Ab-initio calculations of η-nuclear quasi bound states

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Introduction

- moderate attractive ηN interaction with scattering length
  $\Rightarrow \exists$ of η nuclear bound states (starting ^{12}C)
  (Haider, Liu **PLB 172** (1986) 257, **PRC 34** (1986) 1845)

Numerous studies:

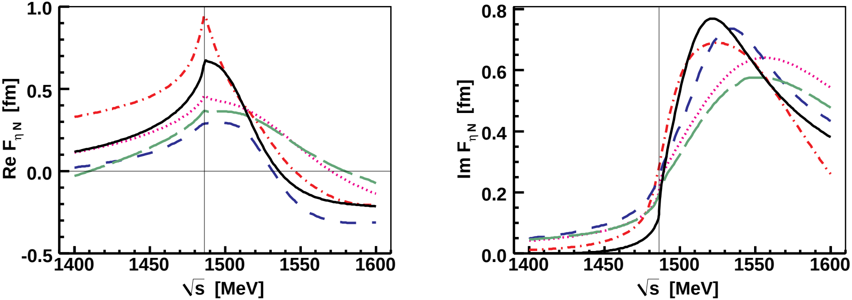
- chiral coupled channel models and K matrix methods
  (fitting πN and γN reaction data in $N^\star$ (1535) resonance region)
  (bound states already in He isotopes)

No decisive experimental evidence so far

- $^{25}_{\eta}\mathrm{Mg}$?, $B_\eta = 13.1 \pm 1.5\mathrm{MeV}$ and $\Gamma_\eta = 10.2 \pm 3.0\mathrm{MeV}$
  (COSY-GEM, **PRC 79** (2009) 012201(R))
- $^{3}_{\eta}\mathrm{He}$ not found
  (MAMI, **PLB 709** (2012) 21; Xie et al., **PRC 95** (2017) 015202)
- $^{4}_{\eta}\mathrm{He}$ not found
  (WASA@COSY, **PRC 87** (2013) 035204; **NPA 959** (2017) 102)

ηN scattering amplitudes

- significant model dependence
- strong energy dependence

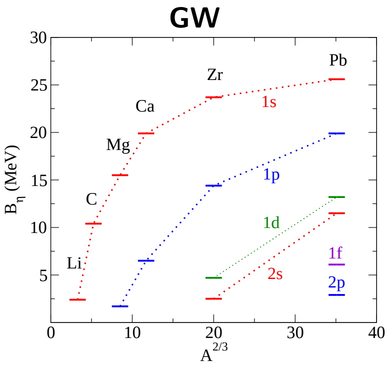


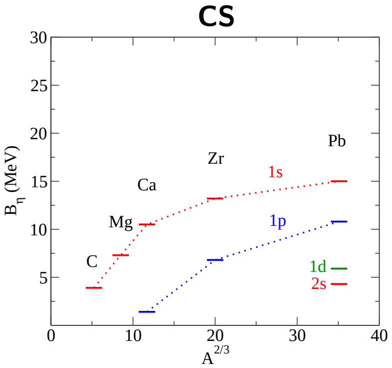
line	$a_{\eta N}$ [fm]	model
dotted	0.46+i0.24	N. Kaiser, P.B. Siegel, W. Weise, **PLB 362** (1995) 23
short-dashed	0.26+i0.25	T. Inoue, E. Oset, **NPA 710** (2002) 354 (GR)
dot-dashed	0.96+i0.26	A.M. Green, S. Wycech, **PRC 71** (2005) 014001 (GW)
long-dashed	0.38+i0.20	M. Mai, P.C. Bruns, U.-G. Meißner, **PRD 86** (2012) 094033 (M2)
full	0.67+i0.20	A. Cieply, J. Smejkal, Nucl. Phys. **A 919** (2013) 334 (CS)

η in many-body systems

- Selfconsistent RMF predictions for the GW (left) and CS (right) models:
  all η quasi-bound states in selected nuclei are shown; small widths - $\Gamma_\eta \lesssim 5$ MeV
  (E. Friedman, A. Gal, J. Mareš, **PLB 725** (2013) 334)

  (A. Cieplý, E. Friedman, A. Gal, J. Mareš, **NPA 925** (2014) 126)





η in few-body systems

Faddeev (AGS) calculations of ηNNN and $\eta NNNN$ systems

- A. Fix, H Arenhövel, Phys. Rev. **C 66** (2002) 024002
- A. Fix, O. Kolesnikov, **PLB 772** (2017) 663
- A. Fix, O. Kolesnikov, Phys. Rev. **C 97** (2018) 044001

Variational calculations

Hyperspherical basis

- N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345

Stochastic Variational Method (SVM)

