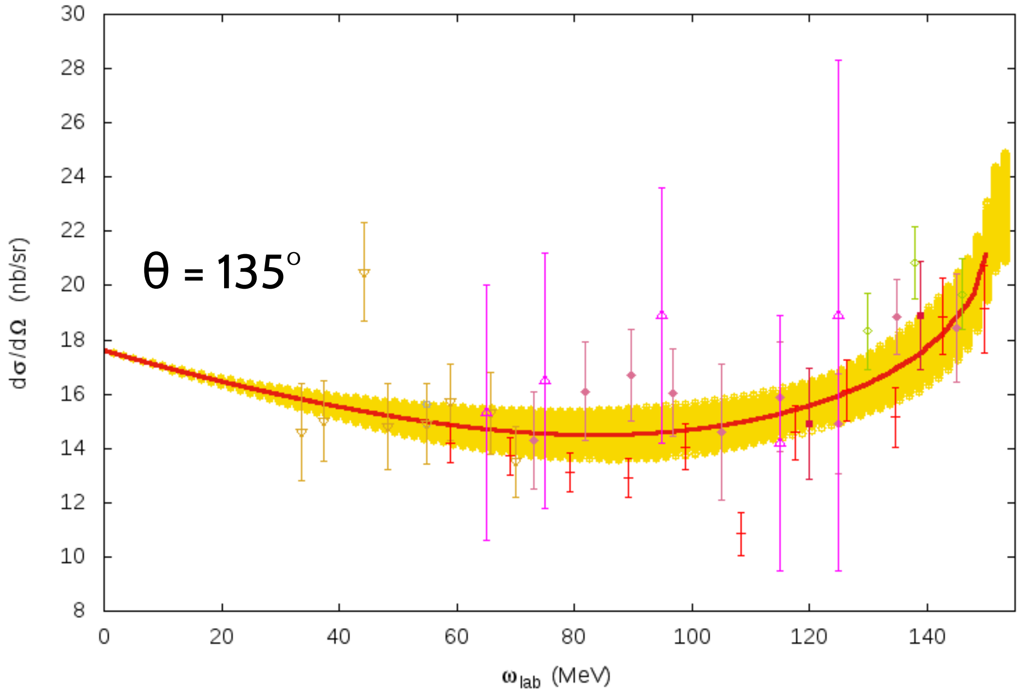
The χ^2 is not the only *quality indicator* → no “definition” of data set

Waiting for new data (MAMI)

