The most correct $\rho^0(770)$ meson mass and width values

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Outline

INTRODUCTION

The determination of $\rho^0(770)$ **meson parameters** m_ρ, Γ_ρ in

C.Patrignani et al. (PDG) Chin. Phys. C40 (2016) 100001

comes from a description of data on

$$\sigma_{tot}(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha^2\beta_\pi^3}{3t} \mid F_\pi^{I=1}(t) + Re^{i\phi}\frac{m_\omega^2}{m_\omega^2 - t - im_\omega\Gamma_\omega} \mid^2 \quad (1)$$

at the elastic region - to be considered up to $1 GeV^2$, where for the charged pion EM FF $F_\pi^{I=1}(t, m_\rho, \Gamma_\rho)$ the **Gounaris-Sakurai (G.-S.)** model

$$F_\pi^{GS}(t) = \frac{m_\rho^2 + m_\rho\Gamma_\rho(\frac{3}{\pi}\frac{m_\pi^2}{q_\rho^2}ln(\frac{m_\rho+2q_\rho}{2m_\pi}) + \frac{m_\rho}{2\pi q_\rho} - \frac{m_\pi^2 m_\rho}{\pi q_\rho^3})}{(m_\rho^2 - t) + \Gamma_\rho(\frac{m_\rho^2}{q_\rho^3})(q^2(h(t) - h(m_\rho^2)) + q_\rho^2 h'(m_\rho^2)(m_\rho^2 - t)) - im_\rho\Gamma_\rho(\frac{q}{q_\rho})^3\frac{m_\rho}{\sqrt{t}}}. \quad (2)$$

has been used.

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In **this presentation it is clearly demonstrated** - the **G.-S.** pion charged EM FF model **is not enough accurate for a correct determination of the $\rho^0(770)$ meson parameters**.
For these investigations we utilize **very precise measurements** of $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$
by BABAR
J.P.Lees et al.(BABAR Collab.) Phys. Rev. D86 (2012) 032013
by BESIII
M.Ablikin et al.(BESIII Collab.) Phys. Lett. B753 (2016) 629

and **independently**, also the **most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data** at the elastic region with theoretical errors,

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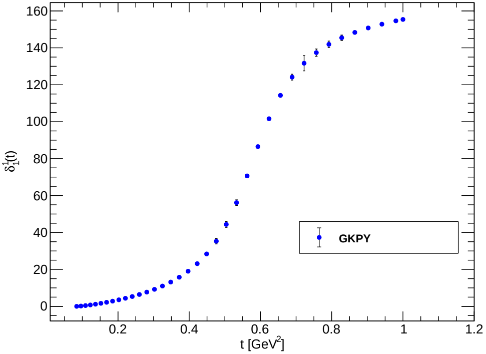

Figure : The most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with theoretical errors.

INTRODUCTION

to be **generated** from the existing inaccurate experimental information **by the Garcia-Martin-Kaminski-Pelaez-Yndurain Roy-like equations**.

For more detail of these investigations see
E.Bartos, S.Dubnicka, A.Liptaj, A.Z.Dubnickova, R.Kaminski,
Phys. Rev. D96 (2017) 113004

ρ^0(770) MASS AND WIDTH FROM BESIII-BABAR DATA ON $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$

First description of BESIII-BABAR $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by **G.-S. charged pion EM FF model** (2) is carried out

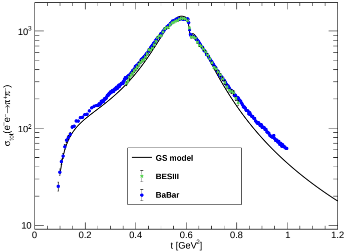

Figure : Description of the unified BESIII-BABAR data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by pion GS FF model with χ^2/ndf=40.634

$\rho^0(770)$ MASS AND WIDTH FROM BESIII-BABAR DATA ON $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$

It gives ρ^0 parameter values

$$\begin{aligned} m_\rho &= (775.73 \pm 0.10) MeV \\ \Gamma_\rho &= (126.51 \pm 0.13) MeV. \end{aligned} \tag{3}$$

however, **different (especially the width) from the parameter values**

$$\begin{aligned} m_\rho &= (775.26 \pm 0.25) MeV \\ \Gamma_\rho &= (149.1 \pm 0.8) MeV. \end{aligned} \tag{4}$$

quoted in

C. Patrignani et al. (PDG), Chin. Phys. C40 (2016) 100001.

$\rho^0(770)$ MASS AND WIDTH FROM BESIII-BABAR DATA ON $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$

Then description of the BESIII-BABAR unified data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ has been carried out **by U&A charged pion EM FF model**.