- N. Barnea, E. Friedman, A. Gal, **NPA 968** (2017) 35
- N. Barnea, B. Bazak, E. Friedman, A, Gal, **PLB 771** (2017) 297

Stochastic Variational Method

(K. Varga et al., Nucl. Phys. **A 571** (1994) 447)
(K. Varga, Y. Suzuki, Phys. Rev. **C 52** (1995) 2885)

optimizes variational basis in a **random trial and error procedure**

Variational basis states

- antisymmetrized correlated Gaussians (assuming L=0)

$$\psi_{SM_S TM_T}(\mathbf{x}, A) = \mathcal{A}\{G_A(\mathbf{x})\chi_{SM_S}\eta_{TM_T}\}, \quad G_A(\mathbf{x}) = e^{-\frac{1}{2}\mathbf{x}A\mathbf{x}}$$

- Jacobi coordinates $\mathbf{x}$, symmetric positive definite matrix of variational parameters A, spin χ_{SM_S} and isospin η_{TM_T} parts
- $\frac{N(N-1)}{2}$ real parameters for one basis state
- explicit antisymmetrization → computational complexity grows with N!

$$\mathcal{A} = \sum_{i=1}^{N!} p_i \mathcal{P}_i$$

Variation procedure

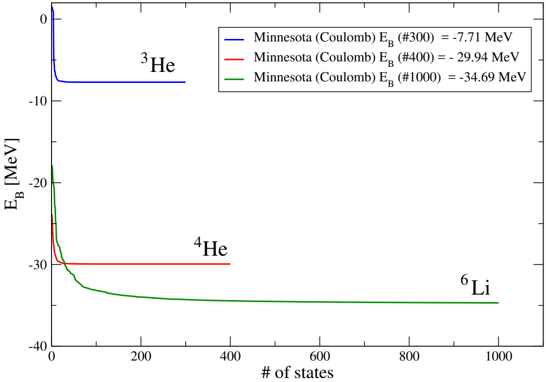
Hamiltonian and wave function

$$\hat{H} = \hat{T}_N + \hat{T}_\eta + \hat{V}_{NN} + \hat{V}_{\eta N} - \hat{T}_{\mathrm{cm}} \ , \qquad \Psi = \sum_{i=1}^{K} c_i \psi_{SM_{S_i} TM_{T_i}}(\mathbf{x}, A_i)$$

Step-by step optimization procedure

- starting with the 1st basis state $\psi_{SM_{S_1} TM_{T_1}}(\mathbf{x}, A_1)$
- optimization loop over variational parameters of the 1st basis state A_1
  - random selection of variational parameters A_1
  - solution of generalized eigenvalue problem
- saving the optimized set of randomly selected variational parameters A_1 (giving the lowest binding energy E_B)
- addition of the 2nd basis state $\psi_{SM_{S_2} TM_{T_2}}(\mathbf{x}, A_2)$
- optimization loop over variational parameters of the 2nd basis state $\psi_{SM_{S_2} TM_{T_2}}(\mathbf{x}, A_2)$
- .....

Variation procedure



Result

- wave function in a basis of optimized correlated Gaussians

$$\Psi = \sum_{i=1}^{K} c_i \psi_{SM_{S\,i}TM_{T\,i}}(\mathbf{x}, A_i)$$

$\mathrm{V_{NN}}$ and $\mathrm{V_{\eta N}}$ potential input

NN two-body potentials

- Argonne AV4' potential
  (R. B. Wiringa, S. C. Pieper, Phys. Lett. **89** (2002) 182501)
- Minnesota MN potential
  (D. R. Thomson, M. LeMere, Y. C. Tang, Nucl. Phys **A 286** (1977) 53)

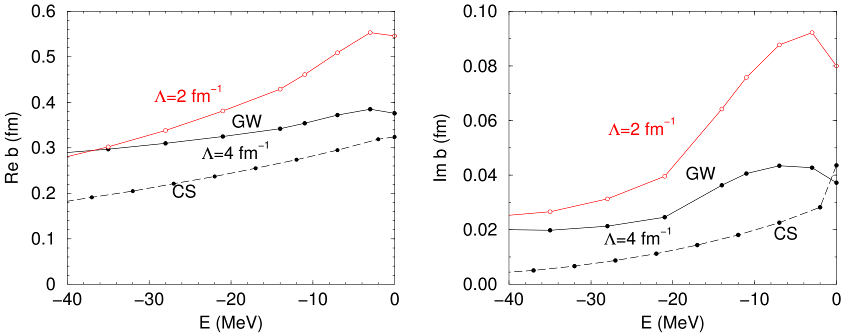
ηN two-body potential

- complex energy-dependent local potential derived from the full chiral coupled-channel model

$$V_{\eta N}(E,r) = -\frac{4\pi}{2\mu_{\eta N}} b(E)\rho_\Lambda(r) \ , \qquad \rho_\Lambda(r) = \left(\frac{\Lambda}{2\sqrt{\pi}}\right)^3 \exp\left\{-\frac{\Lambda^2 r^2}{4}\right\}$$

