Conventional and exotic states in the DSE/BSE framework

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Reviews: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]; Sanchis-Alepuz, Williams, CPC [1710.04903]

Bundesministerium für Bildung und Forschung

HIC for FAIR Helmholtz International Center

Overview

- Introduction: dynamical quark masses

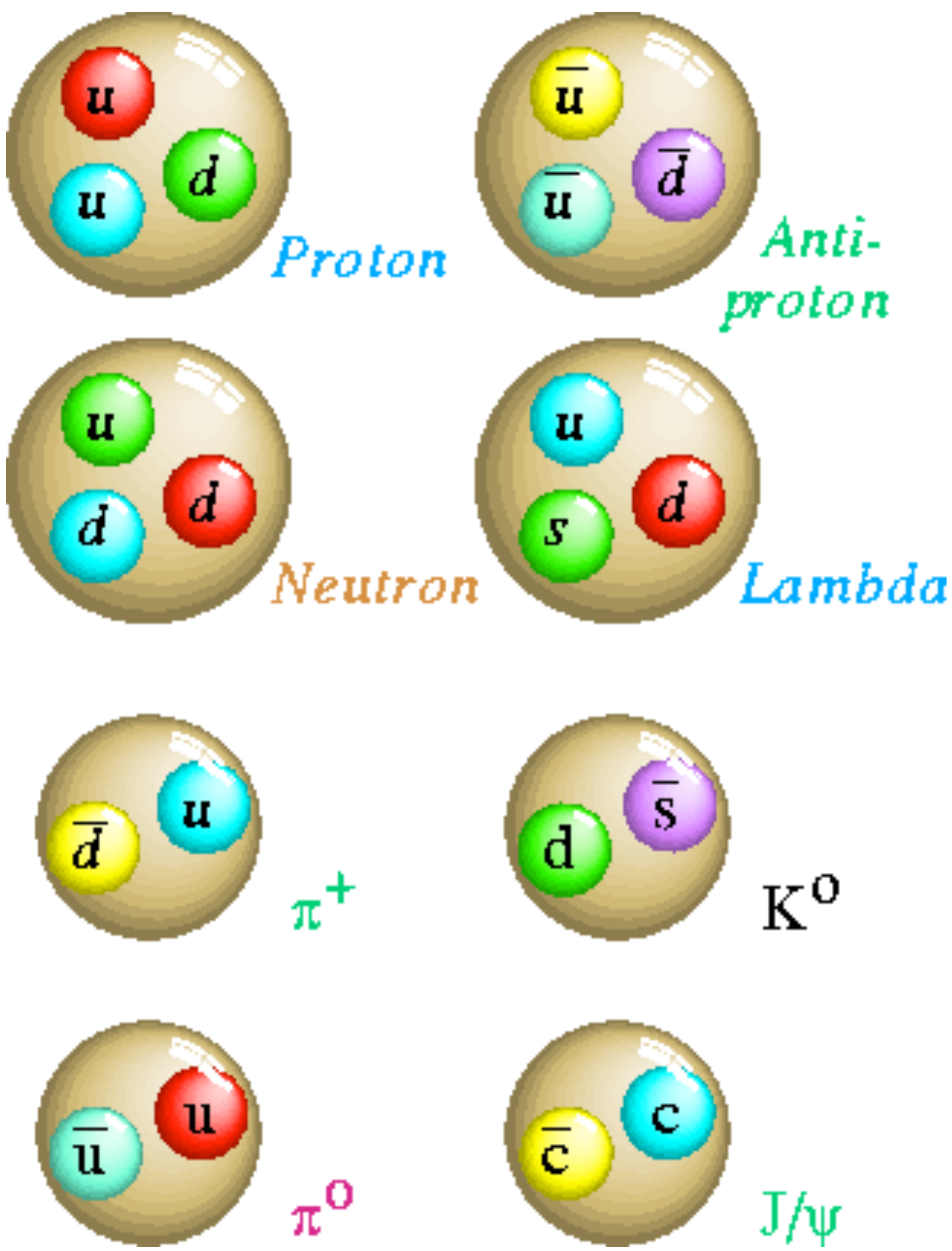
- Conventional and exotic mesons

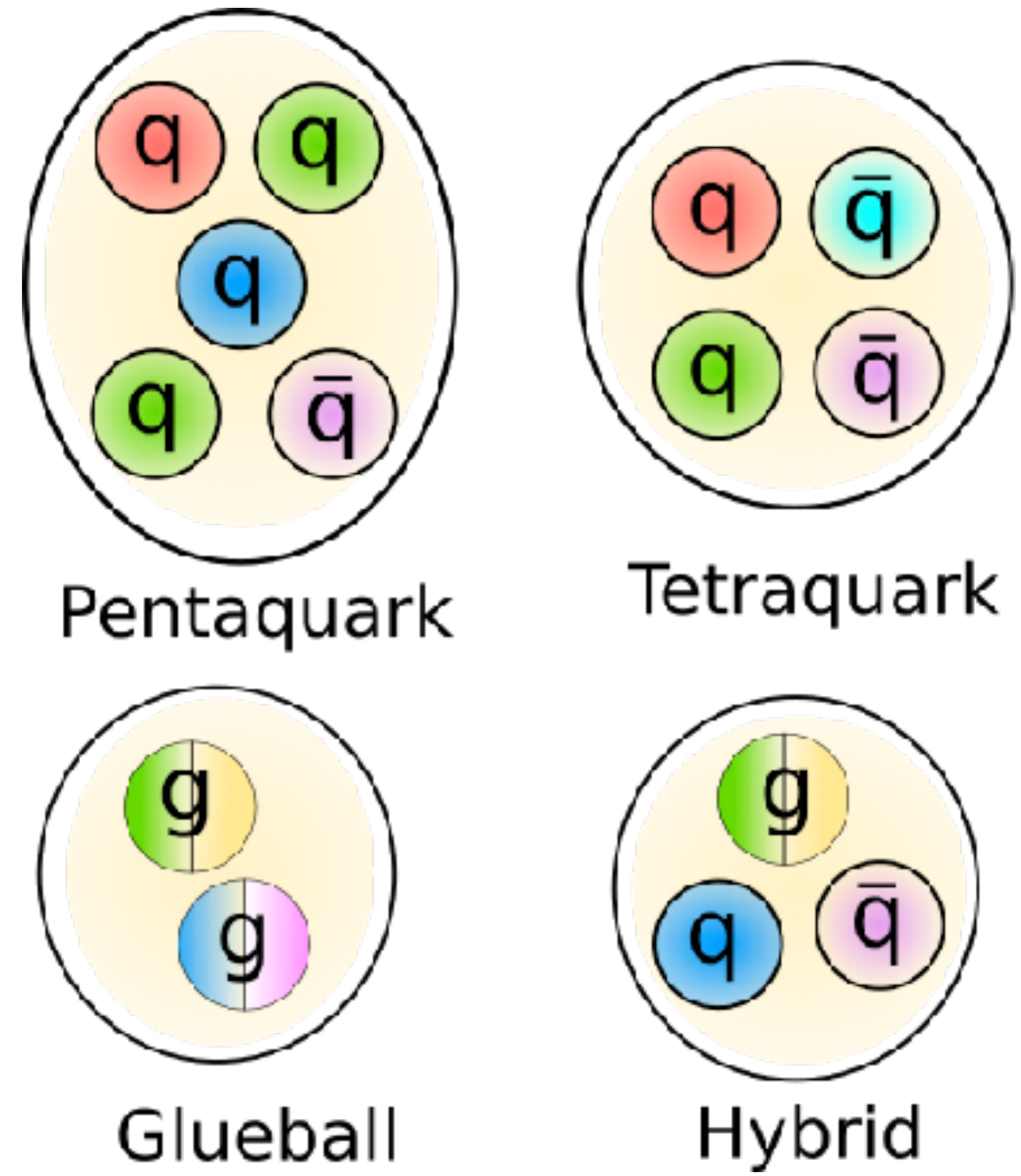
- Transition form factors and decays

Hadrons: baryons, mesons and ... exotics !

Baryons

Mesons

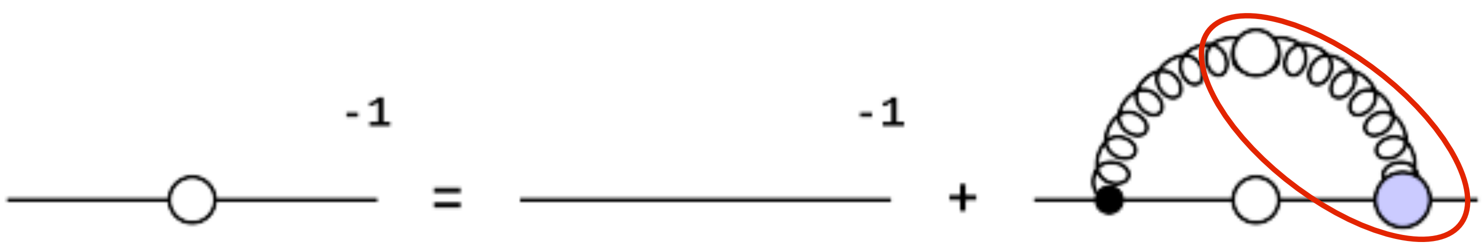


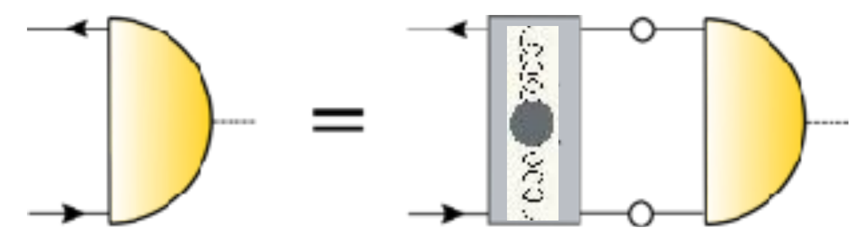


DSE-approach: truncations

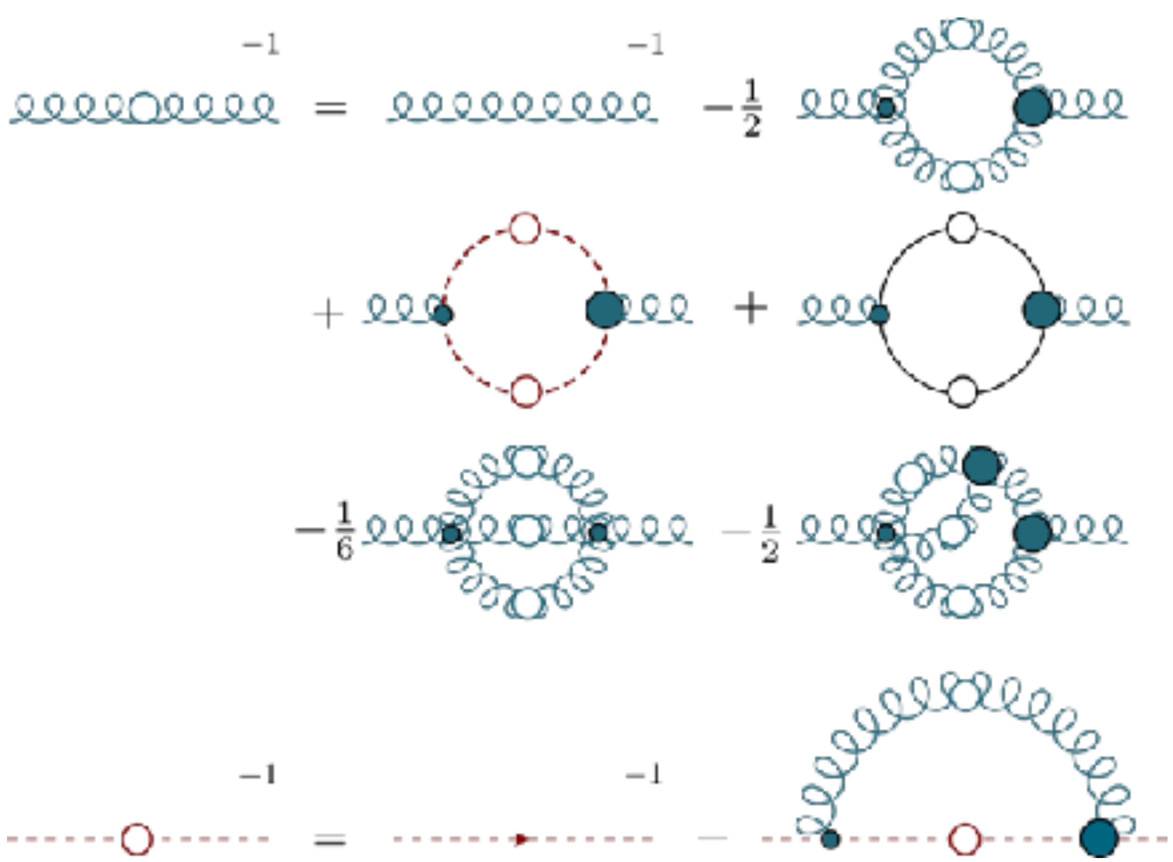
$$[S(p)]^{-1} = [-i\not{p} + M(p^2)]/Z_f(p^2)$$

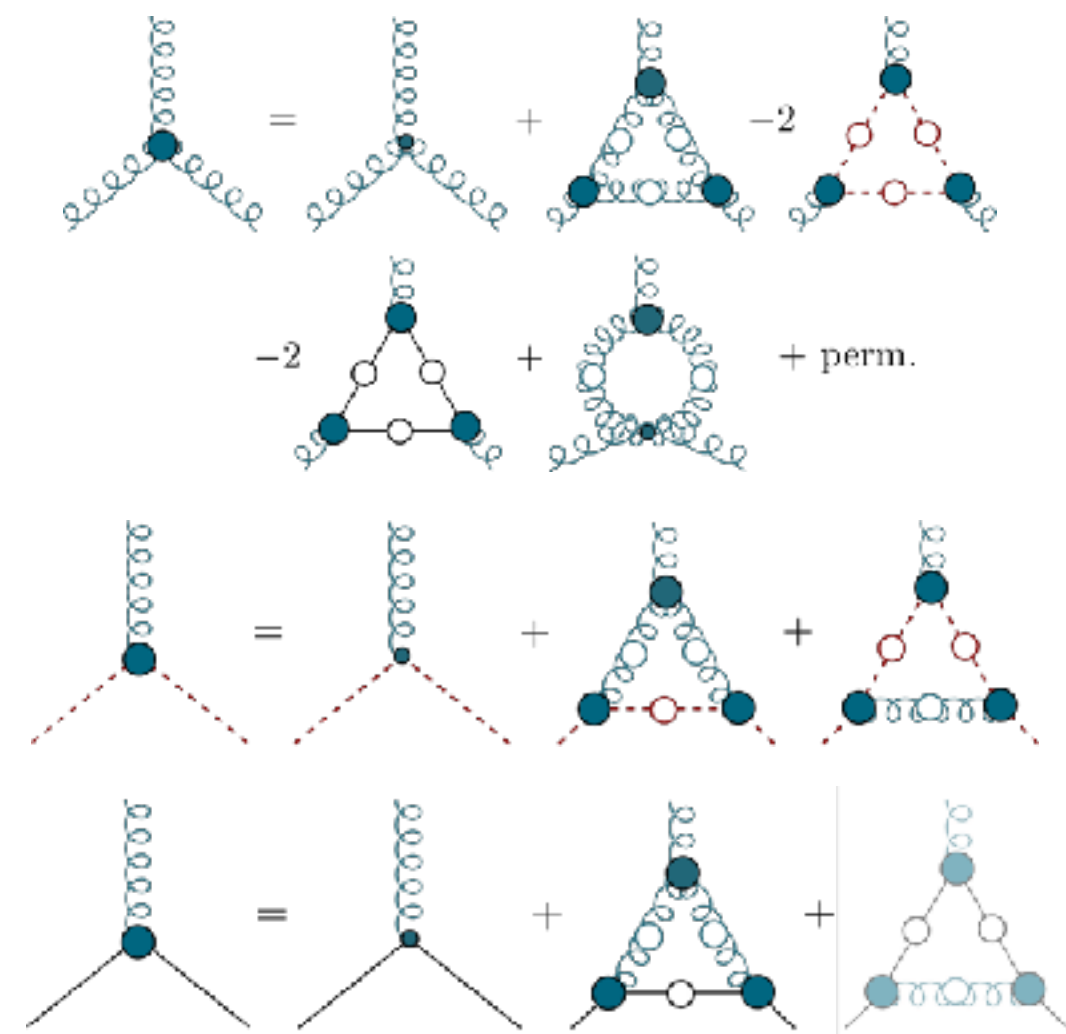
Rainbow-Ladder (RL):





Beyond the rainbow (BRL):