S.Dubnicka and A.Z.Dubnickova, Acta Phys. Slovaca 60 (2010) 1

$$F_\pi^{EM,I=1}(q) = \frac{(q-q_Z)(q_N-q_P)}{(q-q_P)(q_N-q_Z)} \frac{(q_N-q_\rho)(q_N-q_{\bar\rho})}{(q-q_\rho)(q-q_{\bar\rho})} (f_{\rho\pi\pi}/f_\rho). \quad (5)$$

It contains the right-hand unitary cut in t-plane, also a contribution of the left-hand cut from the II. Riemann sheet and generally **it reflects all known theoretical properties of the charged pion EM FF**.

$\rho^0(770)$ MASS AND WIDTH FROM BESIII-BABAR DATA ON $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$

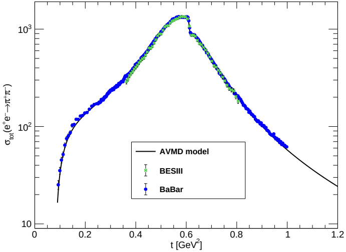

Figure : Description of the unified BESIII-BABAR data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by U&A pion EM FF model with χ^2/ndf=1.544

$\rho^0(770)$ MASS AND WIDTH FROM BESIII-BABAR DATA ON $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$

It provides

$$
\begin{aligned}
m_\rho &= (763.03 \pm 0.10) MeV \\
\Gamma_\rho &= (144.23 \pm 0.13) MeV
\end{aligned} \tag{6}
$$

but also **different (slightly lower) from the parameter values** quoted in

C. Patrignani et al. (PDG), Chin. Phys. C40 (2016) 100001.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

Which of three sets of ρ^0 meson parameters,
-**found by G.-S. FF model**,
-**found by U&A FF model** or
-**presented by newest PDG**,
can be considered to **be correct one?**

The latter problem is solved **by exploiting the most accurate up-to-now $\delta_1^1(t)$ data !**

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

The **basis of G.-S.charged pion EM FF model** is two parametric effective-range formula of Chew-Mandelstam type for $\delta_1^1(t)$

$$\frac{q^3}{\sqrt{t}} \cot \delta_1^1(t) = a + bq^2 + q^2 h(t) \tag{7}$$

where

$$h(t) = \frac{2}{\pi} \frac{q}{\sqrt{t}} ln\left(\frac{\sqrt{t} + 2q}{2m_\pi}\right); \tag{8}$$

$$q = [(t - 4m_\pi^2)/4]^{1/2}. \tag{9}$$

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

If this is used to a description of $\delta_1^1(t)$ data

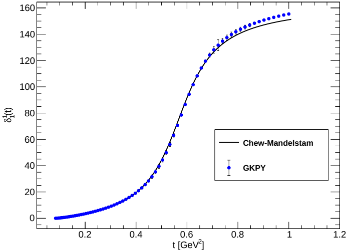

Figure : Description of $\delta_1^1(t)$ data by effective-range formula of Chew-Mandelstam type with χ^2/ndf=2.4499

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

one finds not very well result with numerical values of a and b

$$a = 0.2860 \pm 0.0011; \quad b = -2.7025 \pm 0.0089. \tag{10}$$

However, requiring for effective range formula of Chew-Mandelstam type two natural conditions

$$cotg\delta_1^1(t)|_{t=m_\rho^2} = 0 \tag{11}$$

and

$$\frac{d\delta_1^1(t)}{dt}|_{t=m_\rho^2} = \frac{1}{m_\rho \Gamma_\rho} \tag{12}$$