E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Differential cross section

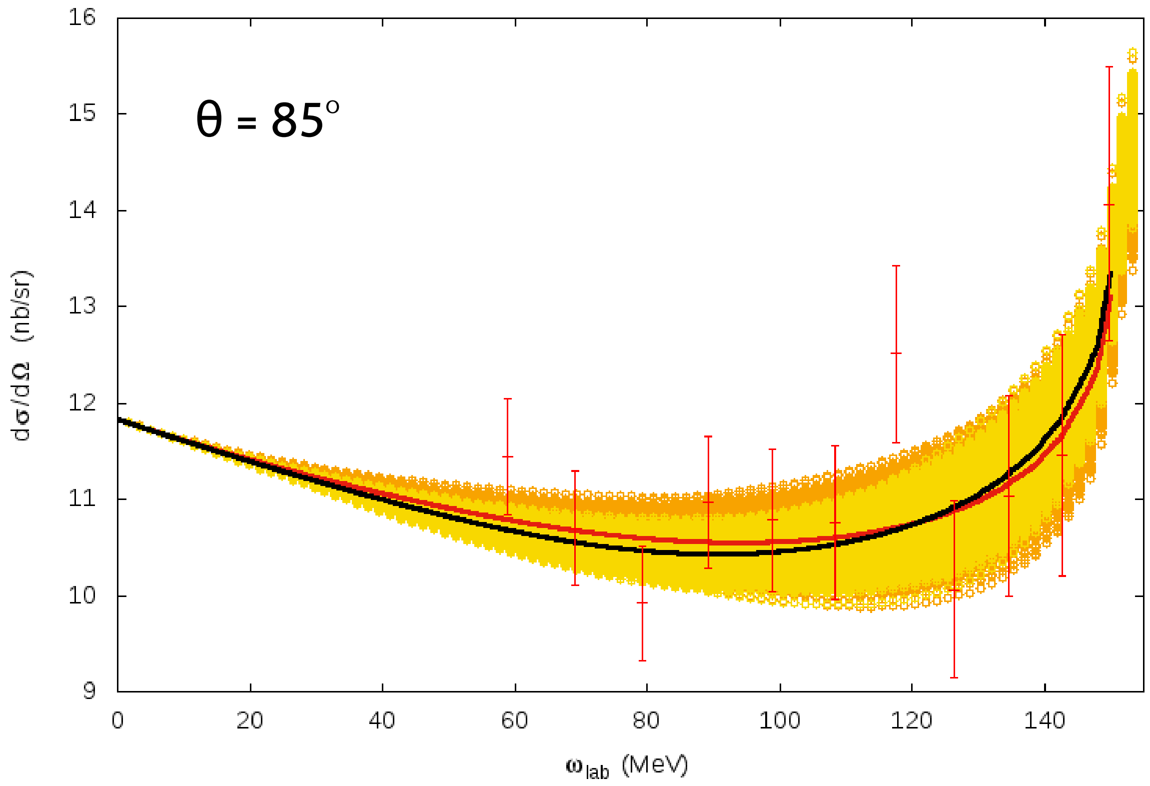




dσ/dΩ VS lab energy

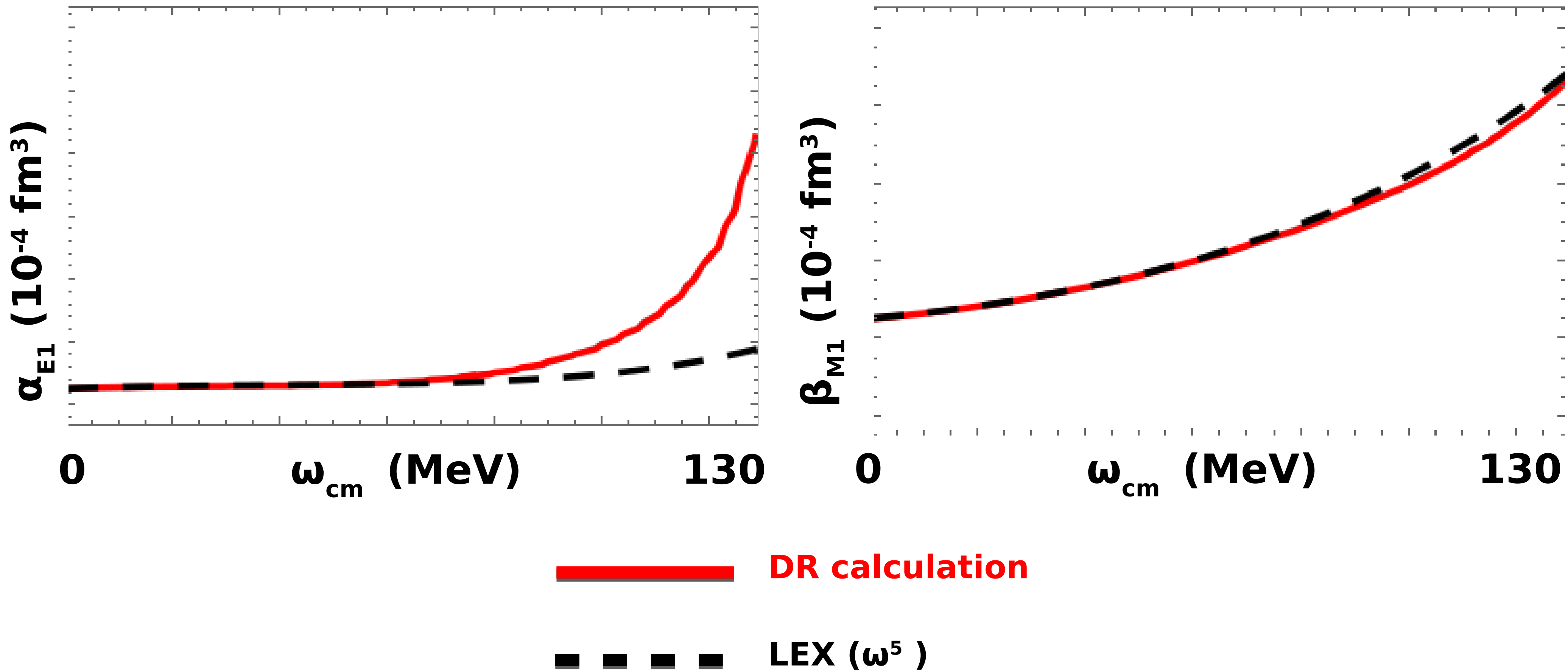
100% error band from the bootstrap fit

TAPS vs FULL data set

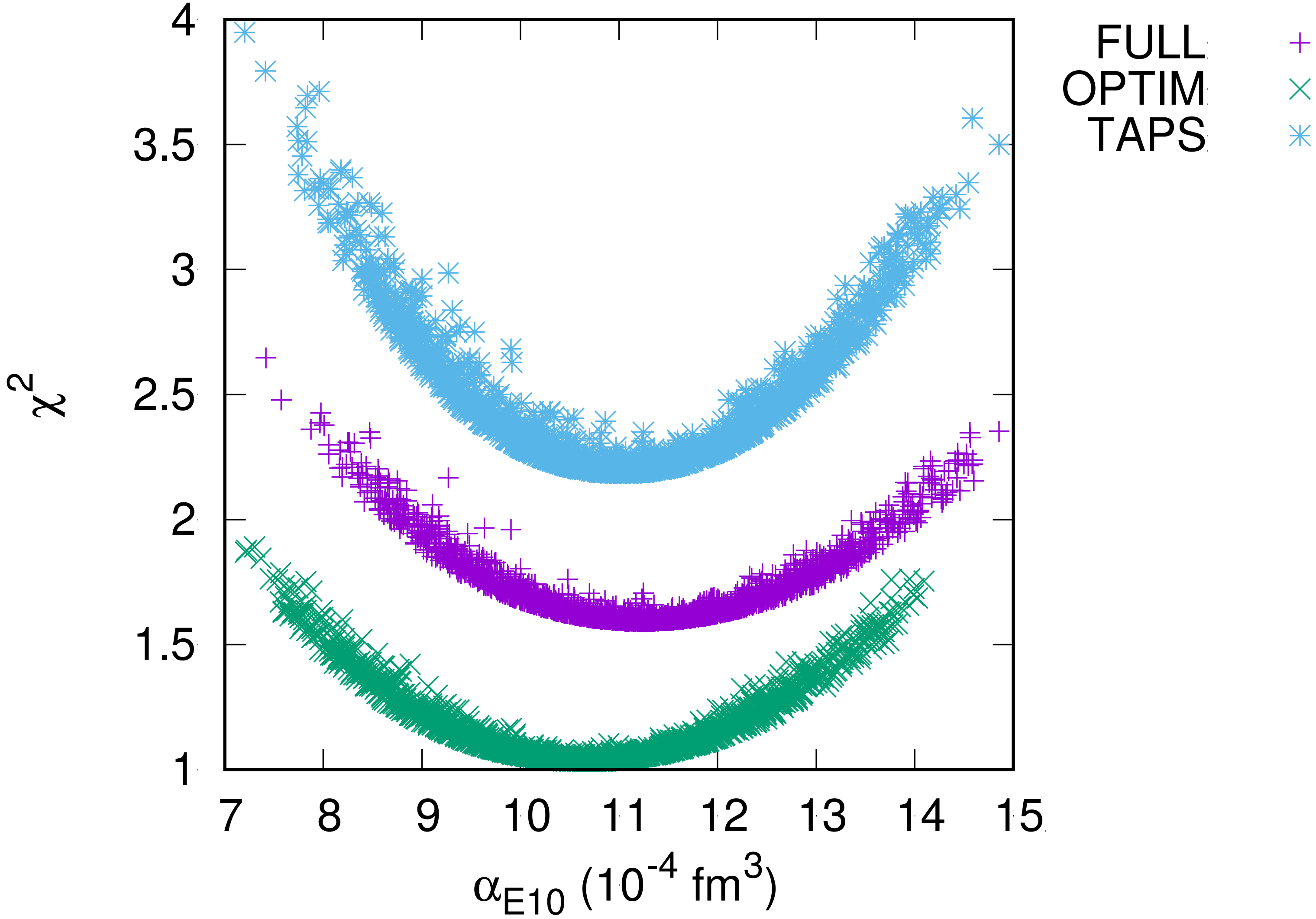