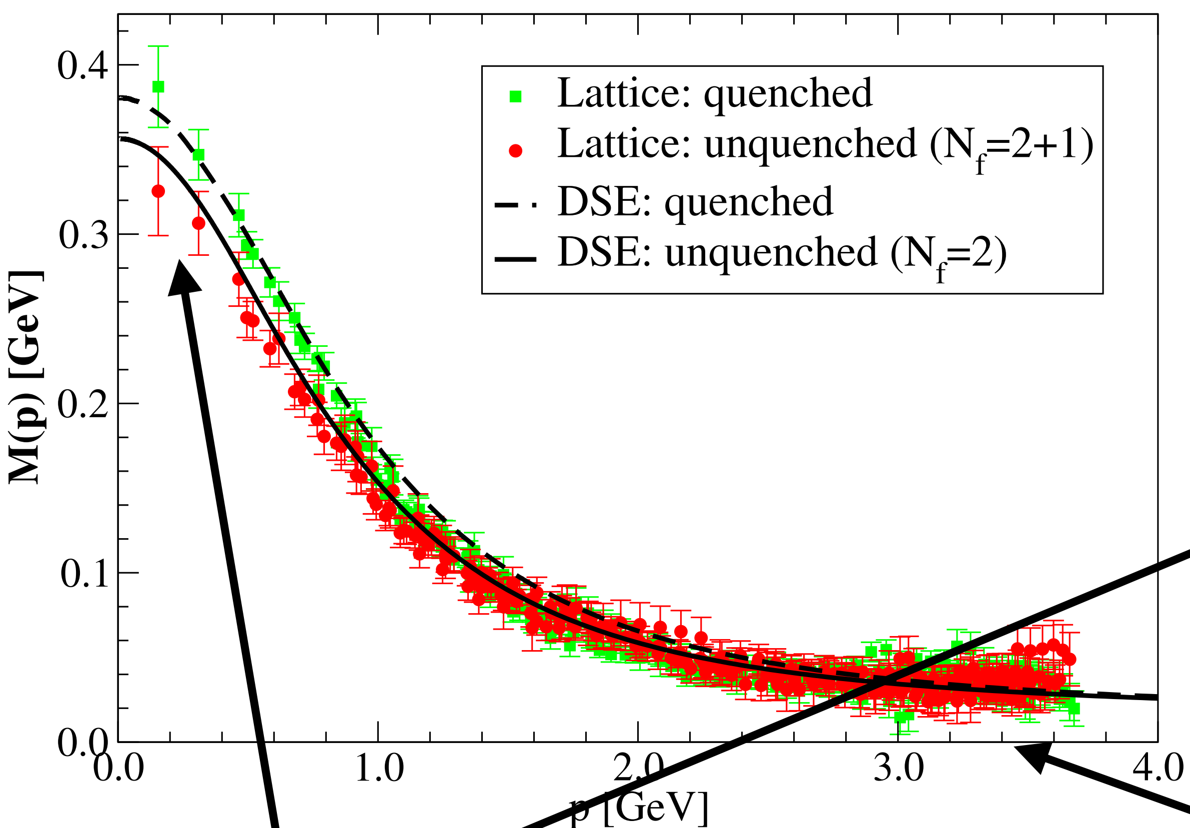


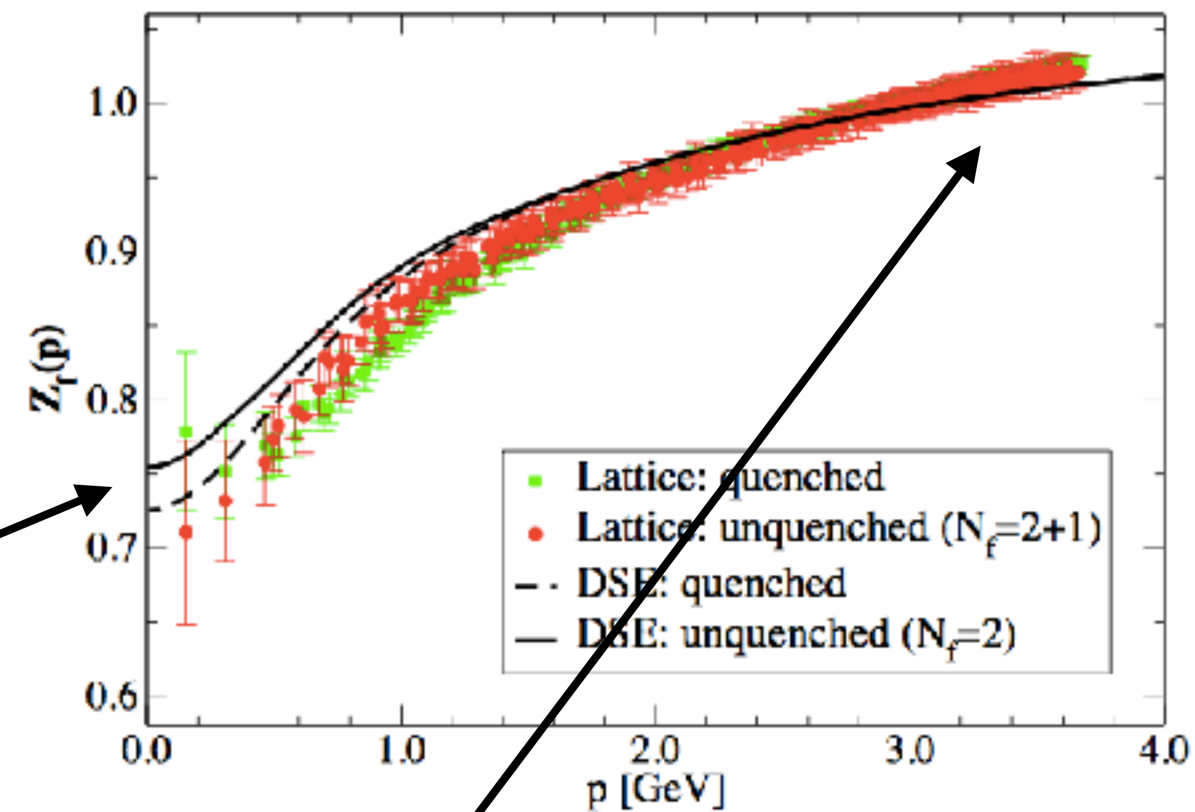
Williams, CF, Heupel, PRD 93 (2016) 034026
CF, Williams, PRL 103 (2009) 122001

Quark dressing - comparison with lattice

$$S(p) = Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507





'constituent quark':
large mass; very composite

'current quark':
- small mass; non-composite

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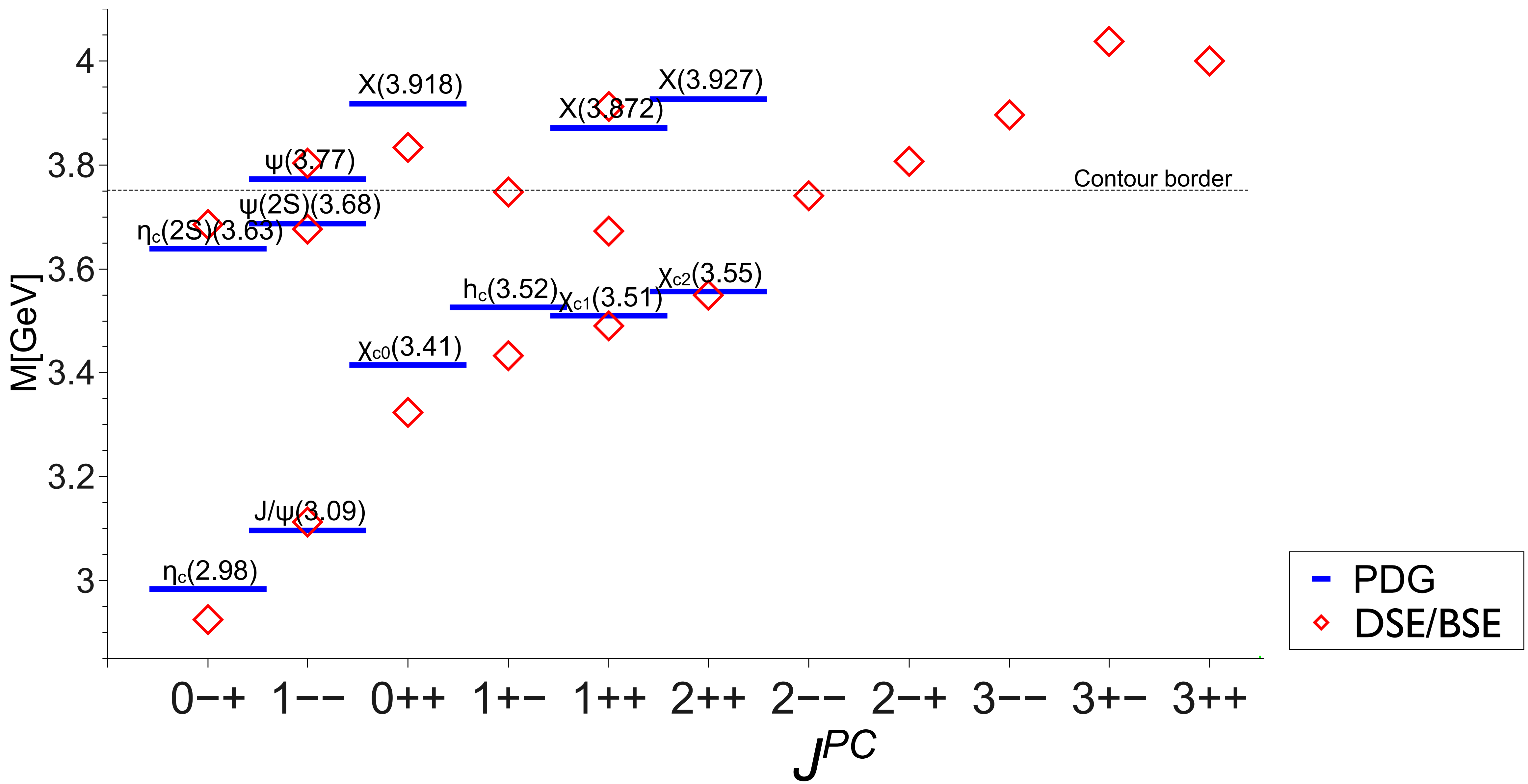
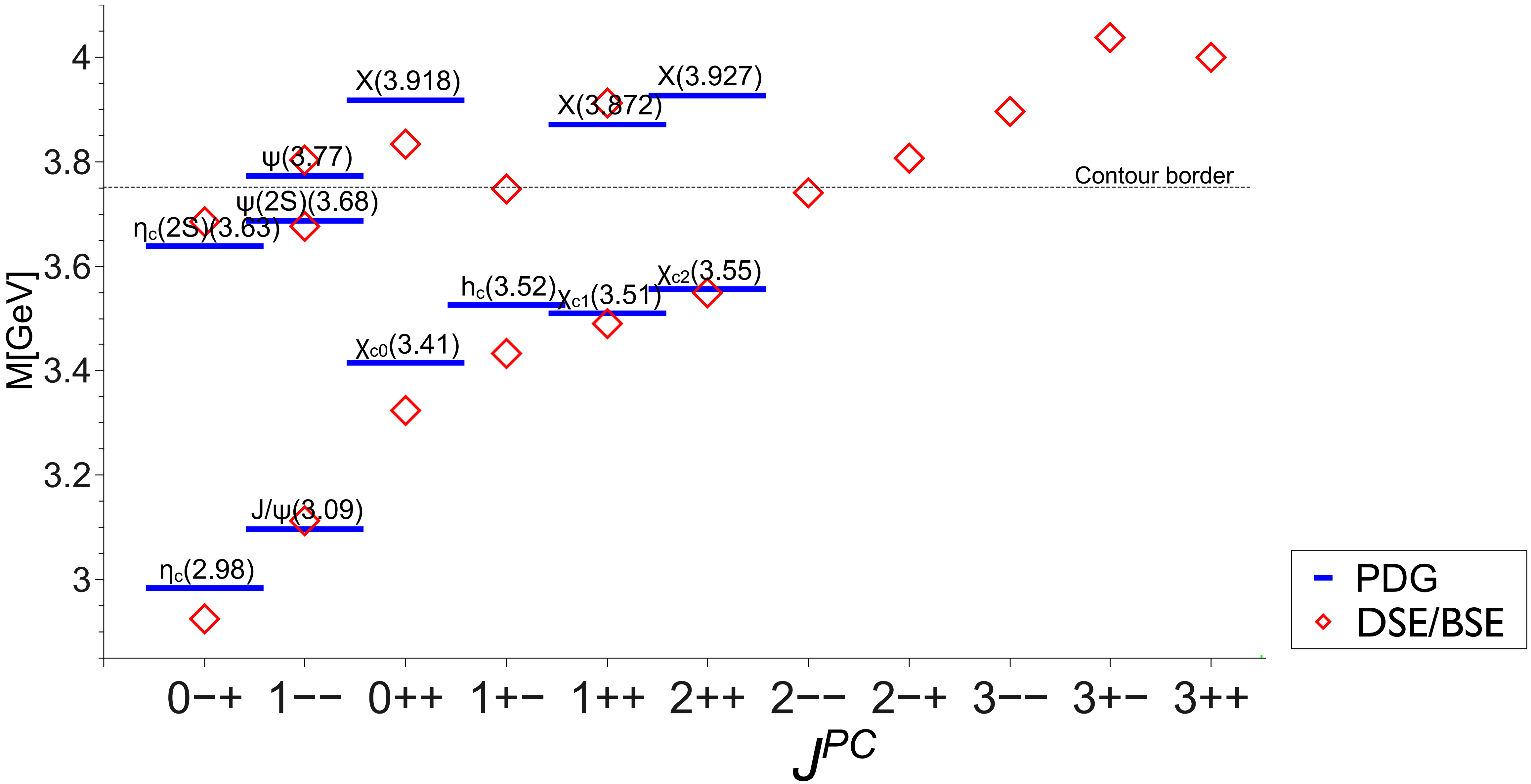
Charmonium spectrum



CF, Kubrak, Williams, EPJA 51 (2015)
Hilger et al. PRD 91 (2015)

- good channels: 1^{--}, 2^{++}, 3^{--},...
- acceptable channels: 0^{-+}
- clear deficiencies in other channels: missing spin-structure
- excited states fine ! (in good channels)

Charmonium spectrum



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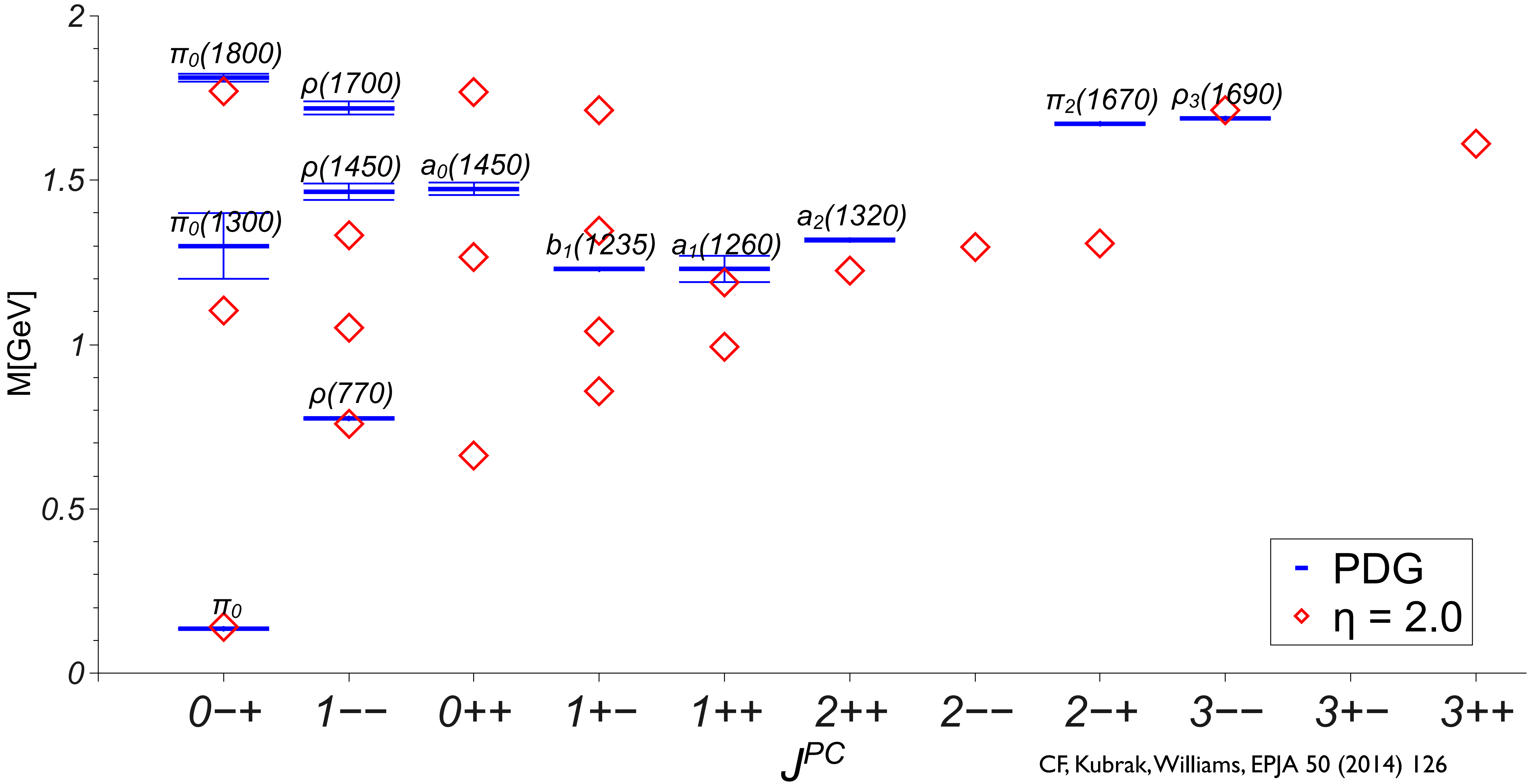
Light meson spectrum



CF, Kubrak, Williams, EPJA 50 (2014) 126

Williams, CF, Heupel, PRD93 (2016) 034026

- good channels (ground state): 0^{-+}, 1^{--}
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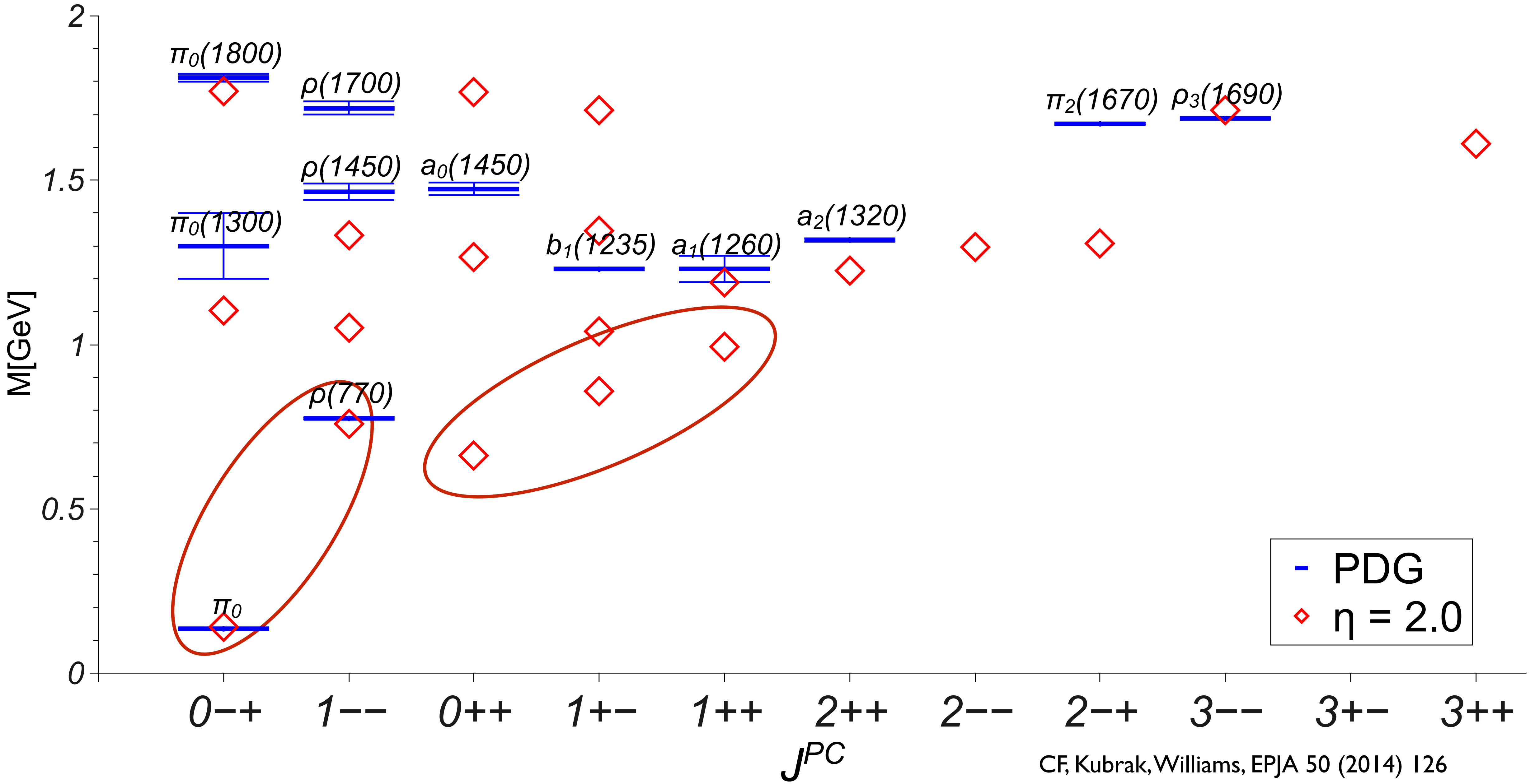