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

one finds a and b to be expressed through ρ^0 meson parameters

$$a = \frac{4q_\rho^5}{m_\rho^2\Gamma_\rho} + 4q_\rho^4 h'(m_\rho^2); \; b = -\frac{4q_\rho^3}{m_\rho^2\Gamma_\rho} - 4q_\rho^2 h'(m_\rho^2) - h(m_\rho^2) \quad (13)$$

which through a and b numerical values give the ρ^0 parameter values

$$\begin{aligned} m_\rho &= (772.42 \pm 0.03) MeV \\ \Gamma_\rho &= (153.85 \pm 0.11) MeV. \end{aligned} \quad (14)$$

They do not coincide, neither with the values obtained by description of the BESIII-BaBaR data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by G.-S. charged pion EM FF model, nor with values of PDG.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

Next a determination of $\rho^0(770)$ mass and width from $\delta_1^1(t)$ data by **fully solvable mathematical scheme**, elaborated in

S. Dubnicka, A. Z. Dubnickova, and A. Liptaj: Phys. Rev. D90 (2014) 114003

S. Dubnicka, A. Z. Dubnickova, R. Kaminski, and A. Liptaj: Phys. Rev. D94 (2016) 054036

has been carried out.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

Starting from the analytic properties of $F_\pi^{I=1}(t)$ and its asymptotic behavior, the **Cauchy formula with one subtraction is written**, which in combination **with the pion EM FF elastic unitarity condition leads to the phase representation**

$$F_\pi^{EM,I=1}(t) = P_n(t) exp\left[\frac{t}{\pi}\int_4^\infty \frac{\delta_1^1(t')}{t'(t'-t)}dt'\right]. \qquad (15)$$

As the branch point $t = 4m_\pi^2$, generating the cut, is a **square-root type $\Rightarrow$ the transformation**

$$q = [(t-4m_\pi^2)/4]^{1/2}$$

maps two-sheeted Riemann surface of $F_\pi^{EM,I=1}(t)$ into one q-plane and if one considers only the elastic region, the **pion EM FF has in it only poles and zeros**.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

The latter leads to a **model independent phase shift** $\delta_1^1(t)$ **representation**

$$\delta_1^1(q) = arctg \frac{A_3 q^3 + A_5 q^5 + ...}{1 + A_2 q^2 + A_4 q^4 + ...} \tag{16}$$

where $A_1 \equiv 0$, in order to secure the threshold behavior of $\delta_1^1(q)$. A **perfect description of the GKPY phase shift** $\delta_1^1(q)$ **data is achieved with four nonzero coefficients** A_2, A_3, A_4, A_5 **and** χ^2/ndf $=$ **0.0244**.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

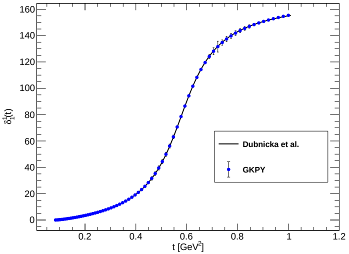

Figure : Optimal description of the most accurate up to now P-wave isovector $\pi\pi$ scattering phase shift $\delta_1^1(t)$ data with its model independent parametrization.

$\rho^0(770)$ MASS AND WIDTH FROM $\delta_1^1(t)$ DATA

Substitution of the **equivalent form**

$$\delta_1^1(q) = \frac{1}{2i} ln \frac{(1 + A_2 q^2 + A_4 q^4) + i(A_3 q^3 + A_5 q^5)}{(1 + A_2 q^2 + A_4 q^4) - i(A_3 q^3 + A_5 q^5)} \qquad (17)$$

into phase representation of $F_\pi^{EM,I=1}(q)$, and explicit calculation lead to the rational function, in the denominator of which two conjugate, according to the negative imaginary axis, ρ-meson poles appear from which the mass and width

$$m_\rho = (763.56 \pm 0.51) MeV \qquad (18)$$
$$\Gamma_\rho = (143.09 \pm 0.82) MeV.$$

are found, **to be almost identical with values determined by U&A charge pion EM FF model**.

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

The BABAR Collab. has measured data on $\sigma(e^+e^- \to \pi^+\pi^-)$ at the region of energies $(0.09 - 9.00) GeV^2$.
Therefore it has a sense to **generalize G.-S. and U&A pion EM FF models** and to carry out fit at least of all three, $\rho^0(770)$, $\rho^0(1450)$ and $\rho^0(1700)$ resonances simultaneously.