- energy dependent $b(E)$ fitted to phase shifts δ derived from $F_{\eta N}$ scattering amplitudes in the GW and CS models
- scale parameter Λ inversely proportional to the $V_{\eta N}$ range
- smaller the range ($\sim 1/\Lambda$) the larger is resulting binding of η

V_{NN} and $V_{\eta N}$ potential input

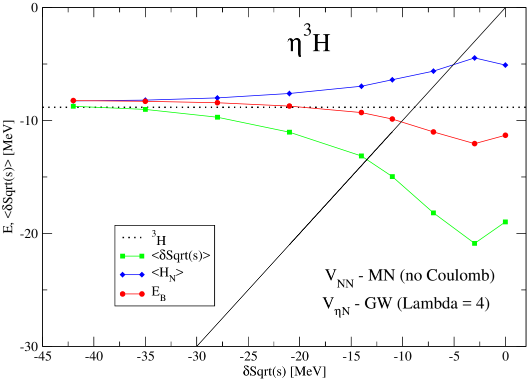


(N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345)

Perturbative estimate of converison widths Γ_η

$$\Gamma_\eta = -2\langle\Psi_{gs}|\mathrm{Im}V_{\eta N}|\Psi_{gs}\rangle$$

Energy dependence of ηN potential



Selfconsistency

We are looking for such solution where $\langle \delta\sqrt{s} \rangle = \delta\sqrt{s}$.

$$\langle \delta\sqrt{s} \rangle = -\frac{B}{A} - \frac{A-1}{A} B_\eta - \xi_N \frac{A-1}{A} \langle T_{N:N} \rangle - \xi_\eta \left(\frac{A-1}{A}\right)^2 \langle T_\eta \rangle$$

(N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345)

ηNN, η^3He and η^4He systems

ηNN

- unbound

(N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345)

η^3He

$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_{sc}}$	B_η	Γ_η
GW, $\Lambda = 2$	MN	-9.385	0.099	1.144
	AV4'	-11.478	-0.028	0.769
GW, $\Lambda = 4$	MN	-13.392	0.990	3.252
	AV4'	-14.881	0.686	2.438
CS, $\Lambda = 2$	MN	-8.388	-0.217	0.057
CS, $\Lambda = 4$	MN	-8.712	-0.161	0.227

(N. Barnea, E. Friedman, A. Gal, **NPA 968** (2017) 35)

ηNN, η^3He and η^4He systems

ηNN

- unbound

(N. Barnea, E. Friedman, A. Gal, **PLB 747** (2015) 345)

		η^3He			η^4He		
$V_{\eta N}$	V_{NN}	$\delta\sqrt{s_{sc}}$	B_η	Γ_η	$\delta\sqrt{s_{sc}}$	B_η	Γ_η
GW, $\Lambda = 2$	MN	-9.385	0.099	1.144	-19.48	0.96	1.98
	AV4'	-11.478	-0.028	0.769	-23.65	0.38	1.21
GW, $\Lambda = 4$	MN	-13.392	0.990	3.252	-29.75	4.69	4.50
	AV4'	-14.881	0.686	2.438	-32.41	3.51	3.62
CS, $\Lambda = 2$	MN	-8.388	-0.217	0.057	-16.70	-0.16	0.13
CS, $\Lambda = 4$	MN	-8.712	-0.161	0.227	-19.25	0.47	0.90

(N. Barnea, E. Friedman, A. Gal, **NPA 968** (2017) 35)

(N. Barnea, B. Bazak, E. Friedman, A. Gal, **PLB 771** (2017) 297)

η^6Li system

- challenging computational problem (computational complexity scales with $N!$)
- large amount of spin-isospin configurations in the case of ^{6}Li nuclear core (45) (only one valid in case of ^{2}H, ^{3}H, ^{3}He, and ^{4}He)