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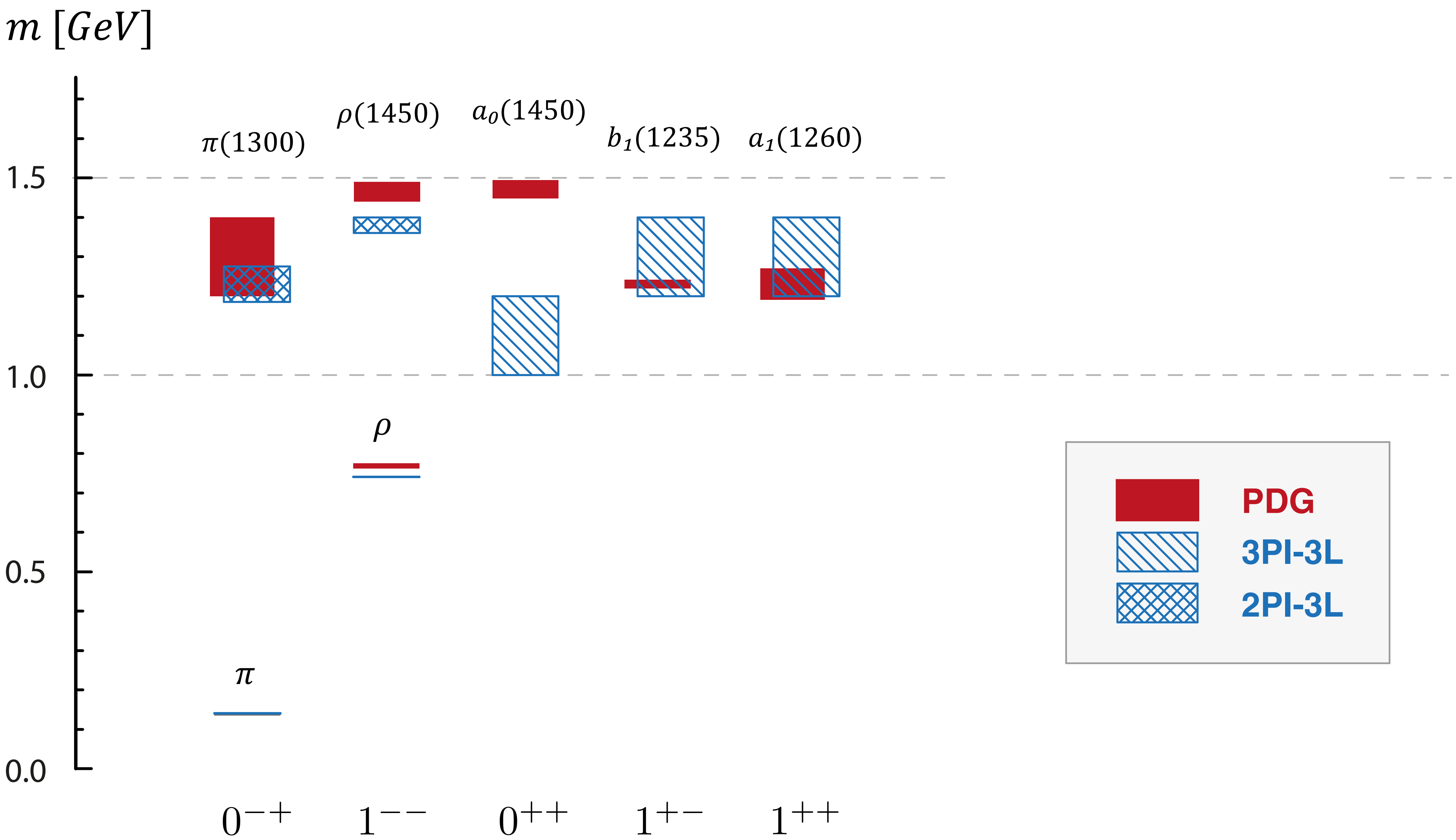

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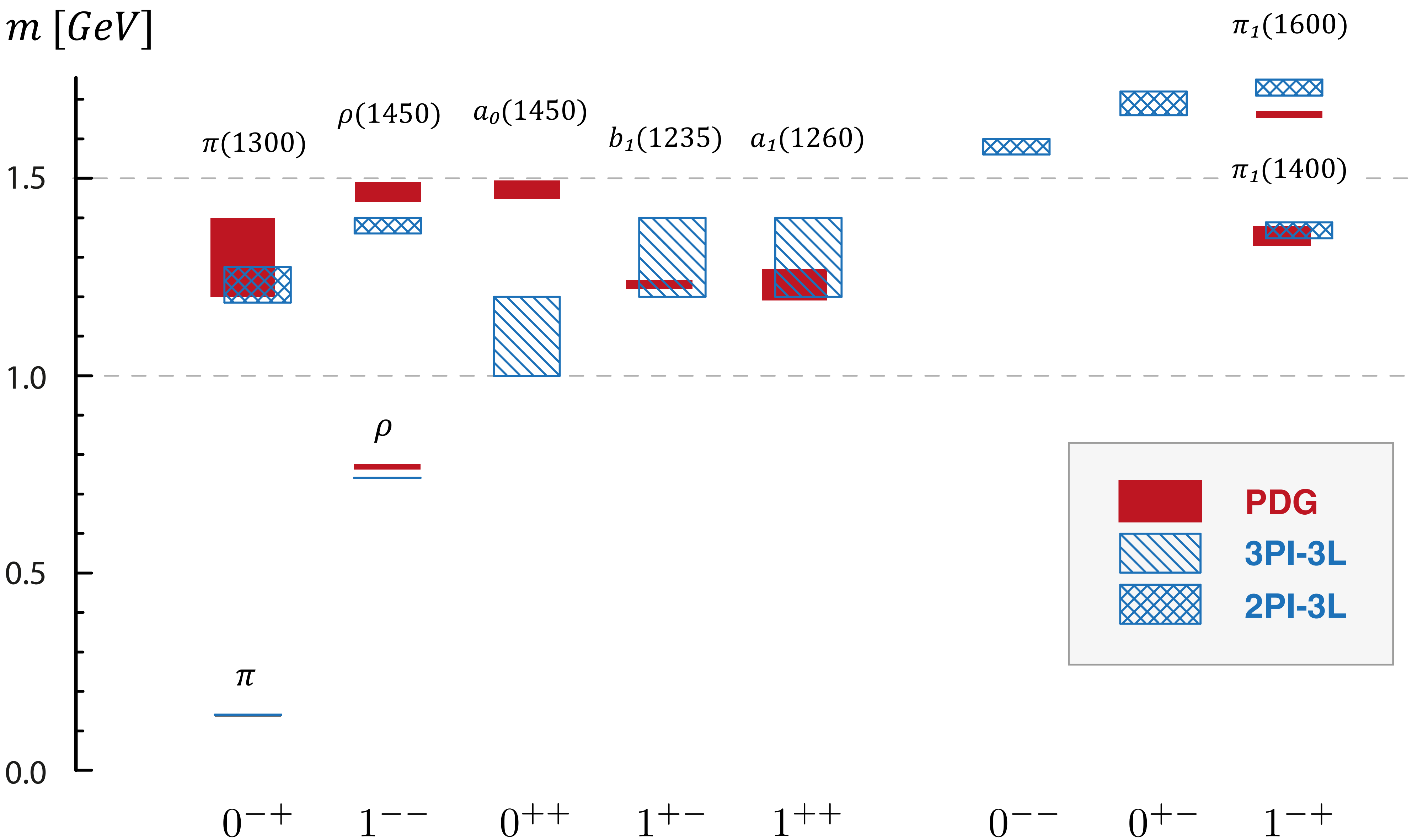

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Quantum numbers: non-relativistic vs relativistic

non-relativistic $q\bar{q}$

$$S: 1/2 \otimes 1/2 \to 0 \oplus 1$$

$$P: (-1)^{L+1}$$

S	L	J^{PC}	
0	0	0^{-+}	
1	0	1^{--}	
0	1	1^{+-}	$^1P_1$
1	1	0^{++}	$^3P_0$
		1^{++}	$^3P_1$
		2^{++}	$^3P_2$

$\boxed{J^{PC}}$ or $\boxed{^{2S+1}L_J}$

relativistic $q\bar{q}$

$$\begin{aligned}\Gamma_\pi(P,p) = \gamma_5\,[&F_1(P,p) \\ &+ F_2(P,p)\, i\not{P} \\ &+ F_3(P,p)\, pP\, i\not{p} \\ &+ F_4(P,p)\, [\not{p}\,, \not{P}]]\end{aligned}$$

s-wave

p-wave

(rest frame of π)

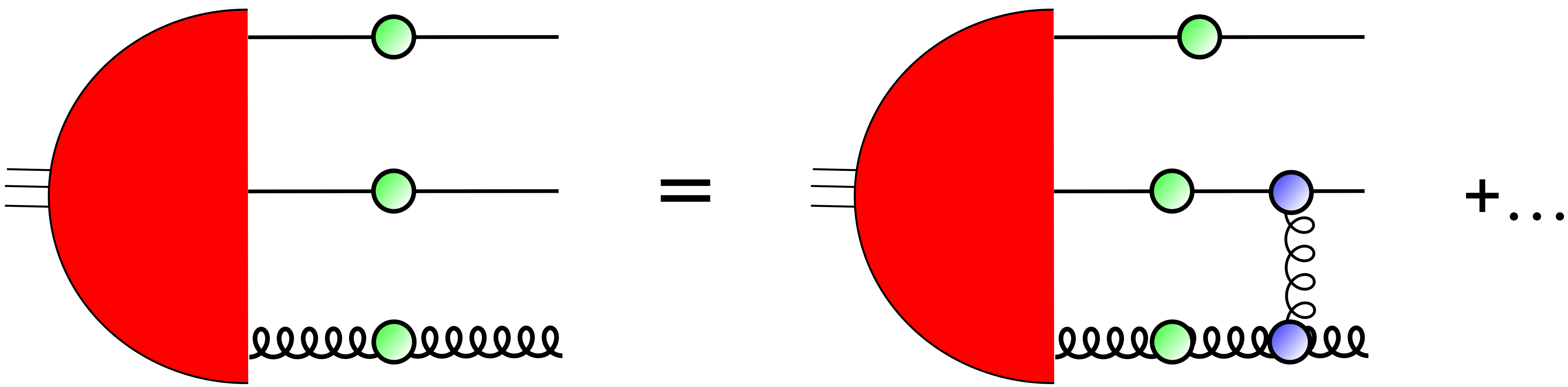
$$\cancel{P: (-1)^{L+1}}$$

Bethe,Salpeter, Llewelyn-Smith 1950ies

- conventional states more complicated
- 'exotic' quantum numbers possible !

$$0^{--}, 0^{+-}, 1^{-+}, 2^{+-} \ldots$$

Hybrids as three-body states

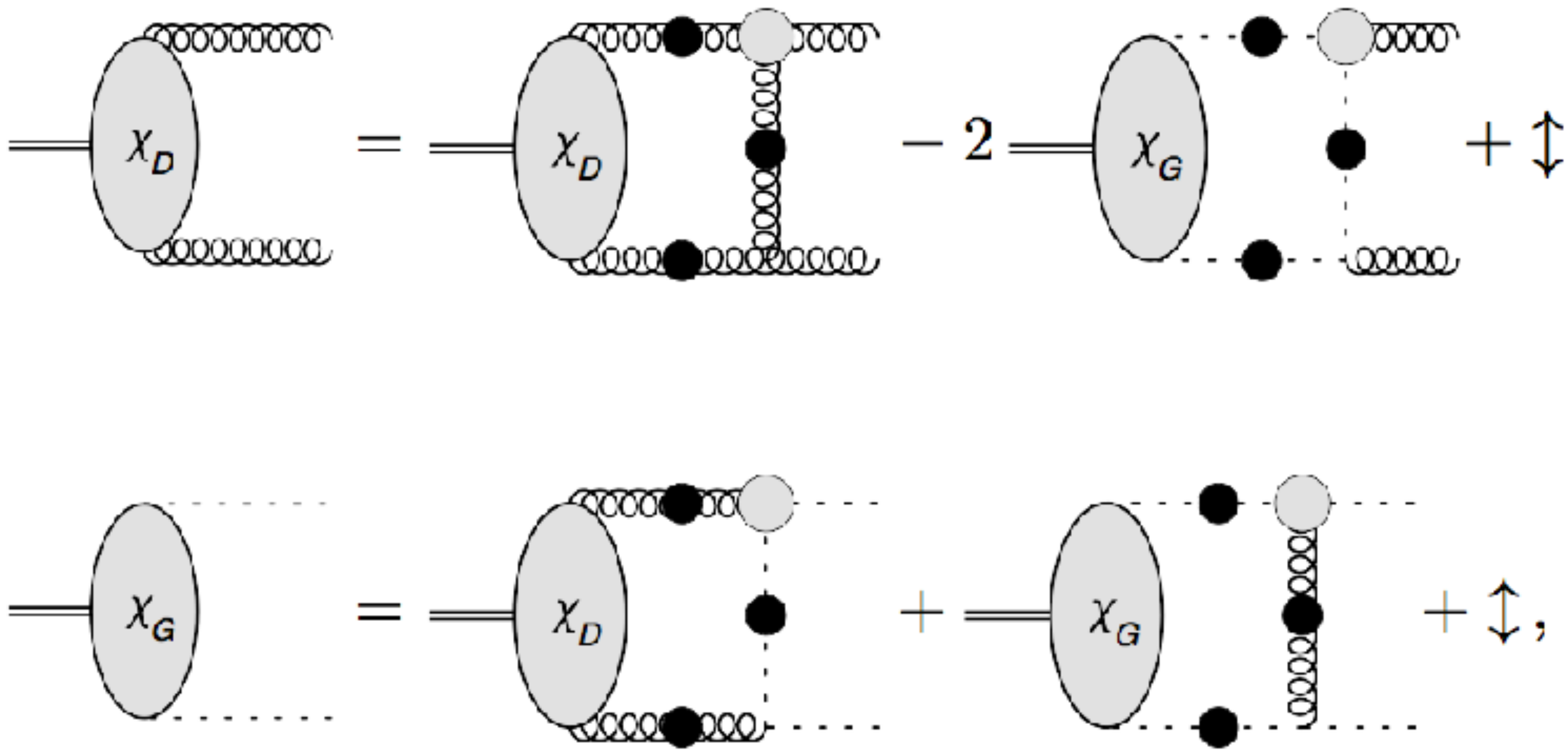


- Similar to Faddeev-eq. for baryons except for glue
- Expectation: bound states around 800 MeV higher than $q\bar{q}$ with same quantum numbers

Liu et al. (HSC), JHEP 1207 (2012) 126

Working hypothesis:
two-body BSE with lots of glue in kernel =
three-body-BSE with glue in valence part

Glueballs from DSE/BSEs

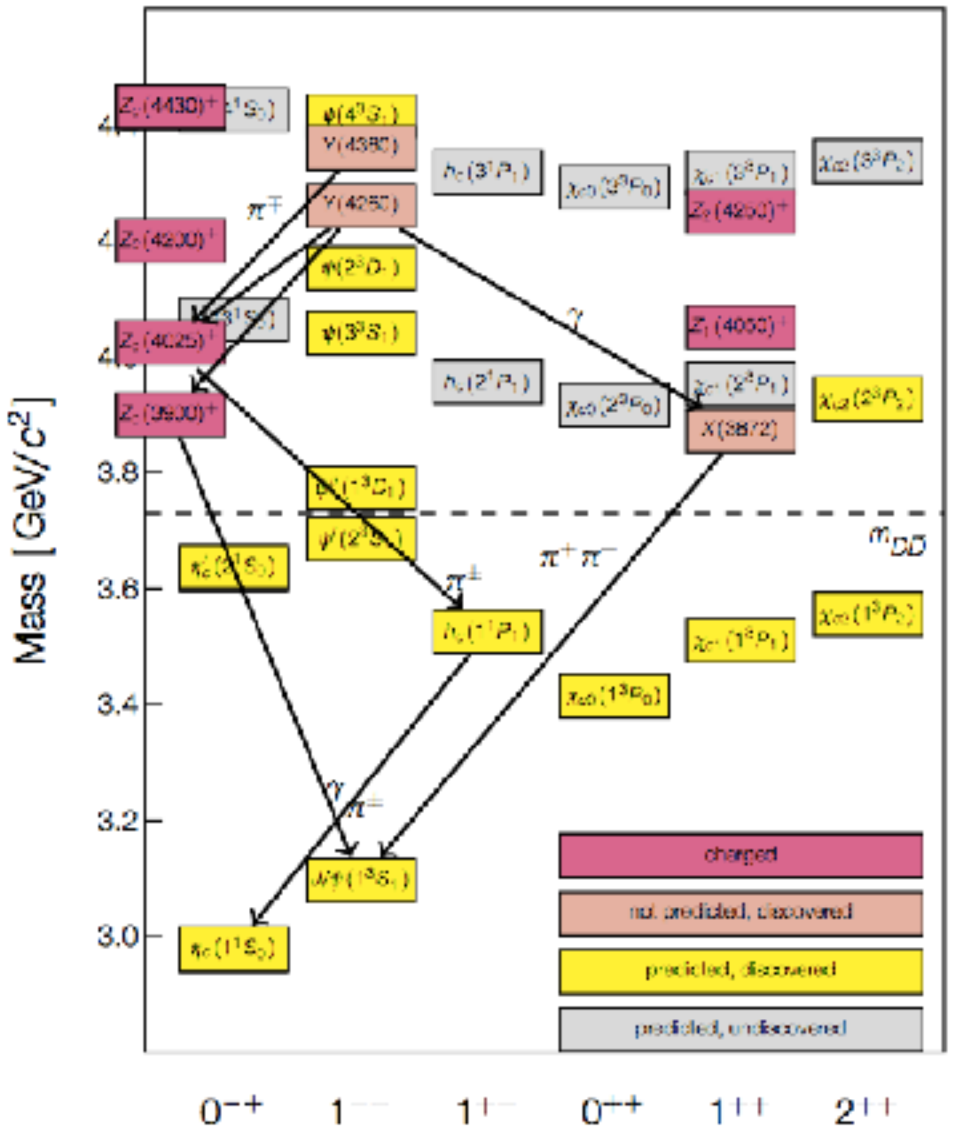


- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

Results: $M(0^{++}) = 1.64\,\mathrm{GeV}$

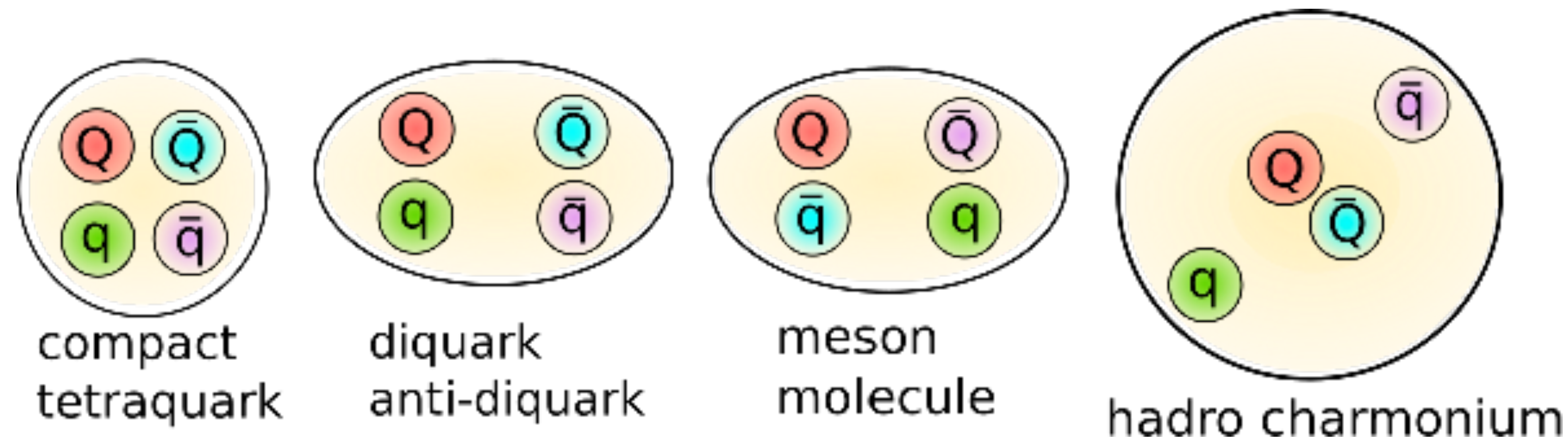
$M(0^{-+}) = 4.53\,\mathrm{GeV}$ ← ghost do not contribute !

Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001



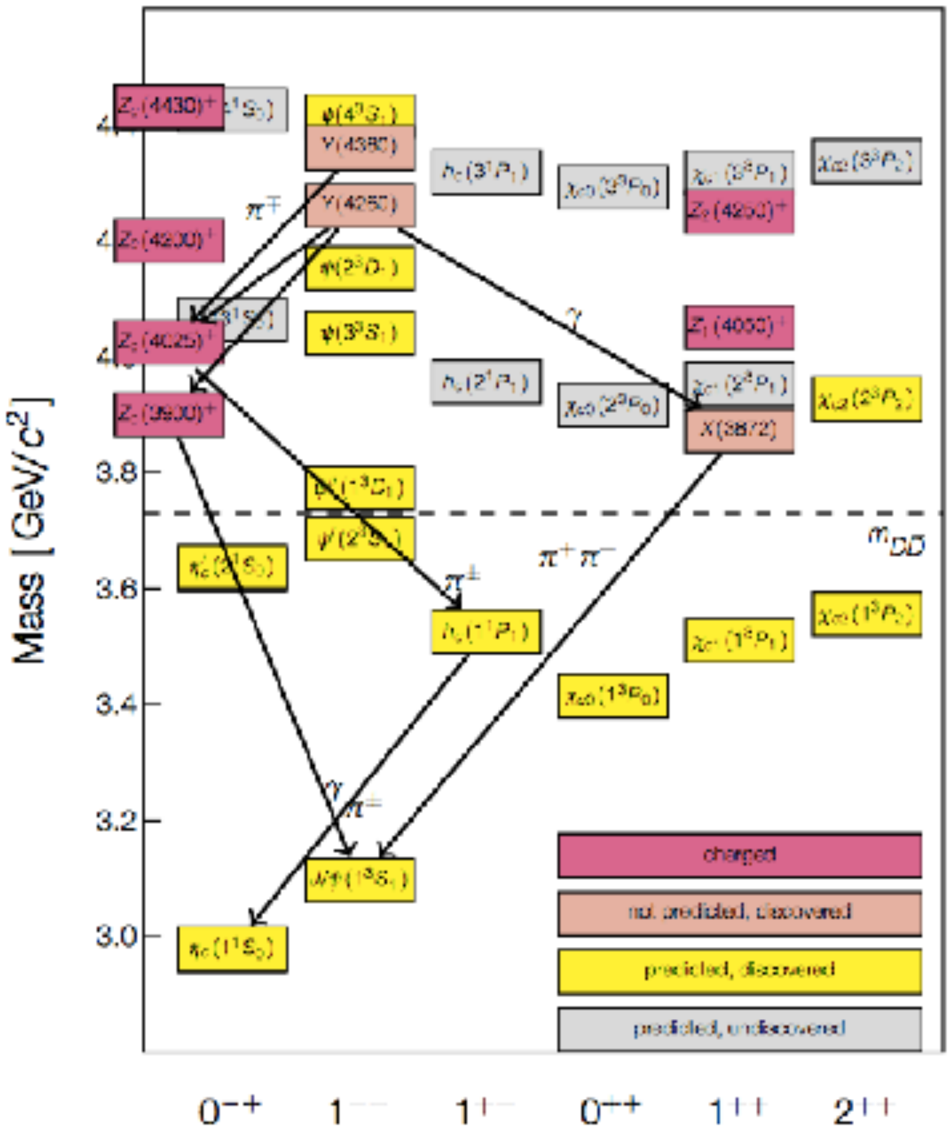
Wolfgang Gradl, BESIII, St Goar 2015

Internal structure ??



Related to details of underlying QCD forces between quarks

Heavy and light tetraquark



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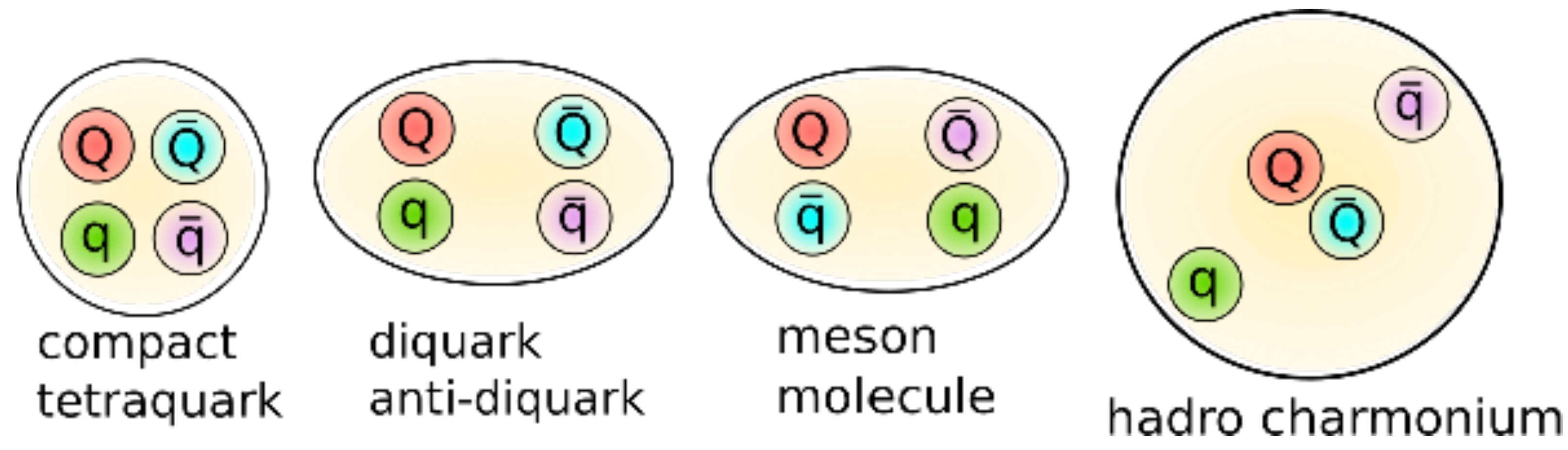
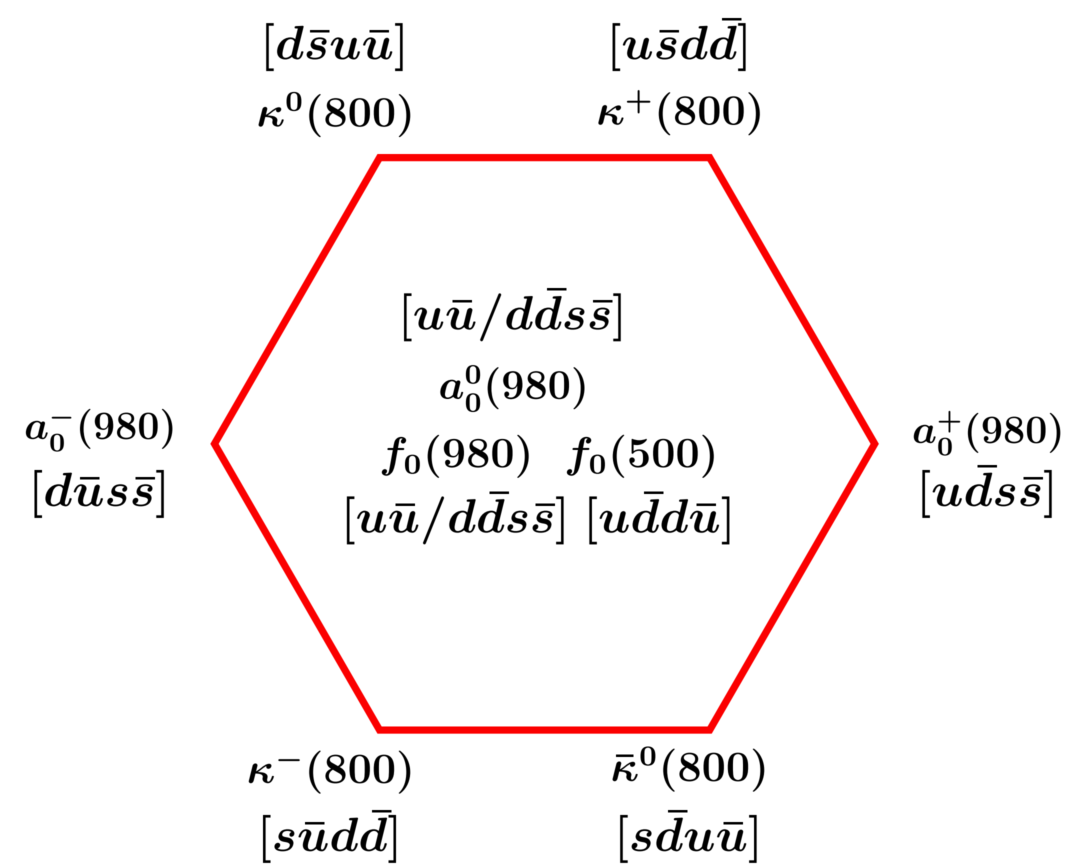
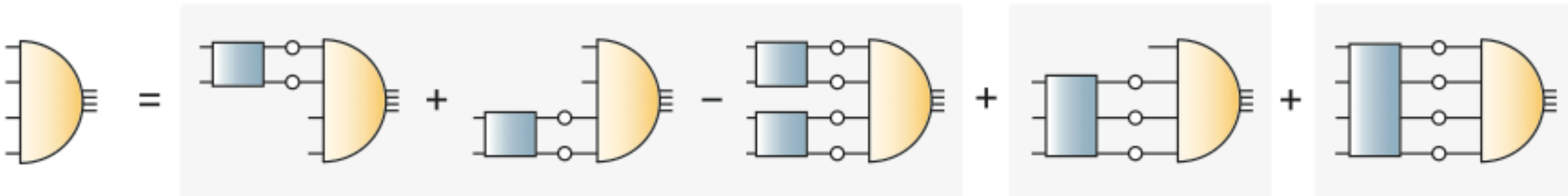




Related to details of underlying QCD forces between quarks

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Basic idea:
  solve four-body equation without any assumption on internal clustering
- Key elements: quark propagator and interaction kernels

Tetraquarks from the four-body equation

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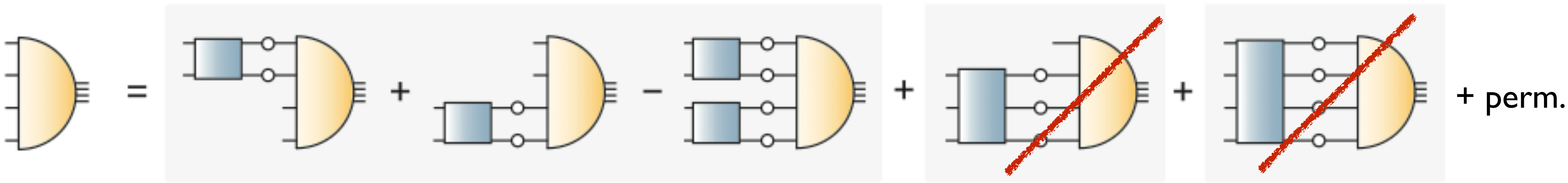

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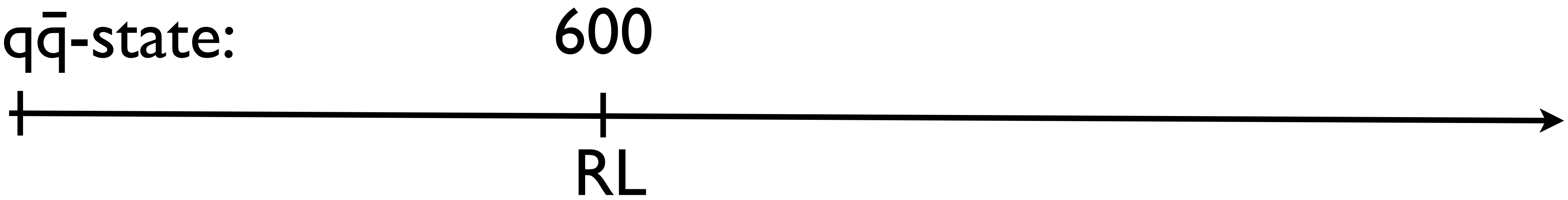
Bound state vs resonance: light scalars

$q\bar{q}$-state:



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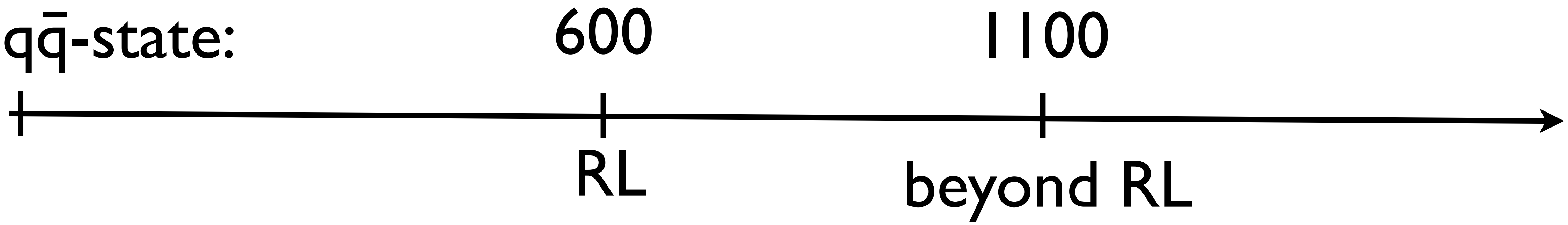

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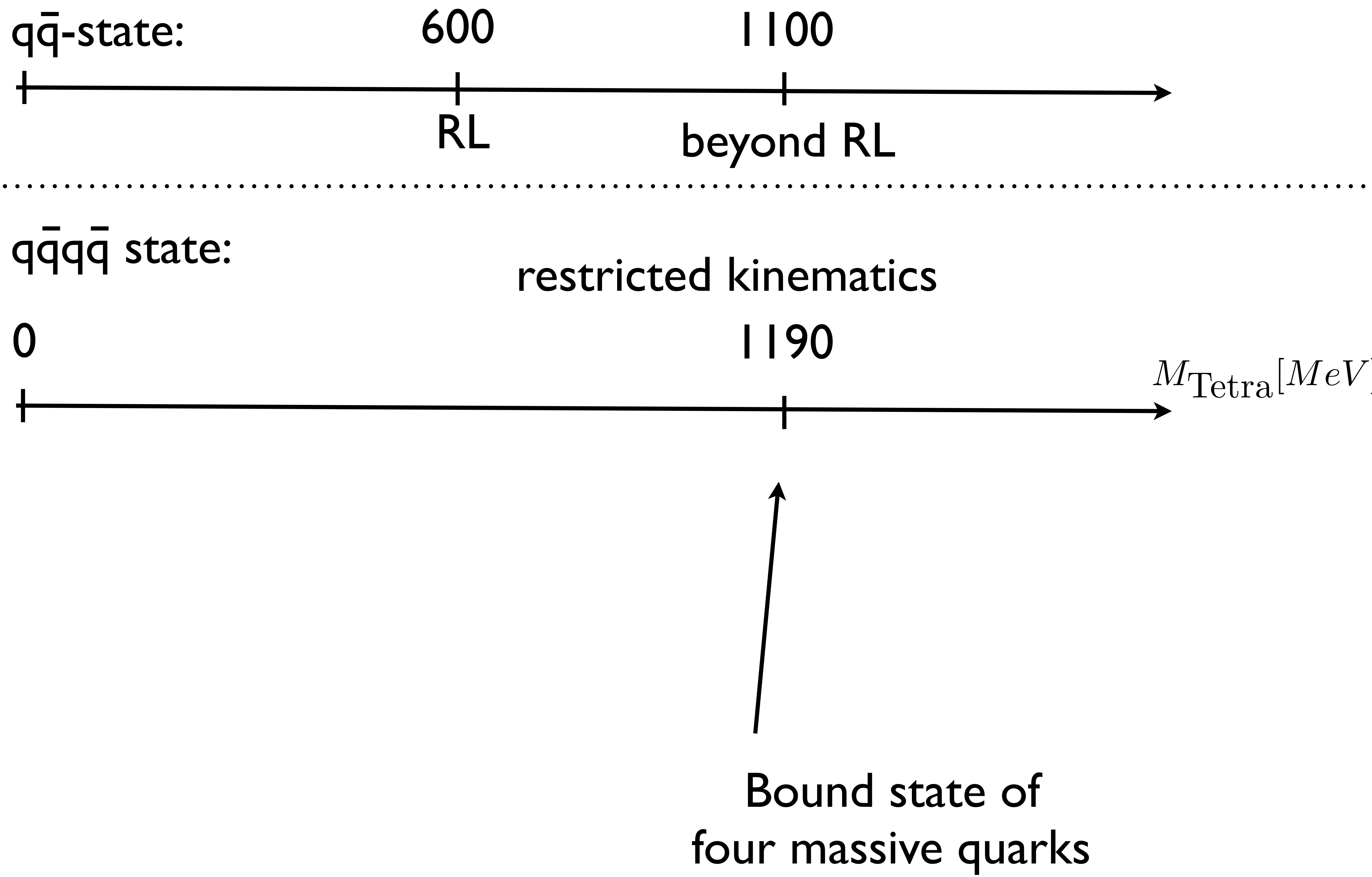


Bound state vs resonance: light scalars

$q\bar{q}$-state:

600 — RL

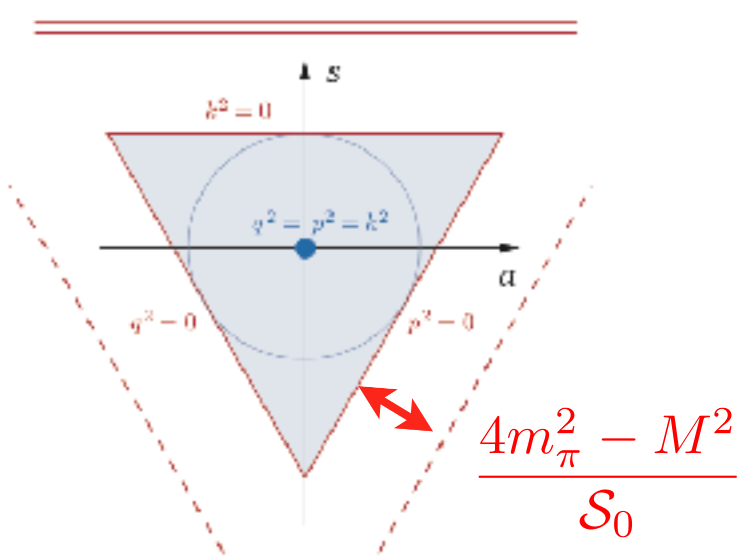
1100 — beyond RL

$q\bar{q}q\bar{q}$ state:

restricted kinematics

0 — 350 — 1190 — $M_{\mathrm{Tetra}}[MeV]$

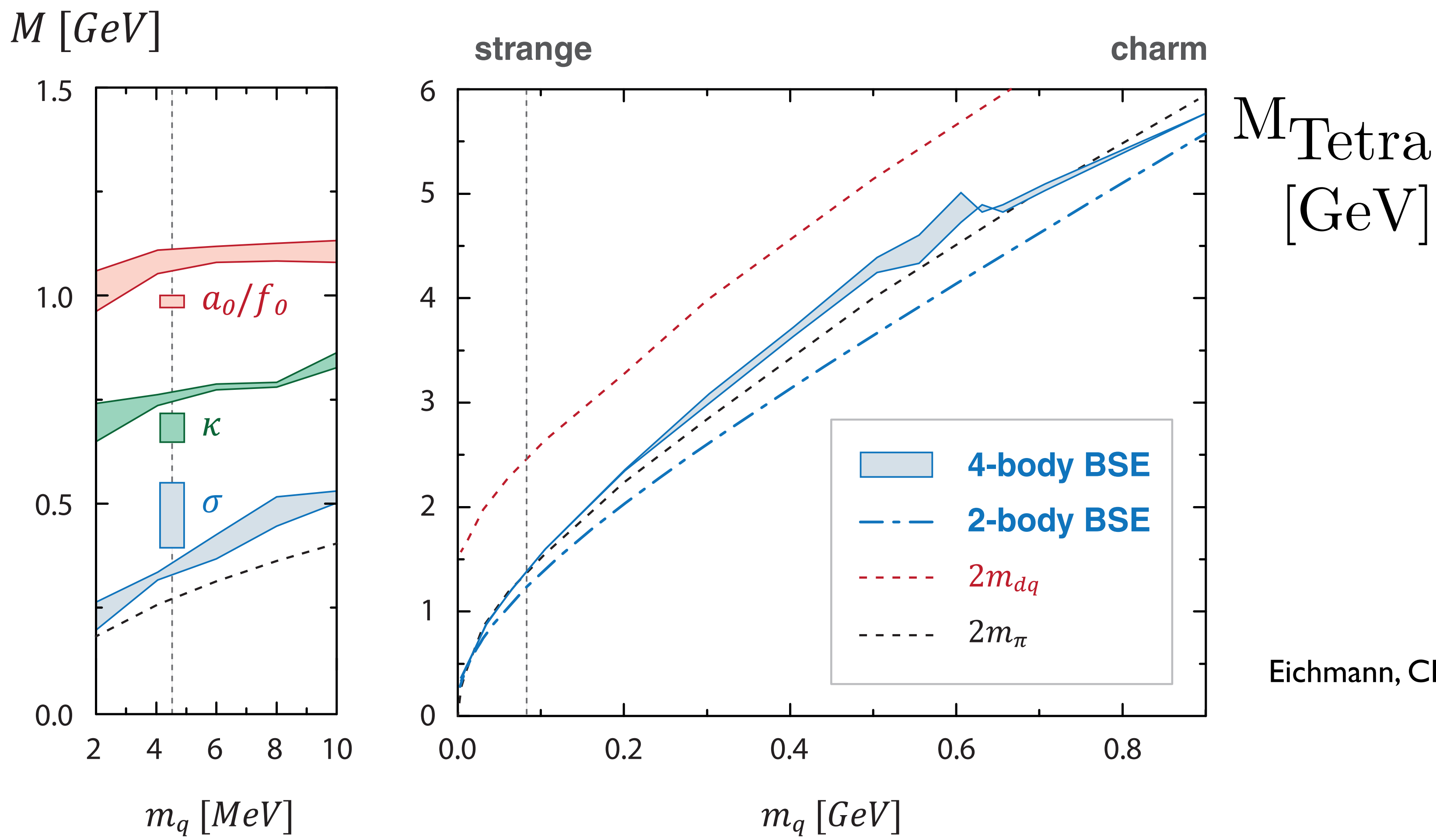
full kinematics



Two-pion resonance

Bound state of four massive quarks

Mass evolution of tetraquark



Eichmann, CF, Heupel, PLB 753 (2016) 282-287

- Resonance becomes bound state for large m_q
- Dynamical decision: meson clusters, not diquarks
- Results:

$$m_\sigma \sim 350\,\mathrm{MeV}$$

$$m_\kappa \sim 750\,\mathrm{MeV} \qquad m_{ss\bar{s}\bar{s}} \sim 1.5\,\mathrm{GeV}$$

$$m_{a_0,f_0} \sim 1080\,\mathrm{MeV} \qquad m_{cc\bar{c}\bar{c}} \sim 5.7\,\mathrm{GeV}$$

qualitatively similar to two-body framework

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

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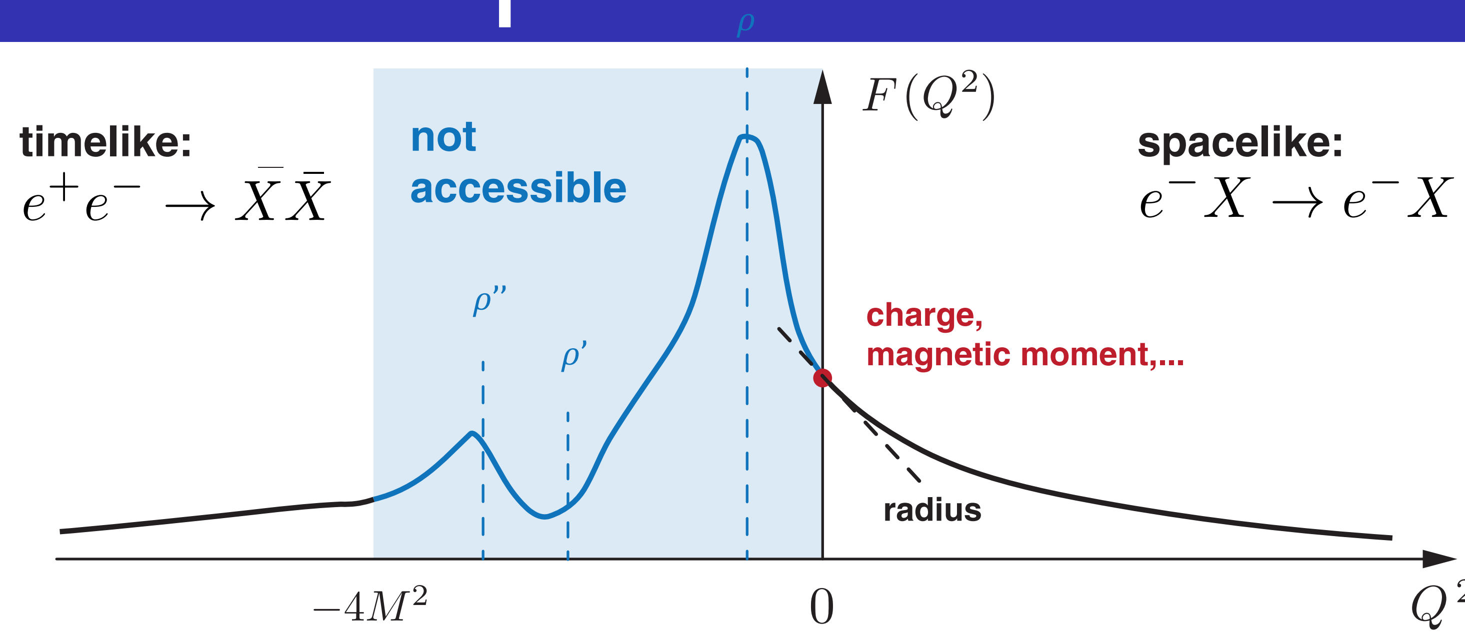
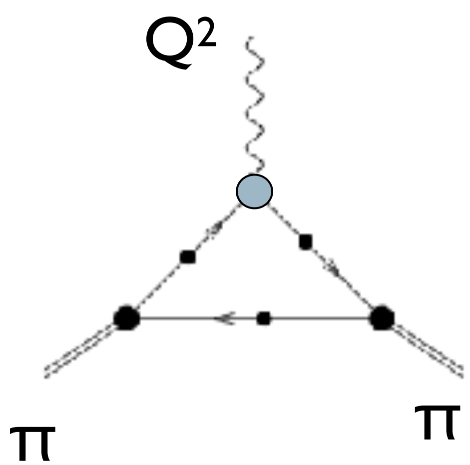
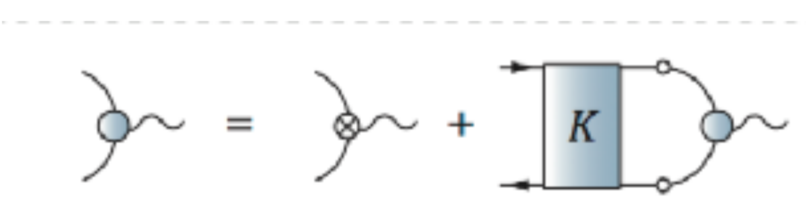
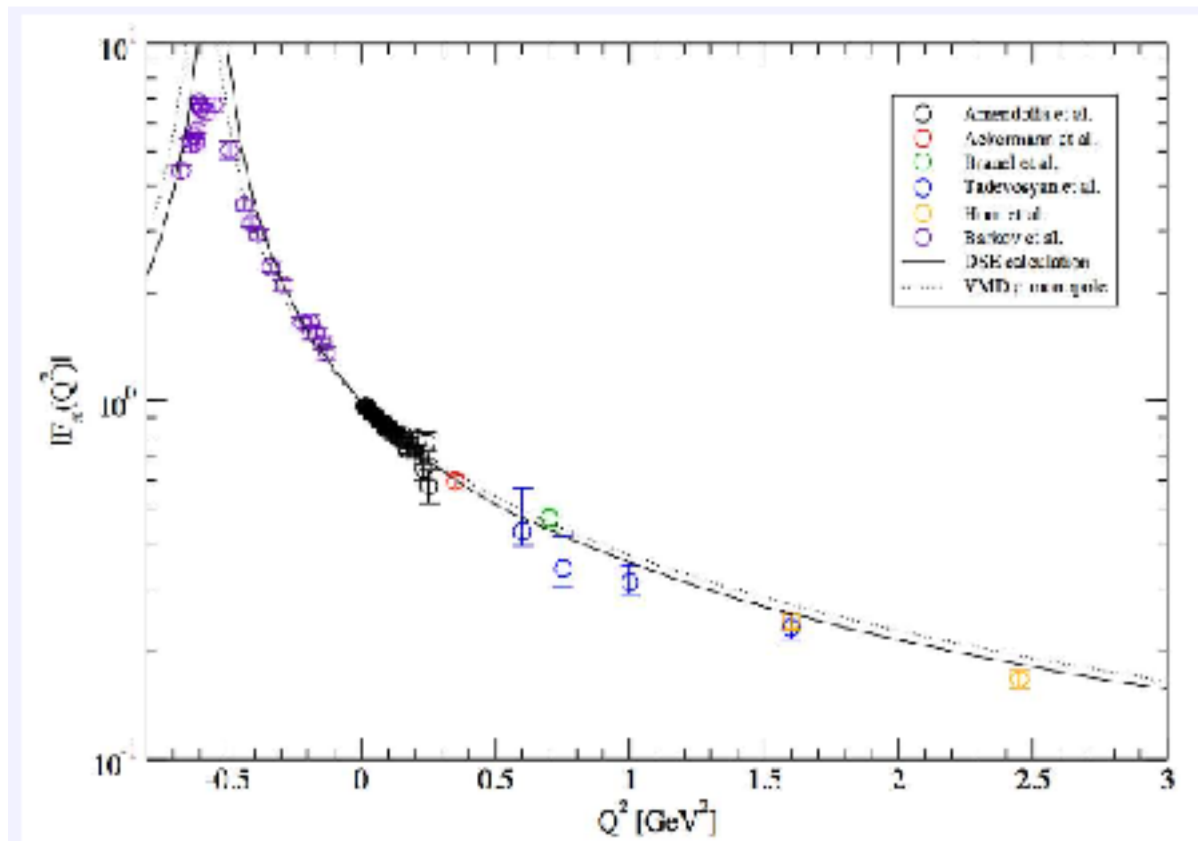
Quark-photon vertex and pion form factors



Pion form factor:



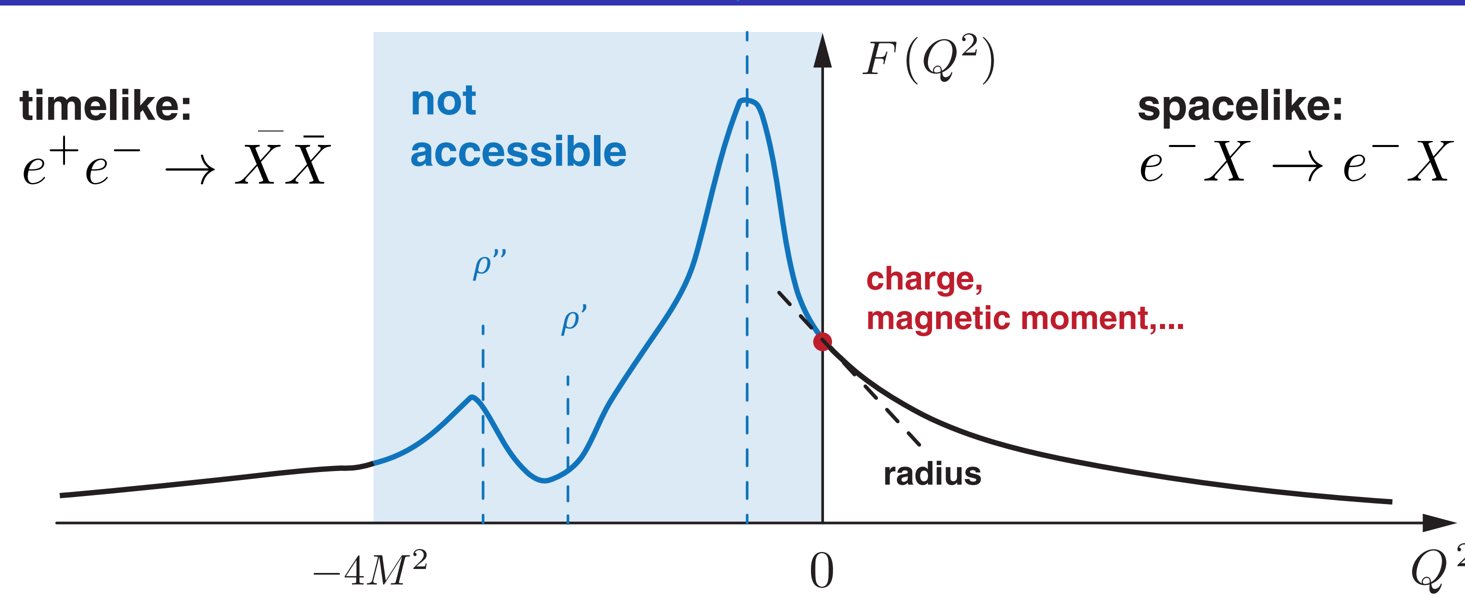




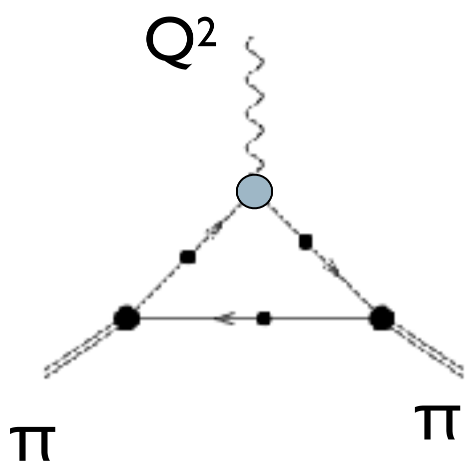
Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

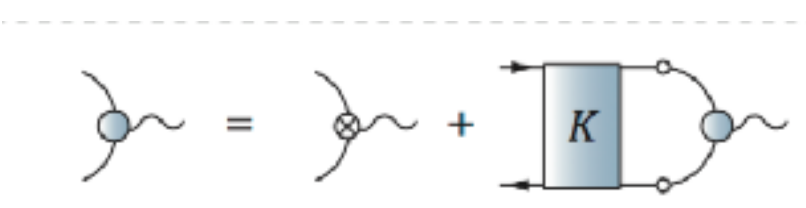
Vector meson poles dynamically generated!

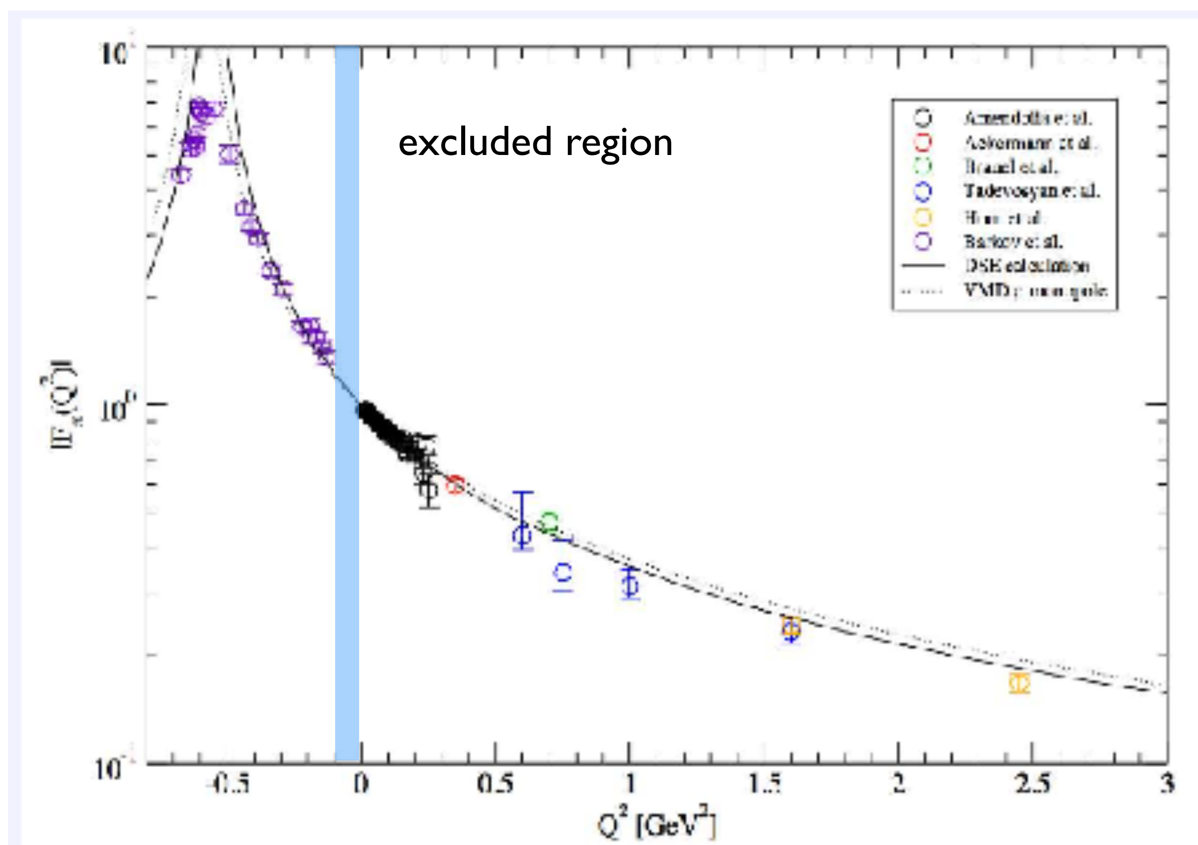
Quark-photon vertex and pion form factors



Pion form factor:





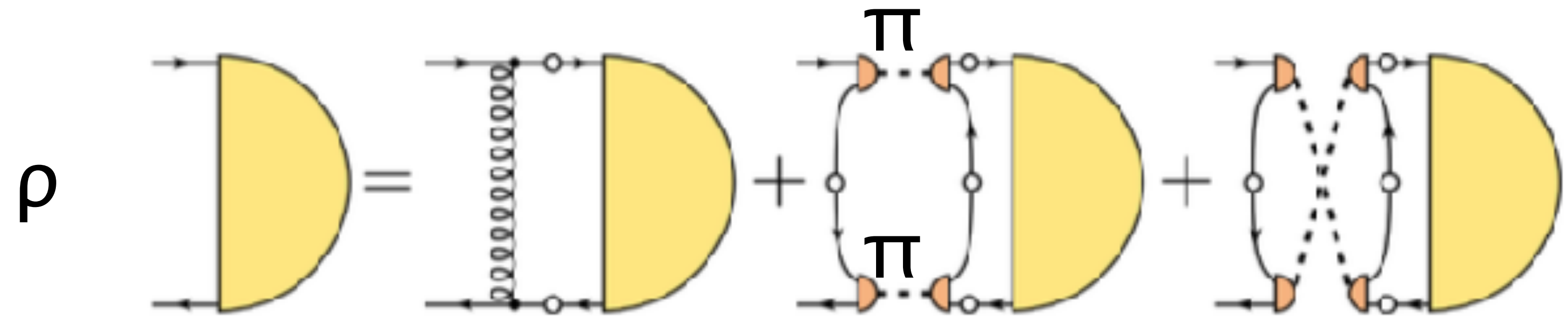


Krassnigg, Schladming 2011; Maris, Tandy NPPS 161, 2006

Vector meson poles dynamically generated!