The pion EM FF G.-S. model was **generalized in various forms**

L. M. Barkov et al., Nucl. Phys. B256 (1985) 365

F. Jegerlehner and R. Szafron, Eur. Phys. J. C71 (2011) 1632

J. P. Lees et al.(BaBaR Collab.), Phys. Rev. D86 (2012) 032013

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

Though such **generalization for G.-S. model is without any deeper physical background**, as the original G.-S. model has been constructed from effective-range formula of the Chew-Mandelstam type to be **evidently valid only in the elastic region up to** $1GeV^2$, let us try to do it.

We have applied the **generalization of J. P. Lees et al** in the form

$$F_\pi(t) = \frac{1}{1+\beta+\gamma}[F^{GS}_{\rho(770)}(t).(1+\delta\frac{t}{m^2_\omega}BW_\omega(t)) + \beta F^{GS}_{\rho(1450)}(t) + \gamma F^{GS}_{\rho(1700)}(t)] \quad (19)$$

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

Its application to a description of the unified BESSIII+BaBaR data up to $9 GeV^2$ is achieved with χ^2/ndf=0.981

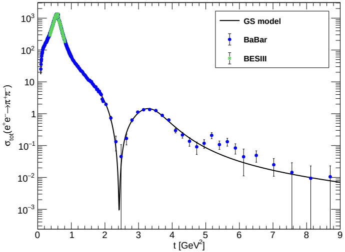

Figure : Optimal description of the unified BESIII-BaBaR complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized G.-S. pion EM FF model.

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

In a **generalization of the U&A pion EM FF model**, in comparison with G.-S. model, one has **totally different situation**. In this case **contributions of all three vector mesons are on equal level**. Only now the **inelastic threshold t_{in} has to be taken into account explicitly**.

Therefore instead of q variable the W variable is considered

$$W(t) = i\frac{\sqrt{\left(\frac{t_{in}-t_0}{t_0}\right)^{1/2} + \left(\frac{t-t_0}{t_0}\right)^{1/2}} - \sqrt{\left(\frac{t_{in}-t_0}{t_0}\right)^{1/2} - \left(\frac{t-t_0}{t_0}\right)^{1/2}}}{\sqrt{\left(\frac{t_{in}-t_0}{t_0}\right)^{1/2} + \left(\frac{t-t_0}{t_0}\right)^{1/2}} + \sqrt{\left(\frac{t_{in}-t_0}{t_0}\right)^{1/2} - \left(\frac{t-t_0}{t_0}\right)^{1/2}}}, \tag{20}$$

in construction of $F_\pi^{I=1}(t)$, which is **mapping the four-sheeted Riemann surface into one W-plane**.

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

Then $F_\pi^{I=1}(t)$ takes the following form

$$F_\pi^{EM,I=1}[W(t)] = \Big(\frac{1-W^2}{1-W_N}\Big)^2 \frac{(W-W_Z)(W_N-W_P)}{(W_N-W_Z)(W-W_P)}.$$

$$\cdot\Big[\frac{(W_N-W_\rho)(W_N-W_\rho^*)(W_N-1/W_\rho)(W_N-1/W_\rho^*)}{(W-W_\rho)(W-W_\rho^*)(W-1/W_\rho)(W-1/W_\rho^*)}(f_{\rho\pi\pi}/f_\rho)+ \tag{21}$$

$$+\sum_{\upsilon=\rho',\rho''}\frac{(W_N-W_\upsilon)(W_N-W_\upsilon^*)(W_N+W_\upsilon)(W_N+W_\upsilon^*)}{(W-W_\upsilon)(W-W_\upsilon^*)(W+W_\upsilon)(W+W_\upsilon^*)}(f_{\upsilon\pi\pi}/f_\upsilon)\Big],$$

with

$$(f_{\rho'\pi\pi}/f_{\rho'}) = -\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}+$$

$$+\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}-(1+2\frac{W_ZW_P}{W_Z-W_P}.Re[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4}-\frac{N_{\rho''}}{|W_{\rho''}|^4}}(f_{\rho\pi\pi}/f_\rho), \tag{22}$$