VERY small difference both in calculation and in error band

LEX is very slow ...



χ^2 curvature close to its minimum

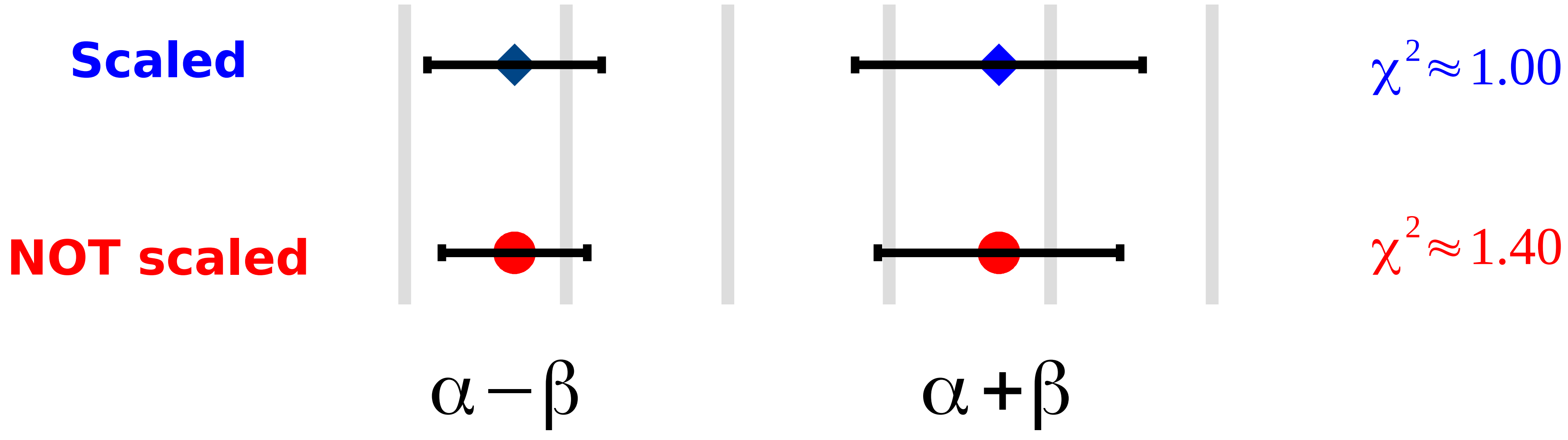


H. W. Grießhammer, J. A. McGovern, D. R. Phillips, and G. Feldman,Prog. Part. Nucl. Phys. 67, 841 (2012).

Outliers identification: rescaling of the errors

Outliers → rescaling of all the statistic uncertainties by a factor

$$\sqrt{\chi^2}$$



Effect: enlarging of errors on fitted parameters (~20%)

O. Behnke, K. Kröninger, G. Schott and T. Schörner-Sadenius, *Data Analysis in High Energy Physics: A Practical Guide to Statistical Methods*, Wiley-VCH, 2013