Decays: ρππ

Beyond rainbow-ladder: pion contributions in BSE-kernel:



Williams, arXiv:1804.11161

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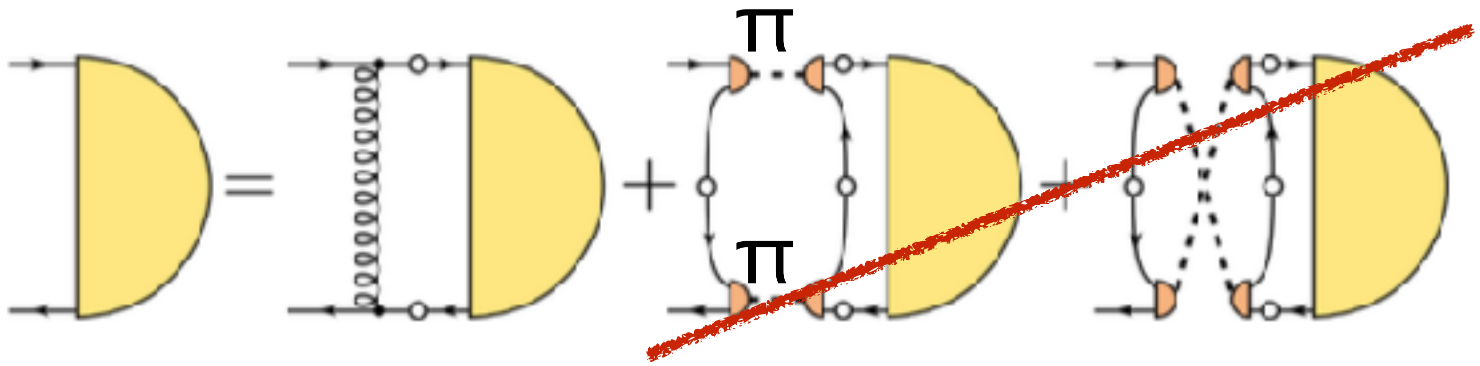

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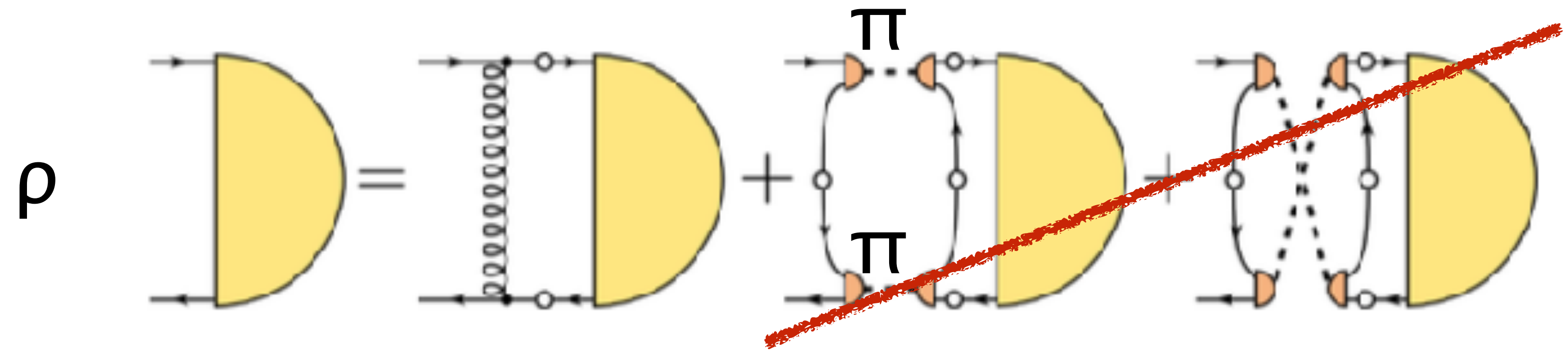

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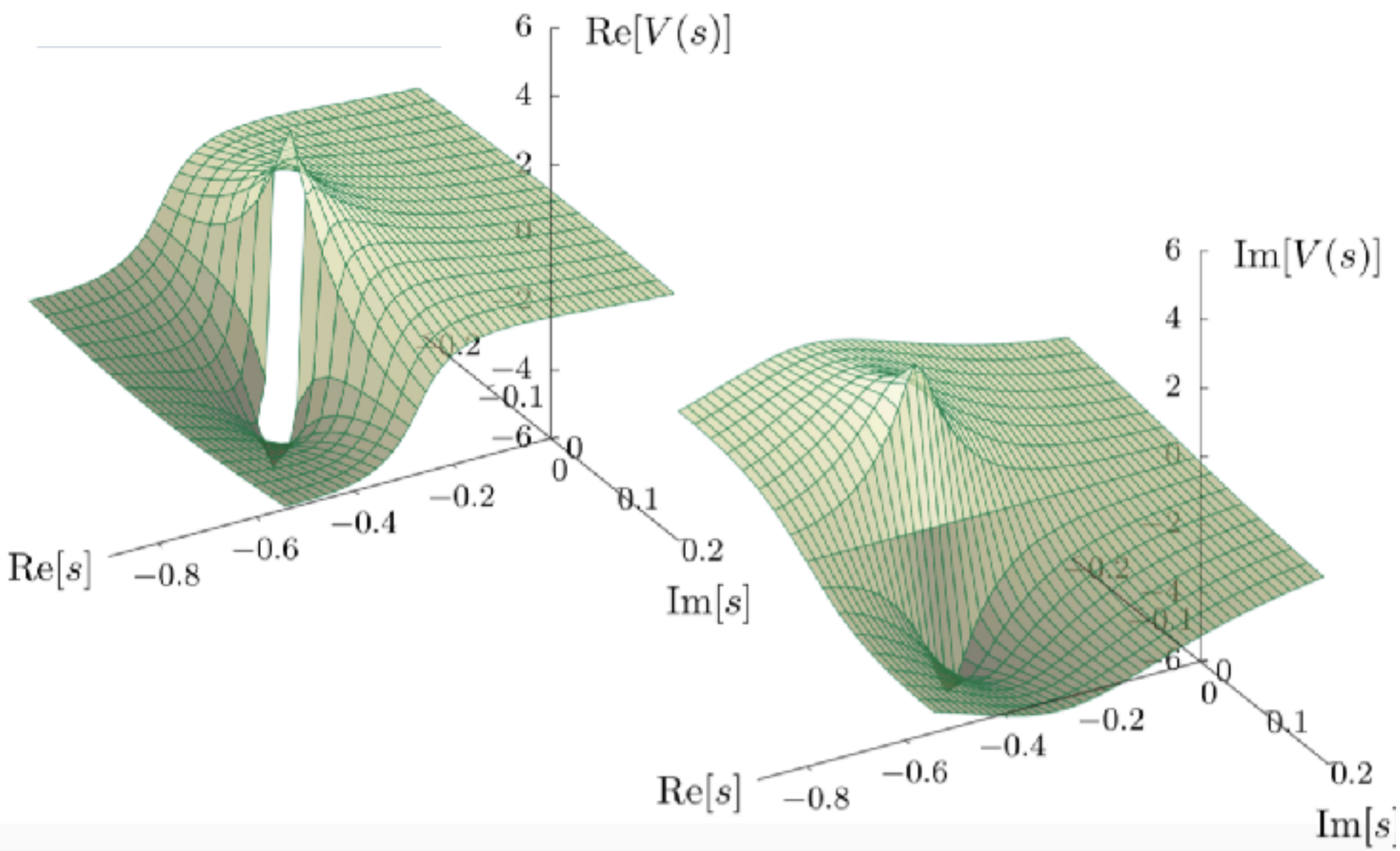

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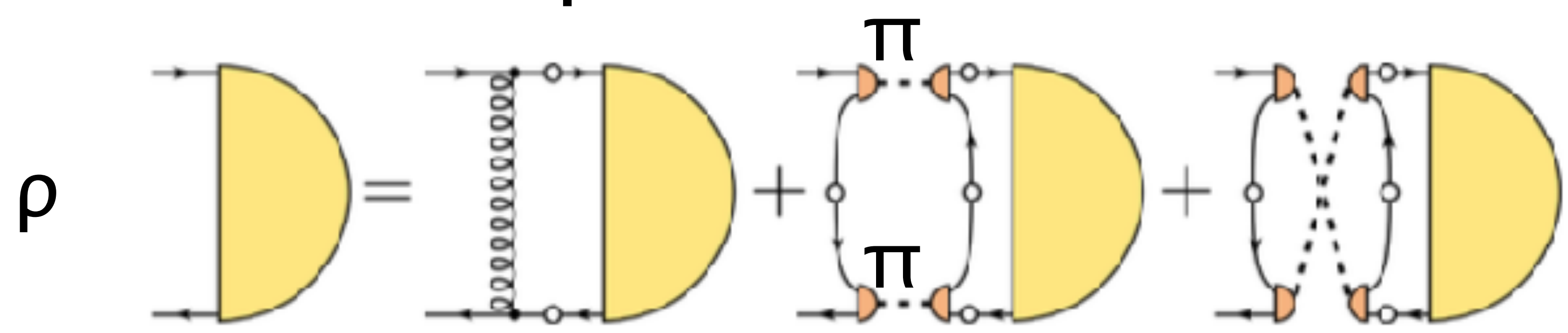

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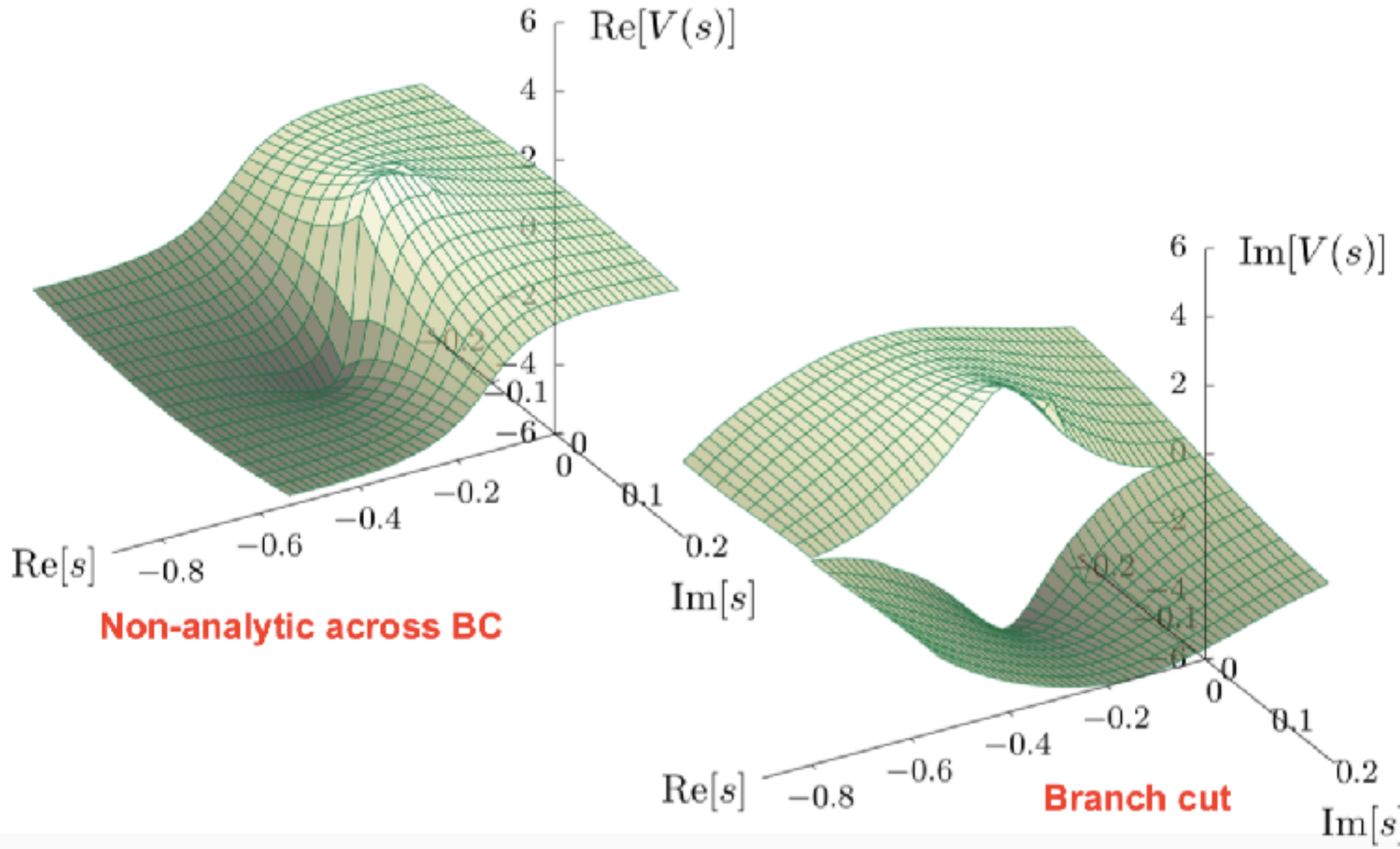

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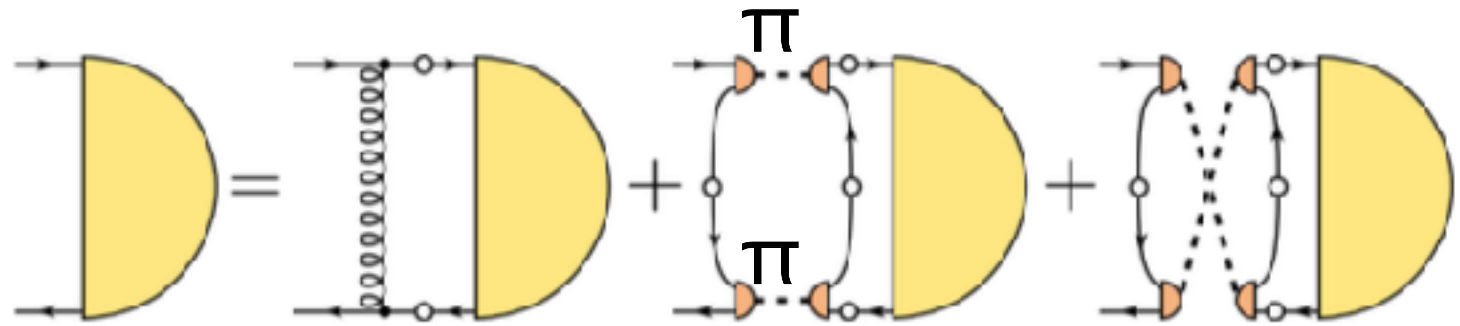
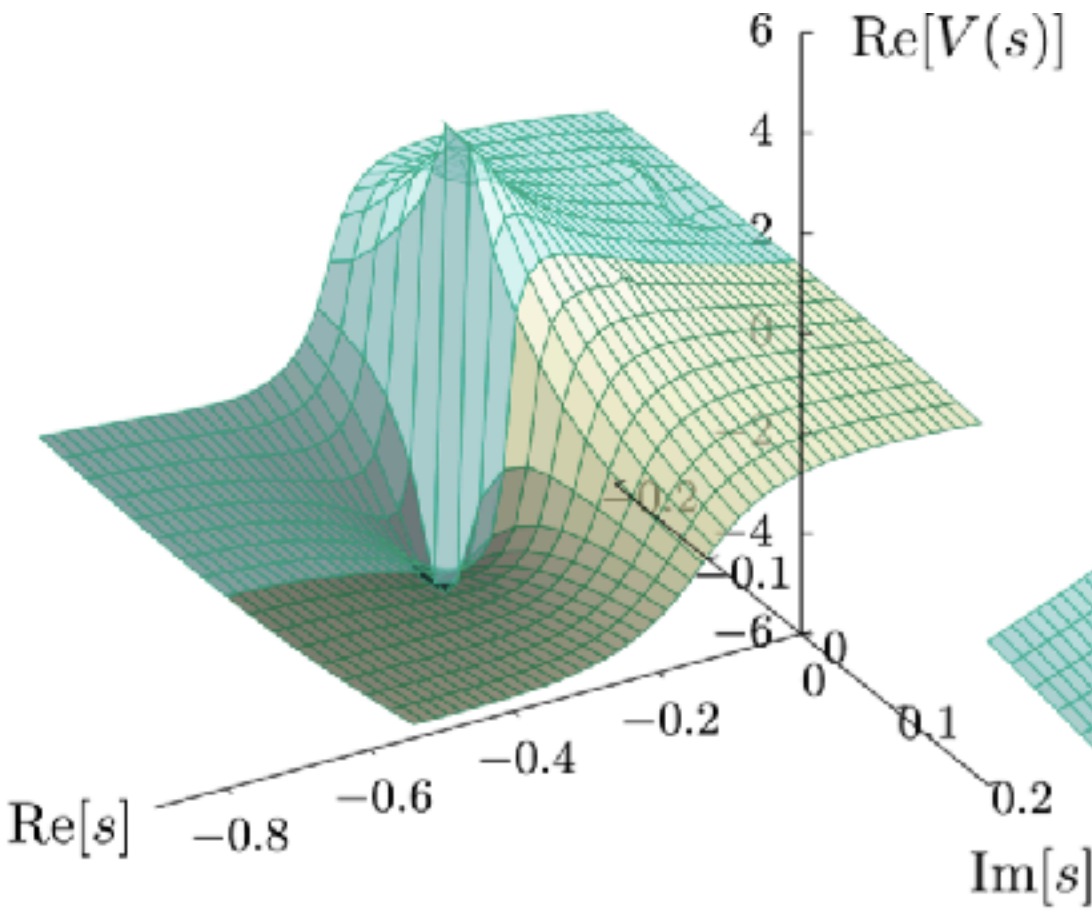
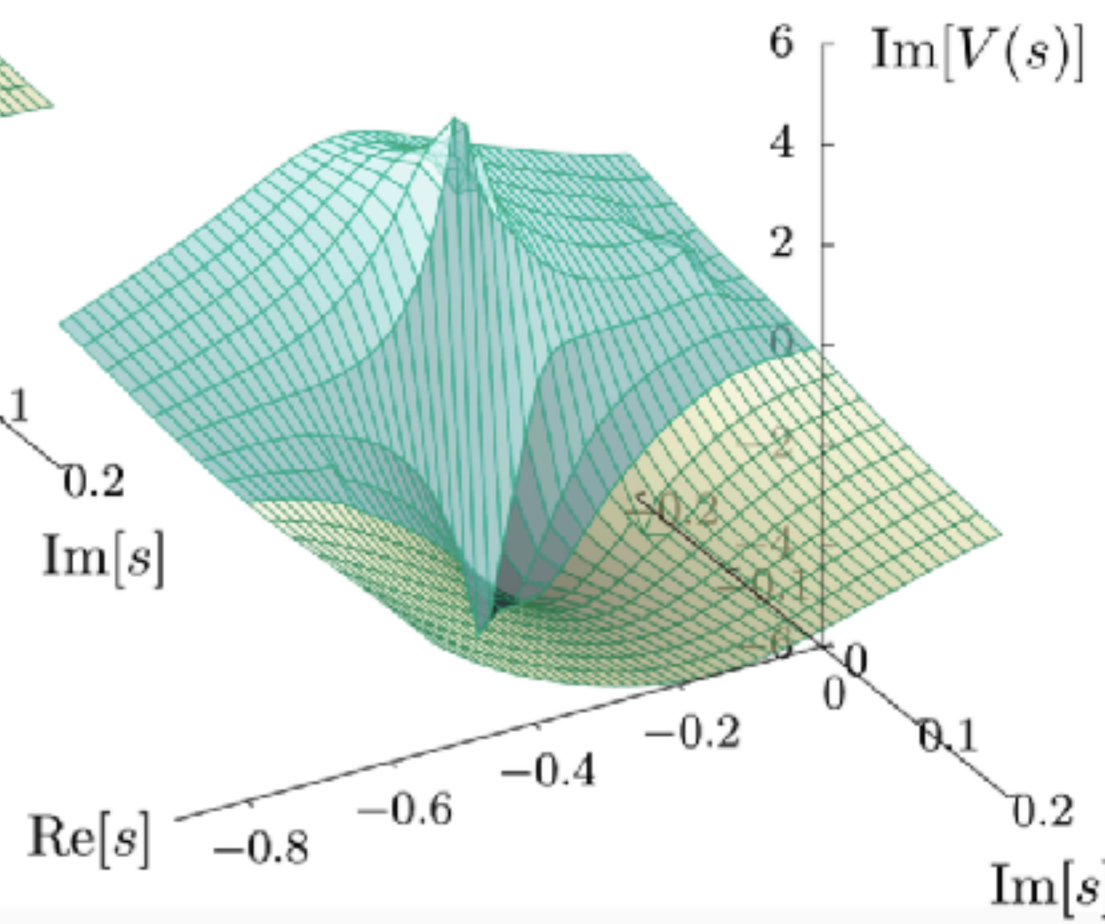