→ new high performance SVM code was developed

One spin-isospin configuration

- Minnesota NN potential, no Coulomb
- $E_B(^6\mathrm{Li})$= -34.70 MeV

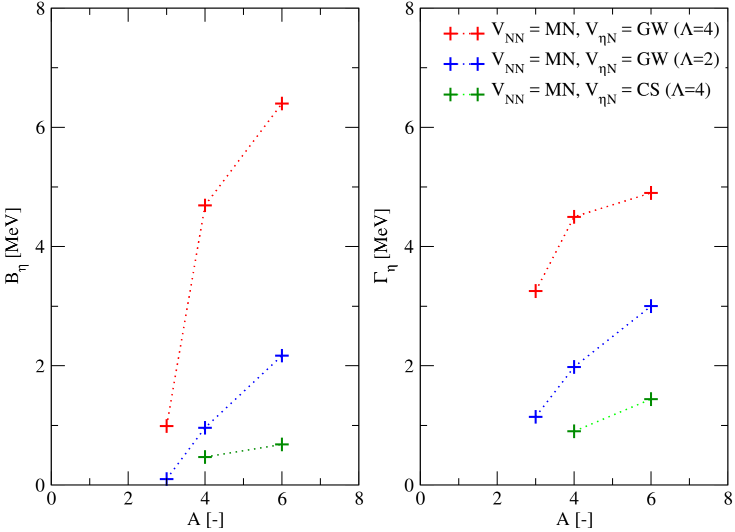
η^6Li	$\delta\sqrt{s_{sc}}$	B_η	Γ_η
GW $\Lambda = 2$	-21.49	2.14	2.92
GW $\Lambda = 4$	-33.55	6.44	4.97
CS $\Lambda = 2$	-16.19	-0.17	0.82
CS $\Lambda = 4$	-20.57	1.06	1.28

All spin-isospin configurations included

- Minnesota NN potential, Coulomb included
- $E_B(^6\mathrm{Li})$= -34.69 MeV

η^6Li	$\delta\sqrt{s_{sc}}$	B_η	Γ_η
GW $\Lambda = 2$	-21.47	2.17	3.00
GW $\Lambda = 4$	-33.11	6.40	4.90
CS $\Lambda = 2$	-16.07	-0.08	0.85
CS $\Lambda = 4$	-21.08	0.68	1.44

Overall results



Imaginary part of the $V_{\eta N}$ potential - preliminary

Mean-value approximation : $\Gamma_\eta = -2\langle \Psi_{gs} | \mathrm{ImV}_{\eta N} | \Psi_{gs} \rangle$

$\rightarrow$

We can go further and solve generalized eigenvalue problem with complex Hamiltonian (imaginary part of the $V_{\eta N}$ included) using variationally selected $|\Psi_{gs}\rangle$ SVM basis states. This yields complex eigenenergy of the ground state $\mathrm{E} = \mathrm{Re(E)} + \mathrm{iIm(E)}$ and width $\Gamma_\eta = -2\mathrm{Im(E)}$.

η^3He	B_η [MeV]	Γ_η [MeV]
GW $\Lambda = 2$	0.11	1.37
GW $\Lambda = 2$ (cmplx)	-0.25	1.32
GW $\Lambda = 4$	1.01	3.32
GW $\Lambda = 4$ (cmplx)	0.36	3.44
η^4He	B_η [MeV]	Γ_η [MeV]
GW $\Lambda = 2$	0.97	2.17
GW $\Lambda = 2$ (cmplx)	0.77	2.22
GW $\Lambda = 4$	4.62	4.38
GW $\Lambda = 4$ (cmplx)	4.40	4.41

Table: Calculations has been performed for 600 basis states for both η^3He and η^4He systems. Coulomb interaction has been included.

Summary

- selfconsistent calculations of ηNN, ηNNN, $\eta NNNN$ systems
  - $\eta \mathrm{d}$ unbound, η bound in $^3\mathrm{He}$ for GW model, and in $^4\mathrm{He}$ for GW model and CS model with $\Lambda = 4$
- new optimized SVM code developed
- selfconsistent ab-initio calculations of $\eta^6\mathrm{Li}$ system using Minnesota V_{NN} potential
  - $\mathrm{B}_\eta = 2.17$ MeV , $\Gamma_\eta = 3.00$ MeV (GW $\Lambda = 2$)
  - $\mathrm{B}_\eta = 6.40$ MeV , $\Gamma_\eta = 4.90$ MeV (GW $\Lambda = 4$)
  - $\mathrm{B}_\eta = 0.68$ MeV , $\Gamma_\eta = 1.44$ MeV (CS $\Lambda = 4$)
  - unbound for CS $\mathrm{V}_{\eta\mathrm{N}}$ potential with $\Lambda = 2$
  - consistent with an RMF calculations
    ($\mathrm{B}_\eta \approx 2.5$ MeV , $\Gamma_\eta \approx 4$ MeV)

Next steps :

- calculations of $\eta^7\mathrm{Li}$ in progress $\rightarrow$ few-body systems with $L > 0$
- realistic interactions (AV6, AV8)
- heavier (9, 10-body) systems