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

$$(f_{\rho''\pi\pi}/f_{\rho''}) = 1 + \frac{\frac{N_{\rho''}}{|W_{\rho''}|^4}}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} -$$

$$-\Big[\frac{\frac{N_{\rho''}}{|W_{\rho''}|^4} - (1 + 2\frac{W_Z W_P}{W_Z - W_P}.Re[W_\rho(1+|W_\rho|^{-2})])N_\rho}{\frac{N_{\rho'}}{|W_{\rho'}|^4} - \frac{N_{\rho''}}{|W_{\rho''}|^4}} - 1\Big](f_{\rho\pi\pi}/f_\rho), \quad (23)$$

and

$$N_\rho = (W_N - W_\rho)(W_N - W_\rho^*)(W_N - 1/W_\rho)(W_N - 1/W_\rho^*), \quad (24)$$

$$N_\upsilon = (W_N - W_\upsilon)(W_N - W_\upsilon^*)(W_N + W_\upsilon)(W_N + W_\upsilon^*). \quad (25)$$

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

Its application to a description of the unified BESSIII+BaBaR data up to $9 GeV^2$ is achieved with χ^2/ndf=1.844

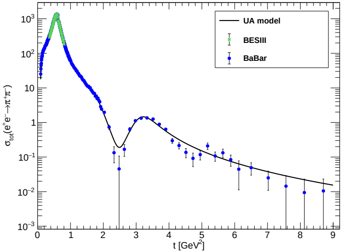

Figure : Optimal description of the unified BESIII-BaBaR complete data on $\sigma_{tot}(e^+e^- \to \pi^+\pi^-)$ by the generalized U&A pion EM FF model.

GENERALIZATION OF G.-S. AND U&A FF MODELS TO $\rho^0(1450)$, $\rho^0(1700)$ REGION

Obtained **vector meson parameters by the generalized G.-S. and U&A pion EM FF models** are presented in this Table

Parameter	PDG value [MeV]	Gounaris-Sakurai [MeV]	Unitary & Analytic [MeV]
m_ρ	775.26 ± 0.25	774.81 ± 0.01	763.88 ± 0.04
$m_{\rho'}$	1465.00 ± 25.00	1497.70 ± 1.07	1326.35 ± 3.46
$m_{\rho''}$	1720.00 ± 20.00	1848.40 ± 0.09	1770.54 ± 5.49
Γ_ρ	149.10 ± 0.80	149.22 ± 0.01	144.28 ± 0.01
$\Gamma_{\rho'}$	400.00 ± 60.00	442.15 ± 0.54	324.13 ± 12.01
$\Gamma_{\rho''}$	250.00 ± 100.00	322.48 ± 0.69	268.98 ± 11.40
χ^2/ndf		0.981 [14 parameters]	1.842 [11 parameters]

Table : The values of ρ-meson parameters obtained from fits of BESIII+BaBar data up to $9 GeV^2$ on $\sigma(e^+e^- \to \pi^+\pi^-)$ with generalized G.-S. and U&A pion EM FF models to be compared to PDG values.

CONCLUSIONS

G.-S.-elastic

$$m_\rho = (775.73 \pm 0.10) MeV$$
$$\Gamma_\rho = (126.51 \pm 0.13) MeV.$$

effective range Chev-Mandelstam for $\delta_1^1(t)$

$$m_\rho = (772.42 \pm 0.03) MeV$$
$$\Gamma_\rho = (153.85 \pm 0.11) MeV.$$

G.-S.-generalized (coincided with PDG)

$$m_\rho = (774.81 \pm 0.01) MeV$$
$$\Gamma_\rho = (149.22 \pm 0.01) MeV.$$

CONCLUSIONS

U&A -elastic

$$m_\rho = (763.03 \pm 0.10) MeV$$
$$\Gamma_\rho = (144.23 \pm 0.13) MeV$$

model independent parametrization of $\delta_1^1(t)$

$$m_\rho = (763.56 \pm 0.51) MeV$$
$$\Gamma_\rho = (143.09 \pm 0.82) MeV.$$

U&A -generalized

$$m_\rho = (763.88 \pm 0.04) MeV$$
$$\Gamma_\rho = (144.28 \pm 0.01) MeV.$$

Thank you for your attention.