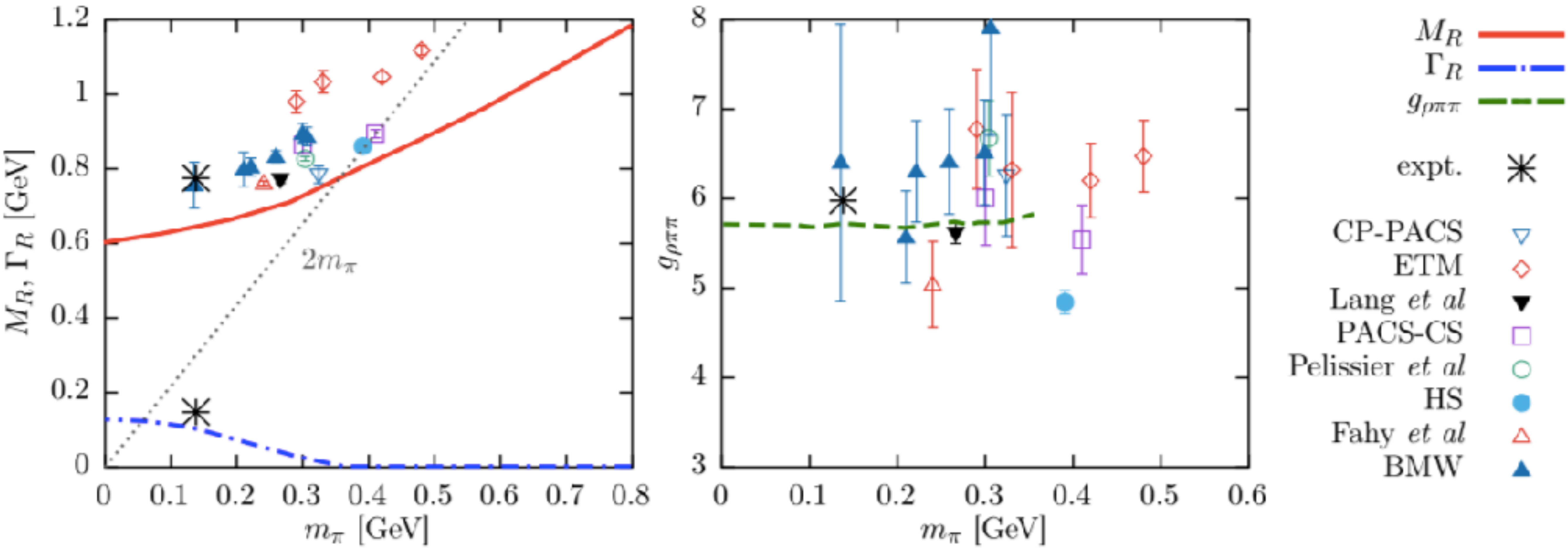
Williams, arXiv:1804.11161





Analytically continue across branch cut to the second Riemann sheet.

Decays: ρππ



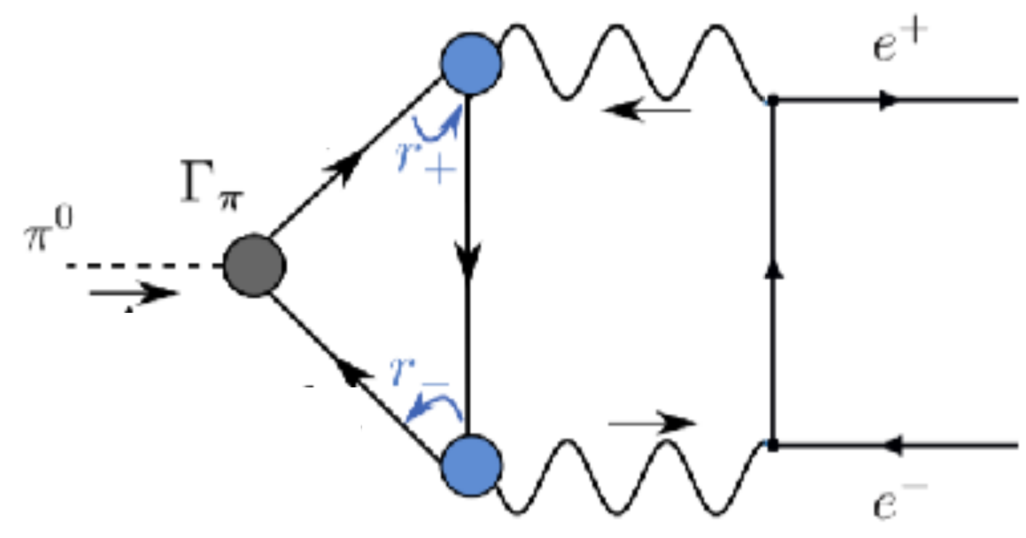
Williams, arXiv:1804.11161

- Additional corrections known to increase mass by O(100) MeV

CF and Williams, PRL 103 (2009), 122001

Rare pion decay

Rare pion decay $\pi^0 \to e^+e^-$:



$$\frac{B(\pi_0 \to e^+e^-)}{B(\pi_0 \to \gamma\gamma)} = 2\left(\frac{m\alpha_{em}}{\pi m_\pi}\right)^2 \sqrt{1-4\frac{m^2}{m_\pi^2}}\;|A(-m_\pi^2/4)|^2$$

Usual: dispersive approach
DSE: direct calculation

Collaboration	B($\pi^0 \to e^+e^-$) [10^{-8}]
Experiment [1, 27, 28]	6.87(36)
Dorokhov et al. [2, 26]	6.23(9)
Husek et al. [35](THS)	6.14(8)
Masjuan et al. [29]	6.23(5)
Our result (DR)	6.21(3)
Our result (direct)	6.22(3)

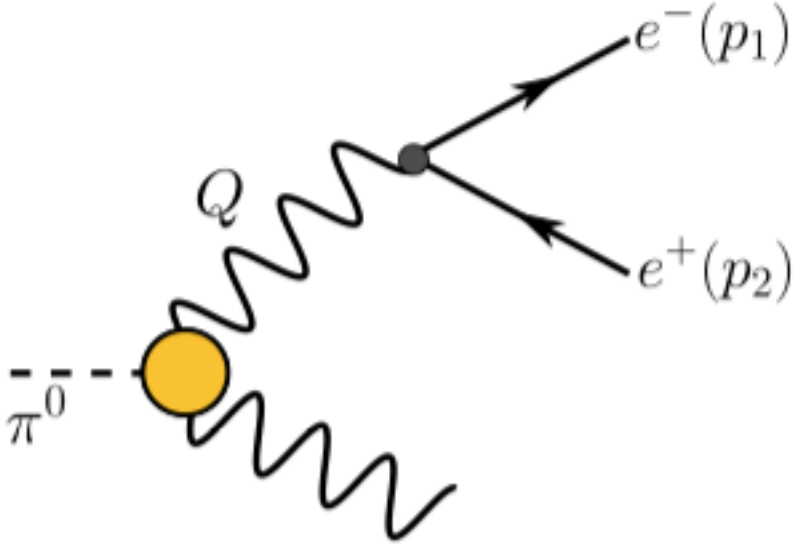
Weil, Eichmann, CF and Williams, PRD96 (2017) no.1, 014021

TFF: Eichmann, CF, Weil and Williams, PLB 774 (2017) 425

- same result as everybody else
- discrepancy with exp. remains

More rare decays

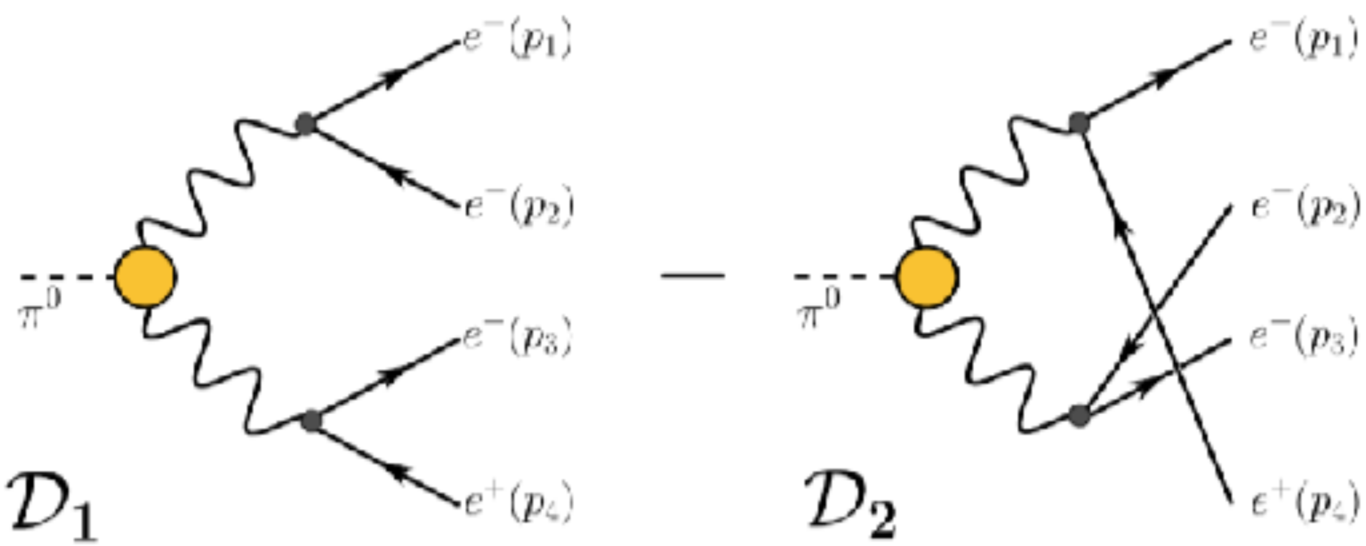
$\pi_0 \to e^+ e^- \gamma$



Collaboration	$\Gamma_{\pi^0 \to e^+ e^- \gamma}$ [10^{-11} GeV]
PDG [21]	9.06(18)
Terschlüsen et al. [22]	9.26
Hoferichter et al. [23]	9.065
Our result	9.11(4)

$\pi_0 \to e^+ e^- e^+ e^-$

Collaboration	$\Gamma_{\pi^0 \to 2e^+ 2e^-}$ [10^{-13}GeV]
PDG [21]	2.58(12)
Terschlüsen et al. [22]	2.68
Escribano et al. [24]	2.62
Our result	2.63(1)



Weil, Eichmann, CF and Williams, Phys.Rev. D96 (2017) no.1, 014021

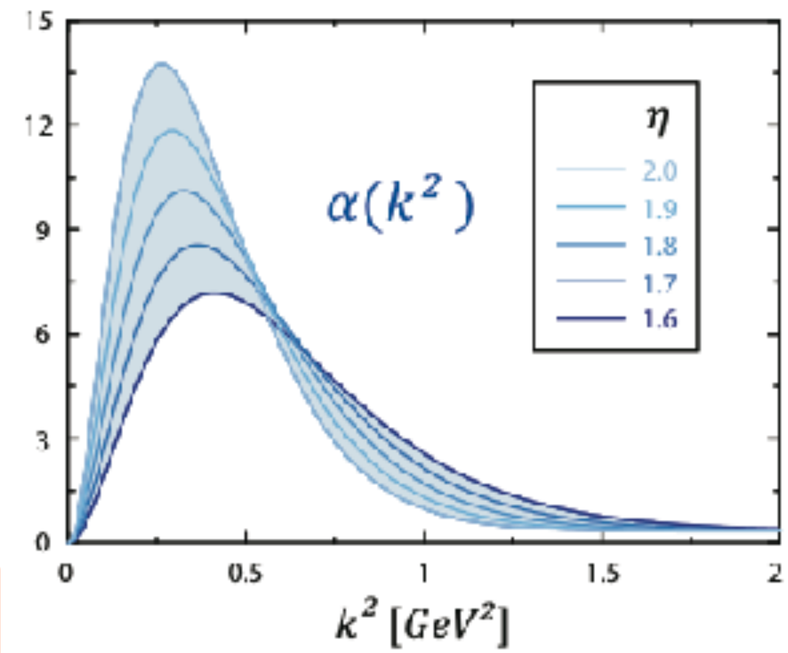
Summary

Internal dynamics very important !!

- Glueballs: $M(0^{++}) = 1.64 \text{ GeV}$
- Hybrids in $q\bar{q}$-BRL
- Four-quarks states dominated by meson-meson configurations
- Dynamical description of σ as π-π resonance
- Dynamical width of rho-meson
- Decays of neutral pion

Backup

Rainbow-ladder model for quark-gluon interaction



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses $m_u = m_d$, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : band of results

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

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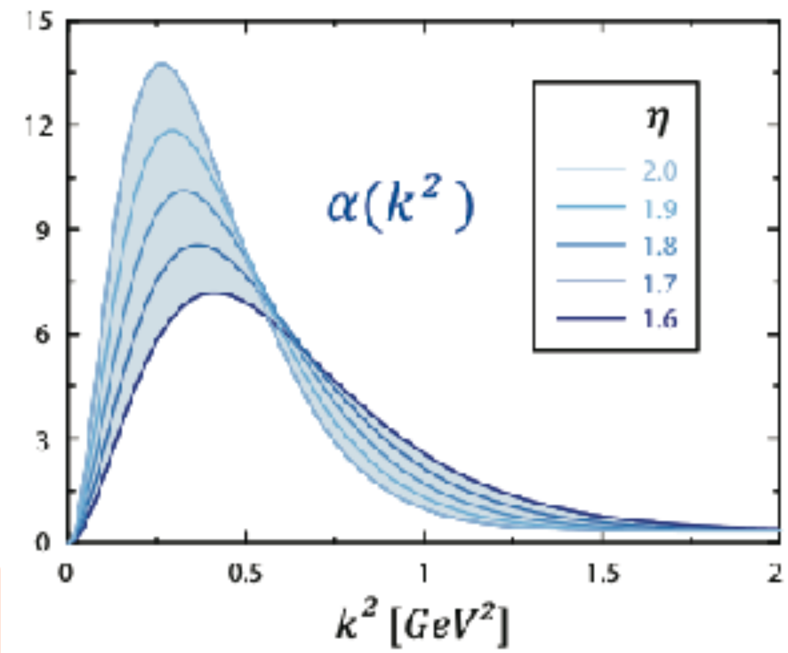

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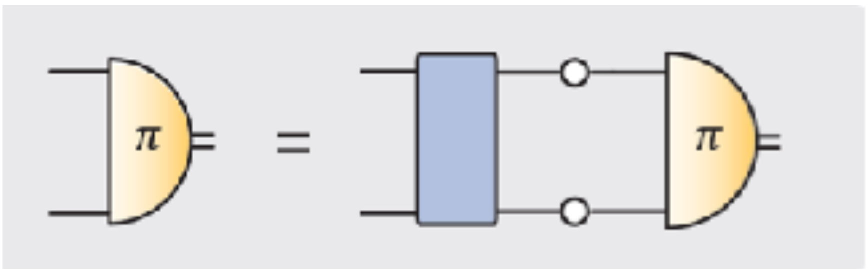

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

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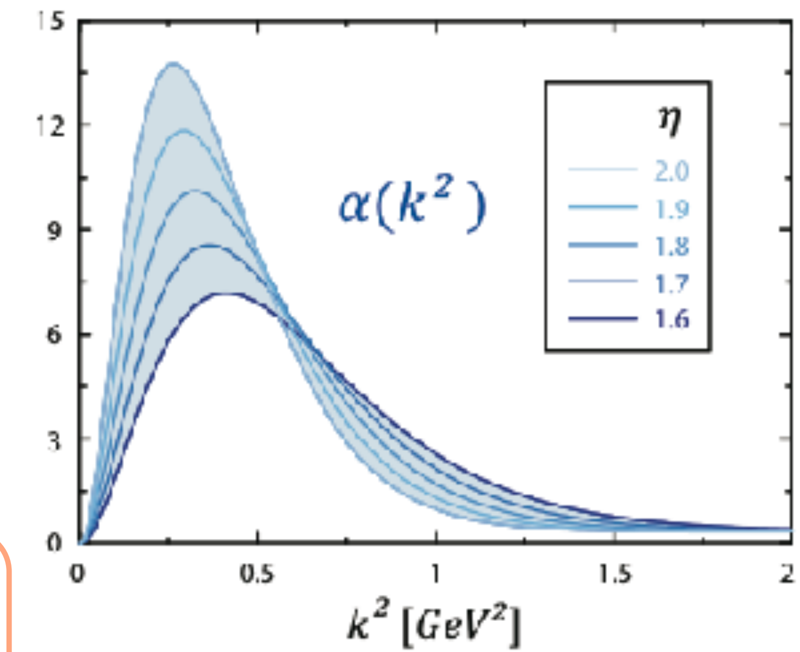

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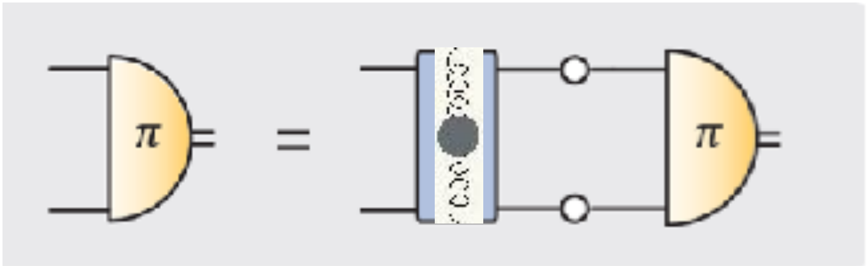

Binosi, Chang, Papavassiliou and Roberts, PLB 742 (2015) 183

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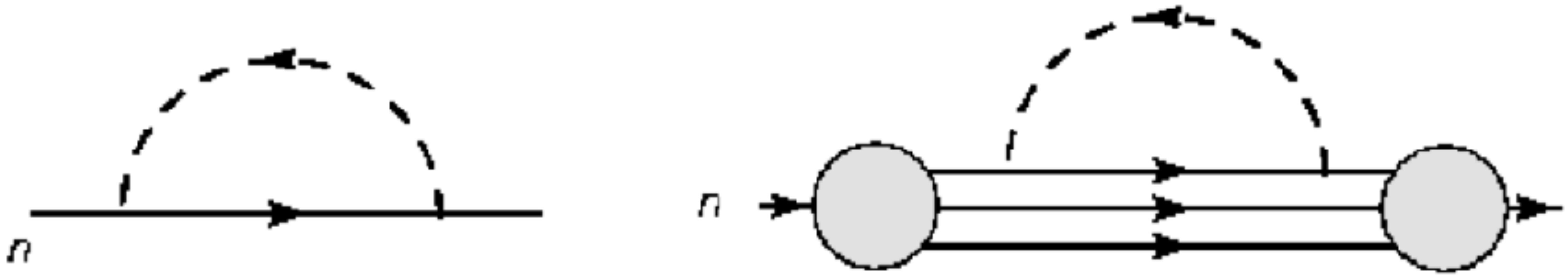
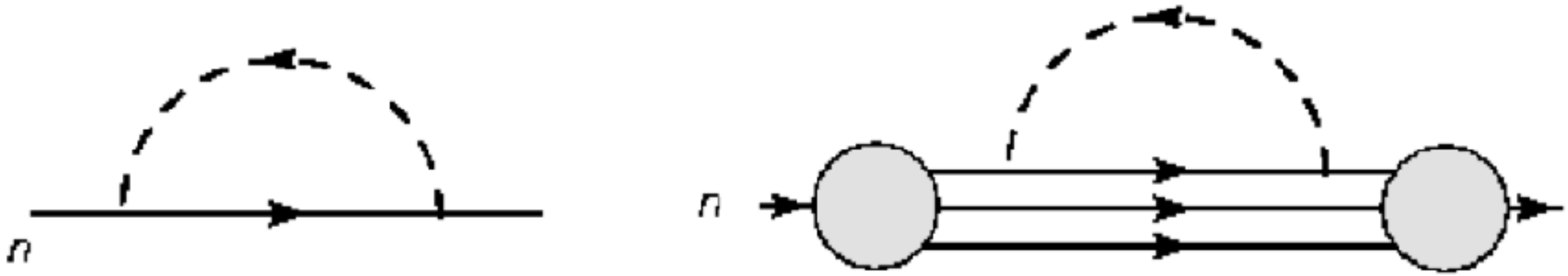
Pion cloud



- Hadron level: πN-contributions to nucleon self-energy
- Quark-level: π-contributions to quark self-energy

Pion cloud



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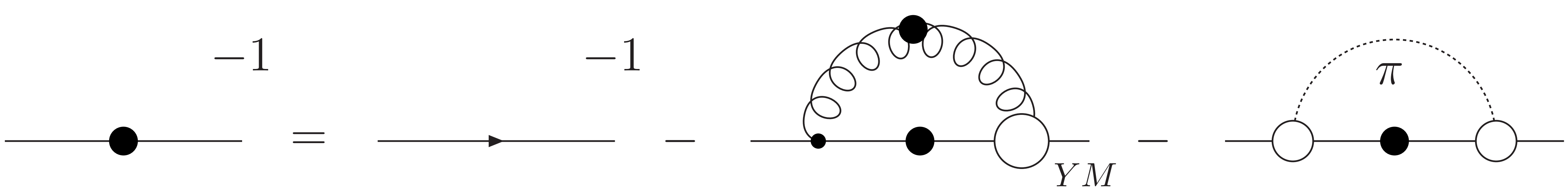
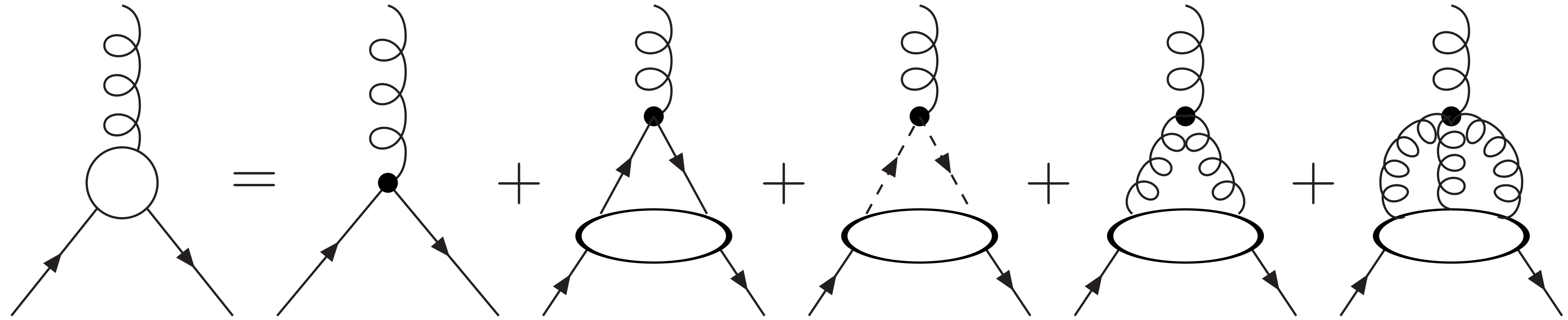
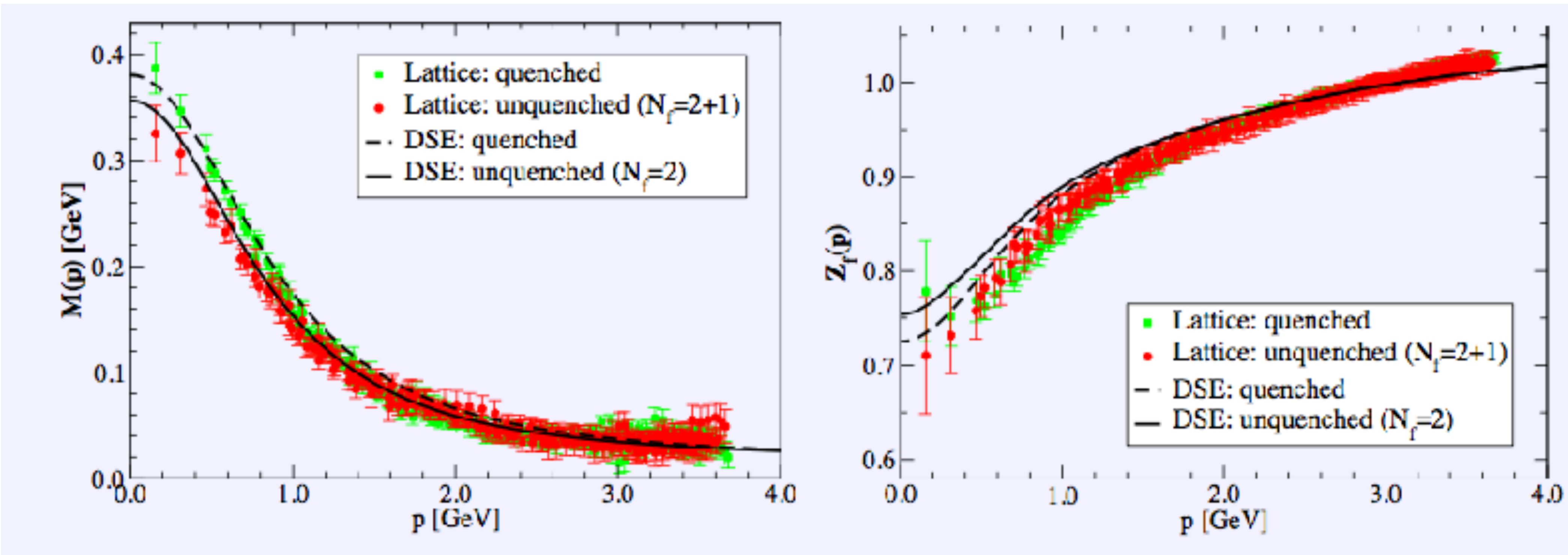


Pion effects in quark-gluon interaction

quark-gluon vertex:



quark:



CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

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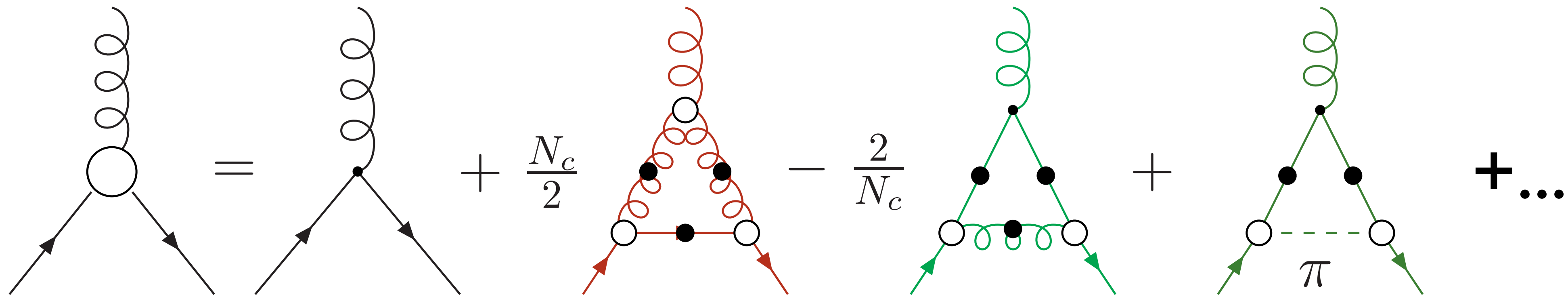
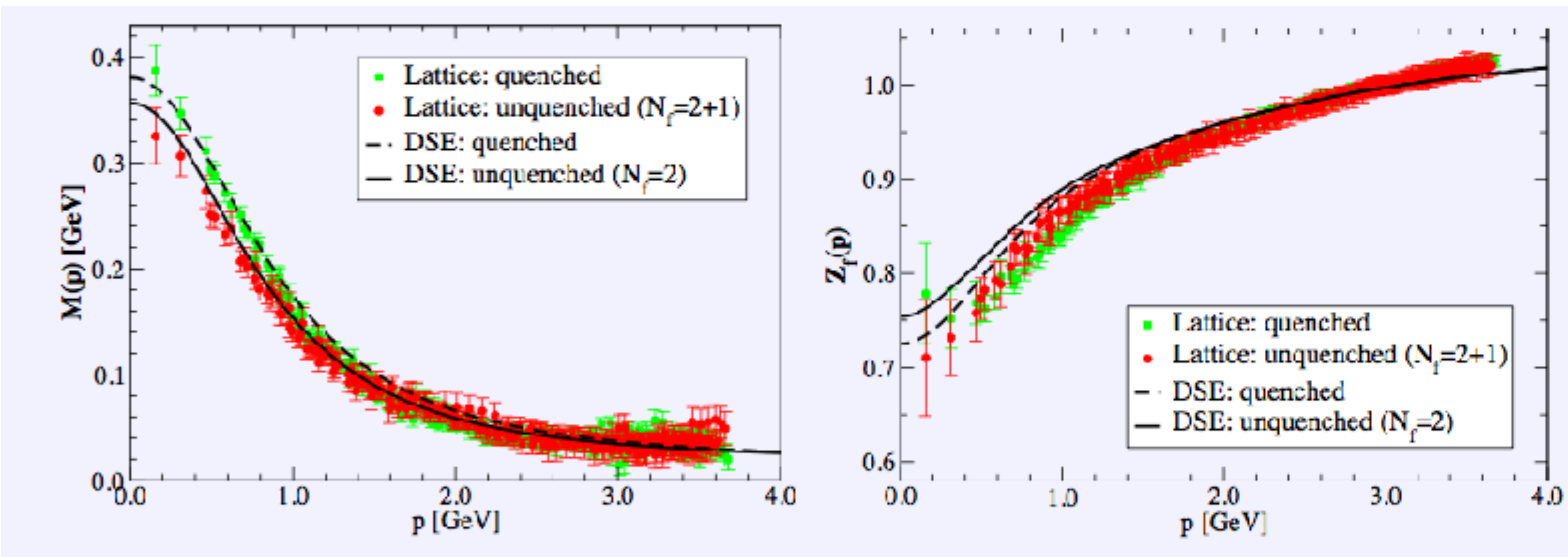


quark:



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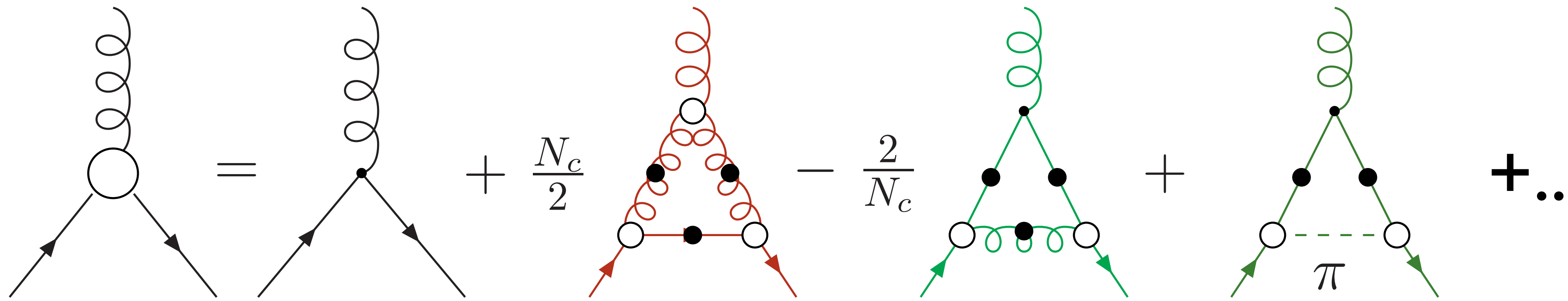
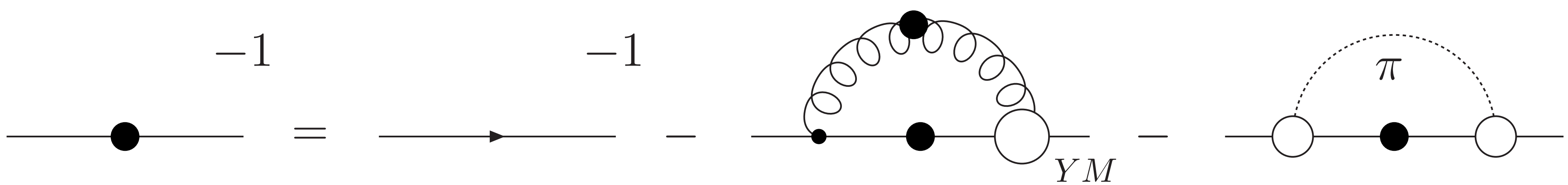
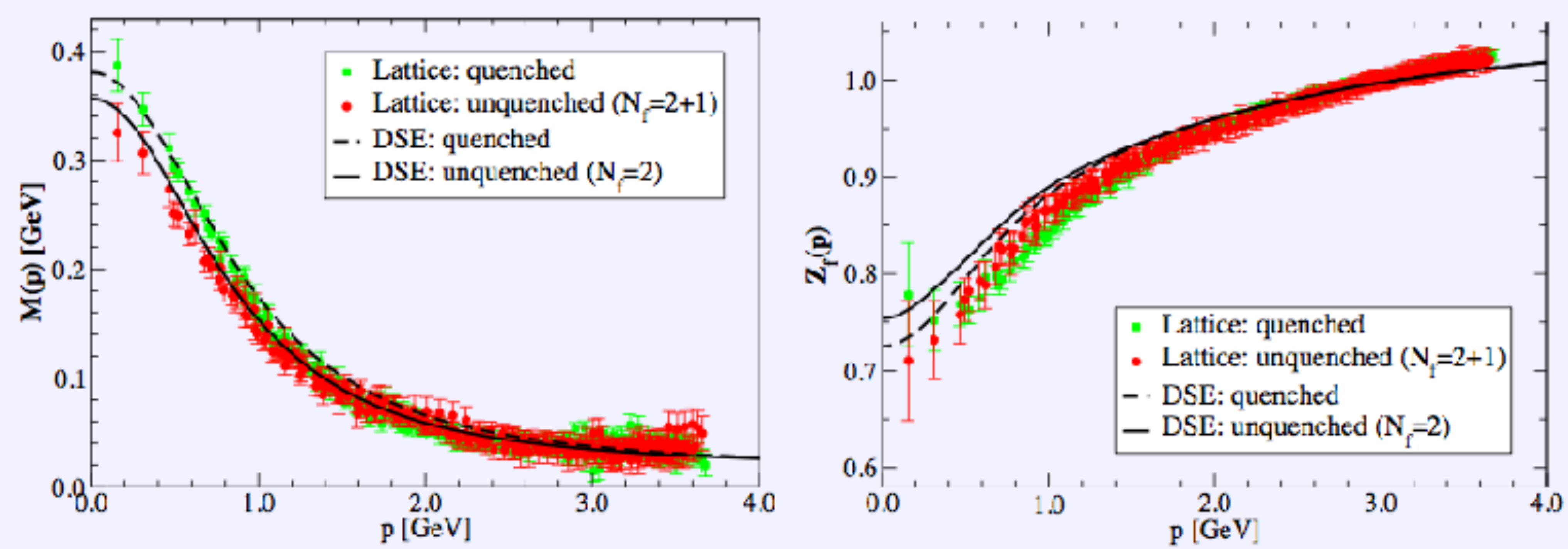


quark:





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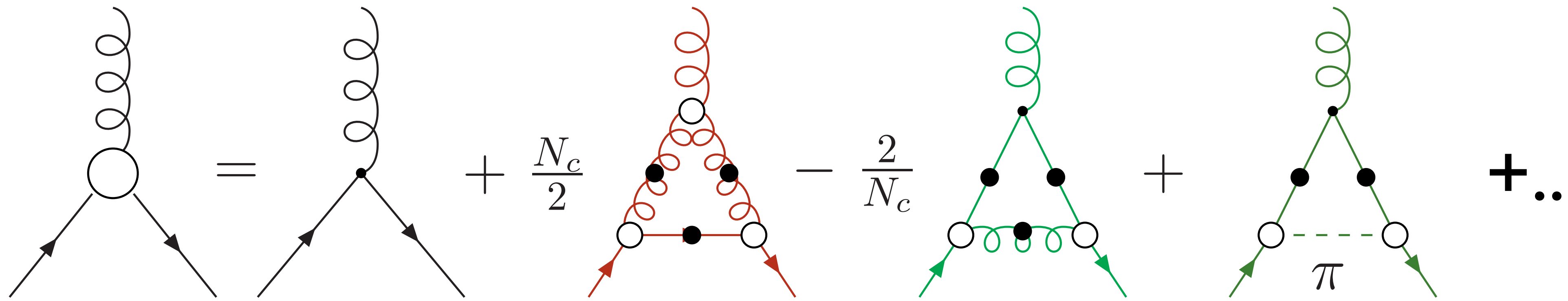

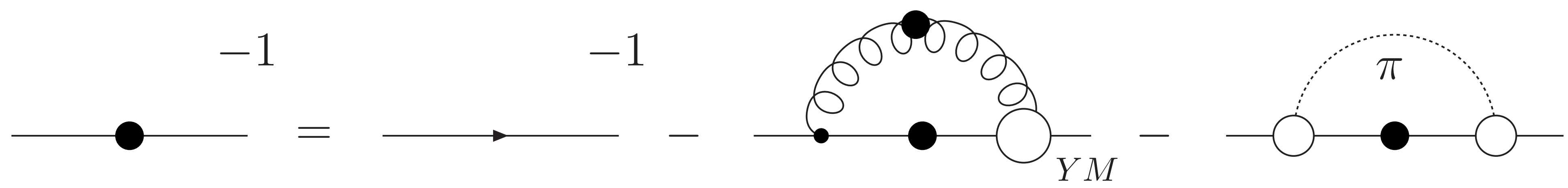

Bethe-Salpeter equation: